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COUPLED PROCESSES OF DEFORMATION AND LONG-TERM DAMAGE OF LAYERED MATERIALS UNDER THERMAL LOADING

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The failure criterion for a microvolume is characterized by its stress-rupture strength. It is determined by the dependence of the time to brittle fracture on the difference between the equivalent stress and its limit, which is the ultimate strength, according to the Schleicher–Nadai failure criterion, and assumed to be a random function of coordinates. An equation of damage (porosity) balance in the layers at an arbitrary time is formulated taking into account the thermal component. Algorithms of calculating the time dependence of microdamage and macrostresses are developed. Corresponding curves are plotted. The effect of temperature on the deformation and microdamage of the layers is studied

Keywords: layered material, thermal effect, long-term damage, porosity, effective characteristics, porosity balance equation

Introduction. High loads cause dispersed microdamages in materials and structural members, which commonly lead to the formation of main cracks. Microdamages are chaotically dispersed damaged microvolumes that have completely or partially lost their load-carrying capacity. They reduce the effective or bearing portion of the material that resists loads. Microdamages may occur during deformation because microstresses may reach local strength limits, which, in turn, may also be reduced due to climatic and radiation factors.

Experimental data on and observation of using structural members and structures suggest that damage can be either short-term (occurring instantaneously after the application of stresses or strains) or long-term (building up with time after the application of load). A structural theory of short-term microdamage of homogeneous and composite materials was proposed in [7, 9]. It employs the mechanics of microinhomogeneous bodies of stochastic structure and models dispersed microdamages by quasispherical micropores [5]. Long-term damage is usually considered as accumulation of dispersed microdamages such as micropores and microcracks. At the microscopic level, the strength of a material is nonuniform, i.e., the ultimate strength and stress-rupture curves for a microvolume are random functions of coordinates with certain distribution density or cumulative distribution. When a macrospecimen is subject to constant tensile stress, some microvolumes whose ultimate strength is less than the applied stress are damaged, i.e., microcracks or micropores form in their place. Microvolumes where the stress is less than, yet close to the ultimate strength are damaged after a lapse of time, which depends on the difference between the applied stress and the ultimate microstrength. The theory of long-term damage of homogeneous, laminated, and fibrous composites was developed based on models and methods of the mechanics of stochastically inhomogeneous materials.

In the present paper, we will study the effect of thermal loads on the deformation and long-term damage of a laminated composite material. The structural theory of long-term damage of composites is based on the mechanics of microinhomogeneous materials of stochastic structure. The damage of the components of a layered material is modeled by dispersed microvolumes destroyed to become randomly arranged micropores. The failure criterion for a single microvolume is determined by its stress-rupture strength described by a fractional or exponential power function, which is, in turn, determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its limit, which characterizes the ultimate strength

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according to the Schleicher–Nadai criterion. The ultimate strength is assumed to be a random function of coordinates whose one-point distribution is described by a power function on some interval or by the Weibull function. The effective elastic properties and the stress–strain state of a laminated composite with randomly arranged microdamages are determined from the stochastic equations of thermoelasticity of porous materials.

We will derive a damage (porosity) balance equation with thermal effect from the properties of the distribution functions and ergodicity of the random field of ultimate microstrength, and the dependence of the time to brittle failure for a microvolume on its stress state and ultimate microstrength for given macrostrains and an arbitrary time. The macrostress–macrostrain relationship and the porosity balance equations for a layered material with porous components describe the coupled and interacting processes of deformation and long-term damage, which cause the macrostresses to decrease at given time-dependent macrostrains. An iteration method will be used to develop algorithms for calculating the microdamage and macrostresses as functions of time and to plot the respective curves in the case of fractional-power and exponential-power microdurability functions. We will analyze the effect of temperature on the macrodeformation and damage curves.

1. Consider a composite with N isotropic layers. Denote the bulk and shear moduli, thermal stress and strain factors of the skeleton of the ith layer by $K_i, \mu_i, \beta_i, \alpha_i$, its porosity by p_i, and the volume fraction of the porous ith layer by c_i $(i = 1, \ldots, N)$. The macrostresses $\langle \sigma_{jk} \rangle$ are related to the macrostrains $\langle \varepsilon_{jk} \rangle$ and temperature θ by

$$\langle \sigma_{jk} \rangle = (\lambda_{11}^* - \lambda_{12}^*)\langle \varepsilon_{jk} \rangle + (\lambda_{12}^* \langle \varepsilon_{rr} \rangle + \lambda_{13}^* \langle \varepsilon_{33} \rangle - \beta_1^* \theta)\delta_{jk},$$

$$\langle \sigma_{33} \rangle = \lambda_{13}^* \langle \varepsilon_{rr} \rangle + \lambda_{33}^* \langle \varepsilon_{33} \rangle - \beta_3^* \theta,$$

$$\langle \sigma_{j3} \rangle = 2\lambda_{44}^* \langle \varepsilon_{j3} \rangle \qquad (j, k, r = 1, 2), \tag{1.1}$$

where the effective moduli $\lambda_{11}^*, \lambda_{12}^*, \lambda_{13}^*, \lambda_{33}^*, \lambda_{44}^*$ and the thermal stress and strain factors $\beta_1^*, \beta_3^*, \alpha_1^*, \alpha_3^*$ of the composite are expressed [2] in terms of those of the porous components $\lambda_{ip}, \mu_{ip}, \beta_{ip}$ $(i = 1, \ldots, N)$ as

$$\lambda_{11}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle^2 + 4\left\langle \frac{\mu_p(\lambda_p + \mu_p)}{\lambda_p + 2\mu_p} \right\rangle,$$

$$\lambda_{12}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle^2 + 2\left\langle \frac{\lambda_p \mu_p}{\lambda_p + 2\mu_p} \right\rangle,$$

$$\lambda_{13}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle, \qquad \lambda_{33}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1}, \qquad \lambda_{44}^* = \left\langle \frac{1}{\mu_p} \right\rangle^{-1},$$

$$\beta_1^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle \left\langle \frac{\beta_p}{\lambda_p + 2\mu_p} \right\rangle + 2\left\langle \frac{\beta_p \mu_p}{\lambda_p + 2\mu_p} \right\rangle,$$

$$\beta_3^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\beta_p}{\lambda_p + 2\mu_p} \right\rangle,$$

$$\alpha_1^* = \frac{\lambda_{33}^* \beta_1^* - \lambda_{13}^* \beta_3^*}{(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2}, \qquad \alpha_3^* = \frac{(\lambda_{11}^* + \lambda_{12}^*)\beta_3^* - 2\lambda_{13}^* \beta_1^*}{(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2}, \tag{1.2}$$

where

$$\langle f^* \rangle = \sum_{i=1}^{N} c_i f_{ip} \qquad (i = 1, \ldots, N). \tag{1.3}$$

Here

$$K_{ip} = \frac{4K_i\mu_i(1-p_i)^2}{4\mu_i + (3K_i - 4\mu_i)p_i}, \qquad \mu_{ip} = \frac{(9K_i + 8\mu_i)\mu_i(1-p_i)^2}{9K_i + 8\mu_i - (3K_i - 4\mu_i)p_i}, \qquad \lambda_{ip} = K_{ip} - 2/3\mu_{ip},$$

$$\beta_{ip} = \frac{4\beta_i\mu_i(1-p_i)^2}{4\mu_i + (3K_i - 4\mu_i)p_i}, \qquad \alpha_{ip} = \frac{\beta_{ip}}{3K_{ip}} = \frac{\beta_i}{3K_i} \qquad (i = 1, \ldots, N) \tag{1.4}$$

according to [2].

We will use the Schleicher–Nadai criterion [3] to describe the short-term damage in a microvolume of the undamaged portion of the ith component:

$$I^{i1}_{\langle\sigma\rangle} + a_i\langle\sigma^{i1}_{rr}\rangle = k_i, \qquad I^{i1}_{\langle\sigma\rangle} = (\langle\sigma^{i1}_{pq}\rangle'\langle\sigma^{i1}_{pq}\rangle')^{1/2} \quad (i = 1, \ldots, N), \tag{1.5}$$

where $\langle\sigma^{i1}_{pq}\rangle'$ and $\langle\sigma^{i1}_{rr}\rangle$ are the average deviatoric and spherical stresses in the undamaged portion of the ith component; a_i is a deterministic constant; k_i is the limiting value of $I^{i1}_{\langle\sigma\rangle} + a_i\langle\sigma^{i1}_{rr}\rangle$ for the ith component, which is a random function of coordinates, the average stresses $\langle\sigma^{i1}_{pq}\rangle$ being defined by the following formula [6]:

$$\langle\sigma^{i1}_{jk}\rangle = \frac{1}{1 - p_i}\langle\sigma^{i}_{jk}\rangle. \tag{1.6}$$

If $I^{i1}_{\langle\sigma\rangle} + a_i\langle\sigma^{i1}_{rr}\rangle$ does not reach the limiting value k_i in some microvolume of the ith component, then, according to the stress-rupture criterion, failure will occur in some time τ^i_k that depends on the difference between $I^{i1}_{\langle\sigma\rangle} + a_i\langle\sigma^{i1}_{rr}\rangle$ and k_i. In the general case, this dependence can be represented as some function:

$$\tau^i_k = \varphi_i(I^{i1}_{\langle\sigma\rangle} + a_i\langle\sigma^{i1}_{rr}\rangle, k_i), \tag{1.7}$$

where $\varphi_i(k_i, k_i) = 0$ and $\varphi_i(0, k_i) = \infty$ according to (1.5).

The one-point distribution function $F_i(k_i)$ for some microvolume in the undamaged portion of the ith component can be approximated by a power function on some interval

$$F_i(k_i) = \begin{cases} 0, & k_i < k_{0i}, \\ \left(\dfrac{k_i - k_{0i}}{k_{1i} - k_{0i}}\right)^{\beta_i}, & k_{0i} \le k_i \le k_{1i}, \\ 1, & k_i > k_{1i} \end{cases} \tag{1.8}$$

or by the Weibull function

$$F_i(k_i) = \begin{cases} 0, & k_i < k_{0i}, \\ 1 - \exp\left[-m_i(k_i - k_{0i})^{\beta_i}\right], & k_i \ge k_{0i}, \end{cases} \tag{1.9}$$

where k_{0i} is the minimum value of k_i from which failure begins in some volumes of the ith component; k_{1i}, m_i, β_i are constants found from strength scatter fitting in the ith component.

Assume that the random field of the ultimate microstrength k_i is statistically homogeneous, which is typical of real materials, and individual microdamages and the distances between them are negligible compared with the inclusions and the distances between them. Then the distribution function $F_i(k_i)$ is ergodic because it defines the content of the undamaged portion of the ith component in which the ultimate microstrength is less than k_i.

Therefore, if the stresses $\langle\sigma^{i1}_{pq}\rangle$ are nonzero, the function $F_i(I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle)$ defines, according to (1.5), (1.8), and (1.9), the content of instantaneously destroyed microvolumes of the *i*th component. Since the destroyed microvolumes are modeled by pores, we can write a balance equation for destroyed microvolumes or porosities of the *i*th component subject to short-term damage:

$$p_i = p_{0i} + (1-p_{0i})F_i(I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle), \tag{1.10}$$

where p_{0i} is the initial porosity of the *i*th component and, according to (1.6),

$$I^{i1}_{\langle\sigma\rangle} = \frac{1}{1-p_i} I^{i}_{\langle\sigma\rangle}, \qquad I^{i}_{\langle\sigma\rangle} = (\langle\sigma^i_{jk}\rangle'\langle\sigma^i_{jk}\rangle')^{1/2}, \tag{1.11}$$

where $\langle\sigma^i_{jk}\rangle$ are related to $\langle\varepsilon_{jk}\rangle$ and θ by

$$\langle\sigma^i_{jk}\rangle = 2\mu_{ip}\langle\varepsilon_{jk}\rangle + \frac{\lambda_{ip}}{\lambda_{ip}+2\mu_{ip}}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}$$

$$\times\left[\left(\left\langle\frac{\lambda_p}{\lambda_p+2\mu_p}\right\rangle + 2\mu_{ip}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle\right)\langle\varepsilon_{rr}\rangle + \langle\varepsilon_{33}\rangle\right]\delta_{jk}$$

$$-\frac{1}{\lambda_{ip}+2\mu_{ip}}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}\left(\lambda_{ip}\left\langle\frac{\beta_p}{\lambda_p+2\mu_p}\right\rangle + 2\mu_{ip}\beta_{ip}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle\right)\theta\delta_{jk},$$

$$\langle\sigma^i_{33}\rangle = \left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}\left(\left\langle\frac{\lambda_p}{\lambda_p+2\mu_p}\right\rangle\langle\varepsilon_{rr}\rangle + \langle\varepsilon_{33}\rangle - \left\langle\frac{\beta_p}{\lambda_p+2\mu_p}\right\rangle\theta\right),$$

$$\langle\sigma^i_{j3}\rangle = 2\left\langle\frac{1}{\mu_p}\right\rangle^{-1}\langle\varepsilon_{j3}\rangle \qquad (j,k,r=1,2, \quad i=1,\ldots,N), \tag{1.12}$$

and the effective moduli $\lambda_{ip}, \mu_{ip}, \beta_{ip}$ are defined by (1.4).

If the stresses $\langle\sigma^{i1}_{pq}\rangle$ act for some time *t*, then, according to the stress-rupture criterion (1.7), those microvolumes of the *i*th component are destroyed that have k_i such that

$$t \ge \tau^i_k = \varphi_i(I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle, k_i), \tag{1.13}$$

where $I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle$ is defined by (1.12).

At low temperatures, the time to brittle fracture τ^i_k for real materials at low temperatures is finite beginning only from some value of $I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle > 0$. In this case, the durability function $\varphi_i(I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle, k_i)$ for a microvolume with instantaneous ultimate strength k_i can be represented as

$$\varphi_i(I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle, k_i) = \tau_{0i}\left(\frac{k_i - I^{i1}_{\langle\sigma\rangle} - a_i\langle\sigma^{i1}_{rr}\rangle}{I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle - \gamma_i k_i}\right)^{n_{1i}}$$

$$(\gamma_i k_i \le I^{i1}_{\langle\sigma\rangle}+a_i\langle\sigma^{i1}_{rr}\rangle \le k_i,\ \gamma_i < 1) \tag{1.14}$$

where some typical time τ_{i0}, exponent n_{1i}, and coefficient γ_i are determined from the fit of experimental durability curves for the *i*th component.

Substituting (1.14) into (1.13), we arrive at the inequality

$$k_i \le I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle \frac{1+\bar{t}_i^{-1/n_{1i}}}{1+\gamma_i \bar{t}_i^{-1/n_{1i}}} \qquad \left(\bar{t}_i = \frac{t}{\tau_{0i}} \right). \tag{1.15}$$

Considering the definition of the distribution function $F_i(k_i)$, we conclude that the function $F_i[(I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle)\psi_i(\bar{t}_i)]$, where

$$\psi_i(\bar{t}_i) = \frac{1+\bar{t}^{-1/n_{1i}}}{1+\gamma_i \bar{t}^{-1/n_{1i}}} \tag{1.16}$$

defines the relative content of the destroyed microvolumes in the undamaged portion of the *i*th component at the time $\bar{t}_i$. Then, in view of (1.6), the porosity balance equation for the *i*th component subject to long-term damage can be represented as

$$p_i = p_{0i} + (1-p_{0i}) F_i \left[\frac{I^{i}_{\langle\sigma\rangle} + a_i \langle \sigma^{i}_{rr} \rangle}{1-p_i} \psi_i(\bar{t}_i) \right], \tag{1.17}$$

where p_i is a function of dimensionless time $\bar{t}_i$, and $I^{i}_{\langle\sigma\rangle} + a_i \langle \sigma^{i}_{rr} \rangle$ is defined by (1.11) and (1.12).

If the time τ^i_k is finite for arbitrary values of $I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle$, which may be observed at high temperatures, then the durability function can be approximated by an exponential power function

$$\varphi_i(I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle, k_i) = \tau_{0i} \left\{ \exp m_{1i} \left[\left(k_i / (I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle) \right)^{n_{1i}} - 1 \right] - 1 \right\}^{n_{2i}} \tag{1.18}$$

which has enough constants τ_{0i}, m_{1i}, n_{1i}, n_{2i} to fit experimental curves. Substituting (1.18) into (1.13), we arrive at the inequality

$$k_i \le (I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle) \left[1 + \frac{1}{m_{1i}} \ln\left(1 + \bar{t}_i^{-1/n_{2i}} \right) \right]^{1/n_{1i}} \qquad \left(\bar{t}_i = \frac{t}{\tau_{0i}} \right). \tag{1.19}$$

Considering the definition of the distribution function $F_i(k_i)$, we conclude that the function $F_i[(I^{i1}_{\langle\sigma\rangle} + a_i \langle \sigma^{i1}_{rr} \rangle)\psi_i(\bar{t}_i)]$, where

$$\psi_i(\bar{t}_i) = \left[1 + \frac{1}{m_{1i}} \ln(1+\bar{t}_i^{-1/n_{2i}}) \right]^{1/n_{1i}}, \tag{1.20}$$

defines the relative content of the destroyed microvolumes in the undamaged portion of the *i*th component at the time $\bar{t}_i$. Then, in view of (1.1), the porosity balance equation for the *i*th component subject to long-term damage (1.6) can be represented in the form (1.17).

At $\bar{t}_i = 0$, the porosity balance equation (1.17) with (1.11), (1.12), (1.16) defines the short-term (instantaneous) damage of the *i*th component. As time elapses, Eqs. (1.17) with (1.12)–(1.15), (1.16) define its long-term damage, which consists of short-term damage and additional time-dependent damage.

2. Let us generalize the above damage model by assuming that the microdamages caused by loading be pores filled with particles of destroyed material that resist deformation. Let these particles yield to shear and uniform tension, but resist uniform compression as the undamaged material does. Then the shear modulus of the destroyed material filling the pores is equal to zero, and the bulk modulus is equal to zero when $\langle \sigma^{i2}_{rr} \rangle \ge 0 \, (i = 1, \ldots, N)$ and equal to the bulk modulus K_i of the undamaged material when $\langle \sigma_{rr}^{i2} \rangle < 0$, where $\langle \sigma_{rr}^{i2} \rangle$ are the stresses in the material filling the pores of the ith component. Then, according to Sec. 1, when $\langle \sigma_{rr}^{i2} \rangle \geq 0$, i.e., the average bulk stresses in the material filling the pores of the ith component are tensile, the effective moduli K_{1p}, μ_{1p} and factors β_{1p}, α_{1p} $(i = 1, \ldots, N)$ are defined by (1.4). If $\langle \sigma_{rr}^{i2} \rangle < 0$, i.e., the stresses are compressive, then, according to [7], we have

$$K_{ip} = K_i, \quad \mu_{ip} = \frac{[9K_i + 8\mu_i(1-p_i)]\mu_i(1-p_i)^2}{9K_i + 8\mu_i - (3K_i + 4\mu_i)p_i - 4\mu_i p_i^2}, \quad \beta_{ip} = \beta_i, \quad \alpha_{ip} = \alpha_i \quad (i = 1, 2). \tag{2.1}$$

Using the Schleicher–Nadai failure criterion (1.5), we arrive at the porosity balance equation (1.10), where $I_{\langle \sigma \rangle}^{i1}$ is defined by (1.11), (1.12), the function $\psi(\bar{t})$ by (1.16) or (1.20), and

$$\langle \sigma_{rr}^{i1} \rangle = \begin{cases} \dfrac{1}{1-p_i} \langle \sigma_{rr}^{i} \rangle, & \langle \sigma_{rr}^{i} \rangle \geq 0, \\ \langle \sigma_{rr}^{i} \rangle, & \langle \sigma_{rr}^{i} \rangle < 0, \end{cases} \tag{2.2}$$

where $\langle \sigma_{rr}^{i} \rangle$ are the average stresses in the ith component defined by (1.12).

Since the macrostrains $\langle \varepsilon_{jk} \rangle$ and temperature θ are given, we can use relation (1.12) to test the conditions $\langle \sigma_{rr}^{i} \rangle \geq 0$ and $\langle \sigma_{rr}^{i} \rangle < 0$. Considering (1.11), we reduce the porosity balance equation (1.10) with $\langle \sigma_{rr}^{i} \rangle \geq 0$ to the form (1.17), where $K_{1p}, \mu_{1p}, \beta_{1p}, \alpha_{1p}$ $(i = 1, \ldots, N)$ are defined by (1.4). If $\langle \sigma_{rr}^{i} \rangle < 0$, the porosity balance equation (1.10) is reduced to the form

$$p_i = p_{0i} + (1-p_{0i})F_i\left(\frac{I_{\langle \sigma \rangle}^{i}}{1-p_i} + a_i \langle \sigma_{rr}^{i} \rangle\right), \tag{2.3}$$

where the average stresses $\langle \sigma_{jk}^{i} \rangle$ are related to the macrostrains $\langle \varepsilon_{jk} \rangle$ by (1.12), and the effective moduli and thermal factors $K_{1p}, \mu_{1p}, \beta_{1p}, \alpha_{1p}$ $(i = 1, \ldots, N)$ are defined by (2.1).

3. Formulas (1.17), (1.2)–(1.4), (1.6), (1.8), (1.9) (1.11), (1.12), (1.16) (or (1.20)) for $\langle \sigma_{rr}^{i} \rangle \geq 0$ and formulas (1.17), (1.2), (1.3), (2.1), (2.2), (1.8), (1.9), (1.11), (1.12), (1.16) (or (1.20)) for $\langle \sigma_{rr}^{i} \rangle < 0$ can be used to develop an iterative algorithm for the determination of the stress-strain state of a laminated composite and the volume fraction of microdamages in its components. To this end, we will use the secant method [1].

Representing the porosity balance equation (1.10) in the form

$$\varphi_i(p_i) = \{p_i - p_{0i} - (1-p_{0i})F_i[(I_{\langle \sigma \rangle}^{i1} + a_i \langle \sigma_{rr}^{i1} \rangle)\psi_i(\bar{t}_i)]\} = 0 \tag{3.1}$$

it is easy to verify that the root p_i falls into the interval $[p_{0i}, 1]$ because of the inequalities

$$\varphi_i(p_{0i}) < 0, \quad \varphi_i(1) > 0. \tag{3.2}$$

Therefore, the zero approximation $p_i^{(0)}$ of the root is given by

$$p_i^{(0)} = \frac{a_i^{(0)} \varphi_i(b_i^{(0)}) - b_i^{(0)} \varphi_i(a_i^{(0)})}{\varphi_i(b_i^{(0)}) - \varphi_i(a_i^{(0)})}, \tag{3.3}$$

where $a_i^{(0)} = p_{0i}$, $b_i^{(0)} = 1$.

The subsequent approximations of the secant method are found in the iterative process

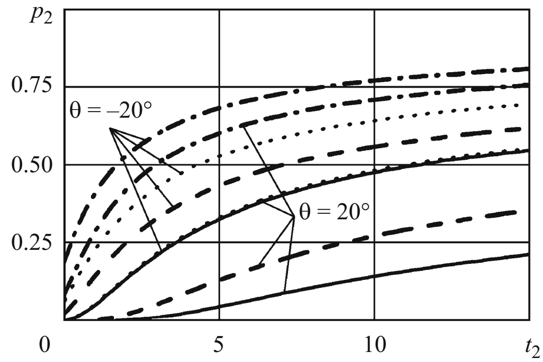

Fig. 1

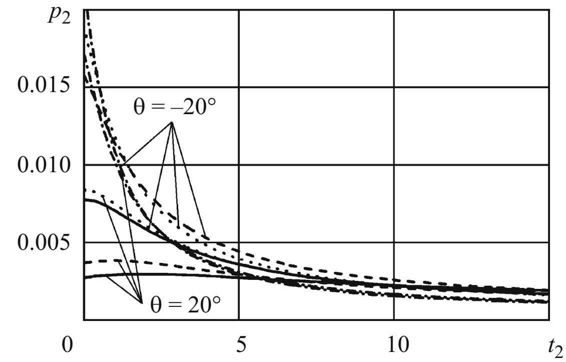

Fig. 2

$$p_i^{(m)} = \frac{a_i^{(m)} \varphi_i(b_i^{(m)}) - b_i^{(m)} \varphi_i(a_i^{(m)})}{\varphi_i(b_i^{(m)}) - \varphi_i(a_i^{(m)})}, \tag{3.4}$$

$$a_i^{(m)} = a_i^{(m-1)}, \quad b_i^{(m)} = p_i^{(m-1)} \quad \text{for} \quad \varphi_i(a_i^{(m-1)})\ \varphi_i(p_i^{(m-1)}) \le 0,$$

$$a_i^{(m)} = p_i^{(m-1)}, \quad b_i^{(m)} = b_i^{(m-1)} \quad \text{for} \quad \varphi_i(a_i^{(m-1)}) \varphi_i(p_i^{(m-1)}) \ge 0 \quad (m = 1, 2, \dots),$$

which proceeds until

$$|\varphi_i(p_i^{(m)})| < \varepsilon, \varphi, \tag{3.5}$$

where ε is the error of the root.

We conducted calculations to plot macrodeformation curves for two-layer composites with microdamaged matrix for Weibull distribution (1.9) and for fractional power durability function $\psi(\bar{t})$ defined by (1.16). Let the composite include a stiff layer made of aluminoborosilicate glass with the following characteristics [2] $E_1 = 70$ GPa, $\nu_1 = 0.2$, $\alpha_1 = 4.9 \cdot 10^{-6}$ °C^{-1} and volume fraction $c_1 = 0$, 0.25, 0.5, 0.75, 1.0 and epoxy matrix with the following characteristics of the undamaged portion [4]: $E_2 = 3$ GPa, $\nu_2 = 0.35$, $\alpha_2 = 45 \cdot 10^{-6}$ °C^{-1}. Here E_1 and E_2 are Young's moduli, ν_1 and ν_2 are Poisson's ratios, α_1 and α_2 are the thermal strain factors of the undamaged portions of the stiff layer and matrix, respectively. Also

$$p_{02} = 0, \quad k_{02} / \mu_2 = 0.01, \quad m_2 \mu_2^{\beta_2} = 1000, \quad \beta_2 = 2,$$

$$\sigma_{2\text{p}} = 0.011 \text{ GPa} \quad (\sigma_{2\text{p}} = \sqrt{1.5} k_{20}), \quad a_2 = 0.02, \quad \gamma_2 = 0.001, \quad n_{12} = 1, \quad \theta = \pm 20\,°\text{C}.$$

If

$$\langle \varepsilon_{33} \rangle = 0.02, \quad \langle \sigma_{11} \rangle = \langle \sigma_{22} \rangle = 0 \tag{3.6}$$

then, according to (1.1), the macrostresses $\langle \sigma_{33} \rangle$ are related to the macrostrains $\langle \varepsilon_{33} \rangle$ and temperature θ by

$$\langle \sigma_{33} \rangle = \frac{1}{\lambda_{11}^* + \lambda_{12}^*} \{[(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2] \langle \varepsilon_{33} \rangle - [(\lambda_{11}^* + \lambda_{12}^*)\beta_3^* - 2\lambda_{13}^* \beta_1^*]\theta\}.$$

In the porosity balance equations (1.17), (1.2)–(1.4), (1.6), (1.8), (1.9), (1.11), (1.12), (1.16) (or (1.20)) for $\langle \sigma_{rr}^2 \rangle \ge 0$ and (1.17), (1.2), (1.3), (2.1), (2.2), (1.8), (1.9), (1.11), (1.12), (1.16) (or (1.20)) for $\langle \sigma_{rr}^2 \rangle < 0$, we have

$$\langle \varepsilon_{11} \rangle = \langle \varepsilon_{22} \rangle = -\frac{\lambda_{13}^* \langle \varepsilon_{33} \rangle - \beta_1^* \theta}{\lambda_{11}^* + \lambda_{12}^*}$$

which is equivalent to (3.6).

Figure 1 shows the porosity p_2 as a function of time $\bar{t}_2$ for different values of θ and for $c_1 = 0$ (solid line), $c_1 = 0.25$ (dashed line), $c_1 = 0.5$ (dotted line), $c_1 = 0.75$ (dash-and-dot line). The same notation is used in Fig. 2. As the temperature θ decreases and the volume fraction c_1 increases, the porosity p_2 increases.

Figure 2 shows the macrostress $\langle \sigma_{33} \rangle / \mu_2$ as a function of time $\bar{t}_2$ for different values of θ and c_1. These curves are descending for all values of temperature and volume fraction. The decrease in macrostresses with time is not a monotonic function of temperature and volume fraction.

Conclusions. We have outlined the theory of long-term damage and deformation of laminated composites under thermal loads. The damage of layers is modeled by randomly arranged micropores. The failure criterion for a single microvolume is determined by its stress-rupture strength determined by the dependence of the time to brittle fracture on the difference between the equivalent stress and its limit, which characterizes the ultimate strength according to the Schleicher–Nadai criterion. An equation of damage (porosity) balance in the layers at an arbitrary time has been formulated taking into account the thermal component. Algorithms of calculating the time dependence of microdamage and macrostresses are developed. Corresponding curves are plotted. The effect of temperature on the macrodeformation and damage curves has been analyzed.

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