International Applied Mechanics, Vol. 45, No. 11, 2009

DEFORMATION OF AN ELASTIC PLATE WITH AN EDGE NOTCH UNDER THE ACTION OF A PLANE SHOCK WAVE: EXPERIMENTAL RESEARCH

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The dynamics of a thin elastic plate with an edge notch under the action of a weak shock wave in air is experimentally studied. It is shown that the notch has a significant effect on the deformation process

Keywords: thin elastic plate, edge notch, weak shock wave, deformation process

Introduction. Special-purpose or process-induced inhomogeneities such as inserts, cover plates, notches, cracks, and holes in structural members (plates or shells) have a strong effect on their behavior under various loads. Such inhomogeneities influence the stress–strain state, critical load, and ultimate strength. Let us mention some recent publications addressing problems for bodies with various stress concentrators under static and dynamic loads.

For example, plates under uniaxial or biaxial tension and shear were considered in the paper [2], which established how the thickness of the plate near holes should vary to decrease the stress concentration. The influence of a notch on the frequency characteristics of a plate was examined in [10]. A number of qualitative mechanical effects in a thin open spherical shell with an oblong elliptic hole were revealed in [11]. The change in the energy of an elastic plate after making a circular hole in it was studied in [12]. The stress–strain state at the boundary of a circular hole and at the interface between a plate and an eccentric circular cover plate was analyzed in [3]. The influence of rectangular plates reinforcing a cylindrical shell on the critical load, frequency, and vibration mode was experimentally examined in [13]. The design of a compound plate weakened by a periodic set of cracks was considered in [4]. Orthotropic plates weakened by a periodic number of collinear cracks were addressed in [7]. The exact solution (as a function of four constants of integration) for and an equation for the critical compressive load of a column with an arbitrary number and arrangement of cracks were obtained in [8]. The flexibility matrix with a crack and stiffness coefficients for a rotor were found in [9]. The deformation of elastic plates with a central notch parallel to two clamped edges under the action of a shock wave was experimentally studied in [6].

The present paper outlines a procedure and results of experimental study of the influence of an edge notch on the nonstationary behavior of a thin elastic plate clamped at two opposite edges and subjected to the action of a normally incident staircase shock wave.

1. Method of Analysis. The shock wave was generated in a diaphragm-type shock pipe. The experimental procedure from [6] was mainly used. A tested rectangular plate is made of SF-1-150 fiberglass plastic (GOST 10316-78) with density ρ = 1.7 g/cm^3 and elastic modulus $E = 2.6 \cdot 10^{10}$ Pa and had a thickness of 2.5 mm, width a = 140 mm, and length l = 210 mm. The 140×250 mm plate was rigidly fixed on the short sides between two steel frames 18 mm thick each. The inside dimensions of the frames coincided with the effective cross-section of the diaphragm-type shock tube; the outside dimensions were 300×230 mm. The plate was fixed with screws and the contact surfaces of the plate and the frames were glued with cold-setting epoxy adhesive. The gaps between the long sides of the test plate and the inside sides of the frame were 0.3 mm approximately.

The plate together with the frames was fixed with screws at the end of the shock tube, which had a cross section of 210×140 along the entire length (chamber–channel–measuring section). The lengths of the chamber and channel were selected so as to produce a staircase shock wave with duration $t \approx 8 \cdot 10^{-3}$ sec, which was enough for the plate to undergo approximately two full oscillations.

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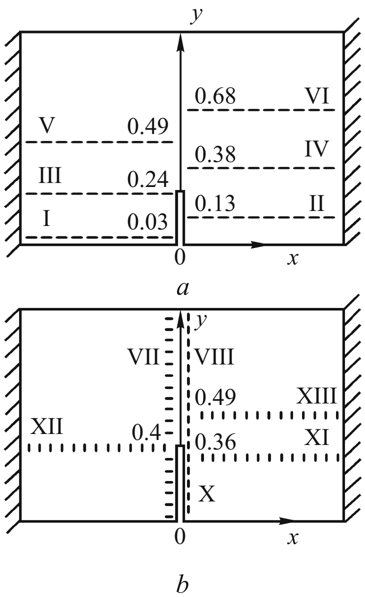

Fig. 1

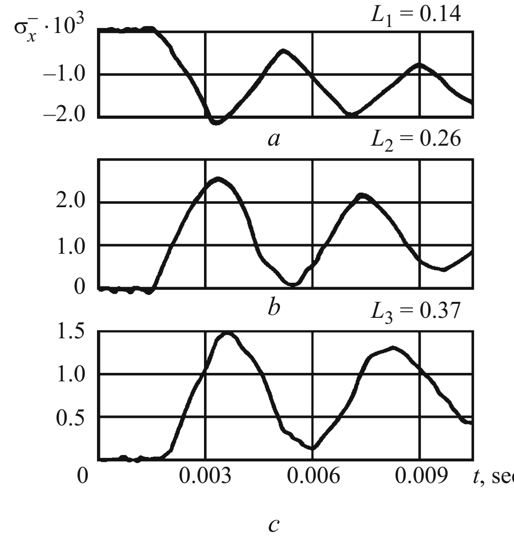

Fig. 2

The wavefront pressure was specified as the pressure in the chamber at the instant the diaphragm was broken and was expressed in terms of the velocity of the incident wavefront, which was determined in each test from the time it took the wave to travel the fixed distance between two piezoelectric elements placed at the end of the measuring section. The signals of the piezoelectric elements were supplied to a C9-8 double-beam storage oscillograph. The relative error of the pressure determined in this way did not exceed ±4% [5].

In all series of tests, the plate was subjected to the action of waves with equal amplitudes that caused a pressure of $(0.1 \cdot 10^5 \pm 5\%)$ Pa on the plate that was equal to the pressure in the shock wave reflected from the plate [5].

A solid plate was tested in the first series. Then a notch was cut from the middle of one of the long (free) edges in parallel to the clamped edges. The notch was approximately 0.5 mm wide, and its dimensionless length L was 0.14, 0.26, and 0.37 times the plate width a in the three series of tests, respectively.

KF5P1-1-200V-12 and KF4P1-3-100V-12 resistance strain gages with gage lengths of 1 and 3 mm and KF5Ts1-1-100B-12 strain-gage networks were bonded with cold-setting epoxy adhesive in parallel to the free (x-axis) and clamped (y-axis) edges. The origin of coordinates was at the middle of the free edge, at the beginning of the notch (the point O in Fig. 1). In Fig. 1, the lines are numbered from I to XIII, and the dimensionless distance $l = y/a$ from the origin to the lines is indicated. Figures 1*a* and 1*b* represent notches with lengths $L_2 = 0.26$ and $L_3 = 0.37$. The strain components along the x-axis were measured on lines I–VII and IX (Fig. 1*a*) and the strains components along the y-axis on lines VIII, X-XIII (Fig. 1*b*). Lines VII–X run along the y-axis at the minimum possible (for the strain gages used) distance from the edges of the notch and farther from the y-axis. The strain gages were bonded on both surfaces of the plate, one under another, which made it possible to measure time-dependent strains on the loaded (ε_x^+, ε_y^+) and free ($\varepsilon_x^\sim$, $\varepsilon_y^\sim$) surfaces of the plate. The bending and membrane strains were determined by the formulas

$$\varepsilon^x = (\varepsilon^+ - \varepsilon^\sim)/2, \qquad \varepsilon^x = (\varepsilon^+ + \varepsilon^\sim)/2. \tag{1}$$

Alternatively, a pair of gages on the opposite faces of the plate was connected so as to record the variation of the sum or difference of their signals with time.

The signals from the strain gages were recorded with a L1250 analog/digital signal processor and displayed on the screen of a Pentium-S computer. In each test, four strains on the plate surfaces or the time-dependent bending (membrane) strains caused by a shock wave incident on one of the surfaces were measured.

2. Test Results. Typical oscillograms of strains along the x-axis of surface elements of the free edge are shown in Fig. 2. The upper oscillogram (Fig. 2*a*) shows the variation of the strain ε_x^- at the notch L_1 measured with the strain gage on line V (Fig. 1*a*) near (~4 mm) the clamped edge. The curve in Fig. 2*b* illustrates the variation of the strains on the continuation of the

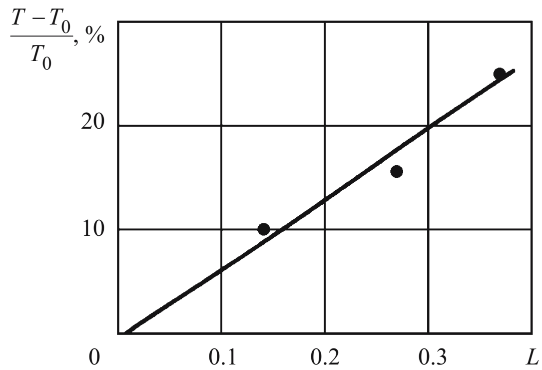

Fig. 3

notch L_2, above its tip ($x = 0, y = 0.28a$). The oscillogram in Fig. 2*c* shows the strain $\varepsilon_x^-(t)$ at the point with coordinates $x = 0, y = 0.4a$ near the notch $L_3 = 0.37$.

An analysis of the oscillograms shows that all elements of the plate with an edge notch (except for the zones near the notch) undergo damped quasiharmonic vibrations after the incidence of a shock wave. As in solid plates [1] and plates with a central notch [6], these vibrations occur about some constant level corresponding to the wavefront pressure rather than about zero. Since even if damping is intensive, the vibration frequency is slightly different from the natural frequency of the corresponding conservative system, we can consider the time between two neighboring peaks to be the period of vibrations T. This time interval was determined for each oscillogram and then the average time over all oscillograms was calculated with an error of ±5%.

Figure 2 demonstrates that the vibration period increases with the notch length. The period T linearly increases with the relative length L of the notch (Fig. 3). The notch with a length of 0.37 times the length of the clamped edge of the plate increases the period by ~25% compared with the solid plate. This value is approximately fourfold greater than that for a central notch with a length equal to half the plate width [6].

The oscillograms of the strains along both axes measured on lines IX and X near the notch represent irregular high-frequency vibrations with amplitudes less than 10% of the maximum strains near the clamped edges.

The strain state of the plate was determined from the time of the first peak on the oscillograms. According to [1], bending in the direction of wave propagation, a solid elastic plate with similar boundary conditions undergoes, along the x-axis, mainly bending and weak tension that is uniform along the whole length of the plate, except for small zones near the clamped edges where it is zero. The tensile strain of the mid-surface of the plate was approximately 10% of the bending strain in its middle.

The edge notch changes the strain state of the plate substantially. The deformation pattern remained symmetric about the straight line coming through the origin, the notch, and its continuation, i.e., about the y-axis. In this connection, the figures show the strains for half the plate on the right or left of the axis of symmetry.

Figures 4*a*–*e* shows the maximum strains ε_x^- on the unloaded side of the plate along lines I–V (Figs. 1*a*, *b*). The lines are numbered in the upper right corner. The solid curve illustrates the strain ε_x^- in the solid plate. The long-dash line corresponds to $L_1 = 0.14$, the dash-and-dot line to $L_2 = 0.26$, and the short-dash line to $L_3 = 0.37$.

It can be seen that the law of variation in strain $\varepsilon_x^-(t)$ on the lines parallel to the free edges of the plate depends on the position of the notch tip. For example, when $l < L$ ($l_1 = 0.03$ (Fig. 4*a*); $l_{11} = 0.13$ (Fig. 4*b*)), the strains on the lines $y = \text{const}$ did not peak in the middle, as was the case in tests on a solid plate. There are two peaks on both sides of the notch that are shifted toward the clamped edges, the peak strains not exceeding the strains in the solid plate. The strains decreased near the notch, tending to zero.

The inflection points (at which the strains reverse sign) also shifted to the middle of the plate. The unloaded side of the plate was compressed beginning from the inflection point to the clamped edge, the curves $\varepsilon_x^-(x)$ differing from those for the solid plate. Somewhere within this interval, the strains in the plate with an edge notch may exceed twofold those in solid plate.

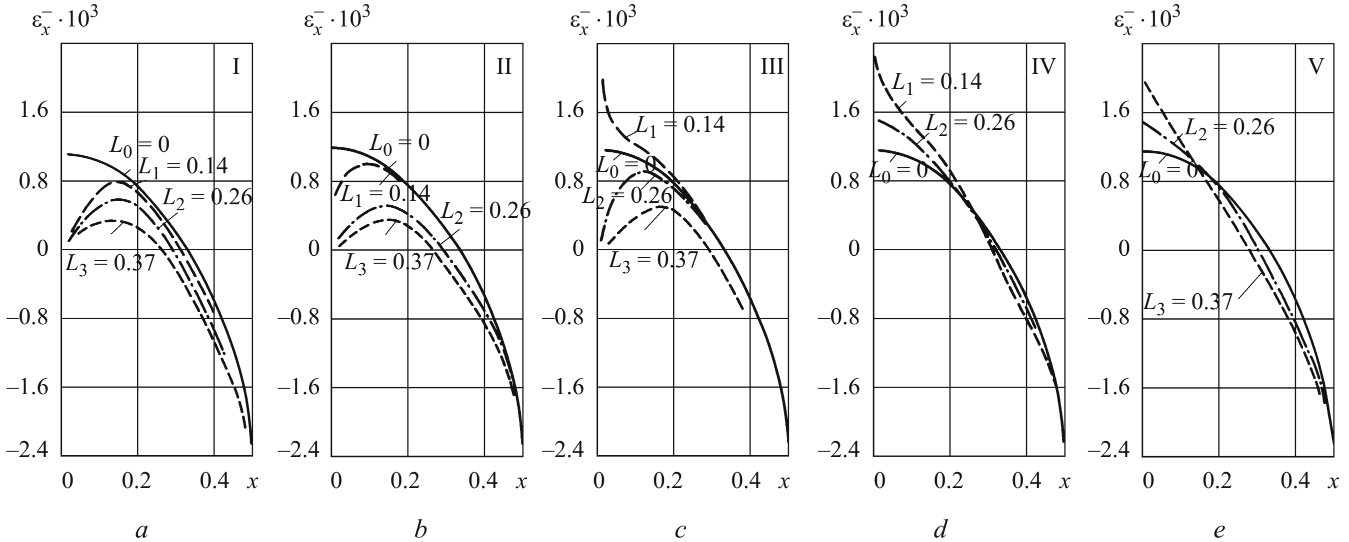

Fig. 4

Figure 4*c* shows the strains along line III that is parallel to the *x*-axis and at distance $l_{\mathrm{III}} = 0.24$ from the origin of coordinates. This line lies below the tip of the notch with length L_2 or L_3 and above the tip of the notch with length $L_1 = 0.14$. In the latter case, beginning from the inflection point, the strain curve goes above the solid line representing the solid plate.

Lines $l_{\mathrm{IV}} = 0.38$ (Fig. 4*d*) and $l_{\mathrm{V}} = 0.49$ (Fig. 4*e*) go above the tips of the notches with $L_2 = 0.26$ and $L_3 = 0.37$. The strains above the notch tip to the right and to the left of the *y*-axis increased considerably (nearly twofold) compared with the solid plate. Comparing Figs. 4*d* and 4*e* reveals that the high-strain zone narrows as the notch tip (i.e., a stress concentrator) is approached.

Figure 5 shows the strain ε_x along line VI ($l_{\mathrm{VI}} = 0.68$) for the notch with $L_3 = 0.37$. The solid curves represent the following strains in the solid plate: $\varepsilon_x^+(L_0)$ on the loaded side and $\varepsilon_x^-(L_0)$ on the free side. The strains $\varepsilon_x^+(L_3)$ and $\varepsilon_x^-(L_3)$ along the same line in the notched plate (dashed lines) are qualitatively the same. The strains on the loaded side coincide only in the middle and the strains on the free side are close from the inflection point to the clamped edge. For other values of *x*, the strains in the notched plate exceed those in the solid plate. For example, at $x = 0$, strains in the notched plate are approximately 20% greater than in the solid plate.

The same figure shows, by almost horizontal solid and dashed lines, the membrane strains $\varepsilon_x^{\mathrm{m}}(L_0)$ and $\varepsilon_x^{\mathrm{m}}(L_3)$ defined by (1). Thus, the uncut side of the plate was subjected to tensile strain (which is approximately double that in the solid plate) constant from one clamped edge to the other along the *x*-axis.

Figure 6 shows the strains ε_x^+ and ε_x^- (solid curves) along line VII, which is the continuation of the notch L_3 and aligned with the *y*-axis. The dashed lines show how the bending ($\varepsilon_x^{\mathrm{b}}$) and membrane ($\varepsilon_x^{\mathrm{m}}$) strains vary along the *y*-axis. Beginning from the uncut free edge of the plate, all the strains increase, first smoothly and then abruptly (severalfold at the notch tip).

When subjected to a shock wave [1], a rectangular plate without notch clamped at two opposite edges undergoes mainly bending and weak uniform tension in the direction of the free edges (*x*-axis). Hence, the plate is compressed in the perpendicular direction (*y*-axis) according to the formula $\varepsilon_y = \mu\varepsilon_x$, where μ is Poisson's ratio ($\mu = 0.3$–0.4 for the material of the plate).

Figure 7 shows how the strains ε_y^+, ε_y^-, $\varepsilon_y^{\mathrm{m}}$, and $\varepsilon_y^{\mathrm{b}}$ vary along line VIII, which is the continuation of the notch with $L_3 = 0.37$. The incident wave causes compression of the uncut side of the plate (unlike the strains along the *x*-axis) and tension of the loaded side. The bending and membrane strains are comparable in magnitude and increase considerably toward the notch tip.

Conclusions. A procedure developed earlier has been used to experimentally examine the effect of an edge notch on the nonstationary behavior of a thin elastic plate clamped at two opposite edges and subjected to a normal weak shock wave.

It has been shown that a shock-wave load causes solid plates, plates with a central notch, and plates with an edge notch to undergo damped, nearly harmonic vibrations about some level corresponding to the load. An edge notch with a length of about 0.4 times the length of the clamped edge increases the period of vibrations by approximately 25%.

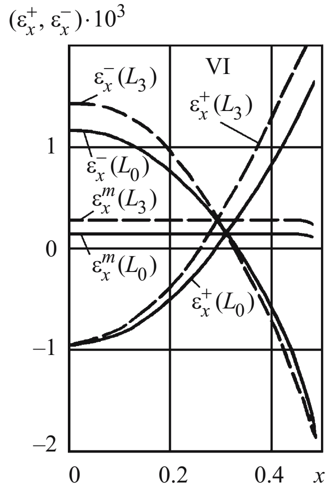

Fig. 5

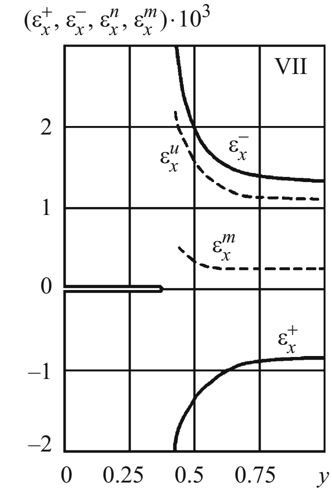

Fig. 6

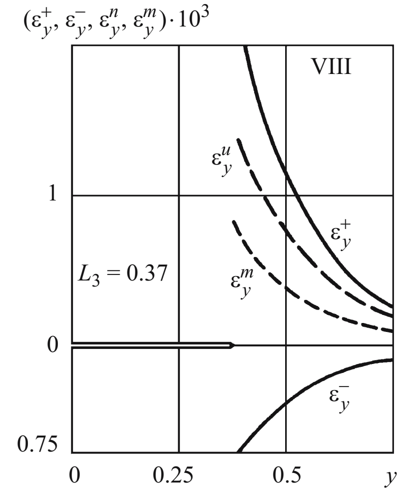

Fig. 7

An edge notch causes qualitative changes in the strain state of the plate. Solid plate undergoes mainly bending and weak tension along the lines connecting the clamped edges. The edge notch changes the bending strains in this direction, on both sides of the notch and in the uncut part of the plate. Moreover, there are substantial bending and membrane strains in the transverse direction in the uncut part of the plate.

The strains found can be used to validate, either numerically or analytically, the design of elastic plates with an edge notch subject to the action of a long shock wave.

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