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DAMPING THE VIBRATIONS OF A CLAMPED PLATE USING THE SENSOR'S READINGS

T. V. Karnaukhova

The problem of active damping of the forced resonant flexural vibrations of a clamped thermoviscoelastic orthotropic plate is solved. It is assumed that the mechanical load is unknown and determined from the sensor's readings. The Bubnov-Galerkin method is used to derive a formula for the voltage that should be applied to the actuator to damp the first vibration mode of the plate. The effect of the dimensions of the sensor and actuator, the dissipative properties, and mechanical boundary conditions on the effectiveness of active damping is analyzed

Keywords: plate, clamped edge, active damping, forced resonant flexural vibrations, sensor, actuator, unknown load

Introduction. The active damping of the vibrations of thin-walled elements with piezoelectric sensors and actuators is based on the assumption that the mechanical load is known [10, 11]. The paper [3] proposed a new approach to the damping of the forced resonant vibrations of thin viscoelastic orthotropic plates with piezoelectric sensors and actuators when the amplitude of the mechanical load on the plate is unknown. Hinged edges as the simplest mechanical boundary conditions were considered in [3]. The voltage applied to the actuator to balance the mechanical load was determined from the sensor's readings. It was shown that with such boundary conditions, active damping will be most effective when the plate is fully covered with sensors and actuators.

In the present paper, we will examine the case of clamped edges of the plate. We will use the method of [3] to damp the first mode of forced resonant flexural vibrations of the plate. We will analyze the effect of the clamped boundary conditions on the effectiveness of the active damping of resonant vibrations. The associated problem will be solved by the Bubnov–Galerkin method. We will derive a formula for the calculation of the voltage that should be applied to the actuator to balance the mechanical load. An analysis of this formula shows that active damping will be most effective when sensors and actuators look like some spots on the plate. We will present formulas to calculate the dimensions of these spots. It will be shown that it becomes impossible to control the vibrations of the plate when the dimensions of the spot decrease or tend to the sizes of the plate. We will also study the influence of viscosity on the effectiveness of active damping.

1. Consider an orthotropic viscoelastic rectangular plate with dimensions $(2a, 2b)$ under harmonic pressure with a frequency close to the first resonant frequency of the plate. The edges of the plate are clamped. The vibrations of the plate are described using the Kirchhoff–Love hypotheses supplemented with analogous hypotheses on the distribution of the electric-field quantities [1]. Let the vibrations be flexural. The orthotropic passive layers may be made of metallic, polymeric, or composite materials. The piezoactive layers are transversely isotropic and polarized throughout the thickness of the plate. If there are no electrodes between layers, then they are in perfect mechanical and electric contact. The dissipative properties of the passive and piezoactive layers are described using the concept of complex characteristics [2]. The equations of the theory of plates with distributed sensors and actuators are presented in [4, 7]. Let us present those of them that will be used below.

2. We choose Cartesian coordinates x, y, z. Let the coordinate surface $z = 0$ coinside with the midsurface of the plate.

The vibrations of an orthotropic viscoelastic plate under mechanical and electric loads are described by the following equation [4]:

National Technical University of Ukraine "KPI," 37 Pobedy Av., Kyiv, Ukraine 03056, e-mail: karn@inmech.kiev.ua.
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$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}-\tilde{\rho}\,\omega^2 w-p_0(x,\ y)-\left(\frac{\partial^2 M_0}{\partial x^2}+\frac{\partial^2 M_0}{\partial y^2}\right)=0, \tag{1}$$

where the notation is the same as in [4].

If the plate is clamped, the mechanical boundary conditions are expressed as

$$w=\frac{\partial w}{\partial x}=0 \quad \text{at} \quad x=\pm a,$$

$$w=\frac{\partial w}{\partial y}=0 \quad \text{at} \quad y=b. \tag{2}$$

The plate is under pressure uniformly distributed over its surface $p_0 e^{i\omega t}$ and varying harmonically with a frequency close to the first resonant one. The amplitude of the mechanical load is unknown. The problem will be solved by the Bubnov–Galerkin method. The displacements are expressed as

$$w=A\tilde{w}, \tag{3}$$

where the first vibration mode is approximated by

$$\tilde{w}=(x^2-a^2)^2(y^2-b^2)^2. \tag{4}$$

The mechanical boundary conditions are satisfied automatically. To damp its resonant vibrations, there is a spot $(2c\times 2d)$ on the plate centered at the origin of coordinates. According to the Bubnov–Galerkin method, expression (3) is substituted into Eq. (1), which, after multiplication by the shape function (4), is integrated over the area of the plate.

Doing so gives the following expression for the complex amplitude of vibrations:

$$A=\Delta_1/\Delta_2, \tag{5}$$

$$\Delta_1=\frac{49}{16}a^2b^2p_0-\frac{735}{256}M_0(a^2+b^2)\psi(s), \qquad \psi(s)=s(1-s)(15-10s+3s^2),$$

$$\Delta_2=8a^2b^2\Delta, \qquad \Delta=\left[7D_{11}b^4+4(D_{12}+2D_{66})a^2b^2+7D_{22}a^4-\frac{2}{9}\tilde{\rho}\omega^2a^4b^4\right], \tag{6}$$

where $s=(l/L)^2$; l and L are the diagonals of the piezoactive inclusions and plate, respectively.

Setting $\Delta_1=0$ in (6), we obtain an expression for the voltage that should be applied to the actuator to balance the external load:

$$V_{\mathrm{A}}=\frac{32a^2b^2}{15(a^2+b^2)\psi(s)\gamma_{31}(h_0+h_1)}p_0. \tag{7}$$

If expression (7) holds, the amplitude of forced vibrations in the principal mode is zero and the plate can undergo only free vibrations.

Note that p_0 in (1)–(7) is unknown. It can be determined from the readings of the sensor. Formula (7) indicates that the mechanical boundary conditions have a significant effect on the effectiveness of active damping. For example, if the edges of the plate are hinged, active damping with sensors and actuators fully covering the plate is the most effective [5, 8]. If the edges are clamped, the actuator will perform best when the value of the function $\psi(s)$ is maximum [6], which is so with $s_{\max}$ being a root of the equation

$$12s^3-39s^2+50s-15=0. \tag{8}$$

Thus, active damping will be the most effective if the diagonal length of the actuator is $l = L\sqrt{s_{\max}}$. It also follows from (7) that the voltage tends to infinity as $s \to 0$ and for $s \to 1$. Thus, it is impossible to control the behavior of the plate when the actuator either fully covers it or is very small. We will show that the sensor performs best when having similar dimensions.

3. Let the arrangement and dimensions of the actuator and sensor be fixed. The approach based on formulas (5) and (7) has the following shortcomings: (i) free vibrations are not damped; (ii) the external mechanical load has to be known.

The latter disadvantage can be resolved with the new approach proposed in [3]. The external load is determined from the readings of the sensor with area S_1. When the electrodes are short-circuited, the charge is determined from the following expression [7–9]:

$$Q = \gamma_{31}(h_0 + h_1) \iint_{(S_1)} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) dxdy. \tag{9}$$

When the electrodes are open-circuited, the voltage is determined from

$$V_{\mathrm{S}} = \frac{h_1 Q}{S_1 \gamma_{33}}. \tag{10}$$

Substituting expression (3) into (9), we obtain the following formula for the sensor's charge:

$$Q = -16\gamma_{31}(h_0 + h_1) A a^3 b^3 (a^2 + b^2) \psi(t) / 15, \tag{11}$$

whence follows that the sensor performs best when its dimensions are the same as those of the actuator determined above.

The sensor's voltage V_{S} is given by (10), (11).

To determine the mechanical load p_0, we will use the expression for the amplitude of vibrations of the plate at a frequency close to the principal resonance. This amplitude is calculated by the formula

$$A = \Pi_1 / \Pi_2, \tag{12}$$

where $\Pi_1 = \frac{49}{64} p_0$, $\Pi_2 = \Delta$.

The first resonant frequency is determined as

$$\omega = \sqrt{\frac{63 D'_{11} b^4 + 36(D'_{12} + 2D'_{66}) a^2 b^2 + 63 D'_{22} a^4}{2\tilde{\rho} a^4 b^4}}. \tag{13}$$

Substituting (12) into (11), we obtain an expression for determining the amplitude and phase of the mechanical load from the sensor's readings:

$$p_0 = -\frac{120}{49} \frac{Q\Delta}{\gamma_{31}(h_0 + h_1) a^3 b^3 (a^2 + b^2) \psi(s)}. \tag{14}$$

Substituting (14) into (7) yields the relation between the sensor's charge and the voltage applied to the actuator to balance the unknown mechanical load:

$$V_{\mathrm{A}} = -\frac{256}{49} \frac{Q\Delta}{ab(a^2 + b^2)^2 (h_0 + h_1)^2 \gamma_{31}^2 \psi^2(s)}. \tag{15}$$

If the sensor reads the voltage, we can obtain a similar formula. To this end, it is necessary to use formula (10).

Our approach assumes applying the voltage determined from the sensor's readings by formula (15) to the actuator. In this case, we only need to know the electromechanical properties and dimensions of the plate.

To solve the problem numerically, we will follow the procedure outlined in [3]. All the problems mentioned in [3] should be solved numerically for clamped boundary conditions.

As with hinged boundary conditions [3], viscosity has a significant effect on the effectiveness of active damping.

Note that the results of [4–9] on the influence of self-heating on the performance of sensors and actuators and on the effectiveness of active damping remain valid.

Conclusions. Using the new approach proposed in [3], we have solved the problem of active damping of the vibrations of a clamped orthotropic viscoelastic rectangular plate subject to an unknown external mechanical load. It is determined from the sensor's readings. We have derived formulas that use only the sensor's readings to calculate the voltage that should be applied to the actuator to balance the unknown mechanical load.

REFERENCES

1. V. T. Grinchenko, A. F. Ulitko, and N. A. Shul'ga, *Electroelasticity*, Vol. 5 of the five-volume series *Mechanics of Coupled Fields in Structural Members* [in Russian], Naukova Dumka, Kyiv (1989).
2. V. G. Karnaukhov and V. V. Mikhailenko, *Nonlinear Thermomechanics of Piezoelectric Inelastic Bodies under Monoharmonic Loading* [in Russian], ZhGTU, Zhitomir (2005).
3. T. V. Karnaukhova, "Active damping of vibrations of plates subject to unknown pressure," *Int. Appl. Mech.*, **46**, No. 5, 562–566 (2010).
4. T. V. Karnaukhova and E. V. Pyatetskaya, "Basic relations for thermoviscoelastic plates with distributed actuators under monoharmonic loading," *Int. Appl. Mech.*, **45**, No. 2, 200–214 (2009).
5. T. V. Karnaukhova and E. V. Pyatetskaya, "Damping the resonant flexural vibrations of a hinged plate with actuators," *Int. Appl. Mech.*, **45**, No. 4, 448–456 (2009).
6. T. V. Karnaukhova and E. V. Pyatetskaya, "Damping the flexural vibrations of a clamped viscoelastic rectangular plate with piezoelectric actuators," *Int. Appl. Mech.*, **45**, No. 5, 546–557 (2009).
7. T. V. Karnaukhova and E. V. Pyatetskaya, "Basic relations of the theory of thermoviscoelastic plates with distributed sensors," *Int. Appl. Mech.*, **45**, No. 6, 660–669 (2009).
8. T. V. Karnaukhova and E. V. Pyatetskaya, "Resonant vibrations of a hinged viscoelastic rectangular plate," *Int. Appl. Mech.*, **45**, No. 7, 762–771 (2009).
9. T. V. Karnaukhova and E. V. Pyatetskaya, "Resonant vibrations of a clamped viscoelastic rectangular plate," *Int. Appl. Mech.*, **45**, No. 8, 904–916 (2009).
10. J. Tani, T. Takagi, and J. Qiu, "Intelligent material systems: Applications of functional materials," *Appl. Mech. Rev.*, **51**, No. 8, 505–521 (1998).
11. H. S. Tzou and L. A. Bergman, *Dynamics and Control of Distributed Systems*, Cambridge University Press, Cambridge (1998).