# REFINED ANALYSIS OF THE STRESS STATE OF ORTHOTROPIC ELLIPTIC CYLINDRICAL SHELLS WITH VARIABLE GEOMETRICAL PARAMETERS

**Ya. M. Grigorenko and S. N. Yaremchenko**

UDC 539.3

**An approach developed earlier to solve boundary-value problems is used to analyze the behavior of the stress–strain state of orthotropic elliptic cylindrical shells with variation in the geometric parameters of their cross section at constant volume (weight)**

**Keywords:** stress–strain state, orthotropic cylindrical shell, refined theory, elliptic cross section, geometric parameter

**Introduction.** In analyzing the stress–strain state of structural members in the form of noncircular cylindrical shells of varying thickness made of composite materials, it is of interest to examine the effect of change in the geometrical parameters of their cross section and thickness at constant volume (weight).

The present paper addresses the stress–strain problems for orthotropic noncircular cylindrical shells with either open or closed elliptic cross-section and circumferentially varying thickness. To solve them, a refined formulation and straight-element model are used [1–3, 6, 9–12].

**1.** Consider a shell described in an orthogonal coordinate system $s, \psi, \gamma$, where $s$ is the axial coordinate of the cylinder, $\psi$ is the angular parameter of ellipse, $\gamma$ is the normal (to the datum surface) coordinate. Then the first quadratic form on the datum surface is

$$dS^2 = ds^2 + \gamma_\psi^2(\psi)d\psi^2, \tag{1}$$

where $\gamma_\psi d\psi = dt$, $t$ is the arc length along the directrix of the datum surface; $\gamma_\psi$ is a transformation coefficient relating the arc length and the angular parameter $\psi$.

The elliptical cross section of the datum surface is described by the parametrical equation

$$x = b\cos\psi, \quad z = a\sin\psi \quad (0 \le \psi \le 2\pi), \tag{2}$$

where $a$ and $b$ are the major and minor semiaxes of the ellipse whose curvature is given by

$$k(\psi) = \frac{ab}{(a^2\cos^2\psi + b^2\sin^2\psi)^{3/2}}, \tag{3}$$

and the transformation coefficient is

$$\gamma_\psi(\psi) = \sqrt{b^2\sin^2\psi + a^2\cos^2\psi}. \tag{4}$$

Using the equations from [3, 7, 8], we write the governing partial differential equations as

---

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, Kyiv. Translated from Prikladnaya Mekhanika, Vol. 44, No. 9, pp. 53–62, September 2008. Original article submitted October 2, 2007.

$$\frac{\partial^2 u}{\partial\psi^2}=b_{11}\frac{\partial^2 u}{\partial s^2}+b_{12}\frac{\partial^2 v}{\partial s\partial\psi}+b_{13}\frac{\partial w}{\partial s}+b_{14}\frac{\partial v}{\partial s}+b_{15}\frac{\partial u}{\partial s}+b_{16}\frac{\partial v}{\partial\psi}+b_{17}w+b_{18}\frac{\partial u}{\partial\psi}+b_{19}q_s,$$

$$\frac{\partial^2 v}{\partial\psi^2}=b_{21}\frac{\partial^2 u}{\partial s\partial\psi}+b_{22}v+b_{23}\frac{\partial^2 v}{\partial s^2}+b_{24}w+b_{25}\frac{\partial w}{\partial\psi}+b_{26}\frac{\partial^2\psi_s}{\partial s\partial\psi}+b_{27}\psi_\psi+b_{28}\frac{\partial^2\psi_\psi}{\partial s^2}$$

$$+b_{29}\frac{\partial\psi_\psi}{\partial s}+b_{2,10}\frac{\partial u}{\partial s}+b_{2,11}\frac{\partial v}{\partial\psi}+b_{2,12}\frac{\partial u}{\partial\psi}+b_{2,13}\frac{\partial v}{\partial s}+b_{2,14}\frac{\partial\psi_s}{\partial\psi}+b_{2,15}q_\psi,$$

$$\frac{\partial^2 w}{\partial\psi^2}=b_{31}\frac{\partial u}{\partial s}+b_{32}v+b_{33}\frac{\partial v}{\partial\psi}+b_{34}w+b_{35}\frac{\partial^2 w}{\partial s^2}$$

$$+b_{36}\frac{\partial\psi_s}{\partial s}+b_{37}\frac{\partial\psi_\psi}{\partial\psi}+b_{38}\psi_\psi+b_{39}\psi_s+b_{3,10}\frac{\partial w}{\partial s}+b_{3,11}\frac{\partial w}{\partial\psi}+b_{3,12}q_\gamma,$$

$$\frac{\partial^2\psi_s}{\partial\psi^2}=b_{41}\frac{\partial u}{\partial\psi}+b_{42}\frac{\partial^2 u}{\partial s^2}+b_{43}\frac{\partial^2 v}{\partial s\partial\psi}+b_{44}\frac{\partial w}{\partial s}+b_{45}\psi_s+b_{46}\frac{\partial^2\psi_s}{\partial s^2}+b_{47}\frac{\partial^2\psi_\psi}{\partial s\partial\psi}+b_{48}\frac{\partial\psi_\psi}{\partial s}$$

$$+b_{49}\frac{\partial u}{\partial s}+b_{4,10}\frac{\partial v}{\partial\psi}+b_{4,11}w+b_{4,12}\frac{\partial v}{\partial s}+b_{4,13}\frac{\partial\psi_s}{\partial s}+b_{4,14}\frac{\partial\psi_\psi}{\partial\psi}+b_{4,15}\frac{\partial\psi_s}{\partial\psi}+b_{4,16}q_s,$$

$$\frac{\partial^2\psi_\psi}{\partial\psi^2}=b_{51}\frac{\partial^2 u}{\partial s\partial\psi}+b_{52}v+b_{53}\frac{\partial v}{\partial\psi}+b_{54}\frac{\partial^2 v}{\partial s^2}+b_{55}w+b_{56}\frac{\partial w}{\partial\psi}+b_{57}\frac{\partial^2\psi_s}{\partial s\partial\psi}+b_{58}\psi_\psi+b_{59}\frac{\partial^2\psi_\psi}{\partial s^2}$$

$$+b_{5,10}\frac{\partial\psi_\psi}{\partial\psi}+b_{5,11}\frac{\partial u}{\partial s}+b_{5,12}\frac{\partial u}{\partial\psi}+b_{5,13}\frac{\partial v}{\partial s}+b_{5,14}\frac{\partial\psi_s}{\partial\psi}+b_{5,15}\frac{\partial\psi_\psi}{\partial s}+b_{5,16}\frac{\partial\psi_s}{\partial s}+b_{5,17}q_\psi$$

$$(0\le s\le L,\quad 0\le\theta\le 2\pi), \tag{5}$$

where $u$, $v$, and $w$ are the displacements of the coordinate surface in the longitudinal, circumferential, and normal directions, respectively; $\psi_s$ and $\psi_\psi$ are the full angles of rotation of the normal; and $b_{ij}$ are coefficients such that

$$b_{11}=-\frac{C_{11}}{\gamma_{22}C_{66}},\quad b_{12}=-\frac{C_{12}+C_{66}}{\gamma_{11}C_{66}},\quad b_{13}=-k(\varphi)\frac{C_{12}}{\gamma_{22}C_{66}},\quad b_{14}=-\frac{1}{\gamma_{11}C_{66}}\frac{\partial C_{66}}{\partial\psi},$$

$$b_{15}=-\frac{1}{\gamma_{22}C_{66}}\frac{\partial C_{11}}{\partial s},\quad b_{16}=-\frac{1}{\gamma_{11}C_{66}}\frac{\partial C_{12}}{\partial s},\quad b_{17}=-\frac{k(\psi)}{\gamma_{22}C_{66}}\frac{\partial C_{12}}{\partial s},$$

$$b_{18}=-\frac{\gamma_{21}}{\gamma_{22}}-\frac{1}{C_{66}}\frac{\partial C_{66}}{\partial\psi},\quad b_{19}=-\frac{1}{\gamma_{22}C_{66}},\quad b_{21}=-\frac{C_{12}+C_{66}-k^2(\psi)D_{66}}{\gamma_{11}C_{22}},\quad b_{22}=\frac{k^2(\psi)K_2}{\gamma_{22}C_{22}},$$

$$b_{23}=-\frac{C_{66}}{\gamma_{22}C_{22}},\quad b_{24}=-\frac{k'(\psi)}{\gamma_{11}}-\frac{k(\psi)}{\gamma_{11}C_{22}}\frac{\partial C_{22}}{\partial\psi},\quad b_{25}=-k(\psi)\frac{C_{22}+K_2}{\gamma_{11}C_{22}},\quad b_{26}=-\frac{k(\psi)D_{66}}{\gamma_{11}C_{22}},$$

$$b_{27}=-\frac{k(\psi)K_2}{\gamma_{22}C_{22}},\quad b_{28}=-\frac{k(\psi)D_{66}}{\gamma_{22}C_{22}},\quad b_{29}=-\frac{k(\psi)}{\gamma_{22}C_{22}}\frac{\partial D_{66}}{\partial s},\quad b_{2,10}=-\frac{1}{\gamma_{11}C_{22}}\frac{\partial C_{12}}{\partial\psi},$$

$$b_{2,11}=-\frac{\gamma_{21}}{\gamma_{22}}-\frac{1}{C_{22}}\frac{\partial C_{22}}{\partial\psi},\quad b_{2,12}=-\frac{1}{\gamma_{11}C_{22}}\left(\frac{\partial C_{66}}{\partial s}-k^2(\psi)\frac{\partial D_{66}}{\partial s}\right),\quad b_{2,13}=-\frac{1}{\gamma_{22}C_{22}}\frac{\partial C_{66}}{\partial s},$$

$$b_{2,14}=-\frac{k(\psi)}{\gamma_{22}C_{22}}\frac{\partial D_{66}}{\partial s},\quad b_{2,15}=-\frac{1}{\gamma_{22}C_{22}},\quad b_{31}=\frac{k(\psi)C_{12}}{\gamma_{22}K_2},\quad b_{32}=\frac{k'(\psi)}{\gamma_{11}}+\frac{k(\psi)}{\gamma_{11}K_2}\frac{\partial K_2}{\partial\psi},$$

$$b_{33}=k(\psi)\frac{K_2+C_{22}}{\gamma_{11}K_2},\quad b_{34}=\frac{k^2(\psi)C_{22}}{\gamma_{22}K_2},\quad b_{35}=-\frac{K_1}{\gamma_{22}K_2},\quad b_{36}=b_{35},\quad b_{37}=\frac{-1}{\gamma_{11}},$$

$$b_{38}=-\frac{1}{\gamma_{11}K_2}\frac{\partial K_2}{\partial\psi},\quad b_{39}=-\frac{1}{\gamma_{22}K_2}\frac{\partial K_1}{\partial s},\quad b_{3,10}=b_{39},\quad b_{3,11}=-\frac{\gamma_{21}}{\gamma_{22}}-\frac{1}{K_2}\frac{\partial K_2}{\partial\psi},$$

$$b_{3,12}=-\frac{1}{\gamma_{22}K_2},\quad b_{41}=k'(\psi)-\frac{k(\psi)}{C_{66}}\frac{\partial C_{66}}{\partial\psi}+\frac{k(\psi)}{D_{66}}\frac{\partial D_{66}}{\partial\psi},\quad b_{42}=-k(\psi)\frac{C_{11}}{\gamma_{22}C_{66}},$$

$$b_{43}=k(\psi)\frac{D_{12}C_{66}-C_{12}D_{66}-C_{66}D_{66}}{\gamma_{11}C_{66}D_{66}},\quad b_{44}=\frac{k^2(\psi)(D_{12}C_{66}-C_{12}D_{66})+K_1C_{66}}{\gamma_{22}C_{66}D_{66}},$$

$$b_{45}=\frac{K_1}{\gamma_{22}D_{66}},\quad b_{46}=-\frac{D_{11}}{\gamma_{22}D_{66}},\quad a_{47}=-\frac{D_{12}+D_{66}}{\gamma_{11}D_{66}},\quad b_{48}=-\frac{1}{\gamma_{11}D_{66}}\frac{\partial D_{66}}{\partial\psi},$$

$$b_{49}=-\frac{k(\psi)}{\gamma_{22}C_{66}}\frac{\partial C_{11}}{\partial s},\quad b_{4,10}=\frac{k(\psi)}{\gamma_{11}D_{66}}\frac{\partial D_{12}}{\partial s}-\frac{k(\psi)}{\gamma_{11}C_{66}}\frac{\partial C_{12}}{\partial s},\quad b_{4,11}=\frac{k^2(\psi)}{\gamma_{22}D_{66}}\frac{\partial D_{12}}{\partial s}-\frac{k^2(\psi)}{\gamma_{22}C_{66}}\frac{\partial C_{12}}{\partial s},$$

$$b_{4,12}=-\frac{k(\psi)}{\gamma_{11}C_{66}}\frac{\partial C_{66}}{\partial\psi},\quad b_{4,13}=-\frac{1}{\gamma_{22}D_{66}}\frac{\partial D_{11}}{\partial s},\quad b_{4,14}=-\frac{1}{\gamma_{11}D_{66}}\frac{\partial D_{12}}{\partial s},\quad b_{4,15}=-\frac{1}{\gamma_{11}D_{66}}\frac{\partial D_{66}}{\partial\psi},$$

$$b_{4,16}=-\frac{k(\psi)}{\gamma_{22}C_{66}},\quad b_{51}=k(\psi)\frac{D_{66}C_{22}-C_{12}D_{22}-C_{66}D_{22}+k^2(\psi)D_{66}D_{22}}{\gamma_{11}C_{22}D_{22}},$$

$$b_{52}=k(\psi)K_2\frac{k^2(\psi)D_{22}-C_{22}}{\gamma_{22}C_{22}D_{22}},\quad b_{53}=k'(\psi)-\frac{k(\psi)}{C_{22}}\frac{\partial C_{22}}{\partial\psi}+\frac{k(\psi)}{D_{22}}\frac{\partial D_{22}}{\partial\psi},\quad b_{54}=-k(\psi)\frac{C_{66}}{\gamma_{22}C_{22}},$$

$$b_{55}=\frac{k(\psi)k'(\psi)}{\gamma_{11}}-\frac{k^2(\psi)}{\gamma_{11}C_{22}}\frac{\partial C_{22}}{\partial\psi}+\frac{k^2(\psi)}{\gamma_{11}D_{22}}\frac{\partial D_{22}}{\partial\psi},\quad b_{56}=K_2\frac{C_{22}-k^2(\psi)D_{22}}{\gamma_{11}C_{22}D_{22}},$$

$$b_{57}=-\frac{k^2(\psi)D_{22}D_{66}+C_{22}D_{12}+C_{22}D_{66}}{\gamma_{11}C_{22}D_{22}},\quad b_{58}=K_2\frac{C_{22}-k^2(\psi)D_{22}}{\gamma_{22}C_{22}D_{22}},\quad b_{59}=-\frac{D_{66}(k^2(\psi)D_{22}+C_{22})}{\gamma_{22}C_{22}D_{22}},$$

$$b_{5,10}=-\frac{\gamma_{21}}{\gamma_{22}}-\frac{1}{D_{22}}\frac{\partial D_{22}}{\partial\psi},\quad a_{5,11}=-\frac{k(\psi)}{\gamma_{11}C_{22}}\frac{\partial C_{12}}{\partial\psi},\quad b_{5,12}=\frac{k^3(\psi)}{\gamma_{11}C_{22}}\frac{\partial D_{66}}{\partial s}-\frac{k(\psi)}{\gamma_{11}C_{22}}\frac{\partial C_{66}}{\partial s}+\frac{k(\psi)}{\gamma_{11}D_{22}}\frac{\partial D_{66}}{\partial s},$$

$$b_{5,13}=-\frac{k(\psi)}{\gamma_{22}C_{22}}\frac{\partial C_{66}}{\partial s},\quad b_{5,14}=-\frac{k^2(\psi)}{\gamma_{11}C_{22}}\frac{\partial D_{66}}{\partial s}-\frac{1}{\gamma_{11}D_{22}}\frac{\partial D_{66}}{\partial s},\quad b_{5,15}=-\frac{k^2(\psi)}{\gamma_{22}C_{22}}\frac{\partial D_{66}}{\partial s}-\frac{1}{\gamma_{22}D_{22}}\frac{\partial D_{66}}{\partial s},$$

$$b_{5,16}=-\frac{1}{\gamma_{11}D_{22}}\frac{\partial D_{12}}{\partial\psi},\quad b_{5,17}=-\frac{k(\psi)}{\gamma_{22}C_{22}},\tag{6}$$

where

$$\gamma_{11}=\frac{1}{\gamma_\psi},\quad \gamma_{22}=\frac{1}{\gamma_\psi^2},\quad \gamma_{21}=\frac{\gamma'_\psi}{\gamma_\psi^3},\quad \gamma'_\psi=\frac{\sin 2\psi}{2\gamma_\psi}(b^2-a^2),$$

$$C_{11}=\frac{E_s h}{1-\nu_s\nu_\psi},\quad C_{12}=\nu_\psi C_{11},\quad C_{22}=\frac{E_\psi h}{1-\nu_s\nu_\psi},\quad C_{66}=G_{s\psi}h,$$

$$D_{11} = \frac{E_s h^3}{12(1-\nu_s \nu_\psi)}, \quad D_{12} = \nu_\psi D_{11}, \quad D_{22} = \frac{E_\psi h^3}{12(1-\nu_s \nu_\psi)}, \quad D_{66} = \frac{G_{s\psi} h^3}{12},$$

$$K_1 = \frac{5}{6} h G_{s\gamma}, \quad K_2 = \frac{5}{6} h G_{\psi\gamma}, \tag{7}$$

where $E_s$, $E_\psi$ and $\nu_s$, $\nu_\psi$ are the elastic moduli and Poisson's ratios in the longitudinal and circumferential directions; $G_{s\psi}$, $G_{s\gamma}$, and $G_{\psi\gamma}$ are the shear moduli; and $h = h(s, \psi)$ is the thickness of the shell.

The stresses are defined by the formulas

$$\sigma_s = \frac{E_s}{1-\nu_s \nu_\psi}[\varepsilon_s + \gamma\kappa_s + \nu_\psi(\varepsilon_\psi + \gamma\kappa_\psi)], \quad \sigma_\psi = \frac{E_\psi}{1-\nu_s \nu_\psi}[\varepsilon_\psi + \gamma\kappa_\psi + \nu_s(\varepsilon_s + \gamma\kappa_s)],$$

$$\tau_{s\psi} = G_{s\psi}(\varepsilon_\psi + 2\gamma\kappa_{s\psi}), \quad \tau_{s\gamma} = G_{s\gamma}\gamma_s, \quad \tau_{\psi\gamma} = G_{\psi\gamma}\gamma_\psi, \tag{8}$$

where

$$\varepsilon_s = \frac{\partial u}{\partial s}, \quad \varepsilon_\psi = \gamma_{11}\frac{\partial v}{\partial \psi} + k(\psi)w, \quad \varepsilon_{s\psi} = \gamma_{11}\frac{\partial u}{\partial \psi} + \frac{\partial v}{\partial s}, \quad \kappa_s = \frac{\partial \psi_s}{\partial s},$$

$$\kappa_\psi = \gamma_{11}\frac{\partial \psi_\psi}{\partial \psi} - k(\psi)\left(\gamma_{11}\frac{\partial v}{\partial \psi} + k(\psi)w\right), \quad 2\kappa_{s\psi} = \gamma_{11}\frac{\partial \psi_s}{\partial \psi} + \frac{\partial \psi_\psi}{\partial s} - k(\psi)\gamma_{11}\frac{\partial u}{\partial \psi},$$

$$\gamma_s = \psi_s - \vartheta_s, \quad \gamma_\psi = \psi_\psi - \vartheta_\psi, \quad \vartheta_s = -\frac{\partial w}{\partial s}, \quad \vartheta_\psi = -\gamma_{11}\frac{\partial w}{\partial \psi} + k(\psi)v. \tag{9}$$

Supplementing the governing equations (5) with boundary conditions on all edges for open shells and boundary conditions on the curvilinear edges and periodicity conditions along the directrix for closed shells, we arrive at a two-dimensional boundary-value problem.

To solve this class of two-dimensional problems, we use spline functions [4, 6, 7] to approximate the desired solution in one coordinate (for example, longitudinal) direction. The resulting one-dimensional boundary-value problem is solved by the stable discrete-orthogonalization method [2]. Such an approach, unlike variational and projective methods, allows us to solve problems with various boundary conditions on the opposite edges of the shell.

Since the system of equations (5) contains no higher than second-order derivatives of unknown functions with respect to the coordinate $s$, it is possible to restrict the approximation of solutions in this coordinate to cubic splines. Then the solution of the boundary-value problem for the system of equations (4) with appropriate boundary conditions can be represented in the form

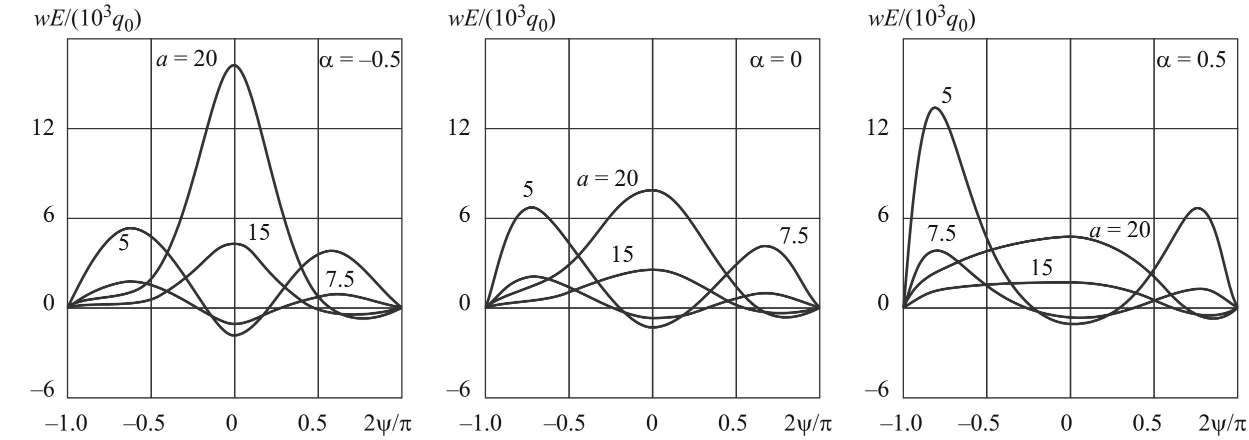
$$u(s,\psi) = \sum_{i=0}^{N} u_i(\psi)\varphi_{1i}(s), \quad v(s,\psi) = \sum_{i=0}^{N} v_i(\psi)\varphi_{2i}(s), \quad w(s,\psi) = \sum_{i=0}^{N} w_i(\psi)\varphi_{3i}(s),$$

$$\psi_s(s,\psi) = \sum_{i=0}^{N} \psi_{si}(\psi)\varphi_{4i}(s), \quad \psi_\theta(s,\psi) = \sum_{i=0}^{N} \psi_{\theta i}(\psi)\varphi_{5i}(s), \tag{10}$$

where $u_i(\psi)$, $v_i(\psi)$, $w_i(\psi)$, $\psi_{si}(\psi)$, and $\psi_{\theta i}(\psi)$ are unknown functions of the variable $\psi$; $\varphi_{ji}(s)$ ($j = \overline{1, 5}$) are linear combinations of cubic B-splines [4] on a uniform mesh $\Delta$: $0 = s_0 < s_1 \ldots < s_N = L$ satisfying the boundary conditions.

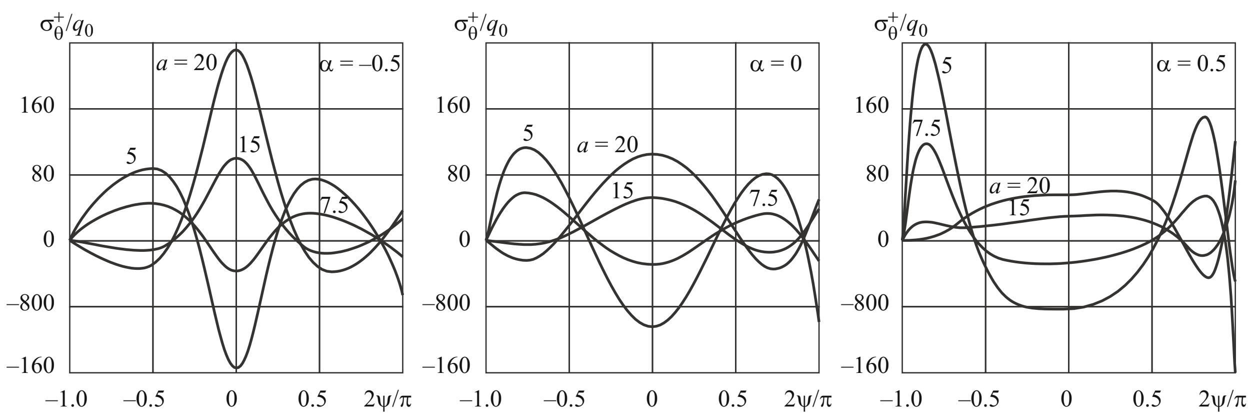
Substituting expressions (10) into the governing equations (5), applying the spline-collocation method [5], and requiring them to be satisfied on $N + 1$ lines $s = \xi_i$ ($i = \overline{1, N+1}$), we obtain a system of ordinary differential equations of order $10(N + 1)$, which can be represented in Cauchy's normal form:

$$\frac{d\bar{R}}{d\psi} = A(\psi)\bar{R} + \bar{f}(\psi), \tag{11}$$



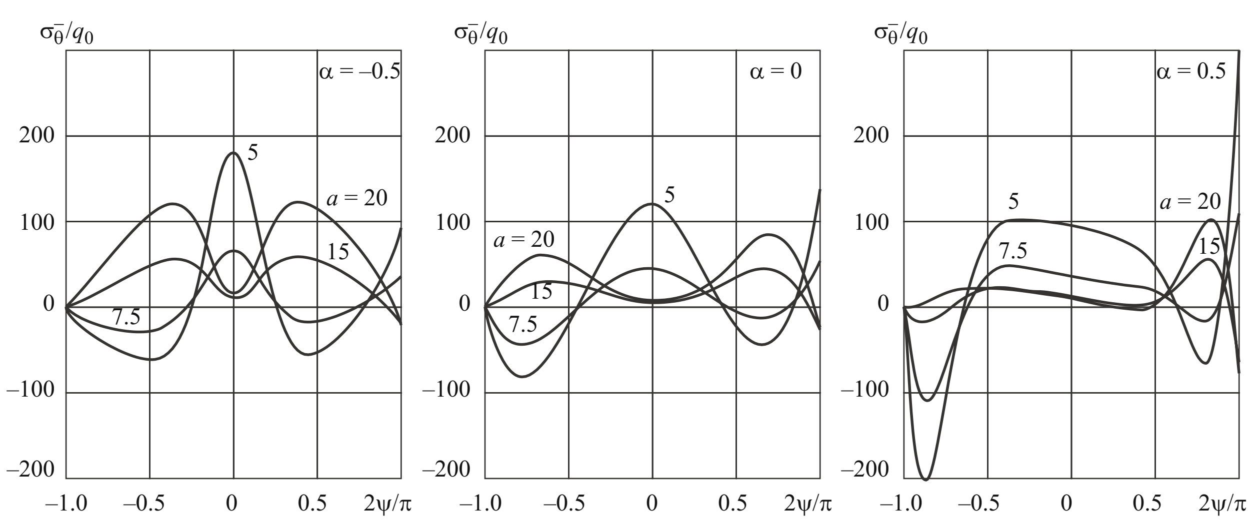

*a* *b* *c*

Fig. 1




*a* *b* *c*

Fig. 2




*a* *b* *c*

Fig. 3

where

$$\overline{R} = \{u_0, u_1, \ldots, u_N, u'_0, u'_1, \ldots, u'_N, v_0, v_1, \ldots, v_N, v'_0, v'_1, \ldots, v'_N, w_0, w_1, \ldots, w_N, w'_0, w'_1, \ldots, w'_N,$$

$$\psi_{s0}, \psi_{s1}, \ldots, \psi_{sN}, \psi'_{s0}, \psi'_{s1}, \ldots, \psi'_{sN}, \psi_{\psi 0}, \psi_{\psi 1}, \ldots, \psi_{\psi N}, \psi'_{\psi 0}, \psi'_{\psi 1}, \ldots, \psi'_{\psi N}\}^{\mathrm{T}} \quad (12)$$

is a vector function of $\psi$; $\bar{f}(\psi)$ is the vector of right-hand sides; $A$ is a square matrix whose elements depend on the parameter $\psi$. If the shell is open in the circumferential direction ($\psi_1 \le \psi \le \psi_2$), the system of equations (11) is supplemented with boundary conditions that are found, as Eqs. (11), from the boundary conditions on the longitudinal edges of the shell:

$$A_1 \overline{R}(\psi_1) = \overline{a}_1, \quad A_2 \overline{R}(\psi_2) = \overline{a}_2, \quad (13)$$

$A_1$ and $A_2$ are rectangular matrices, $\overline{a}_1$ and $\overline{a}_2$ are the corresponding vectors.

If the shell is closed in the circumferential direction, it is possible to use periodicity conditions in $\psi$ or symmetry conditions on some segments of the directrix. In the latter case, we arrive at similar boundary conditions (13).

**2.** We will use this approach to determine the stress–strain state of transversely isotropic open cylindrical shells with elliptic cross-section (2) and thickness varying as

$$h = h_0 (1 + \alpha \cos 2\psi) \quad (-\pi/2 \le \psi \le \pi/2). \quad (14)$$

The load is distributed uniformly: $q_\gamma = q_0 = \text{const}$. Formula (14) indicates that the volume of the shell remains constant as $\alpha$ changes. In this case, one edge of the shell is clamped, i.e.,

$$u = v = w = \psi_s = \psi_\psi = 0, \quad \psi = \pi/2, \quad (15)$$

and the other is hinged, i.e.,

$$u = w = N_\psi = \psi_s = M_\psi = 0, \quad \psi = -\pi/2$$

or

$$u = w = \frac{\partial v}{\partial \psi} = \psi_s = \frac{\partial \psi_\psi}{\partial \psi} = 0, \quad \psi = -\pi/2. \quad (16)$$

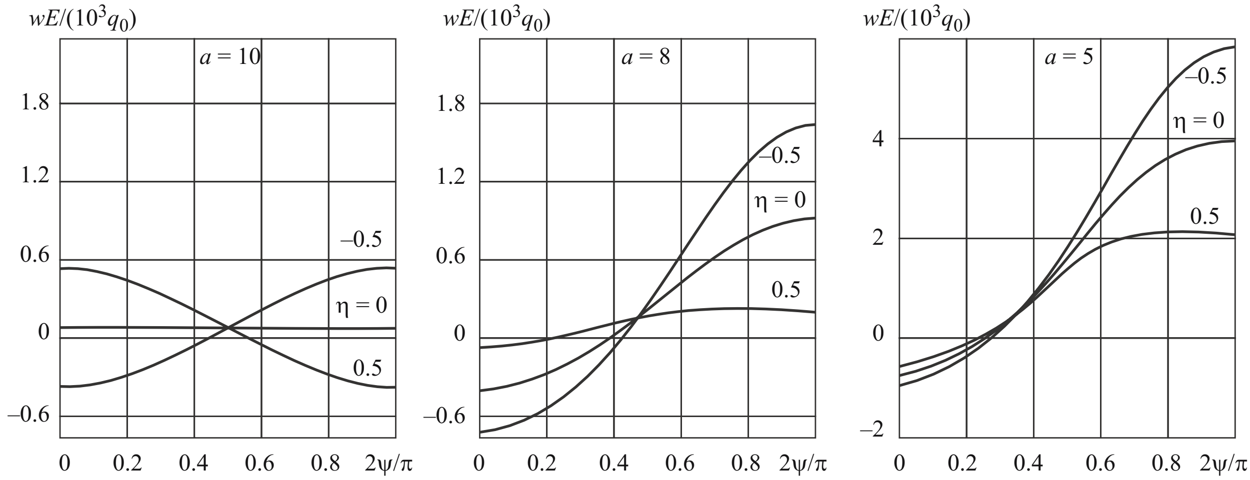
Let $ab = R^2$. The input data: $R = 10$, $L = 30$, $h_0 = 1$, $E_s = E_\psi = E$, $\nu_s = \nu_\psi = \nu = 0.3$, $G_{s\gamma} = G_{\psi\gamma} = G' = E/d$, $d = 40$.

The task is to examine the influence of variation in the thickness of the shell (at constant volume) and the ellipse parameters on the circumferential distribution of the deflection $w$ and stresses $\sigma_\psi^\pm$ at $a = 5, 7.5, 15, 20$ and $\alpha = -0.5, 0, 0.5$.

Figures 1–3 show the deflection $w$ and stresses $\sigma_\psi^\pm$ on the lateral surfaces of the shell in the section $s = L/2$ as functions of thickness and ellipticity.

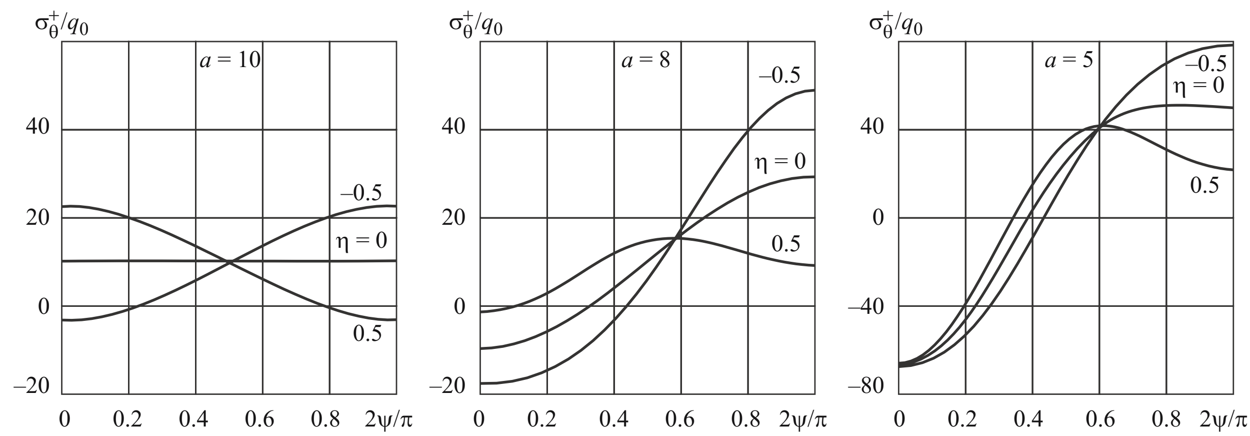
It can be seen that with change in the thickness, the maximum deflection at $a = 20$ decreases by almost half for $0.5 \le \alpha \le 0$ and by a factor of 1.5 for $0 \le \alpha \le 0.5$, which is due to the increase in the thickness in the neighborhood of $\psi = 0$. As $\alpha$ varies within the same limits, the maximum deflection at $a = 5$ increases almost threefold. While the difference between the boundary conditions on the opposite straight edges of the shell has almost no effect on the symmetry of the circumferential distribution of the deflection $w$ about $\psi = 0$ when $\alpha = -0.5$, the difference makes this distribution asymmetric when $\alpha = 0$ and $\alpha = 0.5$, and especially if $a = 5$.

Figures 2 and 3 show the circumferential distribution of the stresses $\sigma_\psi^\pm$ on the lateral surfaces of the shell. It can be seen that if $-0.5 \le \alpha \le 0.5$, the maximum value of $\sigma_\psi^+$ decreases by almost 75% and passes from the shell with $a = 20$ to the shell with $a = 5$ because of the different boundary conditions on its opposite edges. With variation in $\alpha$, the maximum value of $\sigma_\psi^-$ at $a = 5$ decreases reversing sign, which is because the edge at $\psi = \pi/2$ is clamped. It should also be noted that when $\alpha = -0.5$, i.e., when the shell is quite thin in the neighborhood of $\psi = 0$, the difference between the boundary conditions on the opposite edges of the shell has almost no effect on the symmetry of the circumferential distribution of the stresses $\sigma_\psi^\pm$ similarly to the deflection $w$.



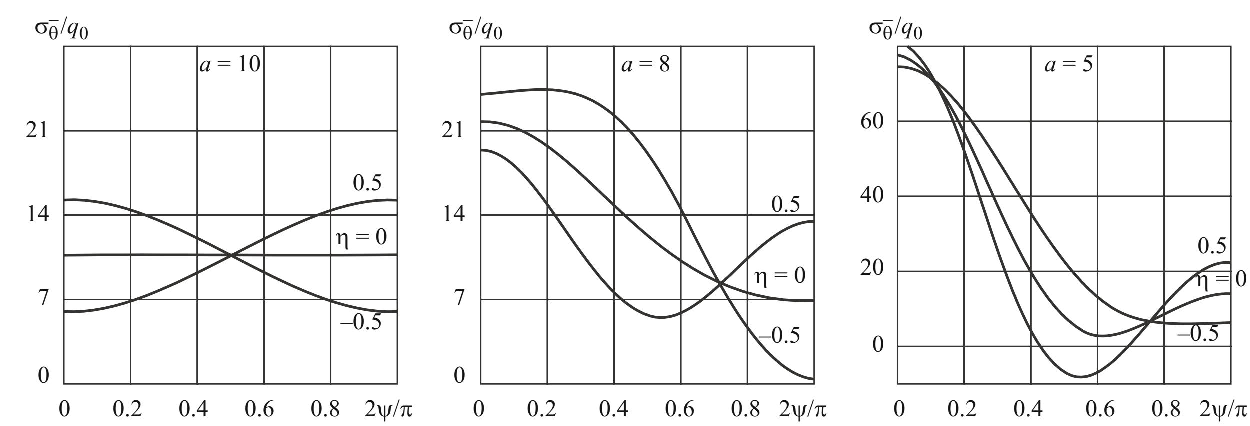

*a* *b* *c*

Fig. 4




*a* *b* *c*

Fig. 5




*a* *b* *c*

Fig. 6

Let us now address circumferentially closed orthotropic cylindrical shells with elliptic cross-section and thickness varying as (14). The ends of the shell are clamped, and the normal load varies as

$$q_\gamma = q_0 (1+\eta \cos 2\psi) \quad (-\pi \le \psi \le \pi). \tag{17}$$

The initial geometrical parameters are the same as in the previous problem, while the mechanical characteristics are the following: $E_s = 5E$, $E_\psi = 1.25E$, $\nu_s = 0.18$, $\nu_\psi = 0.045$, $G_{s\psi} = 0.4E$, $G_{s\gamma} = G_{\psi\gamma} = 0.2E$.

Figures 4–6 show the distribution of the deflection $w$ and stresses $\sigma_\psi^\pm$ on the lateral surfaces of the shell for $\alpha = 0$. In Fig. 4 for $a = 10$ (circular shell), we have $w = \text{const}$ for $\eta = 0$, the deflection is opposite to the load for $\eta = -0.5$ and $\psi = 0$, and the deflection is in the same direction as the load for $\eta = -0.5$ and $\psi = \pi/2$. When $a = 8$ (Fig. 4*b*), the pattern changes at the expense of ellipticity. This change is even greater for $a = 5$, where $a/b = 1/4$. When $\eta = -0.5$, the maximum deflection increases by a factor of four for $a = 8$ and by a factor of 12 for $a = 5$.

The distribution of the stresses $\sigma_\psi^\pm$ (Figs. 5 and 6) shows a similar pattern. As ellipticity and load change, the stress $\sigma_\psi^+$ decreases for $\psi = 0$ and peaks for $\psi = \pi/2$, the curves approaching each other with increasing ellipticity. The pattern for the stress $\sigma_\psi^-$ is similar, yet opposite.

Thus, by varying the ellipticity of the cross-section and the thickness of the shell and keeping its volume (weight) constant, we can choose a more rational stress–strain state.

## REFERENCES


1. Ya. M. Grigorenko, "Approaches to the numerical solution of linear and nonlinear problems in shell theory in classical and refined formulation," *Int. Appl. Mech.*, **32**, No. 6, 409–442 (1996).
2. Ya. M. Grigorenko and A. T. Vasilenko, *Theory of Shells of Variable Stiffness*, Vol. 4 of the five-volume series *Methods of Shell Design* [in Russian], Naukova Dumk, Kyiv (1981).
3. Ya. M. Grigorenko, A. T. Vasilenko, and G. P. Golub, *Statics of Anisotropic Shells with Finite Shear Stiffness* [in Russian], Naukova Dumka, Kyiv (1987).
4. Yu. S. Zav'yalov, Yu. I. Kvasov, and V. L. Miroshnichenko, *Spline-Function Methods* [in Russian], Nauka, Moscow (1980).
5. Ya. M. Grigorenk, "Nonconventional approaches to static problems for noncircular cylindrical shells in different formulations," *Int. Appl. Mech.*, **43**, No. 1, 35–53 (2007).
6. Ya. M. Grigorenko, A. Ya. Grigorenko, and L. I. Zakhariichenko, "Stress analysis of noncircular cylindrical shells with cross section in the form of connected convex half-corrugations," *Int. Appl. Mech.*, **42**, No. 4, 431–438 (2006).
7. Ya. M. Grigorenko and S. N. Yaremchenko, "Stress analysis of orthotropic noncircular cylindrical shells of variable thickness in a refined formulation," *Int. Appl. Mech.*, **40**, No. 3, 266–274 (2004).
8. Ya. M. Grigorenko and S. N. Yaremchenko, "Refined design of corrugated noncircular cylindrical shells," *Int. Appl. Mech.*, **41**, No. 1, 7–13 (2005).
9. Ya. M. Grigorenko and S. N. Yaremchenko, "Influence of orthotropy on displacements and stresses in nonthin cylindrical shells with elliptic cross section," *Int. Appl. Mech.*, **43**, No. 6, 654–661 (2007).
10. I. Yu. Khoma, "Tension of a nonthin transversely isotropic plate with a noncircular cylindrical cavity," *Int. Appl. Mech.*, **42**, No. 11, 1285–1297 (2006).
11. V. G. Piskunov and A. O. Rasskazov, "Evolution of the theory of laminated plates and shells," *Int. Appl. Mech.*, **38**, No. 2, 135–166 (2002).
12. K. P. Soldatos, "Mechanics of cylindrical shells with non-circular cross section. A survey," *Appl. Mech. Rev.*, **52**, No. 8, 237–274 (1999).