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ACADEMICIAN NIKOLAI NIKOLAEVICH BOGOLYUBOV (FOR THE 100TH ANNIVERSARY OF HIS BIRTH)

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This paper is dedicated to the memory of N. N. Bogolyubov in recognition of his towering stature in nonlinear mechanics and theoretical physics, his remarkable many-sided genius, and the originality and depth of his contribution to the world's science. The paper briefly describes Bogolyubov's achievements in nonlinear mechanics, classical statistical physics, theory of superconductivity, quantum field theory, and strong interaction theory

Keywords: nonlinear mechanics, statistical physics, superconductivity, quantum theory, strong interaction

Nikolai Nikolaevich Bogolyubov was born on August 21, 1909, in Nizhny Novgorod. He showed extraordinary mathematical ability at a very early age; when he was 13, he took part in a workshop held by Academician N. M. Krylov, an outstanding scientist and teacher, and wrote his first scientific paper "On the behavior of solutions of linear differential equations at infinity" in 1924. From 1925 to 1951, N. N. Bogolyubov was a researcher at the Mathematical Physics Department of the Institute of Structural Mechanics, Academy of Sciences of the Ukrainian Soviet Republic (now the S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine). When he worked with Academician N. M. Krylov, he conducted fundamental studies of differential equations of mathematical physics, approximate solutions of differential equations, the theory of dynamic systems, and the direct variational method.

Bogolyubov's papers of those years represented a new area in the theory of uniform, almost-periodic functions and, thus, showed a close relationship between this theory and the general theory of the behavior of linear combinations of a random bounded function. In 1930, Bogolyubov was awarded the Merlani prize by the Bologna Academy of Sciences for one of his first papers in this area. The Presidium of the Academy of Sciences of Ukraine awarded him the Honoris Causa degree.

The technological progress introduced new problems in the area of radio and electrical engineering, mechanics of complex vibrating systems, and aircraft construction. In 1932, Bogolyubov and his teacher Krylov started developing a totally new area of mathematical physics—the theory of nonlinear vibrations, which they called nonlinear mechanics.

The first research efforts of Krylov and Bogolyubov in the new area are concerned with the theory of vibrations in electrical machines and mechanical systems. The research in the area of nonlinear mechanics was focused on two aspects: development of methods for the asymptotic integration of nonlinear equations of motion for vibrating systems and their justification based on measure theory.

Krylov and Bogolyubov overcame great fundamental difficulties to apply perturbation theory to general nonconservative systems and developed new asymptotic methods of nonlinear mechanics. Their asymptotic methods are based on a rigorous mathematical justification and permit problem solving not only in the first approximation (as is the case with the Van der Pol method), but also in higher approximations. These methods may be used to study both periodic and quasiperiodic vibrations. In addition, they meet practical requirements for design models to be simple and easy to interpret.

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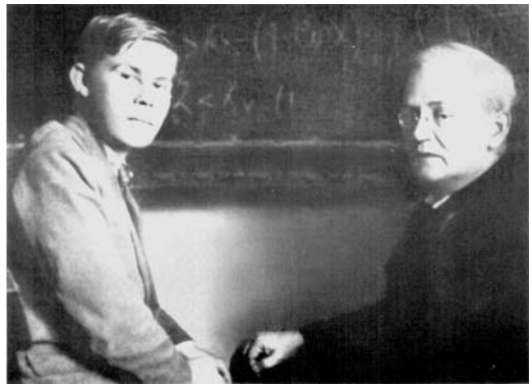


N. N. Bogolyubov and N. M. Krylov

The authors immediately used the asymptotic methods to solve many important problems. They derived second-order formulas to determine the frequency of stationary oscillations in electronic generators that accounted for the effect of overtones on frequency stability, and studied the frequency scaling resonance and internal resonance in systems with many degrees of freedom. Special attention was focused on resonance theory in connection with the use of nonlinear elements to prevent resonance in mechanical engineering. Asymptotic methods were used to solve problems of the directional stability of airplanes and vibrations and stability of rods and various engineering structures.

The general measure theory in nonlinear mechanics elaborated by Bogolyubov and Krylov promoted further development of the theory of dynamic systems and explanation such properties of stationary motion as recurrence, i.e., strong Poisson stability.

They used the Lyapunov–Poincaré theory and Poincaré–Denjoy theory of trajectories on a torus to examine the exact stationary solution near an approximate solution when the parameter is sufficiently small and established theorems proving the existence and stability of quasiperiodic solutions.

Their research findings have become classical in the modern theory of dynamic systems, and fundamental results are associated with efficient methods for the asymptotic integration of a wide class of nonlinear equations. Bogolyubov here develops new mathematical tools to study the behavior of general nonconservative systems with a small parameter to clearly comprehend the stationary solution in the neighborhood of an approximate solution.

Of special significance is Krylov and Bologyubov's research of the resonance of nonlinear oscillations and the accompanying synchronization, demultiplication, and reduction of amplitude of resonant oscillations in the presence of nonlinear elements in the oscillating system.

One of their most significant results was the prediction of a new phenomenon, which they called abnormal excitation and confirmed further by experiment. This phenomenon is loss of stability of a conventionally stable equilibrium point under small sinusoidal disturbing forces.

Bogolyubov's averaging method, when applied to standard equations, solved two problems: (i) establishing conditions under which the standard difference between the solutions of the exact system and the averaged system for a sufficiently small parameter remains small on a finite interval of random length and (ii) establishing correspondence between different properties of the solutions of the exact equations and averaged equations on a finite interval of random length.

As early as 1945, Bogolyubov developed a fundamental theory of the existence and major properties of a one-parameter integral manifold of a standard-form system of nonlinear differential equations. He analyzed periodic and quasiperiodic solutions using a one-dimensional manifold and, thus, laid the foundation for a new method of nonlinear mechanics—integral manifold method.

Bogolyubov obtained major results for differential equations with a rapidly rotating phase. The method of solving averaged equations was based on the decomposition of motion.

In 1963, Bogolyubov advanced an idea that initiated the use of accelerated convergence methods in nonlinear mechanics. As early as 1934, he and Krylov developed a combination of special transformations of variables, using which in the integral manifold method solved the problem of existence of multifrequency conditionally periodic solutions in strict yet not asymptotic sense.

Yu. A. Mitropol’skii and N. N. Bogolyubov

The development of these ideas is currently known as the accelerated convergence method in nonlinear mechanics.

Bogolyubov’s ideas and methods expressed at the Kanev lectures in 1963 were later developed and applied to solve numerous interesting and important problems of nonlinear mechanics (reduction of a nonlinear system to a linear system with constant coefficients, reduction of linear differential equations with quasiperiodic coefficients, behavior of integral curves near analytic and smooth manifolds).

Bogolyubov’s ideas and fundamental results in the area of nonlinear mechanics underlie many modern research methods in general mechanics, continuum mechanics, celestial mechanics, solid mechanics and gyroscopic systems, theory of stability of motion, theory of control, adjustment, and stability, space flight mechanics, vibrations of mechanical systems, mathematical ecology, and other fields of science and technology.

The term “nonlinear mechanics” has come into common use mainly among mechanicians, engineers, and electricians who study and design systems with small perturbations and among mathematicians dealing with differential equations with small perturbations. The term has recently been used in other areas—nonlinear analysis and nonlinear dynamics.

The methods developed by Bogolyubov to study dynamic systems opened up new approaches to problems of classical statistical physics. In 1945, he studied the effect of a random force on a harmonic oscillator and developed and first used the idea of time hierarchy in the statistical theory of irreversible processes.

Bogolyubov’s method of successive equations for single- and multi-particle distribution functions is most effective in modern statistical mechanics of equilibrium and nonequilibrium processes.

Bogolyubov neglected the hypothesis of Boltzmann molecular chaos and proposed a new idea of using boundary conditions to decrease correlation and developed a method to account for higher-order terms of density expansion.

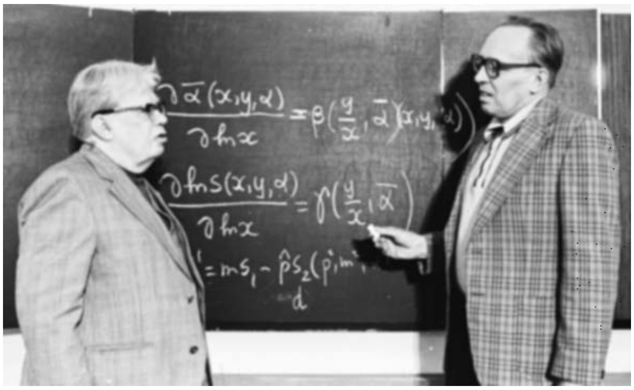
The name of Nikolai Bogolyubov is inseparably connected with the new theory of imperfect quantum macrosystems. The theory is based on his scientific treatment of such important physical phenomena as superfluidity (1946) and superconductivity (1957).

Bogolyubov developed a mathematical tool based on the Bose-amplitude transformation, which is now widely known as the Bogolyubov (u, v)-transformation. This transformation is widely used in theoretical physics, in current studies on the quantum theory of gravity.

The notion of superconductivity was further developed as the superfluidity of Fermi gas, which led Bogolyubov to discover a new fundamental phenomenon—superfluidity of nuclear substance. The notion of the superfluidity of nuclear substance underlies modern nuclear theory.

Bogolyubov studied the stabilization of condensate in imperfect systems and developed the quasi-mean-value method (1961), which is a multi-purpose tool to analyze systems whose principal state is unstable under small perturbations.

In the 1950s, Bogolyubov developed quantum field theory under a new causality condition. This condition is now known as the microcausality condition. Bogolyubov’s axiomatic theory of perturbations under the field theory based on a scattering matrix determines its development so far. Bogolyubov proved a theorem stating that the scattering matrix is successfully determined in any order of the perturbation theory based on the requirements of relativistic invariance, spectrality, unitarity, and causality within quasilocal operators and thus revealed the source of ultraviolet divergences of the scattering matrix and gave recommendations on their successive exclusion, which is called the R-operation (1955). Bogolyubov was one of



N. N. Bogolyubov and D. V. Shirkov

N. N. Bogolyubov at the Joint Institute for Nuclear Research (Dubna)

the first to start developing the axiomatic method of quantum field theory (without the assumption of weak interactions and the perturbation theory). Bogolyubov just partially changed the system of axioms in perturbation theory and added the requirement of stability of vacuum of one particle state and restated the causality condition (1956). To prove this, a special mathematical tool was to be developed for analytical extension of generalized functions of many variables.

Of pure mathematical results, the edge-of-the-wedge theorem is noteworthy, which was proved by Bogolyubov in 1956 and named after him.

Bogolyubov's studies on quantum field theory laid the foundation for a new scientific area—strong interaction theory.

The scope of Bogolyubov's work is not limited to scientific advances. Worthy of mention are fundamental studies on plasma theory and kinetic equations, which are widely used.

Bogolyubov always made extensive efforts to train young scientists. When he headed departments at the Kiev and then Moscow Universities, he regularly delivered lectures, which were of great interest to the audience. The lectures and reports of Academician Bogolyubov were attended by experts in Great Britain, Belgium, Hungary, Italy, India, Poland, USA, Finland, Germany, Yugoslavia, Japan, and many other countries. His speech was always a great event in the scientific life.

Bogolyubov deserves the credit for founding several scientific schools. He founded the school of mathematical physics and nonlinear mechanics in Kiev when working in Ukraine and then founded the school of theoretical physics in Moscow and Dubna. Many well-known scientists call Bogolyubov their teacher with pride and respect.

Academician Bogolyubov was an outstanding institutor of science in the former Soviet Union. As a member of the Presidium and a Secretary Academician of the Mathematical Departments at the Academy of Sciences of the USSR, Bogolyubov promoted mathematical and physical research in Ukraine. He headed the large international scientific center—Joint Institute for Nuclear Research in Dubna and was the founder and first director of the Institute for Theoretical Physics of the Academy of Sciences of Ukraine. He was the founder and chief editor of the Theoretical and Mathematical Physics Journal.

Bogolyubov was occupied with scientific, administrative, and educational work and was also engaged in extensive public work. He was a member of the Supreme Soviet and the peace movement of scientists.

The government of the former Soviet Union highly appreciated Bogolyubov's scientific and public work and awarded him five Lenin Orders and other prizes and medals.

Bogolyubov was an honorary member of many foreign academies and scientific societies, an honorary doctor of many foreign universities, and a winner of many special prizes and medals.

As a scientist, Nikolai Bogolyubov is tripersonal—he is an outstanding mathematician, physicist, and mechanician He could manage subtle problems of mathematical simulation, he was the first to open up new horizons in modern mathematical physics and, undoubtedly, was well aware of both theoretical and practical demands of modern mechanics and engineering.

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