International Applied Mechanics, Vol. 45, No. 2, 2009

RESONANT VIBRATION AND HEATING OF RING PLATES WITH PIEZOACTUATORS UNDER ELECTROMECHANICAL LOADING AND SHEAR DEFORMATION

I. F. Kirichok

The bending vibration and dissipative heating of a viscoelastic isotropic ring plate with piezoceramic actuators under electromechanical loading and shear deformation are studied by solving a coupled problem. The temperature dependence of the complex characteristics of the passive and piezoactive materials is taken into account. The nonlinear problem of thermoviscoelasticity is solved by time stepping with discrete orthogonalization used at each iteration to integrate the equations of elasticity and using an explicit finite-difference scheme to solve the heat-conduction equation with a nonlinear heat source. The effect of shear deformation, fixation conditions for the plate, the geometry of the piezoactuators, and the dissipative-heating temperature on the active damping of the forced vibration of a circular plate subject to uniform transverse monoharmonic compression is studied

Keywords: forced vibration, dissipative heating, piezoactuator, temperature, active damping

Introduction. Inelastic circular and ring plates are widely used in various fields of modern engineering. They are often subject to intensive nonstationary (including harmonic) loading and tranresonant vibrations accompanied by large displacements and heating because of internal dissipation. Along with passive damping of such vibration with the help of highly elastic coatings, active damping with embedded piezoactive elements has recently been used [17, 20]. They either play the role of sensors providing information on the vibratory state or actuators to which an electric potential with certain amplitude and phase and a frequency equal to that of the mechanical load is applied to balance it. The effectiveness of damping is influenced by such factors as the geometrical and electroelastic characteristics of the passive and piezoactive components, electric and mechanical boundary conditions [10, 11], and heating due to hysteresis losses or heat transfer to the environment [4, 5, 7, 8, 14, 15, 23, etc.]. Because of the temperature dependence of the electroelastic characteristics, these factors may considerably affect the effectiveness of damping a thin-walled structural member with piezoactuators and the performance of the system as a whole. Simulating the mechanical behavior of thin-walled inelastic members to analyze the vibration spectra and to study the influence of the factors on active damping was mainly based on the Kirchhoff–Love hypotheses supplemented with assumptions on the electric and temperature fields [6, 9, 13, 15, 19, 20–23, etc.]. However, if the vibrating element is thick and the material properties of the passive and piezoactive layers are very different, it is necessary to use refined mechanical hypotheses and relevant assumptions on the electric and thermal variables [5, 9, 12, 14, 18, etc.].

Here we solve, in a refined formulation, the problem of the forced monoharmonic vibration and dissipative heating of a viscoelastic ring plate with piezoelectric actuators considering that the electroelastic characteristics depend on temperature.

1. Problem Formulation. Consider a ring plate of thickness h described in an orthogonal coordinate system $0, r, \theta, z$ such that $z = 0$ is the midsurface and $r = r_0$ and $r = R$ are the inner and outer radii. The plate is orthotropic and viscoelastic. The outside planes $z = \pm h/2$ of radii $r = r_1$ and $r = r_2$ are divided into three ring zones in each of which thin piezoactive pads (actuators) of equal thickness δ can be attached to the surface $z = \pm h/2$. The actuators are polarized throughout the thickness in opposite (to each other) directions. The polarization of the pads is characterized by a piezoelectric modulus d_{31} in the direction

---

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterov St., Kyiv, Ukraine 03057, e-mail: karn@inmech.kiev.ua. Translated from Prikladnaya Mekhanika, Vol. 45, No. 2, pp. 124–132, February 2009. Original article submitted December 19, 2007.

1063-7095/09/4502-0215

$z > 0$ and by $-d_{31}$ in the opposite direction. The outside surface of the actuators and the surface between them and the passive layer are covered with infinitely thin solid electrodes. The plate is subjected to harmonic (in time t) axisymmetric pressure $P(r)\cos\omega t$ with nearly resonant circular frequency ω. Moreover, an electric potential difference of the same frequency is applied to the external electrodes of the actuators:

$$\psi(h/2+\delta)-\psi(-h/2-\delta)=2\,\mathrm{Re}(Ve^{i\omega t}),\qquad V=V'+iV''.$$

The internal electrodes are maintained at zero potential. On its surfaces, the plate transfer heat by convection to the environment of temperature T_c. The edges of the plate are either both clamped or the internal edge is clamped and the external edge is hinged. Due to the harmonic deformation of the body, the behavior of the passive and piezoactive materials is described by temperature-dependent complex moduli [7].

Simulating the forced vibration of such a plate is based on Timoshenko's straight-line hypothesis for the mechanical variables [2]. As for the electric-field variables, it is assumed that the tangential components D_r and D_θ of electric-flux density can be neglected compared with the normal component D_z. From the simplified equation of electrostatics, it follows that D_z in each piezoactive layer is independent of the thickness coordinate. The tangential components E_r and E_θ of electric-field strength are found from the constitutive equations for the piezoceramics polarized along the z-axis when $D_r = 0$ and $D_\theta = 0$. Considering that the plate is thin and that it is in perfect thermal contact with the thin piezopads, we assume that the dissipative-heating temperature is constant throughout the thickness.

2. Problem Solving. Due to structural symmetry, polarization of piezopads, and loading conditions, the plate undergoes purely bending axisymmetric vibration. After some transformations, we reduce the problem of flexural vibration of the plate to a system of ordinary differential equations in normal form for complex amplitudes Q_r, ψ_r, w, and M_r:

$$\frac{dQ_r}{dr}=-\frac{1}{r}Q_r-\hat{\rho}\omega^2 w-P(r),\qquad \frac{d\psi_r}{dr}=J_D M_r+\frac{\nu_D}{r}\psi_r-J_D M_E,\qquad \frac{dw}{dr}=-\psi_r+\frac{1}{k_s C_{55}}Q_r,$$

$$\frac{dM_r}{dr}=Q_r+\left[\frac{D_{11}}{r}(1-\nu_D^2)-\hat{\rho}_*\omega^2\right]\psi_r-\frac{1+\nu_D}{r}M_r+\frac{1+\nu_D}{r}M_E \tag{1}$$

with the boundary conditions

$$w=0,\quad \psi_r=0\quad (r=r_0,\ r=R) \tag{2}$$

if both edges are clamped and

$$w=0,\quad \psi_r=0\quad (r=r_0);\qquad w=0,\quad M_r=0\quad (r=R) \tag{3}$$

if the internal edge is clamped and the external edge is hinged.

With the assumptions made above and the electric boundary conditions, the electrostatic equations for electric-flux density and electric-field strength become

$$D_z=-b_{33}\frac{V}{\delta}+\frac{1}{2}(h+\delta)b_{31}\kappa\,(\kappa=\kappa_r+\kappa_\theta), \tag{4}$$

$$E_z=-\frac{V}{\delta}+\frac{b_{31}}{b_{33}}\left(\frac{h+\delta}{2}\pm z\right)\kappa\,(-h/2-\delta\le z\le -h/2,\ h/2\le z\le h/2+\delta).$$

To calculate the dissipative-heating temperature, problem (1)–(4) should be supplemented with the energy equation. Since the temperature T is postulated to be independent of the thickness coordinate, the heat-conduction equation averaged over the thickness of the plate and period of vibration has the form

$$\frac{1}{a}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}-\frac{2\alpha_s}{\lambda H}(T-T_c)+\frac{1}{\lambda H}\overline{W} \tag{5}$$

where

$$\overline{W}=\frac{\omega}{2}[D''_{11}(\kappa'^2_r+\kappa''^2_r+\kappa'^2_\theta+\kappa''^2_\theta)+2D''_{12}(\kappa'_r\kappa'_\theta+\kappa''_r\kappa''_\theta)$$

$$+k_sC''_{55}(\varepsilon'^2_{rz}+\varepsilon''^2_{rz})+2(h+\delta)b''_{31}(\kappa''V'+\kappa'V'')+2b''_{33}(V'^2+V''^2)/\delta] \qquad (6)$$

is a nonlinear dissipation function.

The thermal boundary and initial conditions are

$$\frac{\partial T}{\partial r}=\pm\frac{\alpha_{1,2}}{\lambda}(T-T_c) \quad (r=r_0, r=R), \qquad T=T_0 \quad (t=0). \qquad (7)$$

In (1)–(7), the following notation is used:

$$D_{11}=\frac{1}{12}(C_{11}h^3+2C^E_{11}\delta^3_*+2\gamma_{31}\delta^3), \qquad D_{12}=\frac{1}{12}(C_{12}h^3+2C^E_{12}\delta^3_*+2\gamma_{31}\delta^3),$$

$$C_{55}=G_{13}h+C^E_{55}\delta, \quad M_E=b_{31}(h+\delta)V, \quad \gamma_{31}=b^2_{31}/b_{33}, \quad \delta^3_*=4\delta^3+6\delta^2h+3\delta h^2,$$

$$C_{11}=\frac{E}{1-\nu^2}, \quad C^E_{11}=\frac{1}{s^E_{11}(1-\nu^2_E)}, \quad \nu_E=-\frac{s^E_{12}}{s^E_{11}}, \quad C^E_{55}=\frac{1}{s^E_{55}(1-k^2_{15})},$$

$$C_{12}=\nu C_{11}, \quad C^E_{12}=\nu_EC^E_{11}, \quad b_{33}=\varepsilon^T_{33}(1-k^2_p), \quad J_D=1/D_{11}, \quad \nu_D=-J_DD_{12},$$

$$k^2_p=\frac{2d^2_{31}}{s^E_{11}\varepsilon^T_{33}(1-\nu_E)}, \quad k^2_{15}=\frac{d^2_{15}}{s^E_{55}\varepsilon^T_{11}}, \quad b_{31}=\frac{d_{31}}{s^E_{11}(1+\nu_E)}, \quad G_{13}=G=\frac{E}{2(1+\nu)},$$

$$\hat{\rho}=\rho h+2\rho_*\delta, \quad \hat{\rho}_*=\frac{1}{12}(\rho h^3+2\rho_*\delta^3_*), \quad H=h+\delta,$$

$$\varepsilon_{rz}=\psi_r-\vartheta_r, \quad \kappa_r=\frac{d\psi_r}{dr}, \quad \kappa_\theta=\frac{\psi_r}{r}, \quad \vartheta_r=-\frac{dw}{dr},$$

$$s^E_{mn}=s'_{mn}(1-i\delta^s_{mn}), \quad d_{mn}=d'_{mn}(1-i\delta^d_{mn}), \quad \varepsilon^T_{mn}=\varepsilon'_{mn}(1-i\delta^\varepsilon_{mn}) \qquad (8)$$

are complex compliances, piezoelectric moduli, and dielectric permittivities of the piezomaterial, their components being functions of temperature; $G=G'+iG''$ is the temperature-dependent complex shear modulus of the passive material; $i=\sqrt{-1}$, $\nu=$ const is Poisson's ratio assumed real and independent of temperature; ρ and ρ_* are the densities of the passive material and piezoceramics; w, ψ_r, M_r, and Q_r are the complex deflection amplitudes, angle of rotation, bending moment, and shearing force, respectively; λ and a are the coefficients of heat conductivity and thermal diffusivity; $\alpha_{1,2,s}$ are the heat-transfer coefficients on the lateral and face surfaces of the plate; and k_s is the shear coefficient [2].

If the electroelastic moduli are considered temperature dependent, problem (1)–(8) is coupled and nonlinear. To solve it, we will use step-by-step integration over time [7]. At the first step, the linear problem of electroelasticity (1)–(3) is solved with temperature-independent material properties. The mechanical and electric-field variables found are then used to calculate the dissipation function (6) and to solve the heat-conduction problem (5), (7). At the next time step, the electroelastic properties at the temperature found at the first step are calculated, and the process is repeated. At each time step, the complex-valued system of equations (1)–(3) is integrated using the discrete-orthogonalization method and a standard software program for solving systems of ordinary differential equations in normal form [3]. The heat-conduction problem (5), (7) is solved with the explicit finite-difference method.

We will assume that the mechanical load is independent of the spatial coordinate. The potential that should be applied to the actuator to balance the external harmonic pressure is found from the linear relation

$$V=k_AP. \qquad (9)$$

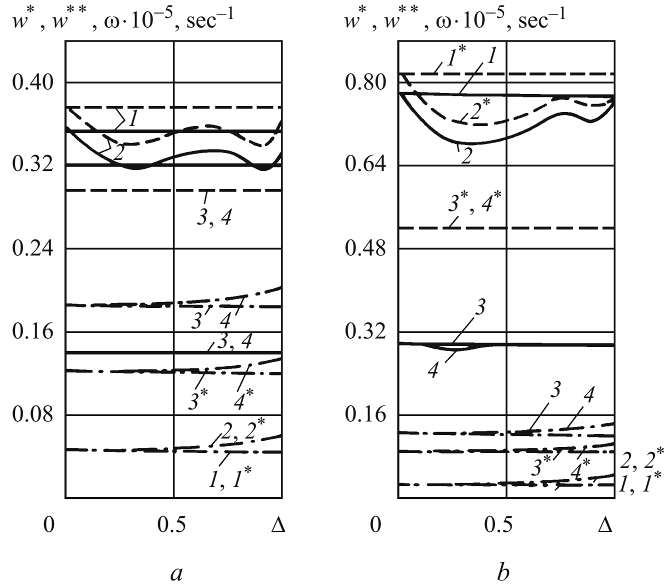


Fig. 1

According to [14], the complex coefficient k_A in (9) is calculated from the integral relations following from a variational problem formulation:

$$k_A = -R^2 \int_{r_0}^{R} w r dr \Bigg/ \left[(h+\delta) \int_{r_1}^{r_2} b_{31} (\kappa_r + \kappa_\theta) r dr \right]. \tag{10}$$

Moreover, if the electroelastic properties of the plate are independent of temperature, k_A can be found as the ratio (calculated at the resonant frequency) of the maximum deflection $w_{p\max}$ caused by a unit mechanical load $P = 1\,\text{Pa}$ $(V = 0)$ to the maximum deflection $w_{E\max}$ caused by a unit electric potential $V' = 1\,\text{V}$ $(V'' = 0, P = 0)$ applied to the actuator:

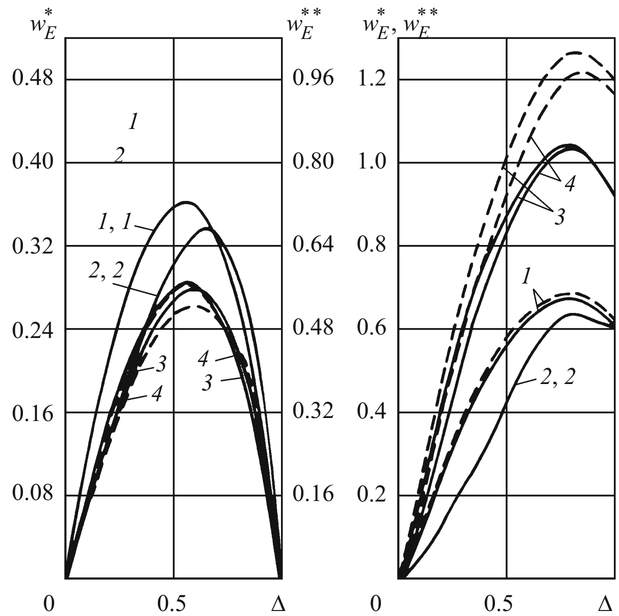
$$k_A = -\frac{w_{p\max}}{w_{E\max}}. \tag{11}$$

The minus sign in (10) and (11) indicates that the electric potential is applied to the actuator in antiphase to the mechanical load to suppress vibration.

3. Numerical Results. The results below represent a plate with a middle-ring actuator ($r_1 \le r \le r_2$) as the most effective to damp the forced vibration of ring plates [22]. Equations (1)–(3) and (5)–(7) were integrated over the dimensionless spatial ($x(r) = (r - R)/L, L = R - r_0$) and time ($\tau = at/L^2$) coordinates using the dimensionless heat-transfer parameters $\gamma_s = \alpha_s L/\lambda$, $\gamma_{1,2} L/\lambda$. The plate is made of passive polymer [16] and the actuator of TsTStBS-2 piezoceramics [1]. The temperature approximations of the electroelastic moduli and the thermal coefficients of these materials are presented in [21, 23]. The plate has radii $r_0 = 0.05$ m, $R = 0.2$ m, its internal boundary is heat-insulated ($\gamma_1 = 0$), and heat-transfer conditions with coefficients $\gamma_s = \gamma_2 = 0.638$ are prescribed on the other surfaces. The shear coefficient $k_s = 5/6$ [2].

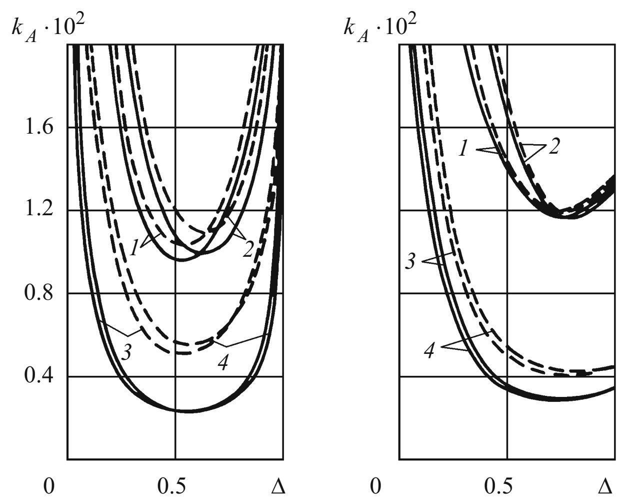
Figures 1–3 present calculated vibration characteristics of the ring plate with temperature-independent material properties ($T = T_R = 20$ °C) and either both edges clamped (Figs. 1*a*–3*a*) or internal edge clamped and external edge hinged (Figs. 1*b*–3*b*) depending on the dimensionless width $\Delta = x_2 - x_1, x_i = (r_i - R)/L$ of the actuator. The parameter Δ was selected so that the mid-radius of the actuator was equal to the radial coordinate $r = (r_0 + R)/2$ of the plate where the deflection amplitude is maximum under electrically excited vibration. Curves *1*, *2*, *3*, and *4* correspond to the following thicknesses of the passive layer and actuator:

(i) $h = 0.01$ m, $\delta = 0$;



a *b*

Fig. 2



a *b*

Fig. 3

(ii) $h = 0.01$ m, $\delta = 0.5 \cdot 10^{-4}$ m;

(iii) $h = 0.04$ m, $\delta = 0$ m;

(iv) $h = 0.04$ m, $\delta = 0.5 \cdot 10^{-4}$ m.

Figure 1 shows (dash-and-dot curves) resonant frequencies of the first flexural mode calculated using Kirchhoff–Love theory (curves *1*–*4*) and allowing for the shear strain (curves 1^*–4^*). The maximum amplitudes $w_p^* = \frac{1}{2h} |w(0.5)| \cdot 10^5$ (curves *1* and *2*) and $w_p^{**} = \frac{1}{2h} |w(0.5)| \cdot 10^7$ (curves *3* and *4*) calculated at these frequencies and caused by a unit mechanical load ($P = 1$Pa) are shown by solid lines in the case of Kirchhoff–Love theory and by dashed lines in the case of accounting for the shear strain. It is seen that the presence of the shear strain increases deflections and natural frequencies of the plate compared with classical

TABLE 1

Δ	$\delta = 0$ m, $h = 0.01$ m		$\delta = 0.5 \cdot 10^{-4}$ m, $h = 0.01$ m		$\delta = 0$ m, $h = 0.04$ m		$\delta = 0.5 \cdot 10^{-4}$ m, $h = 0.04$ m	
	$k_{A1} \cdot 10$	$k_{A2} \cdot 10$	$k_{A1} \cdot 10$	$k_{A2} \cdot 10$	$k_{A1} \cdot 10^2$	$k_{A2} \cdot 10^2$	$k_{A1} \cdot 10^2$	$k_{A2} \cdot 10^2$
0.5	$\frac{0.142}{0.147}$	$\frac{0.138}{0.142}$	$\frac{0.168}{0.176}$	$\frac{0.165}{0.172}$	$\frac{0.354}{0.519}$	$\frac{0.344}{0.507}$	$\frac{0.370}{0.563}$	$\frac{0.360}{0.551}$
0.6	$\frac{0.124}{0.128}$	$\frac{0.125}{0.128}$	$\frac{0.135}{0.141}$	$\frac{0.136}{0.143}$	$\frac{0.310}{0.453}$	$\frac{0.311}{0.454}$	$\frac{0.316}{0.481}$	$\frac{0.317}{0.483}$
0.7	$\frac{0.121}{0.125}$	$\frac{0.118}{0.122}$	$\frac{0.124}{0.130}$	$\frac{0.121}{0.129}$	$\frac{0.302}{0.435}$	$\frac{0.295}{0.425}$	$\frac{0.304}{0.454}$	$\frac{0.296}{0.446}$
0.8	$\frac{0.116}{0.119}$	$\frac{0.116}{0.119}$	$\frac{0.115}{0.120}$	$\frac{0.115}{0.120}$	$\frac{0.289}{0.410}$	$\frac{0.289}{0.410}$	$\frac{0.288}{0.427}$	$\frac{0.289}{0.428}$
0.9	$\frac{0.122}{0.126}$	$\frac{0.120}{0.123}$	$\frac{0.120}{0.125}$	$\frac{0.118}{0.123}$	$\frac{0.305}{0.425}$	$\frac{0.300}{0.419}$	$\frac{0.303}{0.437}$	$\frac{0.298}{0.429}$
1.0	$\frac{0.130}{0.134}$	$\frac{0.129}{0.136}$	$\frac{0.130}{0.135}$	$\frac{0.128}{0.134}$	$\frac{0.326}{0.441}$	$\frac{0.322}{0.435}$	$\frac{0.325}{0.454}$	$\frac{0.321}{0.448}$

theory, a fact known in the theory of elastic plates. The thicker the plate, the greater the increase. At certain thicknesses of the passive layer and the actuator, the maximum deflection may become an ambiguous function of the actuator width because of the spatially nonuniform stiffness of the plate.

Figures 2 and 3 show the maximum deflection amplitudes $w_E^* = \frac{1}{2h}|w(0.5)| \cdot 10^3$ (curves *1* and *2*) and $w_E^{**} = \frac{1}{2h}|w(0.5)| \cdot 10^5$ (curves *3* and *4*) caused by a unit electric potential ($V' = \pm 1$ V, $V'' = 0$) applied to the actuator and the absolute values of the control coefficient k_A, respectively. The solid lines represent Kirchhoff–Love theory, and the dashed line the presence of shear strain.

Table 1 summarizes the absolute values of the control coefficients k_{A1} and k_{A2} calculated from the resonant frequencies by formulas (10) and (11), respectively, for a plate with internal edge clamped and external edge hinged. The values in the numerator and denominator correspond to the cases of using Kirchhoff–Love theory and allowing for shear strain, respectively.

Curves *1–4* demonstrate that the maximum deflection (Fig. 2) and control coefficient (Fig. 3) are ambiguous functions of Δ. It can be seen that the most effective (optimal) are ring actuators that excite maximum deflections. The associated control coefficient is minimum (Fig. 3), which leads to the minimum electric potential needed to balance the mechanical load. The control coefficient for the optimal actuator is weakly dependent on the actuator thickness, especially if the passive layer is thick (curves *2* and *4*). The effect of shear strain on the distribution of electrically excited deflections is very weak for thin plates and stronger for thicker plates. At the same time, the shear strain increases the control coefficient (Fig. 3). The thicker the passive layer, the greater the increase (curves *3* and *4*). Comparing the values of k_{A1} and k_{A2} in Table 1 shows that they are in good agreement over a wide range of actuator widths Δ. This suggests that the control coefficient can satisfactorily be calculated from the local vibration characteristics (11) even when the shear strain is allowed for.

Comparing Figs. 2*a*, 3*a* and Figs. 2*b*, 3*b* reveals that the width Δ of the optimal actuator is greater when the external edge is hinged (Figs. 2*a* and 3*a*). Figures 4 and 5 show the amplitude–frequency characteristics (AFCs) $w_{\max} = \frac{1}{2h}|w(x = 0.5)|$ and

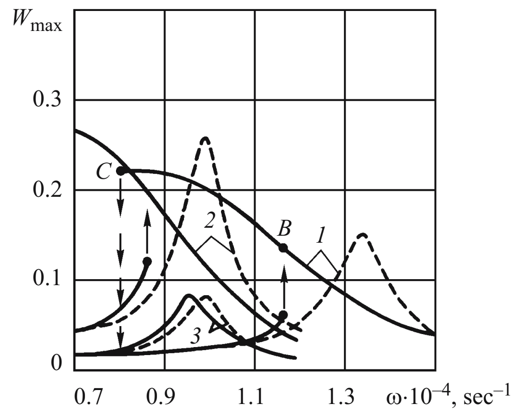


Fig. 4

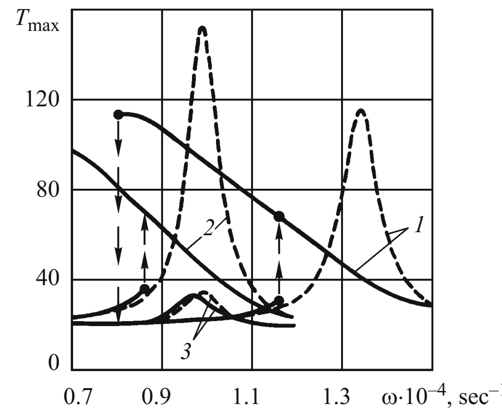


Fig. 5

temperature–frequency characteristics (TFCs) $T_{max} = T(x = 0.5)$ for a plate of thickness $h = 0.04$ m clamped at the internal edge and hinged at the external edge, having an actuator of thickness $\delta = 0.5 \cdot 10^{-4}$ m and optimal width $\Delta = 0.8\,(x_1 = 0.1, x_2 = 0.9)$, and subjected to surface pressure $P = 0.5 \cdot 10^4$ Pa. Curves *1* represent the classical theory of plates and curves *2* shear strain accounted for. The dashed lines represent the case of temperature-independent electroelastic properties, while the solid lines temperature-dependent properties. When the material properties are independent of temperature, the amplitudes of the electric potential balancing the load *P* are equal to $|V_A| = -14.4$ V if the problem is solved using the Kirchhoff–Love hypotheses at the resonant frequency $\omega_p = 0.134 \cdot 10^5$ Hz and $|V_A| = -21.06$ V if the shear strain is taken into account at frequency $\omega_p = 0.990 \cdot 10^4$ Hz. If the material properties are dependent on temperature, then $|V_A| = -14.16$ V and $|V_A| = -21.06$ V, respectively, at the respective frequencies. Curves *3* take into account the shear strain and represent the joint effect of the mechanical load $P = 0.5 \cdot 10^4$ Pa and the electric potential $|V_A| = -14.4$ V applied in antiphase to the actuator and calculated by the Kirchhoff–Love theory of plates.

The calculations show that the AFCs and TFCs due to the electric load with $|V_A|$ coincide with those (curves *1* and *2*) due to the mechanical load $P = 0.5 \cdot 10^4$ Pa. When both loads act simultaneously on the plate, its flexural vibration is suppressed at the first resonant frequencies (AFCs and TFCs degenerate into a straight line). The AFCs and TFCs plotted with allowance for thermomechanical coupling are soft nonlinear ones [7, 8] with ambiguous lower and upper sections. Accounting for shear strain (curve *2*) does not change the curves qualitatively. The processes corresponding to the upper section *BC* occur only if the loading frequency decreases beginning with the point *C*. Here use is made of the parameter continuation method: at each subsequent loading frequency, the electroelastic properties are calculated for the temperature of the plate vibrating at the previous frequency.

The numerical experiments and the plots show that allowing for the shear strain not only decreases the resonant frequency and increases the deflections, but also increases the vibrational heating temperature and the electric potential applied to the piezoactuators to damp the first mode of forced vibration of a ring plate. The thinner the plate, the weaker the effect of this factor. Allowing for the temperature dependence of the piezoactuator has a weak influence on the effectiveness of damping the forced vibration of a viscoelastic ring plate. If the piezoelectric pads are thin ($\delta / h \le 0.5 \cdot 10^{-2}$), the terms with the piezoelectric and dielectric loss moduli can be neglected in formula (6) for the dissipation rate $\overline{W}$.

REFERENCES

1. A. M. Bolkisev, V. L. Karlash, and N. A. Shul'ga, "Temperature dependence of the properties of piezoelectric ceramics," *Int. Appl. Mech.*, **20**, No. 7, 650–653 (1984).
2. Ya. M. Grigorenko and A. T. Vasilenko, *Theory of Shells of Varying Thickness* [in Russian], Naukova Dumka, Kyiv (1981).

3. Ya. M. Grigorenko and A. P. Mukoed, *Solving Linear and Nonlinear Problems in Shell Theory on a Computer* [in Ukrainian], Lybid', Kyiv (1992).
4. V. G. Karnaukhov, "Thermal failure of polymeric structural elements under monoharmonic deformation," *Int. Appl. Mech*., **40**, No. 6, 622–655 (2004).
5. V. G. Karnaukhov, "Refined thermomechanical models of composite pates with distributed sensors and actuators," in: *Mathematical Problems of the Mechanics of Inhomogeneous Structures* [in Ukrainian], Issue 1, Lviv (2006), pp. 82–85.
6. V. G. Karnaukhov, T. V. Karnaujhova, O. V P'yatets'ka, and V. M. Nikitenko, "Influence of the dissipative-heating temperature on the damping of resonant vibration of a circular plate with piezoelectric sensors and actuators," *Visn. Kyiv. Univ.*, *Ser. Fiz.-Mat. Nauky*, No. 4, 84–91 (2005).
7. V. G. Karnaukhov and I. F. Kirichok, *Electrothermoviscoelasticity*, Vol. 4 of the five-volume series *Mechanics of Coupled Fields in Structural Members* [in Russian], Naukova Dumka, Kyiv (1988).
8. V. G. Karnaukhov and I. F. Kirichok, "Forced harmonic vibration and dissipative heating of viscoelastic thin-walled elements," in: *Advances of Mechanics*, Vol. 1, A.S.K., Kyiv (2005), pp. 107–130.
9. V. G. Karnaukhov, I. F. Kirichok, and V. I. Kozlov, "Influence of dissipative-heating temperature on the active damping of the forced flexural vibration of inelastic thin plates with sensors and actuators," in: *Important Aspects of Physical and Mechanical Research: Acoustics and Waves* [in Ukrainian], Naukova Dumka, Kyiv (2007), pp. 127–152.
10. V. G. Karnaukhov, A. V. Kozlov, and E. V. Pyatetskaya, "Damping the vibration of viscoelastic plates with distributed piezoelectric inclusions," *Akust. Vestn*., **5**, No. 4, 15–32 (2002).
11. V. G. Karnaukhov, V. I. Kozlov, and E. V. Pyatetskaya, "Active damping of the vibration of a rectangular plate with distributed sensors and actuators," *Teor. Prikl. Mekh*., **37**, 136–140 (2003).
12. V. G. Karnaukhov, V. I. Kozlov, and T. V. Karnaukhova, "Refined thermomechanical model of composite Timoshenko shells with distributed transversely isotropic actuators under monoharmonic loading," *Prikl. Probl. Mekh. Math*., **4**, 84–95 (2006).
13. I. F. Kirichok, "Flexural vibrations and heating of a circular bimorph piezoplate under electric excitation applied to nonuniformly electroded surfaces," *Int. Appl. Mech*., **41**, No. 9, 1037–1042 (2005).
14. I. F. Kirichok and M. V. Karnaukhov, "Active damping of the forced thermomechanical vibration of circular plates using refined models," in: *Mathematical Problems of the Mechanics of Inhomogeneous Structures* [in Ukrainian], Issue 2, Lviv (2006), pp. 18–192.
15. I. F. Kirichok, O. V. P'yatets'ka, and M. V. Karnaukhov, "Flexural vibration and dissipative heating of a circular viscoelastic plate with piezoelectric actuators under electromechanical loading," *Visn. Kyiv. Univ.*, *Ser. Fiz.-Mat. Nauky*, No. 2, 84–92 (2006).
16. M. P. Nesterenko, O. P. Chervinko, and I. K. Senchenkov, "Simulation of the ultrasonic heating of structural members made of fibrous polymeric composites," *Vestn. Nats. Tekhn. Univ. KhHPI*, No. 9, 3–8 (2002).
17. U. Gabbert and H. S. Tzou, *Smart Structures and Structronic Systems*, Kluwer, Dordrecht–Boston–London (2001).
18. A. Ya. Grigoreko and N. P. Yaremchenko, "Stress–strain state of shallow shells with rectangular planform and varying thickness: Refined formulation," *Int. Appl. Mech*., **43**, No. 10, 1132–1141 (2007).
19. V. L. Karlash, "Evolution of the planar vibrations of a rectangular piezoceramic plate as its aspect ratio is changed," *Int. Appl. Mech*., **43**, No. 7, 786–793 (2007).
20. V. G. Karnaukhov, "Thermomechanics of coupled fields in passive and piezoactive inelastic bodies under harmonic deformations," *J. Therm. Stress*., **28**, No. 6–7, 783–815 (2005).
21. I. F. Kirichok, "Resonant flexural vibrations and dissipative heating of a piezoceramic ring plate with nonuniformly electroded surfaces," *Int. Appl. Mech*., **42**, No. 3, 336–341 (2006).
22. I. F. Kirichok and M. V. Karnaukhov, "Single-frequency vibrations and vibrational heating of a piezoelectric circular sandwich plate under monoharmonic electromechanical loading," *Int. Appl. Mech*., **44**, No. 1, 65–72 (2008).
23. I. F. Kirichok, "Flexural vibrations and vibrational heating of a ring plate with thin piezoceramic pads under single-frequency electromechanical loading," *Int. Appl. Mech*., **44**, No. 2, 200–207 (2008).