International Applied Mechanics, Vol. 46, No. 8, 2011

INFLUENCE OF INITIAL DEFLECTIONS ON THE STABILITY OF COMPOSITE CYLINDRICAL SHELLS INTERACTING WITH A FLUID FLOW

P. S. Koval'chuk and N. P. Podchasov

Composite cylindrical shells interacting with an internal fluid flow are analyzed for stability. It is assumed that the shells have small initial geometrical imperfections. The effect of axisymmetric and nonaxisymmetric initial deflections on the critical speeds of the fluid, which cause static (divergent) or dynamic (flutter) loss of stability, is studied

Keywords: composite cylindrical shell, ideal incompressible liquid, critical speed, initial imperfection, divergence, flutter, loss of stability

Introduction. The quasistatic (divergent) and dynamic (flutter) loss of stability of elastic cylindrical shells interacting with the internal fluid flow was studied in [2–5, 9, 11–13, etc.]. The relevant results obtained both long ago and in the last two decades are analyzed in [8, 9]. It was mainly assumed that the shells are perfect circular cylinders, and their material is described by an isotropic model. Initial geometrical imperfections present in almost every shell structure were neglected. Such imperfections, even if very small, may significantly affect the frequencies of vibrations of shells [6, 10] and the critical speeds of the fluid that cause one type of instability or another [2, 4]. This effect is expected to be stronger in composite shells whose dynamic properties are more sensitive to geometrical imperfections compared with metal shells.

The present paper studies the instability of composite cylindrical shells (orthotropic model) having small geometrical imperfections and interacting with a fluid flow. We will consider axisymmetric and nonaxisymmetric initial deflections. We will analyze the influence of these initial deflections on the critical speeds of the fluid that cause the shell to lose stability through either monotonic bulging (divergent buckling) or bulging with time-dependent amplitude (flutter buckling).

1. To analyze shells with fluid for stability, we will use linearized dynamic equations in mixed form [1–3]:

$$\frac{1}{h}\nabla_D^4 w_1 = \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2\frac{\partial^2 w_0}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{1}{R}\frac{\partial^2 \Phi}{\partial x^2} - \rho\frac{\partial^2 w_1}{\partial t^2} - \varepsilon_0 \rho \frac{\partial w_1}{\partial t} - \frac{P_h}{h},$$

$$\nabla_\delta^4 \Phi = -\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_1}{\partial y^2} - \frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2 w_1}{\partial x^2} + 2\frac{\partial^2 w_0}{\partial x \partial y}\frac{\partial^2 w_1}{\partial x \partial y} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}, \tag{1.1}$$

where $w_1 = w_1(x, y, t)$ and $w_0 = w_0(x, y)$ are dynamic and initial deflections, respectively, that have the order of the shell thickness (x and y are the axial and circumferential coordinates of points on the mid-surface); $\Phi = \Phi(x, y, t)$ is the stress function in the mid-surface; ∇_D^4 and ∇_δ^4 are operators such that

$$\nabla_D^4 = D_1\frac{\partial^4}{\partial x^4} + 2D_3\frac{\partial^4}{\partial x^2 \partial y^2} + D_2\frac{\partial^4}{\partial y^4},$$

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: volna@inmech.kiev.ua. Translated from Prikladnaya Mekhanika, Vol. 46, No. 8, pp. 58–70, August 2010. Original article submitted May 14, 2009.

 1063-7095/11/4608-0902

$$\nabla_{\delta}^{4}=\delta_2\frac{\partial^4}{\partial x^4}+2\delta_3\frac{\partial^4}{\partial x^2\partial y^2}+\delta_1\frac{\partial^4}{\partial y^4}, \tag{1.2}$$

where D_1 and D_2 are the axial and circumferential flexural stifnesses; D_3 is the effective stiffness; D_G is the torsional stiffness;

$$D_{1,2}=\frac{E_{1,2}h^3}{12(1-\mu_1\mu_2)},\qquad D_3=D_1\mu_2+2D_G,\qquad D_G=\frac{Gh^3}{12}, \tag{1.3}$$

E_1 and E_2 are the elastic moduli along the x-and y-axes, respectively; G is the shear modulus; μ_1 and μ_2 are Poisson's ratios $(E_1\mu_2=E_2\mu_1)$;

$$\delta_{1,2}=\frac{1}{E_{1,2}},\qquad 2\delta_3=\frac{1}{G}-\frac{2\mu_1}{E_1}, \tag{1.4}$$

h and R are the radius and thickness of the shell; ρ is the density of the shell's material; ε_0 is the damping factor; P_h is the hydrodynamic pressure on the shell, defined by the following formula [2, 3, 9]:

$$P_h=-\rho_0\left(\frac{\partial\varphi}{\partial t}+U\frac{\partial\varphi}{\partial x}\right)\Bigg|_{r=R}, \tag{1.5}$$

where ρ_0 is the density of the fluid; $\varphi=\varphi(x,r,\theta,t)$ is the perturbed velocity potential of the fluid, which is assumed perfect and incompressible (x,r,θ are cylindrical coordinates); U is the speed of the fluid, which is assumed constant.

Let the shell be simply supported at the ends $x=0$ and $x=l$ (l is the length of the shell). To clarify the fundamental aspect of the effect of the initial deflection w_0 on the stability of the shell, we will use a simplified four-mode approximation of the function w_1 [11]:

$$w_1=f_1\cos sy\sin\lambda_1 x+f_2\sin sy\sin\lambda_1 x+f_3\cos sy\sin\lambda_2 x+f_4\sin sy\sin\lambda_2 x, \tag{1.6}$$

which includes the conjugate and nonconjugate flexural modes [7]. Here f_k $(k=1,\ldots,4)$ are unknown functions of time; $s=n/R$, $\lambda_1=m_1\pi/l$, $\lambda_2=m_2\pi/l$ are wave numbers (n is the number of circumferential waves; m_1 and m_2 are the number of longitudinal half-waves). By analogy with [2], we will set $m_1=1$, $m_2=2$, i.e., we will consider the lowest axial modes that are the earliest (at the minimum speed of the fluid) to cause the shell to lose stability. The initial deflection w_0 is given by

$$w_0=f_{10}\cos s_0 y\sin\lambda_0 x+f_{20}\sin s_0 y\sin\lambda_0 x, \tag{1.7}$$

where $f_{10}, f_{20}=\mathrm{const}$ are constants; $s_0=n_0/R$, $\lambda_0=m_0\pi/l$, the parameters n_0 and m_0 being integers. The stress function Φ can be found from the second equation in (1.1) with (1.6), (1.7):

$$\begin{aligned}\Phi=\sum_{m=1}^{2}\Big[&\Phi_1^{nm}\cos sy\sin\lambda_m x+\Phi_2^{nm}\cos sy\sin\lambda_m x+\Phi_3^{nm}\cos(s-s_0)y\cos(\lambda_m-\lambda_0)x\\&+\Phi_4^{nm}\sin(s-s_0)y\cos(\lambda_m-\lambda_0)x+\Phi_5^{nm}\cos(s+s_0)y\cos(\lambda_m-\lambda_0)x\\&+\Phi_6^{nm}\sin(s+s_0)y\cos(\lambda_m-\lambda_0)x+\Phi_7^{nm}\cos(s-s_0)y\cos(\lambda_m+\lambda_0)x\\&+\Phi_8^{nm}\sin(s-s_0)y\cos(\lambda_m+\lambda_0)x+\Phi_9^{nm}\cos(s+s_0)y\cos(\lambda_m+\lambda_0)x\\&+\Phi_{10}^{nm}\sin(s+s_0)y\cos(\lambda_m+\lambda_0)x\Big],\end{aligned} \tag{1.8}$$

where Φ_j^{nm} $(j=1,\ldots,10)$ are functions expressed in some manner in terms of the displacements $f_1,\ldots,f_4$, wave numbers, physical and geometrical parameters of the shell:

$$\Phi_1^{n1} = \frac{\lambda_1^2 f_1}{R\Delta_\delta(\lambda_1, s)}, \quad \Phi_1^{n2} = \frac{\lambda_2^2 f_1}{R\Delta_\delta(\lambda_2, s)}, \quad \Phi_3^{n1} = -\frac{(\lambda_0 s - \lambda_1 s_0)^2 (f_1 f_{10} + f_2 f_{20})}{4\Delta_\delta(\lambda_1 - \lambda_0, s - s_0)}, \quad \text{etc.} \tag{1.9}$$

Hereafter Δ_δ denotes the operator $\Delta_\delta(A, B) = \delta_2 A^4 + 2\delta_3 A^2 B^2 + \delta_1 B^4$.

Considering $w_1, w_0 << R$ [1, 3], we determine the potential φ by solving the following boundary-value problem [2, 9]:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0,$$

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=R} = -\left(\frac{\partial w_1}{\partial t} + U\frac{\partial w_1}{\partial x}\right), \quad \left.\frac{\partial\varphi}{\partial r}\right|_{r=0} < \infty. \tag{1.10}$$

The hydrodynamic pressure P_h is given by

$$P_\text{h} = \rho_0\left[\left(\frac{K_1}{\lambda_1}\ddot{f}_1 \sin\lambda_1 x + \frac{K_2}{\lambda_2}\ddot{f}_3 \sin\lambda_2 x\right)\cos sy + \left(\frac{K_1}{\lambda_1}\ddot{f}_2 \sin\lambda_1 x + \frac{K_2}{\lambda_2}\ddot{f}_4 \sin\lambda_2 x\right)\sin sy\right.$$

$$+2(K_1\dot{f}_1 \cos\lambda_1 x + K_2\dot{f}_3 \cos\lambda_2 x)U\cos sy + 2(K_1\dot{f}_2 \cos\lambda_1 x + K_2\dot{f}_4 \cos\lambda_2 x)U\sin sy$$

$$\left.-(K_1\lambda_1 f_1 \sin\lambda_1 x + K_2\lambda_2 f_3 \sin\lambda_2 x)U^2 \cos sy - (K_1\lambda_1 f_2 \sin\lambda_1 x + K_2\lambda_2 f_4 \sin\lambda_2 x)U^2 \sin sy\right], \tag{1.11}$$

where

$$K_1 = \frac{I_n(\lambda_1 R)}{I'_n(\lambda_1 R)}, \quad K_2 = \frac{I_n(\lambda_2 R)}{I'_n(\lambda_2 R)}, \tag{1.12}$$

I_n are modified Bessel functions of the nth order (the overbar denotes differentiation).

Substituting (1.6), (1.7), (1.9), (1.11) into the first equation in (1.1) and using the Bubnov–Galerkin method ($z_1 = \sin\lambda_1 x\cos sy$, $z_2 = \sin\lambda_1 x\sin sy$, $z_3 = \sin\lambda_2 x\cos sy$, $z_4 = \sin\lambda_2 x\sin sy$ are chosen to be weight functions), we obtain the following system of equations for the generalized displacements f_k:

$$\ddot{f}_k + (\omega_k^2 - \alpha_k U^2) f_k + \varepsilon_k \dot{f}_k + \sum_{j=1}^{4} c_{kj} f_j + U\sum_{j=1}^{4}\beta_{kj}\dot{f}_j = 0 \quad (k = 1, \ldots, 4), \tag{1.13}$$

where ω_k are the natural frequencies of the shell with the added mass of the fluid,

$$\omega_1^2 = \omega_2^2 = \frac{1}{\rho m_{01}}\left[\frac{\Delta_D(\lambda_1, s)}{h} + \frac{\lambda_1^4}{\Delta_\delta(\lambda_1, s)R^2}\right], \quad \omega_3^2 = \omega_4^2 = \frac{1}{\rho m_{02}}\left[\frac{\Delta_D(\lambda_2, s)}{h} + \frac{\lambda_2^4}{\Delta_\delta(\lambda_2, s)R^2}\right],$$

$$m_{0i} = 1 + \frac{\rho_0}{\rho}\frac{K_i}{\lambda_i h}, \quad \Delta_D(\lambda_i, s) = D_1\lambda_i^4 + 2D_3\lambda_i^2 s_i^2 + D_2 s_i^4 \quad (i = 1, 2), \tag{1.14}$$

ε_k are damping parameters: $\varepsilon_1 = \varepsilon_2 = \dfrac{\varepsilon_0}{m_{01}}$, $\varepsilon_3 = \varepsilon_4 = \dfrac{\varepsilon_0}{m_{02}}$, α_k, β_{jk} are constant coefficients [11, 12],

$$\alpha_1 = \alpha_2 = \frac{\rho_0 K_1\lambda_1}{\rho h m_{01}}, \quad \alpha_3 = \alpha_4 = \frac{\rho_0 K_2\lambda_2}{\rho h m_{02}},$$

$$\beta_{13} = \beta_{24} = -\frac{16\rho_0 K_2}{3\rho h l m_{01}\lambda_2}, \quad \beta_{31} = \beta_{42} = \frac{16\rho_0 K_1}{3\rho h l m_{02}\lambda_1}, \tag{1.15}$$

(the remaining $\beta_{kj} = 0$); c_{kj} are coefficients of integration dependent on the amplitude of initial deflection f_{10}, f_{20} (all $c_{kj} = 0$ for $f_{10} = f_{20} = 0$). The values of c_{kj} are determined by the ratio between the wave numbers of the initial (s_0, λ_0) and dynamic (s, λ_1, λ_2) deflections. Let us consider Eqs. (1.13) for the most typical relations among the parameters.

2. Let $s = s_0 \neq 0$, $\lambda_1 = \lambda_0$, i.e., the initial deflection "resonates" with one of the terms of the dynamic deflection w_1. In this case, the coefficients c_{kj} in (1.13) become

$$c_{11} = \frac{1}{8\rho m_{01}}\left[\frac{\lambda_1^4}{\delta_1}(f_{10}^2 + f_{20}^2) + \frac{s^4}{\delta_2} f_{10}^2\right], \quad c_{12} = \frac{s^4}{8\rho\delta_2 m_{01}} f_{10} f_{20} = c_{21},$$

$$c_{22} = \frac{1}{8\rho m_{01}}\left[\frac{\lambda_1^4}{\delta_1}(f_{10}^2 + f_{20}^2) + \frac{s^4}{\delta_2} f_{20}^2\right], \quad c_{33} = \frac{1}{\rho m_{02}}\left[P(f_{10}^2 + f_{20}^2) + \frac{s^4}{4\delta_2} f_{10}^2\right],$$

$$c_{44} = \frac{1}{\rho m_{02}}\left[P(f_{10}^2 + f_{20}^2) + \frac{s^4}{4\delta_2} f_{20}^2\right], \quad P = \frac{s^4(\lambda_1 + \lambda_2)^4}{16\Delta_\delta(\lambda_1 - \lambda_2, 2s)} + \frac{s^4(\lambda_1 - \lambda_2)^4}{16\Delta_\delta(\lambda_1 + \lambda_2, 2s)},$$

$$c_{13} = c_{14} = c_{23} = c_{24} = c_{31} = c_{32} = c_{34} = c_{41} = c_{42} = c_{43} = 0. \tag{2.1}$$

To determine the critical speeds of the fluid, we set up a characteristic equation for system (1.13) taking (2.1) into account and assuming that

$$f_k = C_k e^{\Omega t}, \quad C_k = \text{const} \quad (k = 1, \ldots, 4). \tag{2.2}$$

Doing so gives an octic equation for Ω:

$$\begin{aligned}&[\beta_1\beta_2 U^2\Omega^2 + (\Omega^2 + \omega_{12} - \alpha_1 U^2 + \varepsilon_1\Omega)(\Omega^2 + \omega_{22} - \alpha_3 U^2 + \varepsilon_3\Omega)]\\ &\times[\beta_1\beta_2 U^2\Omega^2 + (\Omega^2 + \omega_{11} - \alpha_1 U^2 + \varepsilon_1\Omega)(\Omega^2 + \omega_{21} - \alpha_3 U^2 + \varepsilon_3\Omega)]\\ &- c_{12}^2(\Omega^2 + \omega_{21} - \alpha_3 U^2 + \varepsilon_3\Omega)(\Omega^2 + \omega_{22} - \alpha_3 U^2 + \varepsilon_3\Omega) = 0,\end{aligned} \tag{2.3}$$

where

$$\omega_{11} = \omega_1^2 + c_{11}, \quad \omega_{12} = \omega_1^2 + c_{22}, \quad \omega_{21} = \omega_3^2 + c_{33}, \quad \omega_{22} = \omega_3^2 + c_{44},$$

$$\beta_1 = -\beta_{13}, \quad \beta_2 = \beta_{31}. \tag{2.4}$$

The shell is stable as long as the real parts of all characteristic numbers Ω are negative. If at least one of these numbers does not satisfy this condition, the unperturbed configuration of the shell will become unstable.

Let us simplify the expression for the initial deflection w_0 by setting one of its amplitude parameters, say f_{20}, at zero. Then Eq. (2.3) can be decomposed into two quartic equations:

$$\Omega^4 + k_{1j}\Omega^3 + k_{2j}\Omega^2 + k_{3j}\Omega + k_{4j} = 0 \quad (j = 1, 2). \tag{2.5}$$

The coefficients $k_{1j}, \ldots, k_{4j}$ are expressed as

$$k_{1j} = \varepsilon_1 + \varepsilon_3, \quad k_{2j} = q_j - \alpha_0 U^2, \quad k_{3j} = \varepsilon_3(\omega_{1i} - \alpha_1 U^2) + \varepsilon_1(\omega_{2i} - \alpha_3 U^2),$$

$$k_{4j} = (\omega_{1i} - \alpha_1 U^2)(\omega_{2i} - \alpha_3 U^2) \quad (i, j = 1, 2, i \neq j) \tag{2.6}$$

where

$$q_1 = \omega_{12} + \omega_{22} + \varepsilon_1\varepsilon_3, \quad q_2 = \omega_{11} + \omega_{21} + \varepsilon_1\varepsilon_3,$$

TABLE 1

$\bar{f}_0$	n			
	2	3	4	5
0	23.67	17.07	18.87	19.98
1	23.68	17.08	18.87	19.99
2	23.69	17.10	18.90	20.01

$$\alpha_0 = \alpha_1 + \alpha_3 - \beta_1\beta_3. \tag{2.7}$$

Thus, the shell sustains divergent buckling when $U = U_d$, which is the minimum root of the following equations [2]:

$$\omega_{11} - \alpha_1 U^2 = 0, \quad \omega_{12} - \alpha_1 U^2 = 0, \quad \omega_{21} - \alpha_3 U^2 = 0, \quad \omega_{22} - \alpha_3 U^2 = 0. \tag{2.8}$$

Flutter instability occurs when $U = U_f$, which can be found from the equations

$$k_{11}k_{21}k_{31} - k_{31}^2 - k_{41}k_{11}^2 = 0, \quad k_{12}k_{22}k_{32} - k_{32}^2 - k_{42}k_{12}^2 = 0. \tag{2.9}$$

For example, the first equation in (2.9) yields

$$U_f^2 = -\frac{d_2}{2d_1} \pm \sqrt{\left(\frac{d_2}{2d_1}\right)^2 - \frac{d_3}{d_1}}, \tag{2.10}$$

where

$$d_1 = \varepsilon_1\varepsilon_3(\alpha_1 - \alpha_3)^2 - \beta_1\beta_2(\varepsilon_1 + \varepsilon_3)(\varepsilon_1\alpha_3 + \varepsilon_3\alpha_1),$$

$$d_2 = -(\varepsilon_1 + \varepsilon_3)[(\varepsilon_1\alpha_3 + \varepsilon_3\alpha_1)q_2 + (\varepsilon_1\omega_{22} + \varepsilon_3\omega_{12})\alpha_0]$$

$$+2(\varepsilon_1\omega_{22} + \varepsilon_3\omega_{12})(\varepsilon_1\alpha_3 + \varepsilon_3\alpha_1) + (\varepsilon_1 + \varepsilon_3)^2(\alpha_1\omega_{22} + \alpha_3\omega_{12}),$$

$$d_3 = (\varepsilon_1 + \varepsilon_3)q_2(\varepsilon_1\omega_{22} + \varepsilon_3\omega_{12}) - (\varepsilon_1\omega_{22} + \varepsilon_3\omega_{12})^2 - (\varepsilon_1 + \varepsilon_3)^2\omega_{12}\omega_{22}. \tag{2.11}$$

A similar solution follows from the second equation in (2.9). The coefficient d_1 is the same as (2.11), and it is necessary to replace ω_{12} by ω_{11} and ω_{22} by ω_{12} in the expressions for d_2 and d_3, respectively.

Numerical Example. Let the shell be filled with a fluid of density $\rho_0 = 10^3$ kg/m^3 and have the parameters

$$E_1 = 2.15\cdot 10^9 \text{ Pa}, \quad E_2 = 1.23\cdot 10^9 \text{ Pa}, \quad G = 0.21\cdot 10^9 \text{ Pa}, \quad \varepsilon_0 = 0.1 \text{ sec}^{-1},$$

$$\rho = 1.65\cdot 10^3 \text{ kg/m}^3, \quad R = 0.16 \text{ m}, \quad h = R/100, \quad l = 5h, \quad \mu_1 = 0.19. \tag{2.12}$$

An analysis of Eqs. (2.8) shows that when $U = U_d = 17.07$ m/sec and $w_0 = 0$, the divergent buckling mode $n = 3$ is observed the earliest.

An initial deflection similar in form to the dynamic deflection has a weak effect on the critical speeds U_d and U_f. Table 1 summarizes the values of the minimum speed U_d corresponding to different dimensionless amplitudes of initial deflection $\bar{f}_0 = f_{10}/h$ and different values of the wave number n for $f_{20} = 0$.

It can be seen that the difference between the critical speeds U_d of the imperfect shell and perfect shell ($\bar{f}_0 = 0$) for $n = 2,\ldots,5$ does not exceed 0.22%. The same is true of the flutter speeds.

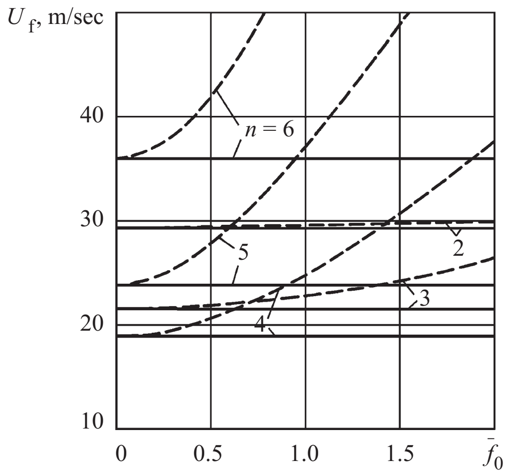


Fig. 1

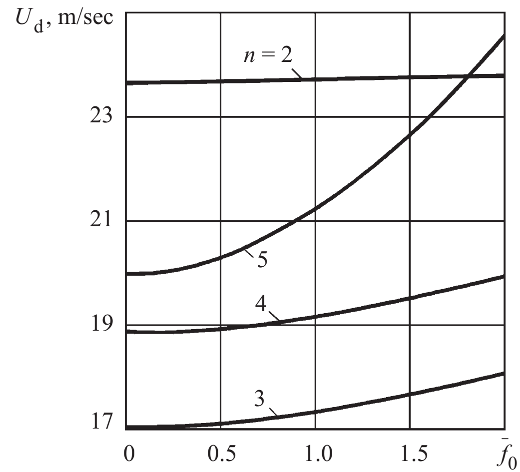


Fig. 2

Figure 1 shows curves of U_f versus $\bar{f}_0$ plotted from Eqs. (2.9) for different values of n (indicated in Fig. 1). The solid lines represent the first equation in (2.9), and the dashed lines the second equation. As is seen, dynamic buckling of the shell occurs the earliest when the circumferential mode $n = 4$ is excited. This happens when $w_0 = 0$ and $U = U_f = 19.06$ m/sec. If the shell has an initial deflection w_0 with $f_{10} = 2h$, then $U_f = 19.09$ m/sec, i.e., the critical speed U_f increases a little.

When $n = 3$, we have $U_f = 21.60$ m/sec if $\bar{f}_0 = 0$ and $U_f = 21.62$ m/sec if $f_{10} = h$. Thus, the shell with the initial deflection "resonating" with the dynamic deflection in circumferential mode appears weakly "sensitive" to " the effect of geometrical imperfections on both divergent and flutter buckling.

3. Let us consider the second case of relations between the wave numbers of dynamic and initial deflections: $s \neq s_0 \neq 0$, $\lambda_1 \neq \lambda_0, \lambda_2 \neq \lambda_0$. Then

$$c_{11} = c_{22} = P_1(f_{10}^2 + f_{20}^2), \qquad c_{33} = c_{44} = P_2(f_{10}^2 + f_{20}^2), \tag{3.1}$$

where

$$P_i = \frac{1}{16\rho m_{0i}} \left\{ (s_0\lambda_i + \lambda_0 s)^4 \left[\frac{1}{\Delta_\delta(\lambda_i - \lambda_0, s + s_0)} + \frac{1}{\Delta_\delta(\lambda_i + \lambda_0, s - s_0)} \right] \right.$$

$$\left. + (s_0\lambda_i - \lambda_0 s)^4 \left[\frac{1}{\Delta_\delta(\lambda_i + \lambda_0, s - s_0)} + \frac{1}{\Delta_\delta(\lambda_i - \lambda_0, s - s_0)} \right] \right\} \quad (i = 1, 2) \tag{3.2}$$

should be set in (1.13).

The other coefficients $c_{kj} = 0$. The characteristic equation of system (1.13) is

$$\Omega^4 + k_1\Omega^3 + k_2\Omega^2 + k_3\Omega + k_4 = 0, \tag{3.3}$$

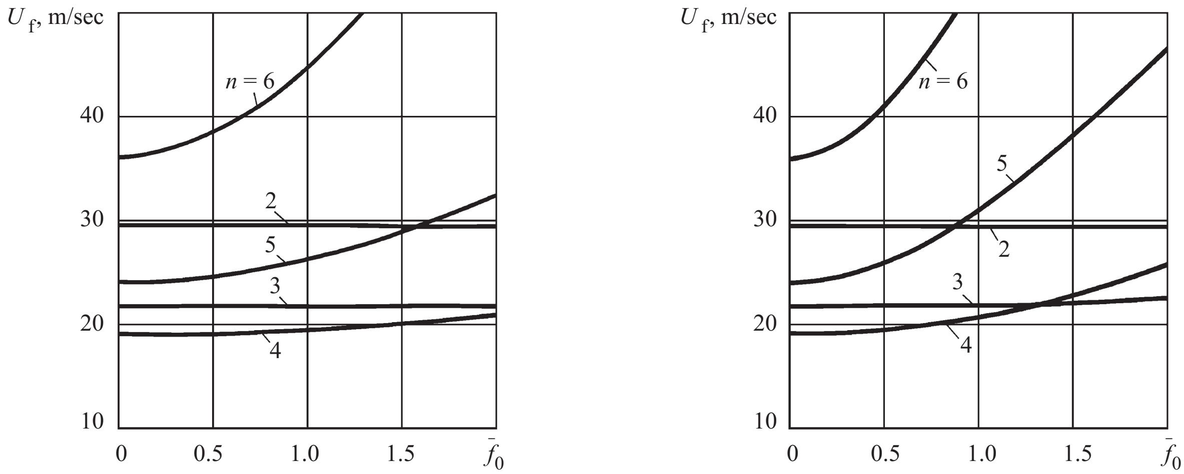
where the coefficients are expressed as

$$k_1 = \varepsilon_1 + \varepsilon_3, \qquad k_2 = \omega_{11} + \omega_{22} - \alpha_0 U^2 + \varepsilon_1\varepsilon_3,$$

$$k_3 = \varepsilon_3(\omega_{11} - \alpha_1 U^2) + \varepsilon_1(\omega_{22} - \alpha_2 U^2),$$

$$k_4 = (\omega_{11} - \alpha_1 U^2)(\omega_{22} - \alpha_2 U^2), \tag{3.4}$$

and $\omega_{11} = \omega_1^2 + c_{11}$, $\omega_{22} = \omega_2^2 + c_{33}$.



a *b*

Fig. 3

The critical speeds U_d and U_f should be determined from the equations

$$(\omega_{11}-\alpha_1 U_d^2)(\omega_{22}-\alpha_2 U_d^2)=0, \tag{3.5}$$

$$(\varepsilon_1+\varepsilon_3)(q-\alpha_0 U_f^2)[\varepsilon_3(\omega_{11}-\alpha_1 U_f^2)+\varepsilon_1(\omega_{22}-\alpha_3 U_f^2)]$$

$$-[\varepsilon_3(\omega_{11}-\alpha_1 U_f^2)+\varepsilon_1(\omega_{22}-\alpha_2 U_f^2)]^2-(\varepsilon_1+\varepsilon_3)^2[(\omega_{11}-\alpha_1 U_f^2)(\omega_{22}-\alpha_2 U_f^2)=0$$

$$(q=\omega_{11}+\omega_{22}+\varepsilon_1\varepsilon_3). \tag{3.6}$$

Figures 2 and 3 show plots of $U_d=U_d(\bar{f}_0)$ and $U_f=U_f(\bar{f}_0)$ for different values of the wave number n.

Figure 2 illustrates curves of minimum critical speed $U_d=U_d(\bar{f}_0)$ found by (3.5) for n = 2, 3, 4, 5 and $0\le \bar{f}_0\le 2$ (as before, the shell has parameters (2.12) and $f_{20}=0$). The wave numbers of the initial deflection are $m_0=3$ and $n_0=7$.

Table 2 gives the width of the divergent instability zone $\Delta_0=U_d^{\max}-U_d^{\min}$ calculated for each parameter n (n = 2,…,5) used in Fig. 2*a*.

Thus, as in the problem of Sec. 2, the divergent buckling of the shell occurs the earliest when the circumferential mode n = 3 is excited. The initial deflection significantly increases the speed U_d compared with the perfect shell for a relatively great wave number n ($n\ge 5$).

When n is small (n = 2, 3), the relative increase in the speed U_d does not exceed 5.9% of its value at $w_0\equiv 0$. As the parameter n increases, the width of the divergent instability zone Δ_0 first decreases a little, reaching the minimum at n = 4, and then abruptly increases.

Figure 3*a* shows the flutter speed U_f as a function of $\bar{f}_0$ for $m_0=3$ and $n_0=7$, as before. It can be seen that oscillatory buckling occurs when the circumferential mode $n=4$ is excited. As the initial deflection increases, the flutter speeds corresponding to $n=3$ and $n=4$ approach each other. Figure 3*b* illustrates a similar pattern for $m_0=8$ and $n_0=7$. While $\bar{f}_0\le 1.33$, the shell buckles in the circumferential mode $n=4$, and $19.06\le U_f\le 22.24$ m/sec. When $\bar{f}_0>1.33$, the buckling mode changes qualitatively: the circumferential mode $n=3$ is excited, along with axial modes ($m_1=1$, $m_2=2$).

4. Let us analyze the effect of an axisymmetric initial deflection on the buckling of the shell. To this end, we set $s_0=0$ in (1.7). The coefficients c_{kj} in (1.13) are now expressed as

$$c_{11}=c_{22}=\gamma_{11}f_0+\gamma_{12}f_0^2,\qquad c_{13}=c_{24}=\gamma_{13}f_0,$$

$$c_{31}=c_{42}=\gamma_{23}f_0,\qquad c_{33}=c_{44}=\gamma_{21}f_0+\gamma_{22}f_0^2, \tag{4.1}$$

where

TABLE 2

$\bar{f}_0$	n			
	2	3	4	5
0	6.83	5.34	0.86	10.14
1	6.83	4.93	2.24	13.83
2	6.82	4.52	3.63	17.51

TABLE 3

f_{10}	n			
	2	3	4	5
0	23.67	17.07	18.87	19.98
$+h$	23.14	16.33	18.75	19.91
$-h$	24.23	17.84	18.97	20.08
$+2h$	22.60	15.60	18.62	19.85
$-2h$	24.78	18.61	19.07	20.18

$$\gamma_{11} = -\frac{\lambda_0\lambda_1 s^2}{R\rho l m_{01}}\left[\frac{4\lambda_1^3}{(4\lambda_1^2-\lambda_0^2)\Delta_\delta(\lambda_1,s)} + \frac{(\lambda_i-\lambda_0)^2}{(2\lambda_1-\lambda_0)\Delta_\delta(\lambda_1-\lambda_0,s)} + \frac{(\lambda_i+\lambda_0)^2}{(2\lambda_1+\lambda_0)\Delta_\delta(\lambda_1+\lambda_0,s)}\right]A,$$

$$\gamma_{12} = \frac{\lambda_0^4 s^4}{4\rho m_{0i}}\left[\frac{1}{\Delta_\delta(\lambda_1+\lambda_0,s)} + \frac{1}{\Delta_\delta(\lambda_1-\lambda_0,s)}\right],$$

$$\gamma_{21} = -\frac{\lambda_0\lambda_1 s^2}{R\rho l m_{02}}\left[\frac{4\lambda_1^3}{(4\lambda_2^2-\lambda_0^2)\Delta_\delta(\lambda_2,s)} + \frac{(\lambda_2-\lambda_0)^2}{(4\lambda_1-\lambda_0)\Delta_\delta(\lambda_2-\lambda_0,s)} + \frac{(\lambda_2+\lambda_0)^2}{(4\lambda_1+\lambda_0)\Delta_\delta(\lambda_2+\lambda_0,s)}\right]A,$$

$$\gamma_{22} = \frac{\lambda_0^4 s^4}{4\rho m_{02}}\left[\frac{1}{\Delta_\delta(\lambda_2+\lambda_0,s)} + \frac{1}{\Delta_\delta(\lambda_2-\lambda_0,s)}\right],$$

$$\gamma_{13} = -\frac{\lambda_0^2\lambda_1 s^2}{R\rho l m_{01}}\left[\frac{8\lambda_1\lambda_0\lambda_2^2}{(\lambda_0^2-\lambda_1^2)(9\lambda_1^2-\lambda_0^2)\Delta_\delta(\lambda_2,s)}\right.$$

$$\left. + \frac{(\lambda_2-\lambda_0)^2}{(\lambda_0-\lambda_1)(3\lambda_1-\lambda_0)\Delta_\delta(\lambda_2-\lambda_0,s)} + \frac{(\lambda_2+\lambda_0)^2}{(\lambda_0+\lambda_1)(3\lambda_1+\lambda_0)\Delta_\delta(\lambda_2+\lambda_0,s)}\right]B,$$

$$\gamma_{23} = -\frac{\lambda_0^2\lambda_2 s^2}{R\rho l m_{02}}\left[\frac{8\lambda_1\lambda_0\lambda_2^2}{(\lambda_0^2-\lambda_1^2)(9\lambda_1^2-\lambda_0^2)\Delta_\delta(\lambda_1,s)}\right.$$

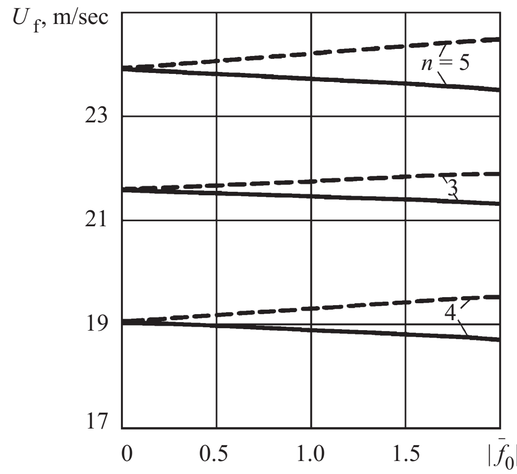


Fig. 4

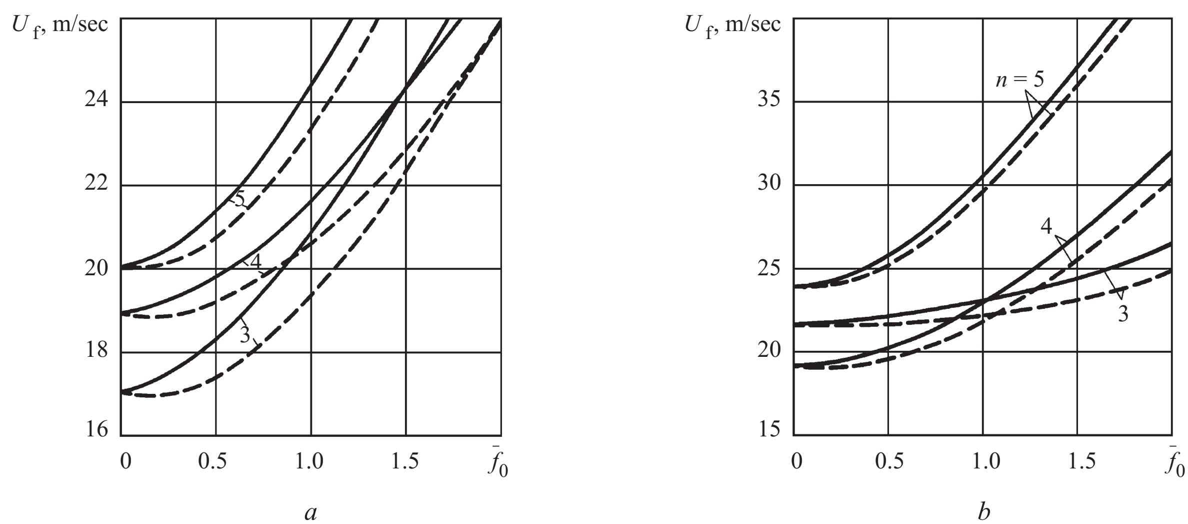


Fig. 5

$$+\frac{(2\lambda_1-\lambda_0)(\lambda_1-\lambda_0)^2}{(\lambda_0+\lambda_1)(3\lambda_1-\lambda_0)\Delta_\delta(\lambda_1-\lambda_0,s)}-\frac{2\lambda_1(\lambda_1+\lambda_0)^2}{(\lambda_1-\lambda_0)(3\lambda_1+\lambda_0)\Delta_\delta(\lambda_1+\lambda_0,s)}\Bigg]B. \quad (4.2)$$

The parameters A and B are given by

$$A = 1-(-1)^{m_0}, \qquad B = 1-(-1)^{\pm m_0 \pm m_1}. \quad (4.3)$$

Let m_0 be odd. Then $\gamma_{13} = \gamma_{23} = 0$, and the problem becomes similar to that considered in Sec. 3. The divergence speeds can be found from Eq. (3.5) with (4.1) and (4.2). The values of these speeds are summarized in Table 3. As usual, the wave number n is varied and the initial deflection amplitude f_{10} ($f_{10} = 0, \pm h, \pm 2h$) is kept constant for $m_0 = 1$.

It can be seen from Table 3 that the critical speed U_d depends on how the axisymmetric initial deflection w_0 is arranged relative to the initial mid-surface of the perfect shell. If, for example, $f_{10} < 0$ (i.e., the initial deflection makes the shell convex, like a barrel), the critical speed U_d increases with $|f_{10}|$. Hence, the shell is more stable against divergent buckling. If, vice versa, $f_{10} > 0$ (the shell is concave, its Gaussian curvature is negative), then the initial deflection decreases U_d compared with the perfect shell.

The initial deflection $w_0(x)$ has a similar effect on the flutter speed U_f: it increases a little (compared with the case of $w_0 = 0$) when $f_{10} < 0$ and decreases when $f_{10} > 0$ (Fig. 4). As follows from Fig. 4, the shell loses stability the earliest when the circumferential mode $n = 4$ is excited (the dashed curves correspond to $f_{10} < 0$ and the solid lines to $f_{10} > 0$).

The more complicated the configuration (say, the greater the wave number m_0) of the axisymmetric deflection $w_0 = w_0(x)$, the stronger its effect on the stability of the shell. In this case, the critical speeds U_d and U_f for $f_{10} < 0$ and $f_{10} > 0$ always exceed those for $f_{10} = 0$. Figures 5*a* and 5*b* show these speeds for $m_0 = 7$ and $0 \le |f_{10}| \le h$. As before, the dashed curves correspond to $f_{10} < 0$, and the solid curves to $f_{10} > 0$.

If the number of waves is even ($f_{10} < 0$), the deflection w_0 has a qualitatively same effect on the stability of the shell as when m_0 is even.

Conclusions. We have solved the buckling problem for imperfect (with initial geometrical imperfections) orthotropic cylindrical shells interacting with an internal fluid flow. We have analyzed the effect of axisymmetric and nonaxisymmetric initial deflections on the critical speeds of the flow that cause quasistatic (divergent) or dynamic (flutter) buckling.

REFERENCES

1. S. A. Ambartsumyan, *General Theory of Anisotropic Shells* [in Russian], Nauka, Moscow (1974).
2. V. V. Bolotin, *Nonconservative Problems of the Theory of Elastic Stability*, Pergamon Press, Oxford (1963).
3. A. S. Vol'mir, *Shells in Fluid Flow: Problems of Hydroelasticity* [in Russian], Nauka, Moscow (1979).
4. J. Horaček and I. A. Zolotarev, "Natural vibrations and stability of cylindrical shells interacting with fluid flow," in: A. N. Guz (ed.), *Dynamics of Bodies Interacting with a Medium* [in Russian], Naukova Dumka, Kyiv (1991), pp. 215–272.
5. P. S. Koval'chuk, "Nonlinear vibrations of a cylindrical shell containing a flowing fluid," *Int. Appl. Mech.*, **41**, No. 4, 405–412 (2005).
6. V. D. Kubenko and P. S. Koval'chuk, "Influence of initial geometric imperfections on the vibrations and dynamic stability of elastic shells," *Int. Appl. Mech.*, **40**, No. 8, 847–877 (2004).
7. V. D. Kubenko, P. S. Koval'chuk, and N. P. Podchasov, *Nonlinear Vibrations of Cylindrical Shells* [in Russian], Vyshcha Shkola, Kyiv (1989).
8. M. Amabili and M. P. Païdoussis, "Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction," *Appl. Mech. Rev.*, **56**, No. 4, 349–381 (2003).
9. M. Amabili, F. Pellicano, and M. P. Païdoussis, "Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid. Part I: Stability," *J. Sound Vibr.*, **225**, No. 4, 655–699 (1999).
10. G. D. Gavrilenko, V. I. Matsner, and O. A. Kutenkova, "Free vibrations of ribbed cylindrical shells with local axisymmetric deflections," *Int. Appl. Mech.*, **44**, No. 9, 1006–1014 (2008).
11. P. S. Koval'chuk and L. A. Kruk, "Nonlinear parametric vibrations of orthotropic cylindrical shells interacting with a pulsating fluid flow," *Int. Appl. Mech.*, **45**, No. 9, 1007–1015 (2009).
12. P. S. Koval'chuk and N. P. Podchasov, "Stability of elastic cylindrical shells interacting with flowing fluid," *Int. Appl. Mech.*, **46**, No. 1, 60–68 (2010).
13. V. D. Kubenko, P. S. Kovalchuk, and L. A. Kruk, "Nonlinear vibrations of cylindrical shells filled with a fluid and subjected to longitudinal and transverse periodic excitation," *Int. Appl. Mech.*, **46**, No. 2, 186–194 (2010).