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DEFORMATION AND LONG-TERM DAMAGE OF PHYSICALLY NONLINEAR PARTICULATE COMPOSITES

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The theory of long-term damage is generalized to particulate composite materials with physically nonlinear components. The damage of the components is modeled by randomly dispersed micropores. The damage criterion for a microvolume is characterized by its stress-rupture strength. It is determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its limit, which is the ultimate strength, according to the Huber–Mises criterion, and assumed to be a random function of coordinates. An equation of damage (porosity) balance in the components at an arbitrary time is formulated. Algorithms of calculating the time dependence of are developed. The effect on the nonlinearity of the matrix on the damage and macrodeformation curves is examined

Keywords: particulate composite, physical nonlinearity, stochastic structure, long-term damage, effective characteristics, damage (porosity) balance equation

Introduction. High loads cause dispersed microdamages in materials and structural members, which commonly lead to the formation of main cracks. Microdamages are chaotically dispersed damaged microvolumes that have completely or partially lost their load-carrying capacity. They reduce the effective or bearing portion of the material that resists loads.

Experimental data on and observation of the behavior of structural members and structures suggest that damage can be either short-term (occurring instantaneously after the application of stresses or strains) or long-term (building up with time after the application of load). The structural theory of short-term microdamage of homogeneous and composite materials [7, 9, 10] employs the mechanics of microinhomogeneous bodies of stochastic structure and models dispersed microdamages by quasispherical micropores [5]. Long-term damage is accumulation of dispersed microdamages such as micropores and microcracks. The strength of a material is micro-nonuniform, i.e., the ultimate strength and stress-rupture curves for a microvolume are random functions of coordinates with certain distribution density or cumulative distribution. Under constant tensile stress, some microvolumes whose ultimate strength is less than the applied stress are damaged, i.e., microcracks or micropores form in their place. Microvolumes where the stress is less than, yet close to the ultimate strength are damaged after a lapse of time, which depends on the difference between the applied stress and the ultimate microstrength. The theory of long-term damage of homogeneous and fibrous materials was developed in [8, 11, 12] based on models and methods of the mechanics of stochastically inhomogeneous materials.

The stress–strain behavior of many materials such as metals and polymers becomes nonlinear at high temperatures.

Therefore, it is important to generalize the theory of long-term damage of fibrous composite materials [11, 12] based on models and methods of the mechanics of stochastically inhomogeneous materials to physically nonlinear particulate materials. The damage of the components of a particulate composite is modeled by dispersed microvolumes destroyed to become randomly arranged micropores. The failure criterion for a single microvolume is determined by its stress-rupture strength described by a fractional or exponential power function, which is, in turn, determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its limit, which characterizes the ultimate strength according to the Huber–Mises

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criterion. The ultimate strength is assumed to be a random function of coordinates whose one-point distribution is described by a power function on some interval or by the Weibull function. The effective elastic properties and the stress–strain state of a particulate composite with randomly arranged microdamages are determined from the stochastic equations of elasticity of particulate materials with porous components. We will derive a damage (porosity) balance equation from the properties of the distribution functions and ergodicity of the random field of ultimate microstrength, and the dependence of the time to brittle failure for a microvolume on its stress state and ultimate microstrength for given macrostrains and an arbitrary time. The macrostress–macrostrain relationship and the porosity balance equations for a particulate material with porous components describe the coupled and interacting processes of deformation and long-term damage. We will use an iteration method to develop algorithms for calculating the microdamage and macrostresses as functions of time and to plot the respective curves. The influence of nonlinearity on the deformation and microdamage of the composite will be analyzed.

1. The physically nonlinear deformation of a particulate composite is described as the dependence of the bulk and shear moduli K_ν, μ_ν $(\nu = 1, 2)$ on strains. The microdamage of the composite components caused by loading is modeled by randomly arranged quasispherical micropores occurring in those microvolumes where the stresses exceed the ultimate microstrength. The macrostresses $\langle \sigma_{ij} \rangle$ and macrostrains $\langle \varepsilon_{ij} \rangle$ in an elementary macrovolume are related by

$$\langle \sigma_{ij} \rangle = (K^* - 2/3\mu^*)\langle \varepsilon_{rr} \rangle \delta_{ij} + 2\mu^* \langle \varepsilon_{ij} \rangle, \tag{1.1}$$

where K^*, μ^* are the effective moduli dependent on the macrostrains $\langle \varepsilon_{ij} \rangle$ due to physical nonlinearity and microdamage.

Denote the porosity of the inclusions and matrix by p_1 and p_2, respectively, the bulk and shear moduli of the skeletons of the inclusions and matrix by K_1, μ_1 and K_2, μ_2, respectively, and the volume fractions of porous inclusions and porous matrix by c_1 and c_2, respectively. The effective moduli of a physically nonlinear particulate composite with porous components can be determined using the following iterative algorithm. The *n*th approximation of the effective moduli $K^{*(n)}$ and $\mu^{*(n)}$ is determined [2, 6] in terms of the *n*th approximation of the respective moduli of the inclusions $(K_{1\mathrm{p}}^{(n)}, \mu_{1\mathrm{p}}^{(n)})$ and matrix $(K_{2\mathrm{p}}^{(n)}, \mu_{2\mathrm{p}}^{(n)})$ as

$$K^{*(n)} = c_1 K_{1\mathrm{p}}^{(n)} + c_2 K_{2\mathrm{p}}^{(n)} - c_1 c_2 \frac{(K_{1\mathrm{p}}^{(n)} - K_{2\mathrm{p}}^{(n)})^2}{c_1 K_{2\mathrm{p}}^{(n)} + c_2 K_{1\mathrm{p}}^{(n)} + n_\mathrm{a}},$$

$$\mu^{*(n)} = c_1 \mu_{1\mathrm{p}}^{(n)} + c_2 \mu_{2\mathrm{p}}^{(n)} - c_1 c_2 \frac{(\mu_{1\mathrm{p}}^{(n)} - \mu_{2\mathrm{p}}^{(n)})^2}{c_1 \mu_{2\mathrm{p}}^{(n)} + c_2 \mu_{1\mathrm{p}}^{(n)} + m_\mathrm{a}} \tag{1.2}$$

$$\left(n_\mathrm{a} = \frac{4}{3}\mu_\mathrm{a}, \quad m_\mathrm{a} = \frac{\mu_\mathrm{a}(9K_\mathrm{a} + 8\mu_\mathrm{a})}{6(K_\mathrm{a} + 2\mu_\mathrm{a})} \right), \tag{1.3}$$

$$K_\mathrm{a} = c_1 K_{1\mathrm{p}} + c_2 K_{2\mathrm{p}}, \qquad \mu_\mathrm{a} = c_1 \mu_{1\mathrm{p}} + c_2 \mu_{2\mathrm{p}}, \tag{1.4}$$

if the porous matrix is stiffer than the porous inclusions and

$$K_\mathrm{a} = \frac{K_{1\mathrm{p}} K_{2\mathrm{p}}}{c_1 K_{2\mathrm{p}} + c_2 K_{1\mathrm{p}}},$$

$$\mu_\mathrm{a} = \frac{\mu_{1\mathrm{p}} \mu_{2\mathrm{p}}}{c_1 \mu_{2\mathrm{p}} + c_2 \mu_{1\mathrm{p}}} \tag{1.5}$$

otherwise. where $\mu_{1\mathrm{p}}, \mu_{2\mathrm{p}}$ are the shear moduli of the components in the linear case. The *n*th approximation of the effective moduli of porous inclusions, $K_{1\mathrm{p}}^{(n)}, \mu_{1\mathrm{p}}^{(n)}$, and porous matrix, $K_{2\mathrm{p}}^{(n)}, \mu_{2\mathrm{p}}^{(n)}$, is defined by the following formulas [2, 6]:

$$K_{\nu p}^{(n)} = \frac{4K_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})(1-p_\nu)^2}{3K_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})p_\nu + 4\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})(1-p_\nu)},$$

$$\mu_{\nu p}^{(n)} = \frac{[9K_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)}) + 8\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})]\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})(1-p_\nu)^2}{3K_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})(3-p_\nu) + 4\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})(2+p_\nu)}$$

$$(\nu = 1,\ 2), \tag{1.6}$$

where $\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)}$ are the average strains in the undamaged portion of the inclusion or matrix in the nth approximation. Their $(n+1)$th approximation is related to the nth approximation $\langle\varepsilon_{ij}^{\nu}\rangle^{(n)}$ of the average strains in the components by

$$\langle\varepsilon_{ij}^{1\nu}\rangle^{(n+1)} = \frac{1}{(1-p_\nu)}\left\{\frac{\mu_{\nu p}^{(n)}}{\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})}\langle\varepsilon_{ij}^{\nu}\rangle^{(n)}\right.$$

$$\left. + \frac{1}{3}\left[\frac{K_{\nu p}^{(n)}}{K_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})} - \frac{\mu_{\nu p}^{(n)}}{\mu_\nu(\langle\varepsilon_{lm}^{1\nu}\rangle^{(n)})}\right]\langle\varepsilon_{rr}^{\nu}\rangle^{(n)}\delta_{ij}\right\}$$

$$(\nu = 1, 2). \tag{1.7}$$

The average strains $\langle\varepsilon_{ij}^{\nu}\rangle^{(n)}$ are determined in terms of the macrostrains $\langle\varepsilon_{ij}\rangle$ by the formulas

$$\langle\varepsilon_{ij}^{\nu}\rangle^{(n)} = (-1)^{\nu+1}\left\{\frac{2\mu^{*(n)}(\mu_{1p}^{(n)} - \mu_{2p}^{(n)})(K^{*(n)} - K_{\chi p}^{(n)}) - 3K^{*(n)}(K_{1p}^{(n)} - K_{2p}^{(n)})(\mu^{*(n)} - \mu_{\chi p}^{(n)})}{6c_\nu\mu^{*(n)}(\mu_{1p}^{(n)} - \mu_{2p}^{(n)})(K_{1p}^{(n)} - K_{2p}^{(n)})}\right.$$

$$\left. \times\langle\varepsilon_{rr}\rangle\delta_{ij} + + \frac{\mu^{*(n)} - \mu_{\chi p}^{(n)}}{c_k(\mu_{1p}^{(n)} - \mu_{2p}^{(n)})}\langle\varepsilon_{ij}\rangle\right\}$$

$$(\chi = 3 - k). \tag{1.8}$$

Given macrostrains $\langle\varepsilon_{ij}\rangle$, the effective moduli are determined as the limits of the iterative process

$$K^* = \lim_{n\to\infty} K^{*(n)},$$

$$\mu^* = \lim_{n\to\infty} \mu^{*(n)}. \tag{1.9}$$

We will use the Huber–Mises criterion [3] as a condition for the formation of a microdamage in a microvolume of the undamaged portion of the material:

$$I_{\langle\sigma\rangle}^{1\nu} = k_\nu \qquad (\nu = 1, 2), \tag{1.10}$$

$I_{\langle\sigma\rangle}^{1\nu} = (\langle\sigma_{ij}^{1\nu}\rangle'\langle\sigma_{ij}^{1\nu}\rangle')^{1/2}$ is the second invariant of the deviatoric average-stress tensor $\langle\sigma_{ij}^{1\nu}\rangle'$ in the undamaged portion of the νth component; k_ν is the ultimate microstrength, which is a random function of coordinates.

Since the average stresses $\langle\sigma_{ij}^{1\nu}\rangle$ in the undamaged portion of the νth component are related to the average stresses $\langle\sigma_{ij}^{\nu}\rangle$ in the component as follows [7]:

$$\langle\sigma_{ij}^{1\nu}\rangle = \frac{1}{1-p_\nu}\langle\sigma_{ij}^{\nu}\rangle \quad (1.11)$$

the invariant $I_{\langle\sigma\rangle}^{1\nu}$ is related to the invariant of the deviatoric average-stress tensor $I_{\langle\sigma\rangle}^{\nu} = (\langle\sigma_{ij}^{\nu}\rangle'\langle\sigma_{ij}^{\nu}\rangle')^{1/2}$ in the component by the formula

$$I_{\langle\sigma\rangle}^{1\nu} = \frac{1}{1-p_\nu} I_{\langle\sigma\rangle}^{\nu}, \quad (1.12)$$

and is related to the invariant of the deviatoric average-strain tensor $I_{\langle\varepsilon\rangle}^{\nu} = (\langle\varepsilon_{ij}^{\nu}\rangle'\langle\varepsilon_{ij}^{\nu}\rangle')^{1/2}$ in the components by the formula

$$I_{\langle\sigma\rangle}^{1\nu} = \frac{2\mu_{\nu p}}{1-p_\nu} I_{\langle\varepsilon\rangle}^{\nu}, \quad (1.13)$$

where $I_{\langle\varepsilon\rangle}^{\nu}$ is related to the invariant $I_{\langle\varepsilon\rangle} = (\langle\varepsilon_{ij}\rangle'\langle\varepsilon_{ij}\rangle')^{1/2}$ for the whole composite by

$$I_{\langle\varepsilon\rangle}^{\nu} = (-1)^{\nu+1}\frac{\mu^* - \mu_{(3-\nu)p}}{c_\nu(\mu_{1p}-\mu_{2p})} I_{\langle\varepsilon\rangle}. \quad (1.14)$$

With (1.13) and (1.14), the failure criterion (1.10) takes the following form in the macrostrain space:

$$(-1)^{\nu+1}\frac{2\mu_{\nu p}(\mu^* - \mu_{(3-\nu)p})}{c_\nu(1-p_\nu)(\mu_{1p}-\mu_{2p})} I_{\langle\varepsilon\rangle} = k_\nu \quad (\nu = 1, 2). \quad (1.15)$$

If the invariant $I_{\langle\sigma\rangle}^{1\nu}$ does not reach the limiting value k_ν in some microvolume of the νth component, then, according to the stress-rupture criterion, failure will occur in some time τ_k^{ν}, which depends on the difference between $I_{\langle\sigma\rangle}^{1\nu}$ and k_ν. In the general case, this dependence can be represented as some function:

$$\tau_k^{\nu} = \varphi_\nu(I_{\langle\sigma\rangle}^{1\nu}, k_\nu), \quad (1.16)$$

where $\varphi_\nu(k_\nu, k_\nu) = 0$ and $\varphi_\nu(0, k_\nu) = \infty$ according to (1.10).

The one-point distribution function $F_\nu(k_\nu)$ for some microvolume in the undamaged portion of the νth component can be approximated by a power function on some interval

$$F_\nu(k_\nu) = \begin{cases} 0, & k_\nu < k_{\nu 0}, \\ \left(\dfrac{k_\nu - k_{\nu 0}}{k_{1\nu} - k_{\nu 0}}\right)^{n_\nu}, & k_{\nu 0} \le k_\nu \le k_{\nu 1}, \\ 1, & k_\nu > k_{\nu 1}, \end{cases} \quad (1.17)$$

or by the Weibull function

$$F_\nu(k_\nu) = \begin{cases} 0, & k_\nu < k_{\nu 0}, \\ 1 - \exp[-m_\nu(k_\nu - k_{\nu 0})^{n_\nu}], & k_\nu \ge k_{\nu 0}, \end{cases} \quad (1.18)$$

where $k_{\nu 0}$ is the minimum value of ultimate microstrength in the component; $k_{\nu 1}, m_\nu, n_\nu$ are deterministic constants describing the behavior of the distribution function and determined by fitting experimental microstrength scatter or stress–strain curves.

Assume that the random field of ultimate microstrength k_ν is statistically homogeneous, which is typical of real materials, and the single microdamages and distance between them are negligible compared with the macrovolume. Then the random field k_ν and the distribution of macrostresses in the component under uniform loading are ergodic, and the distribution function $F_\nu(k_\nu)$ defines the fraction of the undamaged portion of the component in which the ultimate microstrength is less than k_ν.

Therefore, if the stresses $\langle\sigma_{ij}^{1\nu}\rangle$ are nonzero, the function $F_\nu(I_{\langle\sigma\rangle}^{1\nu})$ defines, according to (1.10), (1.17), and (1.18), the content of instantaneously damaged microvolumes of the skeleton of the component. Since the damaged microvolumes are modeled by pores, we can write a balance equation for damaged microvolumes or porosity [7]:

$$p_\nu = p_{\nu 0} + (1 - p_{\nu 0})F_\nu(I_{\langle\sigma\rangle}^{1\nu}), \tag{1.19}$$

where $p_{\nu 0}$ is the initial porosity.

With (1.13) and (1.14), the porosity balance equation (1.19) takes the following form in the macrostrain space:

$$p_\nu = p_{\nu 0} + (1 - p_{\nu 0})F_\nu\left[(-1)^{\nu+1}\frac{2\mu_{\nu p}(\mu^* - \mu_{(3-\nu)p})}{c_\nu(1 - p_\nu)(\mu_{1p} - \mu_{2p})}I_{\langle\varepsilon\rangle}\right], \tag{1.20}$$

where μ^* and μ_{1p}, μ_{2p} are defined by (1.2)–(1.6).

If the stresses $\langle\sigma_{ij}^{\nu}\rangle$ act for some time t, then, according to the stress-rupture criterion (1.10), those microvolumes of the νth component are damaged that have k_ν such that

$$t \ge \tau_k^\nu = \varphi_\nu(I_{\langle\sigma\rangle}^{1\nu}, k_\nu), \tag{1.21}$$

where $I_{\langle\sigma\rangle}^{1\nu}$ is defined by (1.12) or (1.13).

At low temperatures, the time to brittle failure τ_k^ν for real materials at low temperatures is finite beginning only from some value of $I_{\langle\sigma\rangle}^{1\nu} > 0$. In this case, the durability function $\varphi_\nu(I_{\langle\sigma\rangle}^{1\nu}, k_\nu)$ can be represented as

$$\varphi_\nu(I_{\langle\sigma\rangle}^{1\nu}, k_\nu) = \tau_{0\nu}\left(\frac{k_\nu - I_{\langle\sigma\rangle}^{1\nu}}{I_{\langle\sigma\rangle}^{1\nu} - \gamma_\nu k_\nu}\right)^{n_{1\nu}} \qquad (\gamma_\nu k_\nu \le I_{\langle\sigma\rangle}^{1\nu} \le k_\nu,\ \gamma_\nu < 1), \tag{1.22}$$

where some typical time $\tau_{0\nu}$, exponent $n_{1\nu}$, and coefficient γ_ν are determined from the fit of experimental durability curves for the νth component.

Substituting (1.22) into (1.21), we arrive at the inequality

$$k_\nu \le I_{\langle\sigma\rangle}^{1\nu}(1 + \bar{t}_\nu^{-1/n_{1\nu}})/(1 + \gamma_\nu \bar{t}_\nu^{-1/n_{1\nu}}) \qquad \left(\bar{t}_\nu = \frac{t}{\tau_{0\nu}}\right). \tag{1.23}$$

Considering the definition of the distribution function $F_\nu(k_\nu)$, we conclude that the function $F_\nu[I_{\langle\sigma\rangle}^{1\nu}\psi_\nu(\bar{t}_\nu)]$, where

$$\psi_\nu(\bar{t}_\nu) = (1 + \bar{t}_\nu^{-1/n_{1\nu}})/(1 + \gamma_\nu \bar{t}_\nu^{-1/n_{1\nu}}), \tag{1.24}$$

defines the relative content of the destroyed microvolumes in the undamaged portion of the νth component at the time $\bar{t}_\nu$. Then, in view of (1.11), the porosity balance equation for the νth component subject to long-term damage can be represented as

$$p_\nu = p_{0\nu} + (1 - p_{0\nu})F_\nu\left[\frac{I_{\langle\sigma\rangle}^{\nu}}{1 - p_\nu}\psi_\nu(\bar{t}_\nu)\right], \tag{1.25}$$

where p_ν is a function of dimensionless time $\bar{t}_\nu$, and $I_{\langle\sigma\rangle}^{\nu}$ is defined by (1.12)–(1.14).

If the time τ_k^{ν} is finite for arbitrary values of $I_{\langle\sigma\rangle}^{1\nu}$, which may be observed at high temperatures, then the durability function can be approximated by an exponential power function

$$\varphi_{\nu}(I_{\langle\sigma\rangle}^{\nu}, k_{\nu}) = \tau_{0\nu}\left\{\{\exp m_{1\nu}\left[(k_{\nu}/I_{\langle\sigma\rangle}^{\nu})^{n_{1\nu}} - 1\right] - 1\right\}^{n_{2\nu}}, \tag{1.26}$$

which has enough constants $\tau_{0\nu}$, $m_{1\nu}, n_{1\nu}, n_{2\nu}$ to fit experimental curves. Substituting (1.26) into (1.21), we arrive at the inequality

$$k_{\nu} \le I_{\langle\sigma\rangle}^{\nu}\left[1 + \frac{1}{m_{1\nu}}\ln(1 + \bar{t}_{\nu}^{-1/n_{2\nu}})\right]^{1/n_{1\nu}} \qquad \left(\bar{t}_{\nu} = \frac{t}{\tau_{0\nu}}\right). \tag{1.27}$$

Considering the definition of the distribution function $F_{\nu}(k_{\nu})$, we conclude that the function $F_{\nu}[I_{\langle\sigma\rangle}^{1\nu}\psi_{\nu}(\bar{t}_{\nu})]$, where

$$\psi_{\nu}(\bar{t}_{\nu}) = \left[1 + \frac{1}{m_{1\nu}}\ln(1 + \bar{t}_{\nu}^{-1/n_{2\nu}})\right]^{1/n_{1\nu}}, \tag{1.28}$$

defines the relative content of the destroyed microvolumes in the undamaged portion of the νth component at the time $\bar{t}_{\nu}$. Then, in view of (1.11), the porosity balance equation for the νth component subject to long-term damage can be represented in the form (1.25), where p_{ν} is a function of dimensionless time $\bar{t}_{\nu}$, and $I_{\langle\sigma\rangle}^{\nu}$ is defined by (1.12)–(1.14).

At $\bar{t}_{\nu} = 0$, the porosity balance equation (1.25) with (1.12)–(1.14), (1.24) (or (1.28)) defines the short-term (instantaneous) damage of the νth component. As time elapses, Eq. (1.25) with (1.12)–(1.14), (1.24) (or (1.28)) defines its long-term damage, which consists of short-term damage and additional time-dependent damage.

Equations (1.1), (1.2)–(1.8), (1.25), (1.12)–(1.14), (1.17) (or (1.18)), (1.24) (or (1.28)) form a closed-loop system describing the joint processes of statistically homogeneous physically nonlinear deformation and long-term damage of a particulate composite. The physical nonlinearity of its components affects the way pores form during deformation, and the porosity of the material has an effect on its stress–strain curve. This is why the nonlinearity of the stress–strain curve of a particulate composite is determined by the physical nonlinearity of its components and the increase in the porosity during physically nonlinear deformation.

To describe the coupled processes of physically nonlinear deformation and long-term damage, it is necessary to find the macrostrain-dependent effective elastic moduli by the iterative algorithm (1.2)–(1.8) and to determine the porosity from Eq. (1.12)–(1.14), (1.17) (or 1.18)), (1.24) (or (1.28)) also by an iterative method. At the *n*th step of the iterative process (1.2)–(1.8), Eq. (1.25) is represented as

$$f_{\nu}^{(n)} = p_{\nu} - p_{\nu 0} - (1 - p_{\nu 0})F_{\nu}\left[(-1)^{\nu+1}\frac{2\mu_{\nu p}^{(n)}(\mu^{*(n)} - \mu_{(3-\nu)p}^{(n)})}{c_{\nu}(1 - p_{\nu})(\mu_{1p}^{(n)} - \mu_{2p}^{(n)})} I_{\langle\varepsilon\rangle}\psi_{\nu}(\bar{t}_{\nu})\right]. \tag{1.29}$$

Then the root p_{ν} of Eq. (1.29) at the *m*th step of some iterative process can be expressed as

$$p_{\nu}^{(m,n)} = A_{\nu} f_{\nu}^{(n)}(p_{\nu}^{(m-1)}), \tag{1.30}$$

where A_{ν} is an operator on the function $f_{\nu}^{(n)}(p_{\nu})$. The root is found as follows:

$$p_{\nu} = \lim_{\substack{m\to\infty \\ n\to\infty}} p_{\nu}^{(m,n)}. \tag{1.31}$$

2. Let us analyze, as an example, the coupled processes of nonlinear deformation and long-term microdamage of a particulate composite with linear elastic inclusions and nonlinear elastic matrix with linear bulk strains and shear strains described by a linear-hardening diagram

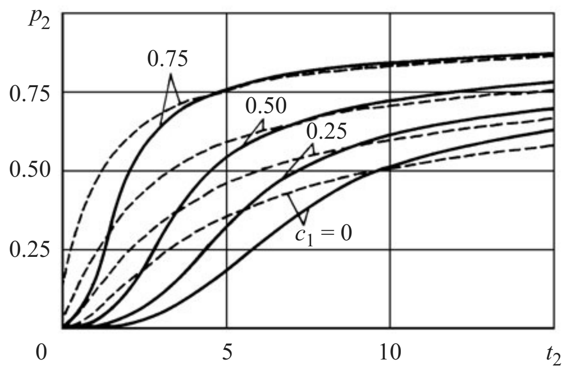

Fig. 1

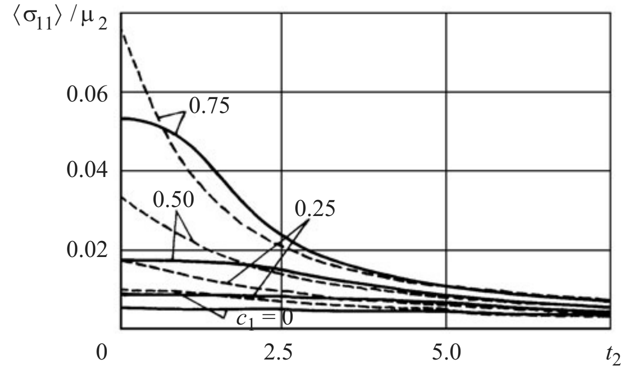

Fig. 2

$$\langle\sigma_{rr}^2\rangle = K_2 \langle\varepsilon_{rr}^2\rangle, \quad \langle\sigma_{ij}^2\rangle' = 2\mu_2(S_2)\langle\varepsilon_{ij}^2\rangle', \tag{2.1}$$

where the bulk modulus K_2 does not depend on the strains, and the shear modulus $\mu_2(S_2)$ is described by

$$\mu_2(S_2) = \begin{cases} \mu_{20}, & T_2 \le T_{20}, \\ \mu_2' + \left(1 - \dfrac{\mu_2'}{\mu_{20}}\right)\dfrac{T_{20}}{2S_2}, & T_2 \ge T_{20}, \end{cases} \tag{2.2}$$

$$(S_2 = (\langle\varepsilon_{ij}^2\rangle'\langle\varepsilon_{ij}^2\rangle')^{1/2}, \quad T_2 = (\langle\sigma_{ij}^2\rangle'\langle\sigma_{ij}^2\rangle')^{1/2}, \quad T_{20} = \sqrt{2/3}\,\sigma_{20}),$$

where $\langle\varepsilon_{ij}^2\rangle'$ and $\langle\sigma_{ij}^2\rangle'$ are the strain and stress deviators in the matrix; σ_{20} is the tensile proportional limit assumed to be independent of the coordinates; μ_{20}, μ_2' are the material constants of the matrix.

The root p_2 of Eq. (1.29) can be found with the secant method [1]. Since the root p_2 falls into the interval $[p_{20}, 1]$, which follows from the inequalities $f_2^{(n)}(p_{20}) \le 0, f_2^{(n)}(1) \ge 0$, the zero approximation $p_2^{(0,n)}$ can be determined with the secant method as

$$p_2^{(0,n)} = \frac{a_2^{(0)} f_2^{(n)}(b_2^{(0)}) - b_2^{(0)} f_2^{(n)}(a_2^{(0)})}{f_2^{(n)}(b_2^{(0)}) - f_2^{(n)}(a_2^{(0)})},$$

where $a_2^{(0)} = p_{20}, b_2^{(0)} = 1$. The subsequent approximations of the secant method are found in the iterative process

$$p_2^{(m,n)} = A_2 f_2^{(n)}(p_2^{(m-1,n)}) \equiv \frac{a_2^{(m)} f_2^{(n)}(b_2^{(m)}) - b_2^{(m)} f_2^{(n)}(a_2^{(m)})}{f_2^{(n)}(b_2^{(m)}) - f_2^{(n)}(a_2^{(m)})},$$

$$a_2^{(m)} = a_2^{(m-1)}, \quad b_2^{(m)} = p_2^{(m-1,n)} \quad \text{for} \quad f_2^{(n)}(a_2^{(m-1)}) f_2^{(n)}(p_2^{(m-1,n)}) \le 0,$$

$$a_2^{(m)} = p_2^{(m-1,n)}, \quad b_2^{(m)} = b_2^{(m-1)} \quad \text{for} \quad f_2^{(n)}(a_2^{(m-1)}) f_2^{(n)}(p_2^{(m-1,n)}) \ge 0$$

$$(m = 1, 2, \ldots),$$

which proceeds until $|f_2^{(n)}(p_2^{(m,n)})| < \delta$, where δ is the error of the root.

We analyzed the coupled processed of nonlinear deformation and long-term damage of a particulate composite for Weibull distribution (1.18) and for both fractional power durability function $\psi_2(\bar{t}_2)$ defined by (1.24) and exponential power durability function $\psi_2(\bar{t}_2)$ defined by (1.28). The inclusions are linear elastic particles with the following characteristics [4]:

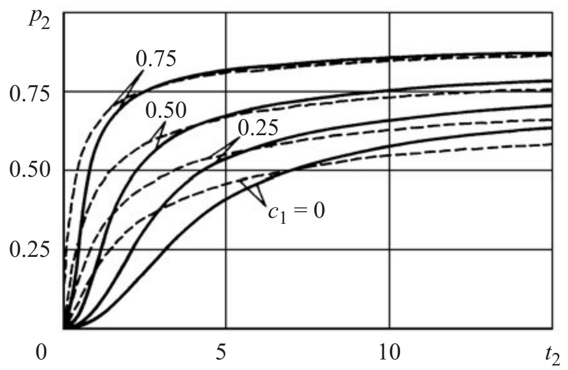

Fig. 3

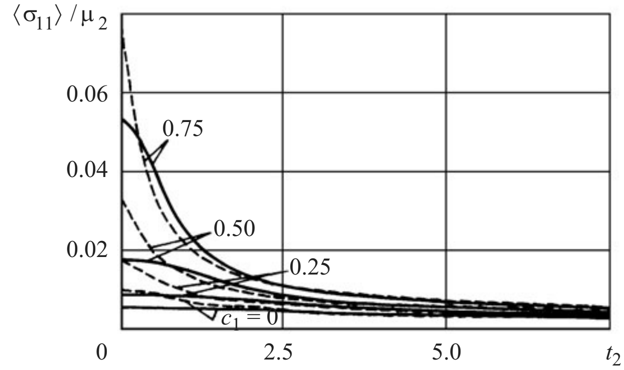

Fig. 4

$K_1 = 38.89$ GPa, $\mu_1 = 29.17$ GPa, $c_1 = 0, 0.25, 0.5, 0.75, 1.0$. The matrix is described by the linear-hardening diagram (2.1), (2.2) with the following constants [2, 4]: $K_2 = 3.33$ GPa, $\mu_{20} = 1.11$ GPa, $\mu'_2 = 0.331$ GPa. Its limits of proportionality and minimum microtensile strength $\sigma_{2p} = \sqrt{3/2}\,k_{20}$ are: $\sigma_{20} = 0.003$ GPa, $\sigma_{2p} = 0.011$ GPa. Moreover, $p_{02} = 0$, $k_{02} / \mu_2 = 0.01$, $m_2 = 1000$, $\gamma_2 = 0.05$, $n_{12} = 1$.

If $\langle\varepsilon_{11}\rangle \neq 0$, $\langle\sigma_{22}\rangle = \langle\sigma_{33}\rangle = 0$, then, according to (1.1), the macrostress $\langle\sigma_{11}\rangle$ is related to the macrostrain $\langle\varepsilon_{11}\rangle$ by

$$\langle\sigma_{11}\rangle = \frac{3K^*\mu^*}{K^* + 1/3\mu^*}\langle\varepsilon_{11}\rangle.$$

In (1.20)

$$I_{\langle\varepsilon\rangle} = \sqrt{\frac{2}{3}}\frac{3K^*\langle\varepsilon_{11}\rangle}{2(K^* + 1/3\mu^*)},$$

which is equivalent to (2.12).

Figure 1 shows (solid lines) the porosity p_2 of the linear-hardening matrix as a function of time $\bar{t}_2$ for fractional-power function $\psi_2(\bar{t}_2)$ defined by (1.24) and for different values of c_1. For comparison, the figure shows (dashed lines) p_2 versus $\bar{t}_2$ for the linear elastic matrix. The same notation is used in Figs. 2–4. As is seen, the physical nonlinearity of the matrix has a significant effect on the microdamage of the particulate composite. The microdamage of the composite with linear-hardening matrix sets in at greater values of $\bar{t}_2$ and occurs more intensively than in the composite with linear elastic matrix, i.e., at great values of $\bar{t}_2$, the porosity of the composite with linear-hardening matrix is higher than that with linear elastic matrix.

Figure 2 shows the macrostress $\langle\sigma_{11}\rangle / \mu_2$ in composites with linear-hardening and linear elastic matrices as a function of time $\bar{t}_2$ for fractional-power function $\psi_2(\bar{t}_2)$ defined by (1.24) and for different values of c_1. As is seen, at small values of $\bar{t}_2$, the physical nonlinearity of the matrix has a significant effect on the stress state of the particulate composite as well. At great values of $\bar{t}_2$, the effect of nonlinearity on the stress state is weak.

Figures 3 and 4 show the porosity of the matrix p_2 of and the macrostress $\langle\sigma_{11}\rangle / \mu_2$ in particulate composites with linear-hardening and linear elastic matrices as functions of time $\bar{t}_2$ for fractional-power function $\psi_2(\bar{t}_2)$ defined by (1.28) and for different values of c_1. As is seen, the curves are qualitatively similar to those for the function $\psi_2(\bar{t}_2)$ defined by (1.24).

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