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STABILITY OF LAYERED COATINGS UNDER BIAXIAL THERMOMECHANICAL LOADING

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The three-dimensional problem of stability of layered coatings at normal and high temperatures and small precritical strains is formulated and solved and the characteristic equations are derived. The temperature dependence of the physical and mechanical parameters of specific layered coatings is established experimentally. Three-layer coatings on a homogeneous substrate at different temperatures are considered as an example. Recommendations for establishing the optimal service conditions for layered structural members with coatings are formulated

Keywords: stability, strains, critical temperature, loading, layered coating, three-dimensional theory, waveformation composite material

Introduction. The stability of layered tribotechnical coatings is analyzed in the papers [2, 4, 23], which indicate specific operation conditions and main types of loading for such coatings, propose design models, and give a three-dimensional problem formulation. Exact and approximate methods for solving problems for metal, polymer, and elastomer coatings on a structurally homogeneous substrate are outlined. The main types of loading in the contact region of friction elements are distributed compressive surface loads that are either follower or dead. They model the interaction of these elements in the presence of dry-film or fluid-film lubrication in the contact region. On the mid-surface of the layers, the coating and substrate are subjected to a uniformly distributed compressive load. It is assumed that the stationary temperature field in the contact region is higher than outside it. The increased temperature in the contact region was described in [4] by changing the physical and mechanical constants of the layers keeping the coating temperature constant. Each material has its specific melting temperature at which the material loses its load-bearing capacity and goes over into another aggregate state. Therefore, coatings consisting of several layers have different limiting temperatures at which there is no risk for such coatings to lose stability and, hence, to lose load-bearing capacity.

The present paper gives a problem formulation, considers problem-solving methods, and solves specific problems of surface instability of layered metal coatings on a structurally homogeneous substrate under biaxial thermomechanical loading. A piecewise-homogeneous material model for linear elastic bodies is used and precritical strains are assumed small. Since we will use [1] the three-dimensional linearized theory of stability and piecewise-homogeneous model, it is expedient to mention other results obtained in solving related problems of solid mechanics in a similar formulation.

The approaches to setting up theories and the basic results on the three-dimensional linearized theory of stability of deformable bodies and the three-dimensional linearized theory of elastic waves in bodies with initial (residual) stresses were analyzed in the scientific literature [8, 9]. A contemporary analysis of the results on stability of layered and fibrous composites obtained by the approaches of [8, 9] can be found in [5, 3, 11, 19], including analysis of results on near-surface instability. It should be noted that the publication [12] was one of the first studies on the strict theory of surface instability of materials with periodic structure. The approaches of [8, 9] were also used to analyze results obtained in some other research areas such as the theory of stability of mine workings [3, 11] and the theory of folding in the Earth's stratified crust [3] for homogeneous and

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essentially inhomogeneous precritical states; exact solutions of plane mixed problems of linearized solid mechanics [18]; ultrasonic nondestructive stress analysis of solids [20].

The results on the three-dimensional theory of stability of layered and fibrous materials with certain interface conditions were analyzed in [3, 5, 12, 19]. Note that results on the three-dimensional theory of stability of composites with interface cracks have recently been obtained [13–17, 21, 22]. Failure criteria for solids compressed along parallel cracks are proposed in [6, 7].

We will solve the three-dimensional problem of stability of layered coatings at normal and high temperatures assuming small precritical strains. We will derive the basic characteristic equations, analyze the temperature dependence of physical and mechanical parameters of specific layered coatings, consider examples of three-layer coatings on a homogeneous substrate at different temperatures, and formulate recommendations for establishing optimal operation conditions for structural elements with layered coatings.

1. Problem Formulation. Let us consider a layered coating bonded to a homogeneous half-space modeling an elastic substrate for tribotechnical elements. Distributed compressive loads p_{ii} $(i=1,2)$ are applied to the midsurface of layers of such a medium, and a load p_{33} modeling the interaction between triboelements is applied to the surface of the coating. The temperature T^* on the coating surface is assumed constant. The area of this surface is such that quite many bulges may form on it. Therefore, surface instability will be analyzed within one buckling half-wave. It will be assumed that perturbations of the displacement components $\vec{u}$ decay with normal distance from the surface of the coating. To solve the problem, we will use the approach outlined in [1, 2]. The quasistatic approach [1] will be used to solve the problem of thermal stability. To analyze stability at normal temperature T_0, we will use equations for the perturbations of the components of the vector $\vec{u}$. The linearized equations of stability within each element of the layered medium are

$$\left((a_{11}+\sigma_{11})\frac{\partial^2}{\partial x_1^2}+(G_{12}+\sigma_{22})\frac{\partial^2}{\partial x_2^2}+(G_{13}+\sigma_{33})\frac{\partial^2}{\partial x_3^2}\right)u_1$$

$$+(a_{12}+G_{12})\frac{\partial^2 u_2}{\partial x_1\partial x_2}+(a_{13}+G_{13})\frac{\partial^2 u_3}{\partial x_1\partial x_3}=0 \quad (1, 2, 3, \text{Curl}), \tag{1.1}$$

where u_i are the perturbations of the components of the vector $\vec{u}$; a_{ij} and G_{ij} are coefficients describing the physical and mechanical properties of the layers and substrate at set temperature; x_i $(i=1,2,3)$ are the local Lagrange coordinates that are used to describe elements of the layered medium and that coincide with Cartesian coordinates prior to deformation. The interface conditions between elements of the medium in perturbed state are

$$P_i^{(k)}(x_1^{(k)},x_2^{(k)},x_3^{(k)})\Big|_{x_3^{(k)}=-h_k}=P_i^{(t)}(x_1^{(t)},x_2^{(t)},x_3^{(t)})\Big|_{x_3^{(t)}=0},$$

$$u_i^{(k)}(x_1^{(k)},x_2^{(k)},x_3^{(k)})\Big|_{x_3^{(k)}=-h_k}=u_i^{(t)}(x_1^{(t)},x_2^{(t)},x_3^{(t)})\Big|_{x_3^{(t)}=0} \qquad (t=k+1), \tag{1.2}$$

where $P_i^{(k)}$ $(i=\overline{1,3})$ denote the components of the principal stress vector in the kth element of the medium. They are defined by the formula

$$P_j^{(k)}=(G_{j3}+\sigma_{33}^{(0)})_{(k)}\frac{\partial u_j^{(k)}}{\partial x_3^{(k)}}+G_{j3}^{(k)}\frac{\partial u_3^{(k)}}{\partial x_j^{(k)}} \quad (j=1,2),$$

$$P_3^{(k)}=(a_{i3}+\delta_{i3}\sigma_{33}^{(0)})_{(k)}\frac{\partial u_i^{(k)}}{\partial x_i^{(k)}}, \tag{1.3}$$

where the summation over repeated indices is omitted.

The boundary conditions on the coating surface $(x_3^{(1)}=0)$ and at infinity $(x_3^{(T+1)}\to-\infty)$ are

$$P_i^{(1)}(x_1^{(1)}, x_2^{(1)}, 0) = \breve{P}_i, \tag{1.4}$$

$$u_i^{(T+1)}(x_1^{(T+1)}, x_2^{(T+1)} x_3^{(T+1)}) \to 0, \tag{1.5}$$

where T is the number of layers in the coating ($k = \overline{1,T}$); $\breve{P}_i$ are the perturbation components of the surface load, which are zero if p_{33} is a dead load and are defined by the following formula if p_{33} is a follower load:

$$\breve{P}_i = -p_{33}\left(N_i \frac{\partial u_\alpha}{\partial x_\alpha} - N_\alpha \delta_{\beta i} \frac{\partial u_\alpha}{\partial x_\beta}\right). \tag{1.6}$$

The perturbation components of the stress tensor are given by

$$\sigma_{ij} = \delta_{ij} a_{ij} \frac{\partial u_\kappa}{\partial x_\kappa} + (1-\delta_{ij}) G_{ij}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{1.7}$$

where δ_{ij} is the Kronecker delta; N_i is a unit normal vector to the undeformed surface to which the dead or follower load is applied. The coordinate x_3 runs along the normal to the surface of the coating. The summation in (1.6) and (1.7) is over repeated indices.

Thus, the original problem is reduced to the eigenvalue analysis of the boundary-value problem (1.1)–(1.5) for the parameters p_{11} and p_{22}. To this end, we use Euler's static method and associated failure criteria. In the general case, the follower load is not conservative and, hence, the corresponding boundary-value problem is self-adjoint. Therefore, if p_{33} is a follower load, the sufficient applicability conditions for the static approach in the theory of stability should be tested (see [1] for the explicit form of these conditions).

To describe the influence of the limiting temperature on the equilibrium state of coatings for some metals and polymers used in friction engineering, we have experimentally established the dependence of their physical and mechanical properties on the limiting (for each material) temperature using a BRP-5 tensile-testing machine in combination with a heat chamber reliable in the temperature range from –60 to +1000 °C. The testing machine included an electronic loadmeter with 12 ranges, a maximum load of 5000 kgf, and strip-chart record at various scales. In setting up the experiment, we used the strain gauging system (SVD-1) intended for accurate measurement and record of small strains. The samples were produced in accordance with GOST 11262-76 or GOST 12423-66 (Russian standard). At least five samples were used in each series of experiments. Preliminarily, the samples were conditioned at low and high temperatures for at least 30 min in accordance with GOST 12423-66. In the experiments, the samples were loaded at a prescribed rate to failure. The stress level and displacement of the moving clamp were automatically recorded on a chart strip. Only those data were used in calculations that were measured on samples destroyed within the test portion. Stress–strain curves samples under tension and compression at different temperatures were plotted and used to find the elastic moduli, proportional limits, breaking stresses, and other material characteristics. Poisson's ratio was determined for each sample. The measured data were processed and the material characteristics were determined as per GOST 9550-81. Thus, we have all necessary data to study the surface instability of layered tribotechnical coatings.

2. Research Technique and Solution of Specific Problems. To identify the common specific features and deformation patterns of coatings at high temperatures, we will solve a model problem for a coating consisting of three layers on a homogeneous substrate. We will analyze the instability of layered structural materials with a special coating that works under extreme thermomechanical loading. The coating consists of three metal layers Kh18N10T, AS1, and AMg4 on an AMg6 aluminum alloy substrate. The physical, mechanical, and geometrical properties of such a sandwich are characterized by n_i, ν_i, ρ_i, y. Here: $n_i = E_i / E_{T+1}, \rho_i = h_i / H, y = p_{33} / p_{11}, H = \sum_{k=1}^{T} h_k$; h_i is the thickness of the ith layer; T is the number of layers in the coating; E_i and ν_i are Young's modulus and Poisson's ratio of the ith layer; H is the total thickness of the coating.

Let us consider the plane $x_1^{(k)} O x_3^{(k)}$ ($k = \overline{1,4}$). Therefore, we should set $u_2^{(k)} = 0$, $u_{1,3}^{(k)} = u_{1,3}^{(k)}(x_1^{(k)}, x_3^{(k)})$ in (1.1)–(1.7). According to [1, 6], Eqs. (1.1) can be rearranged into

$$\left(\frac{\partial^2}{\partial x_3^2}+\eta_3^2\frac{\partial^2}{\partial x_1^2}\right)\left(\frac{\partial^2}{\partial x_3^2}+\eta_1^2\frac{\partial^2}{\partial x_1^2}\right)\Psi=0, \quad (2.1)$$

where the coefficients $\eta_{1,3}^2$ are defined by

$$\eta_{1,3}^2=c\pm\sqrt{[c^2-(a_{11}+\sigma_{11}^0)(G_{13}+\sigma_{11}^0)(a_{33}+\sigma_{33}^0)^{-1}(G_{13}+\sigma_{33}^0)^{-1}]},$$

$$2c(a_{33}+\sigma_{33}^0)(G_{13}+\sigma_{33}^0)=(a_{11}+\sigma_{11}^0)(a_{33}+\sigma_{33}^0)+(G_{13}+\sigma_{11}^0)(G_{13}+\sigma_{33}^0)-(a_{13}+G_{13})^2. \quad (2.2)$$

The perturbation components of the displacement vector $u_{1,3}^{(k)}$ for kth element of the medium are determined in terms of a potential Ψ:

$$u_1=-(a_{13}+G_{13})\frac{\partial^2\Psi}{\partial x_1\partial x_3},\quad u_3=\left[(a_{11}+\sigma_{11})\frac{\partial^2}{\partial x_1^2}+(G_{13}+\sigma_{33})\frac{\partial^2}{\partial x_3^2}\right]\Psi. \quad (2.3)$$

In (2.1)–(2.3), the index k is omitted. In the case of a piecewise-homogeneous subcritical stress state, the solution of Eq. (2.1) is sought in the form

$$\Psi_j^{(k)}=\left[A_j^{(k)}+\sinh\left(\frac{\pi}{l}\eta_j^{(k)}x_3^{(k)}\right)+B_j^{(k)}+\cosh\left(\frac{\pi}{l}\eta_j^{(k)}x_3^{(k)}\right)\right]\exp\left(i\frac{\pi}{l}x_1^{(k)}\right)\quad (i=\sqrt{-1}), \quad (2.4)$$

where $j = 1, 3$; $A_j^{(k)}$ and $B_j^{(k)}$ are the unknown constants of integration of Eq. (2.1). To determine the constants $A_j^{(k)}$ and $B_j^{(k)}$, we use the matrix method outlined in [3, 23, 24]. As a result, we arrive at a system of homogeneous algebraic equations for these constants. We use the following notation: h_4 is the thickness of the substrate; F_k and $\widetilde{S}_k$ are the matrices of the fourth rank (see [2, 5] for their explicit expressions). The column vector of arbitrary constants of integration is denoted by $\vec{R}_k=||A_1^{(k)},A_3^{(k)},B_1^{(k)},B_3^{(k)}||^*$. The system of homogeneous algebraic equations has the following matrix-vector form:

$$D_1\vec{R}_1=0,$$

$$F_1\widetilde{S}_1\vec{R}_1=F_2\vec{R}_2,$$

$$-------$$

$$F_3\widetilde{S}_3\vec{R}_3=F_4\vec{R}_4,$$

$$(D_4\widetilde{S}_4\vec{R}_4)\to 0\quad \text{as } h_4\to-\infty. \quad (2.5)$$

The values of the matrices $D_n=[d_{ij}^{(n)}]\,(i=1,2,\ j=\overline{1,4},\ n=1,3)$ depend on the type of load $\widetilde{P}_i$ on the coating surface and the conditions of decay at infinity. If the surface load p_{33} is follower, then $d_{ij}^{(n)}=0$ for $i=1,\ j=3,4,\ i=2,\ j=1,2$ and

$$d_{1i}=\eta_t[b_{4i}+p_{33}(b_{1i}-b_{2i})],\quad d_{2j}=(G_{13}-p_{33})(b_{2i}-\eta_t^2b_{1i}) \quad (2.6)$$

for $n = 1$. Here $j=i+2,\ t=1$ if $i=1$ and $t=3$ if $i=2$. Using the existence condition for nontrivial solutions of system (2.5) and performing the transformations as in [3], we arrive at the following characteristic equation:

$$(q_{11}+q_{13})(q_{22}+q_{24})-(q_{21}+q_{23})(q_{12}+q_{14})=0, \quad (2.7)$$

where q_{ij} are determined from the matrix equality

TABLE 1

T, °C	Materials of layers: E (MPa)/ν			
	Kh18N10T	AD1	AMg4	AMg6
600	$18.7\cdot10^4/0.3$	$1.3\cdot10^4/0.45$	$1.54\cdot10^4/0.41$	$1.7\cdot10^4/0.4$
615	$18.7\cdot10^4/0.3$	$1.95\cdot10^3/0.47$	$8\cdot10^3/0.42$	$1.3\cdot10^4/0.41$
620	$18.7\cdot10^4/0.3$	$1.3\cdot10^3/0.47$	$8\cdot10^3/0.42$	$1.3\cdot10^4/0.41$
650	$18.2\cdot10^4/0.3$	$5\cdot10^2/0.49$	$6\cdot10^3/0.43$	$1.15\cdot10^4/0.41$

$$D_1\left(\prod_{k=1}^{T-1}(\tilde{S}_k^{-1}F_k^{-1}F_{k+1})\right)=[q_{ij}]_{i=1,2}^{j=\overline{1,4}}. \tag{2.8}$$

The minimum roots of the characteristic equation (2.7) determine the critical loads and wave numbers that are responsible for the surface instability of the layered coating. The precritical stresses at normal temperature T_0 appearing in (1.1), (1.3), (2.2), and (2.3) are defined by the following formulas [23]:

$$\sigma_{11,i}^0=-p_{11}\left[\left(1-y\frac{a_{13}}{a_{33}}\right)_{(T+1)}\frac{(a_{11}-a_{13}^2a_{33}^{-1})_{(i)}}{(a_{11}-a_{13}^2a_{33}^{-1})_{(T+1)}}-y\frac{a_{13}^{(i)}}{a_{33}^{(i)}}\right],$$

$$\sigma_{33,i}^0=-p_{33}=-p_{11}y. \tag{2.9}$$

Formulas (2.4) were derived using the equality of precritical strains $\varepsilon_{11,1}^0=\varepsilon_{11,k}^0=\varepsilon_{11,T+1}^0$. The minimum roots of Eq. (2.7) were found numerically on a PC. Table 1 collects Poisson's ratios (denominator) and Young's moduli (numerator) of the layers and half-space at various temperatures T^*.

A distributed dead load p_{33} act on the surface of the layered medium, and its layers are in perfect contact. The perturbation components of the displacement vector $\vec{u}$ decay at infinity. The medium is compressed by a distributed force p_{11} in the plane of layers. The matrices of the characteristic equation (2.7) have the following elements:

$$b_{11}=b_{12}=1,\quad b_{21}(a_{13}+G_{13})=\eta_1^2(G_{13}+\sigma_{33}^0)-\sigma_{11}^0-a_{11},\quad b_{32}=G_{13}(b_{24}-b_{12}\eta_3^2),$$

$$b_{22}(a_{13}+G_{13})=\eta_3^2(G_{13}+\sigma_{33}^0)-\sigma_{11}^0-a_{11},\quad b_{31}=G_{13}(b_{24}-b_{11}\eta_1^2),$$

$$b_{41}=b_{11}a_{13}+b_{21}a_{33},\quad b_{42}=b_{12}a_{13}+b_{22}a_{33},$$

$$b_{51}=b_{11}a_{11}+b_{21}a_{13},\quad b_{52}=b_{12}a_{11}+b_{22}a_{13}. \tag{2.10}$$

The coefficients of Eq. (2.1) are defined by the formulas

$$\eta_{1,3}=c\pm\sqrt{c^2-\gamma},$$

$$2c(a_{33}+\sigma_{33}^0)(G_{13}+\sigma_{33}^0)=(a_{11}+\sigma_{11}^0)(a_{33}+\sigma_{33}^0)+(G_{13}+\sigma_{11}^0)(G_{13}+\sigma_{33}^0)-(a_{13}+G_{13})^2,$$

$$\gamma=(a_{11}+\sigma_{11}^0)(G_{31}+\sigma_{11}^0)(a_{33}+\sigma_{33}^0)^{-1}(G_{13}+\sigma_{33}^0)^{-1}. \tag{2.11}$$

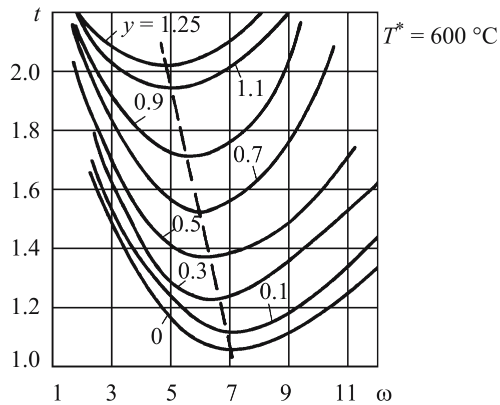

Fig. 1

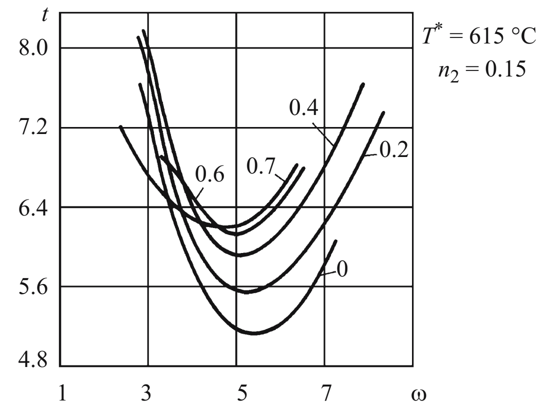

Fig. 2

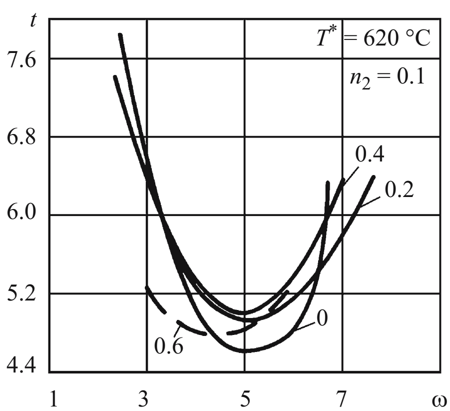

Fig. 3

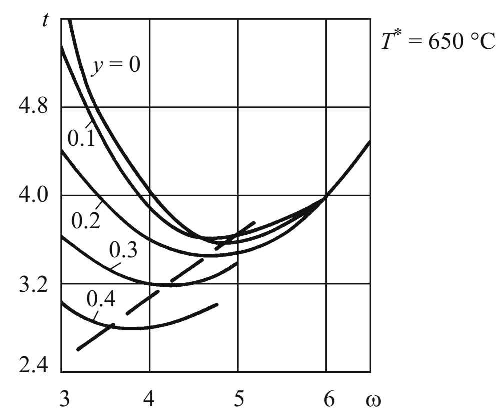

Fig. 4

Figures 1–4 show the solution of the problem for four temperatures indicated in Table 1, where $t = 10^2 (p_{11} / E_4)$, $\omega = (\pi H) / l$; l is the buckling half-wavelength for the whole laminate. The values of the parameter y are indicated near the curves. Figure 5 shows the dependence of the critical load $10^{-2}\, t_{cr}$ on y. The average temperature T^* of the coating is indicated near the curves. The results (Fig. 5) allow us not only to analyze the equilibrium state of a layered coating under compression, but also to formulate recommendations on the service conditions for structural elements with such coatings. Figure 1 demonstrates that at temperature T^* = 600 °C, the parameter t_{cr} strongly increases with y. Hence, the higher the load on the coating, the greater the critical load t_{cr} compared with the case $p_{33} = 0$ in the entire range of y. In the context of surface instability of elements of a laminate, the angle between the straight line and the ω-axis that passes through the critical points of the t–vs–ω curve is constant at various ratios of p_{33} and p_{11}.

However, an increase in temperature changes significantly the properties of the second layer. There is an average steady-state critical temperature of layers and near-surface portion of the substrate at which this angle abruptly changes. This fact may suggest that the effect of surface forces on the stable equilibrium of the surface layer has changed. Figures 2 and 3 show that the straight line passing through the critical points of the t–vs–ω curve becomes perpendicular to the ω-axis at temperature T^* = 615–620 °C. This suggests existence of a transition state in which the substrate still has an effect on the surface stability of the coating. When (Fig. 4) the AD1 layer becomes almost plastic at T^* = 650 °C, increasing the pressure p_{33} strongly decreases the critical load t_{cr} and wave number ω_{cr}, followed cessation of waveformation. In this case, this structural element cannot be used because its instability cannot be predicted. If there is a subsurface layer with very low stiffness, which forms at the limiting stage in the transition state of the so-called “incompetent” layer, an individual layer as a structural element may lose stability earlier

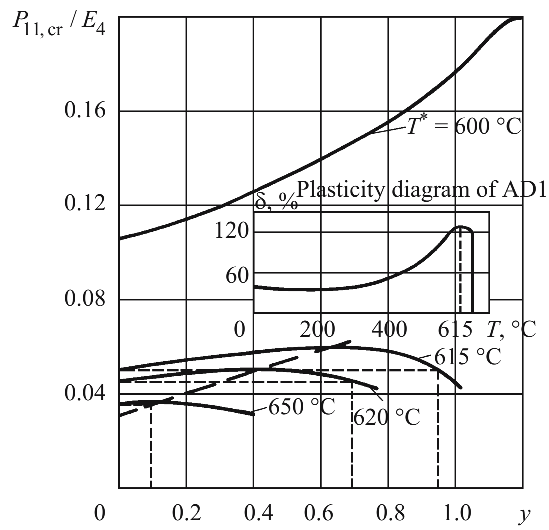

Fig. 5

than surface instability of the half-space as a piecewise-homogeneous medium occurs. In this case, the design model of the problem should be changed, which is beyond the scope of the paper.

Conclusions. The problems considered here well fall into and supplement the range of fundamental problems mentioned in the introduction and solved based on linearized solid mechanics [1]. Instability at critical temperatures should be taken into account in establishing operating conditions for tribotechnical mechanisms and structural elements.

The abrupt decrease in the stiffness of the subsurface layer alters the waveformation mechanism in the interface zone at temperatures $T^* \geq T^*_{\text{cr}}$ and may initiate surface instability of the layered material and coating and, consequently, may cause fracture at short-term overheating.

REFERENCES

1. A. N. Guz, *Stability of Three-Dimensional Deformable Bodies* [in Russian], Naukova Dumka, Kyiv (1971).
2. A. N. Guz, E. A. Tkachenko, and V. N. Chekhov, "Surface instability of layered tribotechnical composite coatings," *Dokl. NAN Ukrainy*, No. 8, 44–49 (1996).
3. A. N. Guz and V. N. Chekhov, "Problems of folding in the Earth's stratified crust," *Int. Appl. Mech.*, **43**, No. 2, 127–159 (2007).
4. É. A. Tkachenko and V. N. Chekhov., "Combined effect of temperature and compressive surface loads on the stability of elastic multilayer coatings with small subcritical strains," *Int. Appl. Mech.*, **34**, No. 8, 729–735 (1998).
5. I. Yu. Babich, A. N. Guz, and V. N. Chekhov, "The three-dimensional theory of stability of fibrous and laminated materials," *Int. Appl. Mech.*, **37**, No. 9, 1103–1141 (2001).
6. A. N. Guz, "A criterion of solid body destruction under compression along cracks (a 3-dimensional problem)," *Dokl. Akad. Nauk SSSR*, **261**, No. 1, 42–45 (1981).
7. A. N. Guz, "A criterion of solid body destruction under compression along cracks (two-dimensional problem)," *Dokl. Akad. Nauk SSSR*, **259**, No. 6, 1315–1318 (1981).
8. A. N. Guz, "Constructing the three-dimensional theory of stability of deformable bodies," *Int. Appl. Mech.*, **35**, No. 7, 641–670 (1999).
9. A. N. Guz, "Elastic waves in bodies with initial (residual) stresses," *Int. Appl. Mech.*, **38**, No. 1, 23–59 (2002).
10. A. N. Guz, "Establishing the fundamentals of the theory of stability of mine workings," *Int. Appl. Mech.*, **39**, No. 1, 20–48 (2003).

11. A. N. Guz and V. N. Chekhov, “On the determination of loads upon the ground support in underground mine workings,” *Int. Appl. Mech*., **41**, No. 11, 1239–1245 (2005).
12. A. N. Guz, V. N. Chekhov, and N. A. Shulga, “Surface instability of a fiber halfspace of periodic structure,” *Dokl. Akad. Nauk SSSR*, **266**, No. 6, 1306–1310 (1982).
13. A. N. Guz and I. A. Guz, “Analytical solution of stability problem for two composite half-planes compressed along interfacial cracks,” *Composites*, *Part B*, **31**, No. 5, 405–418 (2000).
14. A. N. Guz and I. A. Guz, “The stability of the interface between two bodies compressed along interface cracks. 1. Exact solutions for the case of unequal roots,” *Int. Appl. Mech*., **36**, No. 4, 482–491 (2000).
15. A. N. Guz and I. A. Guz, “The stability of the interface between two bodies compressed along interface cracks. 2. Exact solutions for the case of equal roots,” *Int. Appl. Mech*., **36**, No. 5, 615–622 (2000).
16. A. N. Guz and I. A. Guz, “The stability of the interface between two bodies compressed along interface cracks. 3. Exact solutions for the combined case of equal and unequal roots,” *Int. Appl. Mech*., **36**, No. 6, 759–768 (2000).
17. A. N. Guz and I. A. Guz, “Stability of two different half-planes in compression along interfacial cracks: Analytical solutions,” *Int. Appl. Mech*., **37**, No. 7, 906–912 (2001).
18. A. N. Guz and I. A. Guz, “Mixed plane problems in linearized solid mechanics: Exact solutions,” *Int. Appl. Mech*., **40**, No. 1, 1–29 (2004).
19. A. N. Guz and Yu. N. Lapusta, “Three-dimensional problems of the near-surface instability of fiber composites in compression (model of a piecewise-uniform medium) (survey),” *Int. Appl. Mech*., **35**, No. 7, 641–670 (1999).
20. A. N. Guz and F. G. Makhort, “The physical fundamentals of the ultrasonic nondestructive stress analysis of solids,” *Int. Appl. Mech*., **36**, No. 9, 1119–1149 (2000).
21. I. A. Guz, “Stability of the composite under compression along the crack situated on the boundary between layers,” *Dokl. Akad. Nauk SSSR*, **325**, No. 3, 455–458 (1992).
22. I. A. Guz, “Stability of the composite under compression along 2 microcracks situated in the interlayers boundary,” *Dokl. Akad. Nauk SSSR*, **328**, No. 4, 437–439 (1993).
23. E. A. Tkachenko, “Effect of geometric and mechanical characteristics of antifriction coatings on their stability,” *Int. Appl. Mech*., **35**, No. 9, 94–99 (1999).
24. E. A. Tkachenko and V. N. Chekhov, “Stability of a laminate between two homogeneous half-spaces under inelastic deformation,” *Int. Appl. Mech*., **41**, No. 5, 481–489 (2005).