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DEFORMATION AND LONG-TERM DAMAGE OF FIBROUS MATERIALS WITH THE STRESS-RUPTURE MICROSTRENGTH OF THE MATRIX DESCRIBED BY A FRACTIONAL-POWER FUNCTION

L. P. Khoroshun and E. N. Shikula

The theory of long-term damage is generalized to unidirectional fibrous composites. The damage of the matrix is modeled by randomly dispersed micropores. The damage criterion for a microvolume is characterized by its stress-rupture strength. It is determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its limit, which is the ultimate strength, according to the Huber–Mises criterion, and assumed to be a random function of coordinates. An equation of damage (porosity) balance in the matrix at an arbitrary time is formulated. Algorithms of calculating the time dependence of microdamage and macrostresses or macrostrains are developed and corresponding curves are plotted in the case of stress-rupture microstrength described by a fractional power function

Keywords: composite materials, stochastic structure, stress–strain state, long-term damage, porosity of matrix, effective characteristics, porosity balance equation, fractional power function

Introduction. One of the possible failure mechanisms in materials and structural members is the occurrence and development of dispersed microdamages, which commonly lead to the formation of main cracks. Physically, the damage of a material can be considered as dispersed defects such as microcracks, microvoids or damaged microvolumes. They reduce the effective or bearing portion of the material, which resists loads.

There are three types of damage models. The models of the first type proceed from the micrononuniformity of the elastic and strength properties of the material, resulting in dispersed microdamages under loading, which are modeled by microcracks or micropores. The damage equations are derived from the theory of deformation of structurally inhomogeneous media and certain failure criteria for microvolumes of the material. The models of the second type formally introduce a damage parameter as a measure of discontinuity of the material but do not indicate its physical meaning and postulate an evolutionary equation that relates the damage rate and the applied stress. The models of the third type describe damage by thermodynamic (rather than structural) parameters, which contribute, together with stresses and strains, to the laws of thermodynamics. This gives formal relationships among stresses, strains, and damage parameters.

It is obvious that the models of the first type most adequately represent real damage processes. The ideas and methods of the mechanics of stochastically inhomogeneous media make it possible to describe the coupled processes of deformation and short-term damage of both homogeneous [11, 12] and composite [14–17] materials and to study these processes over a wide range of mechanical properties, including thermal effects [10, 12, 18–21] and physically nonlinear deformation [22–35].

However, experimental data on and observations of the behavior of structural members and structures suggest that damage can be either short-term (occurring instantaneously after the application of stresses or strains) or long-term (building up with time after the application of load). Long-term damage is usually considered to result from the accumulation of dispersed microdamages such as micropores and microcracks. At the microscopic level, the strength of a material is inhomogeneous, i.e.,

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the ultimate strength and the stress-rupture strength for a material microvolume are random functions of coordinates described by certain distribution densities or distribution functions. When a macrospecimen is subjected to a constant stress, the microvolumes whose ultimate strength is less than the applied stress are destroyed, i.e., they give rise to microcracks or microcavities. On those microareas where the stress is less than yet close to the ultimate strength, failure occurs in some time that depends on how close the stress is to the ultimate microstrength.

The theory of long-term damage of homogeneous materials developed in [13] based on models and methods of the mechanics of stochastically microinhomogeneous media is generalized here to fibrous materials. The damage of the matrix of a fibrous material is modeled by dispersed microvolumes destroyed to become randomly arranged micropores. The failure criterion for a single microvolume is determined by its stress-rupture strength described by a fractional power function, which is, in turn, determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its limit, which characterizes the ultimate strength according to the Huber–Mises criterion. The ultimate strength is assumed to be a random function of coordinates whose one-point distribution is described by a power function on some interval or by the Weibull function. The effective elastic properties and the stress–strain state of a fibrous material with randomly arranged microdamages in the matrix are determined from the stochastic equations of elasticity of porous materials. We will derive the damage (porosity) balance equation from the properties of the distribution functions and ergodicity of the random field of ultimate microstrength, and the dependence of the time to brittle failure for a microvolume on its stress state and ultimate microstrength for given macrostress or macrostrain and an arbitrary time. The macrostress–macrostrain relationship for a fibrous material with porous matrix and the porosity balance equations for the matrix describe the coupled and interacting processes of deformation and long-term damage, which cause the macrostrain to increase at given macrostress or the macrostress to decrease at given macrostrain. An iteration method is used to develop algorithms for calculating the time dependence of microdamage, macrostress, and macrostrain and to plot the corresponding curves in the case of stress-rupture microstrength described by a fractional power function.

1. Consider a unidirectional fibrous material of stochastic structure. Let the fibers be transversely isotropic and directed normally to the isotropy plane x_1x_2. Denote the elastic moduli of fibers by $\lambda_{11}^1, \lambda_{12}^1, \lambda_{13}^1, \lambda_{33}^1, \lambda_{44}^1$; the bulk and shear moduli for the undamaged portion of the matrix by K_2 and μ_2, and the volume fractions of fibers and porous matrix by c_1 and c_2, respectively. The macrostresses $\langle\sigma_{jk}\rangle$ and the macrostrains $\langle\varepsilon_{jk}\rangle$ are related by

$$\langle\sigma_{jk}\rangle = (\lambda_{11}^* - \lambda_{12}^*)\langle\varepsilon_{jk}\rangle + (\lambda_{12}^*\langle\varepsilon_{rr}\rangle + \lambda_{13}^*\langle\varepsilon_{33}\rangle)\delta_{jk},$$

$$\langle\sigma_{33}\rangle = \lambda_{13}^*\langle\varepsilon_{rr}\rangle + \lambda_{33}^*\langle\varepsilon_{33}\rangle, \qquad \langle\sigma_{j3}\rangle = 2\lambda_{44}^*\langle\varepsilon_{j3}\rangle \qquad (j,k,r=1,2), \tag{1.1}$$

where the effective elastic moduli $\lambda_{11}^*, \lambda_{12}^*, \lambda_{13}^*, \lambda_{33}^*, \lambda_{44}^*$ are defined as follows [2, 7, 9]:

$$\lambda_{11}^* + \lambda_{12}^* = c_1(\lambda_{11}^1 + \lambda_{12}^1) + 2c_2(\lambda_{2p} + \mu_{2p}) - \frac{c_1c_2(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p} - 2\mu_{2p})^2}{2c_1(\lambda_{2p} + \mu_{2p}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m},$$

$$\lambda_{11}^* - \lambda_{12}^* = c_1(\lambda_{11}^1 - \lambda_{12}^1) + 2c_2\mu_{2p} - \frac{c_1c_2(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p})^2}{2c_1\mu_{2p} + c_2(\lambda_{11}^1 - \lambda_{12}^1) + 2n},$$

$$\lambda_{13}^* = c_1\lambda_{13}^* + c_2\lambda_{2p} - \frac{c_1c_2(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p} - 2\mu_{2p})(\lambda_{13}^1 - \lambda_{2p})}{2c_1(\lambda_{2p} + \mu_{2p}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m},$$

$$\lambda_{33}^* = c_1\lambda_{33}^* + c_2(\lambda_{2p} + 2\mu_{2p}) - \frac{2c_1c_2(\lambda_{13}^1 - \lambda_{2p})^2}{2c_1(\lambda_{2p} + \mu_{2p}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m},$$

$$\lambda_{44}^* = c_1\lambda_{44}^1 + c_2\mu_{2p} - \frac{c_1c_2(\lambda_{44}^1 - \mu_{2p})^2}{c_1\mu_{2p} + c_2\lambda_{44}^1 + s}, \tag{1.2}$$

here

$$2m = c_1(\lambda_{11}^1 - \lambda_{12}^1) + 2c_2\mu_{2p}, \quad 2n = c_1(\lambda_{11}^1 + \lambda_{12}^1) + 2c_2(\lambda_{2p} + \mu_{2p}), \quad s = c_1\lambda_{44}^1 + 2c_2\mu_{2p} \tag{1.3}$$

if the matrix is stiffer than fibers, and

$$2m = \left(\frac{c_1}{\lambda_{11}^1 - \lambda_{12}^1} + \frac{c_2}{2\mu_{2p}}\right)^{-1}, \quad 2n = \left(\frac{c_1}{\lambda_{11}^1 + \lambda_{12}^1} + \frac{c_2}{2(\lambda_{2p} + \mu_{2p})}\right)^{-1}, \quad s = \left(\frac{c_1}{\lambda_{44}^1} + \frac{c_2}{2\mu_{2p}}\right)^{-1}, \tag{1.4}$$

otherwise.

According to [6, 8], the effective moduli K_{2p} and μ_{2p} of the porous matrix are defined by

$$K_{2p} = \frac{4K_2\mu_2(1-p_2)^2}{4\mu_2 + (3K_2 - 4\mu_2)p_2}, \quad \mu_{2p} = \frac{(9K_2 + 8\mu_2)\mu_2(1-p_2)^2}{9K_2 + 8\mu_2 - (3K_2 - 4\mu_2)p_2}. \tag{1.5}$$

We will use the Huber–Mises criterion [3] to describe the short-term damage in a microvolume of the undamaged portion of the matrix:

$$I_\sigma^2 = k_2, \quad I_\sigma^2 = (\bar{\sigma}'^2_{pq}\,\bar{\sigma}'^2_{pq})^{1/2}, \tag{1.6}$$

where $\bar{\sigma}'^2_{pq}$ is the average-stress deviator for the undamaged portion of the matrix; k_2 is the limiting value of the invariant I_σ^2 for the matrix, which is a random function of coordinates. The average stresses $\bar{\sigma}^2_{jk}$ are defined by the following formula [9]:

$$\bar{\sigma}^2_{jk} = \frac{1}{1-p_2}\langle\sigma^2_{jk}\rangle. \tag{1.7}$$

If the invariant I_σ^2 does not reach the limiting value k_2 in some microvolume of the matrix, then, according to the stress-rupture criterion, failure will occur in some time τ_k^2, which depends on the difference between I_σ^2 and k_2. In the general case, this dependence can be represented as some function:

$$\tau_k^i = \varphi_i(I_\sigma^i, k_i), \tag{1.8}$$

where $\varphi_2(k_2, k_2) = 0$ and $\varphi_2(0, k_2) = \infty$ according to (1.6).

The one-point distribution function $F_2(k_2)$ for some microvolume in the undamaged portion of the matrix can be approximated by a power function on some interval

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{02}, \\ \left(\dfrac{k_2 - k_{02}}{k_{12} - k_{02}}\right)^{\alpha_2}, & k_{02} \le k_2 \le k_{12}, \\ 1, & k_2 > k_{12} \end{cases} \tag{1.9}$$

or by the Weibull function

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{02}, \\ 1 - \exp\left[-m_2(k_2 - k_{02})^{\alpha_2}\right], & k_2 \ge k_{02}, \end{cases} \tag{1.10}$$

where k_{02} is the minimum value of k_2 that initiates failure in some volumes of the matrix; k_{12}, m_2, and α_2 are constants found from strength scatter fitting in the matrix.

Assume that the random field of the ultimate microstrength k_2 is statistically homogeneous, which is typical of real materials, and microdamages and the distances between them are negligible compared with the inclusions and the distances between them. Then the distribution function $F_2(k_2)$ is ergodic because it defines the content of the undamaged portion of the matrix in which the ultimate microstrength is less than k_2. Therefore, if the stresses $\overline{\sigma}_{jk}^2$ are nonzero, the function $F_2(I_{\overline{\sigma}}^2)$ defines, according to (1.6), (1.9), and (1.10), the content of instantaneously damaged microvolumes in the matrix. Since the damaged microvolumes are modeled by pores, we can write a balance equation for damaged microvolumes or porosities of the matrix subject to short-term damage:

$$p_2 = p_{02} + (1 - p_{02})F_2(I_{\overline{\sigma}}^2), \tag{1.11}$$

where p_{02} is the initial porosity of the matrix and, according to (1.7),

$$I_{\overline{\sigma}}^2 = \frac{1}{1-p_2} I_{\langle\sigma\rangle}^2, \qquad I_{\langle\sigma\rangle}^2 = (\langle\sigma_{jk}^2\rangle'\langle\sigma_{jk}^2\rangle')^{1/2}. \tag{1.12}$$

Here the average stresses $\langle\sigma_{jk}^2\rangle$ in the matrix are related to the average strains $\langle\varepsilon_{jk}^2\rangle$ as follows:

$$\langle\sigma_{jk}^2\rangle = \lambda_{2p}\langle\varepsilon_{rr}^2\rangle\delta_{jk} + 2\mu_{2p}\langle\varepsilon_{jk}^2\rangle. \tag{1.13}$$

Given macrostrains $\langle\varepsilon_{jk}\rangle$, the average stresses $\langle\varepsilon_{ij}^2\rangle$ in the matrix are related to them as follows [2, 7, 9]:

$$\langle\varepsilon_{jk}^2\rangle = A_0\langle\varepsilon_{jk}\rangle - \frac{1}{\Delta_2}(A_1\langle\varepsilon_{rr}\rangle + A_2\langle\varepsilon_{33}\rangle)\delta_{jk},$$

$$\langle\varepsilon_{33}^2\rangle = -\frac{1}{\Delta_2}(A_3\langle\varepsilon_{rr}\rangle + A_4\langle\varepsilon_{33}\rangle) \qquad \langle\varepsilon_{j3}^2\rangle = \frac{\lambda_{44}^* - \lambda_{44}^1}{c_2(\mu_{2p} - \lambda_{44}^1)}\langle\varepsilon_{j3}\rangle$$

$$(j,k,r = 1,2), \tag{1.14}$$

where

$$\Delta_2 = c_2(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p})[(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p} - 2\mu_{2p})(\lambda_{33}^1 - \lambda_{2p} - 2\mu_{2p}) - 2(\lambda_{13}^1 - \lambda_{2p})^2],$$

$$A_0 = \frac{\lambda_{11}^* - \lambda_{12}^* - \lambda_{11}^1 + \lambda_{12}^1}{c_2(2\mu_{2p} - \lambda_{11}^1 + \lambda_{12}^1)}, \qquad A_1 = (\lambda_{11}^* - \lambda_{11}^1)a_1 - (\lambda_{12}^* - \lambda_{12}^1)a_2 - (\lambda_{13}^* - \lambda_{13}^1)a_3,$$

$$A_2 = (\lambda_{13}^* - \lambda_{13}^1)(a_1 - a_2) - (\lambda_{33}^* - \lambda_{33}^1)a_3,$$

$$A_3 = (\lambda_{13}^* - \lambda_{13}^1)a_4 - (\lambda_{11}^* + \lambda_{12}^* - \lambda_{11}^1 - \lambda_{12}^1)a_3,$$

$$A_4 = (\lambda_{33}^* - \lambda_{33}^1)a_4 - 2(\lambda_{13}^* - \lambda_{13}^1)a_3,$$

$$a_1 = (\lambda_{13}^1 - \lambda_{2p})^2 - (\lambda_{12}^1 - \lambda_{2p})(\lambda_{33}^1 - \lambda_{2p} - 2\mu_{2p}),$$

$$a_2 = (\lambda_{13}^1 - \lambda_{2p})^2 - (\lambda_{11}^1 - \lambda_{2p} - 2\mu_{2p})(\lambda_{33}^1 - \lambda_{2p} - 2\mu_{2p}),$$

$$a_3 = (\lambda_{13}^1 - \lambda_{2p})(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}), \qquad a_4 = (\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p} - 2\mu_{2p})(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}). \tag{1.15}$$

Given macrostresses $\langle\sigma_{jk}\rangle$, the average stresses $\langle\varepsilon_{ij}^2\rangle$ in the matrix are related to them as

$$\langle\varepsilon_{11}^2\rangle+\langle\varepsilon_{22}^2\rangle=\frac{1}{\Delta^*}\left\{\left(\lambda_{33}^*A_0+\frac{-\lambda_{33}^*A_1+\lambda_{13}^*A_2}{\Delta_2}\right)\langle\sigma_{rr}\rangle+\left[-2\lambda_{13}^*A_0+\frac{2\lambda_{13}^*A_1-(\lambda_{11}^*+\lambda_{12}^*)A_2}{\Delta_2}\right]\langle\sigma_{33}\rangle\right\},$$

$$\langle\varepsilon_{11}^2\rangle-\langle\varepsilon_{22}^2\rangle=\frac{A_0}{\lambda_{11}^*-\lambda_{12}^*}(\langle\sigma_{11}\rangle-\langle\sigma_{22}\rangle)=\frac{\lambda_{11}^*-\lambda_{12}^*-\lambda_{11}^1+\lambda_{12}^1}{c_2(\lambda_{11}^*-\lambda_{12}^*)(2\mu_{2p}-\lambda_{11}^1+\lambda_{12}^1)}(\langle\sigma_{11}\rangle-\langle\sigma_{22}\rangle),$$

$$\langle\varepsilon_{33}^2\rangle=\frac{1}{\Delta^*\Delta_2}\{(-\lambda_{33}^*A_3+\lambda_{13}^*A_4)\langle\sigma_{rr}\rangle+[2\lambda_{13}^*A_3-(\lambda_{11}^*+\lambda_{12}^*)A_4]\langle\sigma_{33}\rangle\},$$

$$\langle\varepsilon_{j3}^2\rangle=\frac{\lambda_{44}^*-\lambda_{44}^1}{2c_2\lambda_{44}^*(\mu_{2p}-\lambda_{44}^1)}\langle\sigma_{j3}\rangle\qquad(j,k,r=1,2),\tag{1.16}$$

where

$$\Delta^*=(\lambda_{11}^*+\lambda_{12}^*)\lambda_{33}^*-2(\lambda_{13}^*)^2,\tag{1.17}$$

the effective moduli λ_{2p} and μ_{2p} being defined by (1.5).

If the stresses $\overline{\sigma}_{jk}^2$ act for some time t, then, according to the stress-rupture criterion (1.8), those microvolumes of the matrix are damaged that have k_2 such that

$$t\ge\tau_k^2=\varphi_2(I_{\sigma}^2,k_2),\tag{1.18}$$

where the invariant I_{σ}^2 is defined by (1.13)–(1.15) or by (1.13), (1.16), and (1.17) depending on whether the macrostrains $\langle\varepsilon_{jk}\rangle$ or macrostresses $\langle\sigma_{jk}\rangle$ are given.

At low temperatures, the time to brittle failure τ_k^2 for the matrix of real materials is finite beginning only from some value of $I_{\sigma}^2>0$. In this case, the durability function $\varphi_2(I_{\sigma}^2,k_2)$ can be represented as

$$\varphi_2(I_{\sigma}^2,k_2)=\tau_{02}\left(\frac{k_2-I_{\sigma}^2}{I_{\sigma}^2-\gamma_2k_2}\right)^{n_{12}}\qquad(\gamma_2k_2\le I_{\sigma}^2\le k_2,\gamma_2<1),\tag{1.19}$$

where some typical time τ_{20}, exponent n_{12}, and coefficient γ_2 are determined from the fit of experimental durability curves for the matrix.

Substituting (1.19) into (1.18) yields

$$k_2\le I_{\sigma}^2\frac{1+\bar{t}_2^{-1/n_{12}}}{1+\gamma_2\bar{t}_2^{-1/n_{12}}}\qquad\left(\bar{t}_2=\frac{t}{\tau_{02}}\right).\tag{1.20}$$

Considering the definition of the distribution function $F_2(k_2)$, we conclude that the function $F_2[I_{\sigma}^2\psi_2(\bar{t}_2)]$, where

$$\psi_2(\bar{t}_2)=\frac{1+\bar{t}_2^{-1/n_{12}}}{1+\gamma_2\bar{t}_2^{-1/n_{12}}}\tag{1.21}$$

defines the relative content of the damaged microvolumes in the undamaged portion of the matrix at the time $\bar{t}_2$. Then, in view of (1.9), the porosity balance equation for the matrix subject to long-term damage can be represented as

$$p_2 = p_{02} + (1 - p_{02}) F_2 \left[\frac{I^2_{\langle\sigma\rangle}}{1 - p_2} \psi_2(\bar{t}_2) \right], \tag{1.22}$$

where the porosity p_2 is a function of the dimensionless time $\bar{t}_2$, and the invariant $I^2_{\langle\sigma\rangle}$ is defined by (1.13)–(1.15) or by (1.13), (1.16), and (1.17), respectively, depending on whether the macrostrains $\langle\varepsilon_{jk}\rangle$ or microstresses $\langle\sigma_{jk}\rangle$ are given.

At $\bar{t}_2 = 0$, the porosity balance equation (1.22) with (1.13)–(1.17), (1.21) defines the short-term (instantaneous) damage of the matrix. As time elapses, Eqs. (1.22), (1.13)–(1.17), (1.21) define its long-term damage, which consists of short-term damage and additional time-dependent damage.

2. Relations (1.2)–(1.5), (1.22), (1.13)–(1.17), and (1.21) can be used to set up an iterative algorithm to determine the volume fraction of microdamages in the components and the stress–strain state of the composite. To this end, we will use the secant method [1].

Representing Eq. (1.18) as

$$\varphi_2(p_2) = \left\{ p_2 - p_{02} + (1 - p_{02}) F_2 \left[\frac{I^2_{\langle\sigma\rangle}}{1 - p_2} \psi_2(\bar{t}_2) \right] \right\} = 0, \tag{2.1}$$

it is easy to verify that the root p_2 is within $[p_{02}, 1]$ since

$$\varphi_2(p_{02}) < 0, \qquad \varphi_2(1) > 0. \tag{2.2}$$

Therefore, the zero approximation $p_2^{(0)}$ of the root is given by

$$p_2^{(0)} = \frac{a_2^{(0)} \varphi_2(b_2^{(0)}) - b_2^{(0)} \varphi_2(a_2^{(0)})}{\varphi_2(b_2^{(0)}) - \varphi_2(a_2^{(0)})}, \tag{2.3}$$

where $a_2^{(0)} = p_{02}$ and $b_2^{(0)} = 1$. The subsequent approximations in the secant method are determined in the iterative process

$$p_2^{(m)} = \frac{a_2^{(m)} \varphi_2(b_2^{(m)}) - b_2^{(m)} \varphi_2(a_2^{(m)})}{\varphi_2(b_2^{(m)}) - \varphi_2(a_2^{(m)})}, \tag{2.4}$$

$$a_2^{(m)} = a_2^{(m-1)}, \quad b_2^{(m)} = p_2^{(m-1)} \quad \text{for} \quad \varphi_2(a_2^{(m-1)}) \varphi_2(p_2^{(m-1)}) \le 0,$$

$$a_2^{(m)} = p_2^{(m-1)}, \quad b_2^{(m)} = b_2^{(m-1)} \quad \text{for} \quad \varphi_2(a_2^{(m-1)}) \varphi_2(p_2^{(m-1)}) \ge 0$$

$$(m = 1, 2, \ldots),$$

which proceeds until

$$|\varphi_2(p_2^{(m)})| < \varepsilon. \tag{2.5}$$

Here ε is the accuracy with which the root is calculated.

We used the above calculations to plot macrodeformation curves for fibrous composites with microdamaged matrix for the Weibull distribution (1.10) and the function $\psi_2(\bar{t}_2)$ defined by (1.21), when either macrostresses $\langle\sigma_{jk}\rangle$ or macrostrains $\langle\varepsilon_{jk}\rangle$ are given. The composite has aluminoborosilicate-glass inclusions with the following characteristics [2] and volume fractions:

$$E_1 = 70 \text{ GPa}, \quad \nu_1 = 0.2, \qquad c_1 = 0, 0.25, 0.5, 0.75, 1.0 \tag{2.6}$$

and an epoxy matrix with the following characteristics of the undamaged portion [4]:

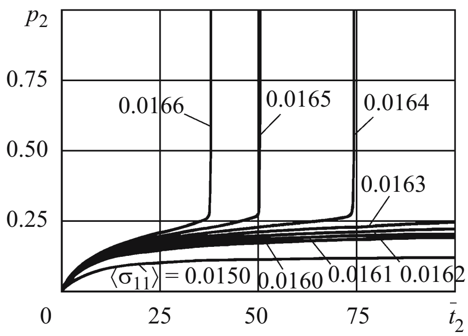

Fig. 1

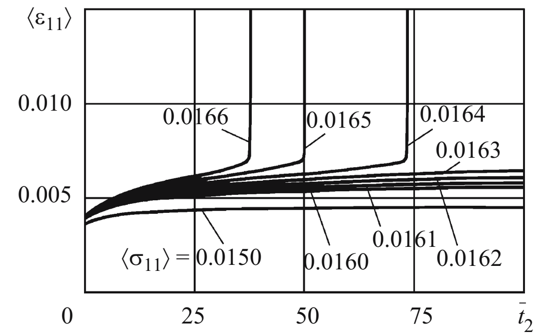

Fig. 2

$$E_2 = 3 \text{ GPa}, \quad \nu_2 = 0.35, \tag{2.7}$$

where E_1 and E_2 are Young's moduli; ν_1 and ν_2 are Poisson's ratios of the undamaged portion of the inclusions and matrix, respectively; and

$$p_{02} = 0, \quad k_{02} / \mu_2 = 0.01, \quad m_2 = 1000,$$

$$\alpha_2 = 2, \quad \sigma_{2p} = 0.011 \text{ GPa} \quad (\sigma_{2p} = \sqrt{3/2}\, k_{20}), \quad \gamma_2 = 0.5, \quad n_{12} = 1. \tag{2.8}$$

If

$$\langle \sigma_{33} \rangle \neq 0, \quad \langle \sigma_{11} \rangle = \langle \sigma_{22} \rangle = 0, \tag{2.9}$$

according to (1.1), the macrostress $\langle \sigma_{33} \rangle$ is related to the macrostrain $\langle \varepsilon_{33} \rangle$ by

$$\langle \sigma_{33} \rangle = \frac{1}{\lambda_{11}^* + \lambda_{12}^*} [(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2] \langle \varepsilon_{33} \rangle. \tag{2.10}$$

In the porosity balance equation defined by (1.22), (1.13), (1.16), and (1.17), we use

$$\langle \sigma_{11} \rangle = \langle \sigma_{22} \rangle = 0, \tag{2.11}$$

which is equivalent to (2.9).

Figures 1 and 2 show the porosity p_2 and macrostrain $\langle \sigma_{33} \rangle$, respectively, as functions of time $\bar{t}_2$ for $c_1 = 0.25$ and different values of $\langle \varepsilon_{33} \rangle$. Plots of p_2 versus $\bar{t}_2$ and of $\langle \varepsilon_{33} \rangle$ versus $\bar{t}_2$ for $c_1 = 0$ are given in [36]. As can be seen, the macrostrain and porosity approach horizontal asymptotes when $c_1 = 0$ and $\langle \sigma_{33} \rangle < 0.0115$ GPa and when $c_1 = 0.25$ and $\langle \sigma_{33} \rangle < 0.0163$ GPa, i.e., they show behavior similar to that of the experimental curves for polymers [5]. When the macrostress is higher than these values, the macrostrain and porosity reach, at some values of $\bar{t}_2$, the critical levels at which failure begins.

If

$$\langle \varepsilon_{11} \rangle \neq 0, \quad \langle \sigma_{22} \rangle = \langle \sigma_{33} \rangle = 0, \tag{2.12}$$

according to (1.1), the macrostress $\langle \sigma_{11} \rangle$ is related to the macrostrain $\langle \varepsilon_{11} \rangle$ by

$$\langle \sigma_{11} \rangle = \frac{\lambda_{11}^* - \lambda_{12}^*}{\lambda_{11}^* \lambda_{33}^* - (\lambda_{13}^*)^2} [(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2] \langle \varepsilon_{11} \rangle. \tag{2.13}$$

In the porosity balance equation defined by (1.22) and (1.13)–(1.15), we use

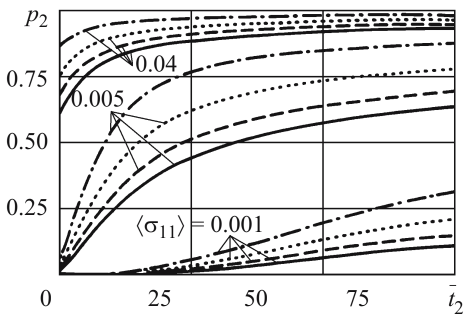

Fig. 3

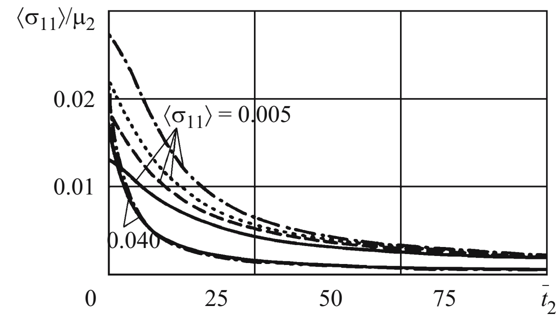

Fig. 4

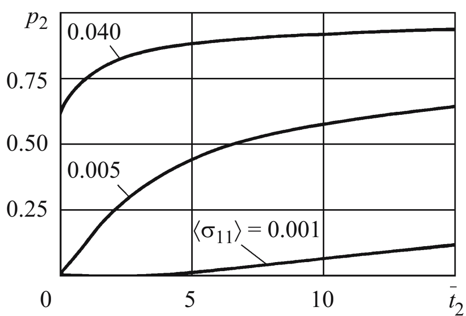

Fig. 5

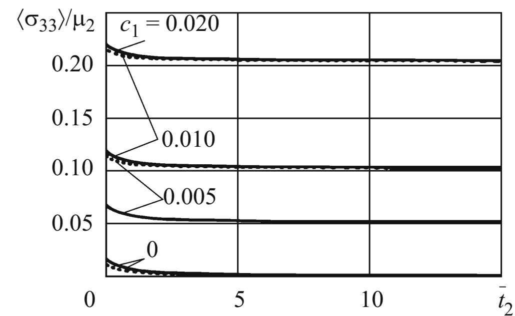

Fig. 6

$$\langle\varepsilon_{22}\rangle = \frac{(\lambda_{13}^*)^2 - \lambda_{12}^*\lambda_{33}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2}\langle\varepsilon_{11}\rangle, \quad \langle\varepsilon_{33}\rangle = \frac{(\lambda_{12}^* - \lambda_{11}^*)\lambda_{13}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2}\langle\varepsilon_{11}\rangle, \tag{2.14}$$

which is equivalent to (2.12).

Figure 3 shows the porosity p_2 as a function of time $\bar{t}_2$ for $\langle\varepsilon_{11}\rangle = 0.001, 0.005, 0.04$ and $c_1 = 0$ (solid line), $c_1 = 0.25$ (dashed line), $c_1 = 0.5$ (dotted line), and $c_1 = 0.75$ (dash-and-dot line). The notation is the same in Fig. 4. It can be seen that the porosity p_2 increases with the macrostrain $\langle\varepsilon_{11}\rangle$. Here damage builds up with time, whereas the experiments on polymers [5] indicate that damage does not change noticeably at constant strain. This disagreement may be attributed to either stress relaxation in polymers due to creep, which is neglected here, or the approximateness of the finite-time damage model.

Figure 4 shows the macrostress $\langle\sigma_{11}\rangle/\mu_2$ as a function of time $\bar{t}_2$ for $\langle\varepsilon_{11}\rangle = 0.005, 0.04$ and different values of c_1. As is seen, the curves are descending for all values of $c_1 < 1$. As the macrostrain $\langle\varepsilon_{11}\rangle$ increases, the influence of the volume fraction c_1 on $\langle\sigma_{11}\rangle/\mu_2$ as a function of $\bar{t}_2$ becomes weaker. As is seen, the curves are descending for all values of $c_1 < 1$. However, it should be noted that the decrease in the stresses with time is not a monotonic function of strains.

If

$$\langle\varepsilon_{33}\rangle \neq 0, \quad \langle\sigma_{11}\rangle = \langle\sigma_{22}\rangle = 0, \tag{2.15}$$

according to (1.1), the macrostress $\langle\sigma_{33}\rangle$ is related to the macrostrain $\langle\varepsilon_{33}\rangle$ by (2.10). In the porosity balance equation defined by (1.22), (1.13)–(1.15), we use

$$\langle\varepsilon_{11}\rangle = \langle\varepsilon_{22}\rangle = -\frac{\lambda_{13}^*}{\lambda_{11}^* + \lambda_{12}^*}\langle\varepsilon_{33}\rangle, \tag{2.16}$$

which is equivalent to (2.14).

Figure 5 shows the porosity p_2 as a function of time $\bar{t}_2$ for $\langle \varepsilon_{33} \rangle = 0.001$, 0.005, 0.04 and different values of c_1. Calculations reveal that the curves of p_2 versus $\bar{t}_2$ are similar for all values of c_1. It can be seen that the porosity p_2 increases with the macrostrain $\langle \varepsilon_{33} \rangle$ at arbitrary $\bar{t}_2$.

Figure 6 shows the macrostress $\langle \sigma_{33} \rangle / \mu_2$ as a function of time $\bar{t}_2$ for $\langle \varepsilon_{33} \rangle = 0.005$ (solid line), $\langle \varepsilon_{33} \rangle = 0.04$ (dashed line), and different values of c_1. As is seen, the curves are descending for all values of $c_1 < 1$.

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