International Applied Mechanics, Vol. 46, No. 3, 2010

RESONANT VIBRATIONS OF A CLAMPED THERMOVISCOELASTIC RECTANGULAR PLATE WITH SENSORS AND ACTUATORS

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The paper discusses the active damping of the resonant flexural vibrations of a clamped thermoviscoelastic rectangular plate with distributed piezoelectric sensors and actuators. The thermoviscoelastic behavior of the passive and active materials is described using the concept of complex characteristics. The interaction of the mechanical and thermal fields is taken into account. The Bubnov–Galerkin method is used. The effect of self-heating, the dimensions of the piezoelectric inclusions, and the feedback factor on the effectiveness of active damping of the resonance vibrations of the plate is studied

Keywords: resonant vibrations, thermoviscoelastic rectangular plate, clamped edges, active damping, sensors and actuators, self-heating

Introduction. Thin viscoelastic composite plates made of passive (without piezoelectric effect) and piezoactive materials are widely used in many fields of modern science and engineering [1–4, 9–12, 18, 19]. They are very frequently subjected to harmonic mechanical loads of nearly resonant frequency. Recently, active-damping methods have effectively been used to decrease the amplitude of resonant vibrations [11, 12, 18, 19]. One of the main methods for the active damping of resonant vibrations employs both piezoelectric sensors and actuators [11, 12, 18, 19]. The effectiveness of this method depends on many factors, including self-heating temperature, mechanical boundary conditions, arrangement and dimensions of sensors and actuators, etc. The basic equations of the thermomechanics of viscoelastic plates with distributed sensors and actuators that take into account self-heating are presented in [6, 7, 15]. The influence of self-heating on the active damping of the vibrations of a hinged rectangular plate with both sensors and actuators was analyzed in [7], and it was shown that with this type of boundary conditions, the effectiveness of damping will be highest if the plate is fully covered with piezoelectric inclusions.

The present paper analyzes the influence of self-heating on the active damping of the forced resonant vibrations of a clamped thin viscoelastic rectangular plate with sensors and actuators. The associated problem will be solved by the Bubnov–Galerkin method. By analyzing solutions of specific problems, we will study how the effectiveness of the active damping of the plate depends on the self-heating temperature, the dimensions of the piezoinclusions, and the feedback factor.

1. Problem Formulation. The general theory of active damping of forced vibrations of thin plates with piezoelectric sensors and actuators taking into account self-heating is outlined in [6]. The active-damping problem for a hinged rectangular plate was solved in [7]. Let us present those equations of [6, 7] that will be used below.

We will restrict ourselves to the case of flexural vibrations of a sandwich plate with a passive core of thickness h_0 and piezoactive face layers having thickness h_1 each and polarized in opposite directions. The properties of these piezolayers only differ by the sign of the piezoelectric constant.

If the passive core is isotropic, the problem is reduced to a nonlinear system of partial differential equations [6, 7]:

$$\frac{\partial^2}{\partial x^2}\left[D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]+\frac{\partial^2}{\partial y^2}\left[D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\right]$$

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$$+\frac{\partial^2}{\partial x\partial y}\left[2(1-\nu)D\left(\frac{\partial^2 w}{\partial x\partial y}\right)\right]-(\gamma h)\omega^2 w-p_0(x,y)-\left(\frac{\partial^2 M_0}{\partial x^2}+\frac{\partial^2 M_0}{\partial y^2}\right)=0,$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}-\frac{2\alpha}{(\lambda h)}(T-T_{\text{ex}})+\frac{\omega E''(T)h^3}{24a^2(1-\nu^2)(\lambda h)}\left\{\left(\frac{\partial^2 w'}{\partial x^2}\right)^2+\left(\frac{\partial^2 w''}{\partial x^2}\right)^2+\left(\frac{\partial^2 w'}{\partial y^2}\right)^2+\left(\frac{\partial^2 w''}{\partial y^2}\right)^2\right.$$

$$\left.+2\nu\left(\frac{\partial^2 w'}{\partial x^2}\frac{\partial^2 w'}{\partial y^2}+\frac{\partial^2 w''}{\partial x^2}\frac{\partial^2 w''}{\partial y^2}\right)+2(1-\nu)\left[\left(\frac{\partial^2 w'}{\partial x\partial y}\right)^2+\left(\frac{\partial^2 w''}{\partial x\partial y}\right)^2\right]\right\}=0, \tag{1.1}$$

where the notation is the same as in [6, 7].

When both sensors and actuators are used, forced vibrations are additionally damped as described above: the actuator voltage V_{a} is proportional to the rate of variation in the sensor voltage $\dot{V}_{\text{S}}$ or the current:

$$V_{\text{a}}=-G_{\text{S}}\dot{V}_{\text{S}},\quad V_{\text{a}}=-G_{\text{S}}I,\quad G_{\text{S}}=GG_{\text{e}}. \tag{1.2}$$

If vibrations are harmonic, M_0 in (1.1) is defined by the formulas

$$M_0=m_0 FAe^{i\omega t}, \tag{1.3}$$

$$m_0=G_{\text{S}}\gamma^2{}_{31}(h_0+h_1)^2,\quad F=\iint\limits_{(S_1)}\left(\frac{\partial^2\tilde{w}}{\partial x^2}+\frac{\partial^2\tilde{w}}{\partial y^2}\right)dxdy, \tag{1.4}$$

where $\tilde{w}(x,y)$ described the vibration mode.

Thus, in addition to the damping caused by hysteresis losses in the material, there is active damping caused by sensors and actuators and dependent on the feedback factor. For the sake of simplicity, we will proceed as in [6, 7]: in the solution that disregards the additional damping [13–17], we will replace the imaginary part of the complex shear modulus with that modified to account for the additional damping.

2. Problem-Solving Method. To analyze the effect of the feedback factor G_{S} and self-heating temperature on the effectiveness of damping of the forced resonant vibrations with both sensors and actuators, it is first necessary to plot amplitude–frequency characteristics (AFCs) regardless of these factors, then to calculate these characteristics taking into account the self-heating temperature, and to compare the results. The above system of nonlinear differential equations is quite complicated and can be solved by iterative methods in combination with numerical methods (for example, the finite-element or discrete-orthogonalization method [3–5]). As in [7], we will use the Bubnov–Galerkin method. In solving problems where the material properties depend on temperature, we will restrict ourselves to the case where the real and imaginary parts of the shear modulus are linearly dependent on temperature:

$$G=G'+G'',\quad G'=G'_0-G'_1T,\quad G''=G''_0-G''_1T, \tag{2.1}$$

where G'_0, G'_1, G''_0, and G''_1 are determined experimentally. These constants for polyethylene in the temperature range $20\le T\le 80$ °C are presented in [8].

Polyethylene has the following characteristics: Poisson's ratio $\nu = 0.3227$, thermal-conductivity factor $\lambda = 0.47$ W(m·°C), density $\rho = 938$ kg/m^3.

Since the piezoactuators are thin, losses in them can be neglected.

The edges $x=\pm a$ and $y=\pm b$ of the plate are clamped, i.e.,

$$w=\frac{\partial w}{\partial x}=0 \qquad \text{at} \qquad x=\pm a,$$

$$w=\frac{\partial w}{\partial y}=0 \qquad \text{at} \qquad y\pm b.$$

The coupled problem of resonant vibrations of a clamped rectangular plate with actuators was solved in [14] by a variational method regardless of the additional damping caused by sensors and actuators. For the principal vibration mode, the transverse deflection is approximated as traditionally done for such boundary conditions:

$$w = A(x^2 - a^2)^2 (y^2 - b^2)^2. \tag{2.2}$$

Using approximation (2.2) and the variational and Bubnov–Galerkin methods, we obtain the following expression for the complex amplitude:

$$A = \Delta_1 / \Delta_2, \tag{2.3}$$

where

$$\Delta_1 = p_0, \tag{2.4}$$

$$\Delta_2 = \left(\frac{16}{21}\right)^2 (\gamma h \omega^2) a^4 b^4 - \frac{128}{7} a^2 b^2 D \left[\left(\frac{a}{b}\right)^2 + \left(\frac{b}{a}\right)^2 + \frac{4}{7}\right]. \tag{2.5}$$

The dissipation function is defined by

$$D = \frac{\omega}{2} D'' 16 |A|^2 f(x, y), \tag{2.6}$$

where

$$\begin{aligned} f(x, y) = {} & (a^4 - 6a^2x^2 + 9x^4)(b^8 - 4b^6y^2 + 6b^4y^4 - 4b^2y^6 + y^8) \\ & + (a^8 - 4a^6x^2 + 6a^4x^4 - 4a^2x^6 + x^8)(b^4 - 6b^2y^2 + 9y^4) \\ & + 2\nu(a^6 - 5a^4x^2 + 7a^2x^4 - 3x^6)(b^6 - 5b^4y^2 + 7b^2y^4 - 3y^6) \\ & + 32(1-\nu)(a^4x^2 - 2a^2x^4 + x^6)(b^4y^2 - 2b^2y^4 + y^6). \end{aligned} \tag{2.7}$$

With (2.3)–(2.7), the energy equation for $\theta = T - T_{\mathrm{ex}}$ (T_{ex} is the temperature of the environment) becomes

$$\nabla^2 \theta - \left(\frac{2\alpha}{\lambda h}\right)\theta + \frac{\omega}{2} D'' \cdot 16 |A|^2 f(x, y) = 0. \tag{2.8}$$

If the boundary of the plate is heat-insulated, then

$$\begin{aligned} \frac{\partial \theta}{\partial x} &= 0 \qquad \text{at} \qquad x = \pm a, \\ \frac{\partial \theta}{\partial y} &= 0 \qquad \text{at} \qquad y = \pm b. \end{aligned} \tag{2.9}$$

For the purpose of using the Bubnov–Galerkin method, temperature is approximated as follows:

$$\theta = \theta_0 + \theta_1 \left[\left(a^2x^2 - \frac{1}{2}x^4\right) + \left(b^2y^2 - \frac{1}{2}y^4\right)\right]. \tag{2.10}$$

Expression (2.10) satisfies the boundary conditions (2.9).

Expressions for θ_0, θ_1 were obtained in [14] using the Bubnov–Galerkin method.

To ensure reliability of results, we will use the following approximation, in addition to (2.10):

$$\theta=\theta_0+\theta_1\left(a^2x^2-\frac{1}{2}x^4\right)\left(b^2y^2-\frac{1}{2}y^4\right). \tag{2.11}$$

Results obtained with these two approximations of temperature agree to three decimal places. These results are also in good agreement with those produced by the finite-element method.

If the boundary of the plate is maintained at constant temperature T_{ex}, the temperature is approximated as follows:

$$\theta=T-T_{ex}=T_1(a^2-x^2)(b^2-y^2). \tag{2.12}$$

An expression for T_1 was obtained in [14] using the Bubnov–Galerkin method.

The plate is most heated at the center. Therefore, the maximum temperature $\theta_{max}=\theta_0$ (θ_0 is the temperature at the point $x=y=0$) in both cases of thermal boundary conditions. Once the self-heating temperature has reached the Curie point T_C, the actuator no longer performs its function. Equating the maximum temperature to T_C, we find the maximum load parameter exceeding which makes the active damping of vibrations of the plate impossible. The amplitude of resonant vibrations is determined from the equation

$$|A|=\frac{A_{01}}{A_{02}},$$

$$A_{01}=q_0a^2b^2,\quad A_{02}=\frac{128}{7}D''\left[\frac{4}{7}+\left(\frac{a}{b}\right)^2+\left(\frac{b}{a}\right)^2\right]. \tag{2.13}$$

If the boundary of the plate is heat-insulated, the maximum temperature is given by

$$\theta_{max}=\theta_0=\frac{A_4\overline{A}_7-\overline{A}_3A_8}{\overline{A}_2\overline{A}_7-\overline{A}_3\overline{A}_6}, \tag{2.14}$$

$$\overline{A}_2=\left(\frac{2\alpha}{\lambda h}\right)A_2,\quad \overline{A}_3=\frac{2\alpha}{\lambda H}A_3-2A_5,\quad \overline{A}_6=\frac{2\alpha}{\lambda h}A_6,\quad \overline{A}_7=\frac{2\alpha}{\lambda h}A_7-2A_5. \tag{2.15}$$

Setting $\theta_0=\theta_C$ and using (2.14) and (2.15), we find the maximum load parameter q exceeding which causes the actuator to lose its function,

$$q=\frac{8\omega q_0^2}{(\lambda H)D''}=\left(\frac{128}{7}\right)^2a^4b^4\,\frac{\overline{A}_2\overline{A}_7-\overline{A}_3\overline{A}_6}{A_4\overline{A}_7-\overline{A}_3A_8}\left[\left(\frac{a}{b}\right)^2+\left(\frac{b}{a}\right)^2+\frac{4}{7}\right]^2\theta_C. \tag{2.16}$$

A coupled problem with physical nonlinearity due to the dependence of the material properties on temperature and the nonlinear dependence of the dissipation function on strains and temperature was also solved in [14]. Given constant temperature T_{ex} on the boundary of the plate, the deflection was approximated as in (2.2). The following expression for the complex amplitude of vibrations has been obtained:

$$A_0=Aa^4b^4=\frac{q_0}{A_1+iA_2}. \tag{2.17}$$

The following expression for T_1 has been derived using the Bubnov–Galerkin method:

$$T_1=\frac{C_1X}{C_0+C_1X},\quad X=|A_0|^2, \tag{2.18}$$

where the notation is the same as in [14].

The squared amplitude can be found from the formula

$$X = \frac{q_0^2}{(B_1^2 + B_2^2) - 2(b_1 B_1 + b_2 B_2) T_1 + (b_1^2 + b_2^2) T_1^2}. \tag{2.19}$$

Eliminating X from (2.18) and (2.19) and using the notation of [6, 7], we obtain a cubic equation for the dimensionless temperature $y = T_1 / T_{\text{ex}}$:

$$y^3 - e_2 y^2 + e_1 y - e_0 = 0. \tag{2.20}$$

Once the self-heating temperature has been determined, the frequency dependence of the amplitude $\sqrt{X}$ can be established by formula (2.19).

If the boundary of the plate is heat-insulated, all the above formulas remain the same, except that

$$C_0 = \frac{2\alpha ab}{\lambda h}, \quad C_1 = \frac{4G_0'' \omega h^3}{3(1-\nu)(\lambda h) a^2} c_{11}, \quad C_2 = \frac{4G_1'' \omega h^3}{3(1-\nu)(\lambda h) a^2} c_{11},$$

$$c_{11} = \frac{b}{a} \left\{ \frac{4}{5} \left(\frac{128}{315} \right) \left[1 + \left(\frac{a}{b} \right)^4 \right] + 32 \left(\frac{8}{15} \right)^2 \left(\frac{a}{b} \right)^2 \right\}. \tag{2.21}$$

The plate is most heated at the center. Therefore, the maximum temperature $T_{\max} = T_1$ in both cases of thermal boundary conditions. Equating the expressions for T_1 to the Curie temperature θ_{C}, we obtain an expression for the critical load parameter $W_{0\text{C}}^2$ above which the sensors and actuators no longer perform their function:

$$W_{0\text{C}}^2 = \frac{4\alpha \theta_{\text{C}}}{a_0 \omega}, \quad W_{0\text{C}}^2 = \frac{(\lambda h) \pi^4 \theta_{\text{C}}}{8\omega a^2} \frac{1 + \left(\frac{a}{b} \right)^2 + \frac{2\alpha a^2}{\lambda h \pi^2}}{a_0 + \frac{2}{3} a_1 + \frac{1}{9} a_2} \tag{2.22}$$

for the heat-insulated edges and constant temperature set at the edges, respectively.

As mentioned above, the above solution for the amplitude of vibration and self-heating temperature disregards the additional damping due to the joint operation of sensors and actuators.

$$\widetilde{B}_2 = B_2 + G_2. \tag{2.23}$$

The additional damping is characterized by the parameter G_2, which can be determined from the following expression for a clamped rectangular plate:

$$G_2 = 64 G_{\text{S}} \gamma_{31}^2 (h_0 + h_1)^2 \left(\frac{b}{a} \right)^2 \left(\frac{c}{a} \right)^2 \left(\frac{d}{b} \right)^2 \left\{ \left[1 - \left(\frac{c}{a} \right)^2 \right] \left[1 - \frac{2}{3} \left(\frac{d}{b} \right)^2 + \frac{1}{5} \left(\frac{d}{b} \right)^4 \right] \right.$$

$$\left. + \left(\frac{a}{b} \right)^2 \left[1 - \left(\frac{d}{b} \right)^2 \right] \left[1 - \frac{2}{3} \left(\frac{c}{a} \right)^2 + \frac{1}{5} \left(\frac{c}{a} \right)^4 \right] \right\}^2. \tag{2.24}$$

Thus, in addition to the damping caused by hysteresis losses in the material, there is active damping described by the parameter G_2 that can be controlled over a wide range. To calculate the self-heating temperature, it is necessary to substitute the expression for the amplitude that accounts for this additional damping into the expression for the dissipation function.

To plot AFCs, it is necessary to introduce (2.23) and (2.24) into the above formulas and to calculate new amplitude– and temperature–frequency characteristics. By comparing them, we can assess the effect of the feedback factor and self-heating temperature. The additional damping affects the self-heating temperature only through the amplitude of vibrations.

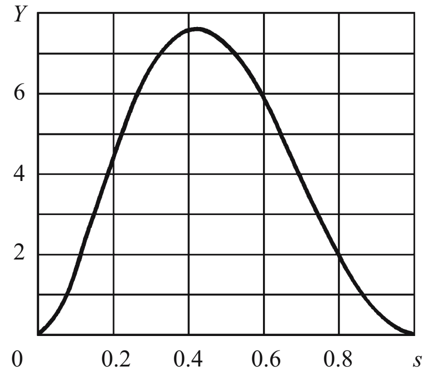

Fig. 1

3. Analysis of the Results. To analyze the effect of the temperature dependence of the properties of the passive material on the effectiveness of the active damping of forced vibrations of a polyethylene plate with both sensors and actuators. The sensors and actuators are made of TsTS$_T$BS-2 piezoceramics (see [9] for its thermoelectromechanical properties). It follows from (2.24) that the effectiveness of active damping depends on the dimensions of the sensors and actuators. This formula can be rearranged into

$$G_2 = \frac{64}{225} G_S \gamma_{31}^2 (h_0 + h_1)^2 \left(\frac{b}{a}\right)^2 \left(\frac{l}{L}\right)^4 \left(1 + \left(\frac{a}{b}\right)^2\right)^2 \times \left[1 - \left(\frac{l}{L}\right)^2\right]^2 \left[15 - 10\left(\frac{l}{L}\right)^2 + 3\left(\frac{l}{L}\right)^4\right]^2, \tag{3.1}$$

where l and L are the diagonals of the piezoelements and plate, respectively. The parameter G_2 is maximum at the value of $s = (l / L)^2$ determined by solving the algebraic equation

$$12s^3 - 39s^2 + 50s - 15 = 0. \tag{3.2}$$

Figure 1 shows the dependence of $Y = G_2 \Bigg/ \left\{ \frac{64}{225} G_S \gamma_{31}^2 (h_0 + h_1)^2 \left(\frac{b}{a}\right)^2 \left[1 + \left(\frac{a}{b}\right)^2\right]^2 \right\}$ on the relative diagonal $s = l / L$.

As is seen, this parameter tends to zero as the diagonal of the piezoinclusion decreases and tends to the diagonal of the plate.

It can be seen from (3.1) that once the temperature has reached the Curie point θ_C, when $\gamma_{31} = 0$, it is no longer possible to control the amplitude of vibrations by active damping because $G_2(\theta_C) = 0$ and the additional damping factor is equal to zero. Equating the self-heating temperature to T_C, we obtain the maximum amplitude of loading above which the amplitude of vibrations cannot be controlled. For temperature-independent properties, the critical amplitude of mechanical loading is defined by (2.18).

We will now analyze the influence of the parameter G_2 on the effectiveness of damping for a square sandwich plate with polyethylene core and TsTS$_T$BS-2 piezoceramic face layers of opposite polarization. The plate is clamped and a constant temperature is maintained at its boundary. The characteristics of the plate are: $\alpha = 0.5$ W/m$^2\cdot$°C is the coefficient of heat transfer to the environment of temperature $T_{ex} = 20$ °C; p_0 = 2500 Pa is the amplitude of normal surface pressure. If the plate has side length a = 0.01 m, then the linear-resonance frequency ω_r = 16,432 Hz. Figure 2 shows AFCs in the case of temperature-independent mechanical properties of the passive material for different values of G_2 ($G_2 = 0$ for the upper curve, $G_2 = 0.5B_2$ for the middle curve, and $G_2 = 1.5B_2$ for the lower curve).

Figure 3 shows how the physical nonlinearity due to the temperature dependence of the material properties affects the AFCs.

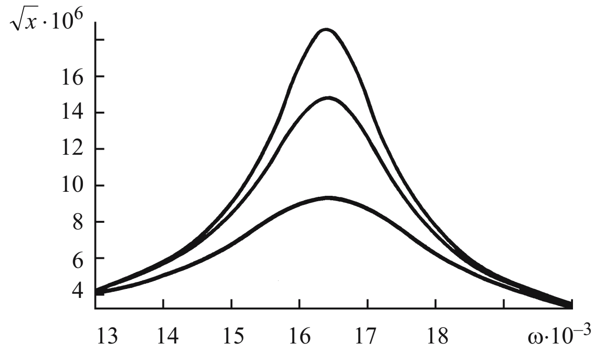

Fig. 2

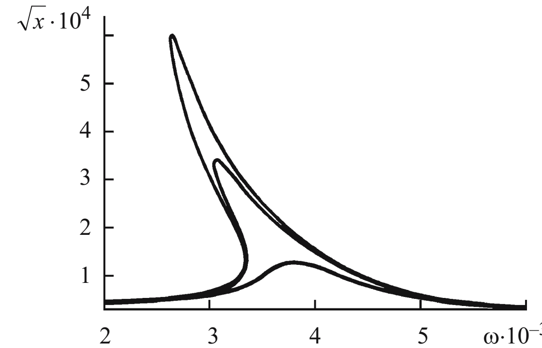

Fig. 3

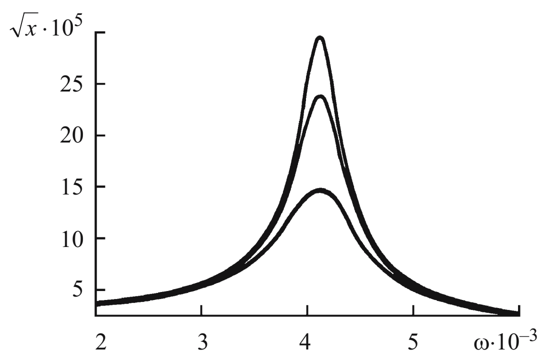

Fig. 4

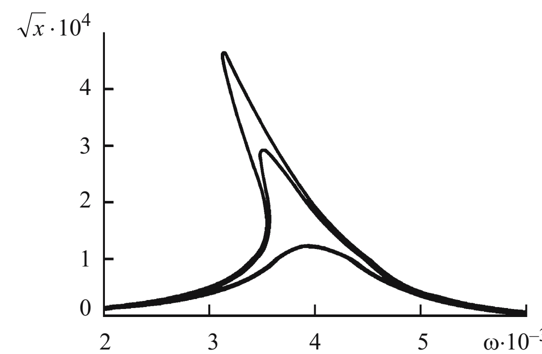

Fig. 5

Figures 4 (properties are independent of temperature) and 5 (properties are dependent on temperature) shows similar characteristics for a five-fold greater heat-transfer coefficient ($\alpha = 2.5$ W/(m^2·°C)).

As is seen, such a physical nonlinearity is responsible for the ambiguity of the AFCs and hysteresis.

The figures indicate that the effect of the nonlinearity weakens with increase in the feedback factor and heat-transfer coefficient temperature, the self-heating temperature decreasing.

Conclustions. We have solved the problem of active damping of the forced flexural resonant vibrations of a clamped thin rectangular thermoviscoelastic plate with piezoelectric sensors and actuators, self-heating being taken into account. The nonlinear problem has been solved by the Bubnov–Galerkin method. The numerical results obtained show that self-heating has a significant effect on the effectiveness of the active damping of resonant vibrations for both temperature-dependent and temperature-independent properties of the piezoactive and passive materials. For example, after the temperature reaches the Curie point, the element keeps its integrity, but no longer performs its functions, which is a specific type of thermal failure. The maximum mechanical load that causes this type of failure has been determined. The numerical results illustrate the influence of the self-heating temperature, feedback factor, and the dimensions of the sensors and actuators on the effectiveness of the active damping of resonant vibrations of a thermoviscoelastic plate.

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