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MONOHARMONIC VIBRATIONS AND VIBRATIONAL HEATING OF AN ELECTROMECHANICALLY LOADED CIRCULAR PLATE WITH PIEZOELECTRIC ACTUATORS SUBJECT TO SHEAR STRAIN

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The paper addresses the forced flexural vibrations and dissipative heating of a circular viscoelastic plate with piezoactive actuators under axisymmetric loading. A refined formulation of this coupled problem is considered. The viscoelastic behavior of materials is described using the concept of complex moduli dependent on the temperature of dissipative heating. The electromechanical behavior of the plate is modeled based on the Timoshenko hypotheses for the mechanical variables and analogous hypotheses for the electric-field variables in the piezoactive layers of the actuator. The temperature is assumed constant throughout the thickness. The nonlinear problem is solved by a time stepping method using, at each step, the discrete-orthogonalization and finite-difference methods to solve the elastic and heat-conduction equations, respectively. A numerical study is made of the effect of the shear strain, the temperature dependence of the material properties, fixation conditions, and geometrical parameters of the plate on the vibrational characteristics and the electric potential applied to the actuator electrodes to balance the mechanical load

Keywords: forced vibrations, dissipative heating, actuator, temperature, damping

Introduction. Recent trends have been toward the wider use of active damping by incorporating piezoelectric components into thin-walled structural members to restrict the amplitudes of their forced vibrations. An electric potential difference of appropriate amplitude and phase is applied to them to balance the most intensive modes of mechanical vibrations [9, 15, 16]. The efficiency of damping thin-walled elements is greatly dependent on the fixation conditions of elements, the shape and dimensions of the actuator, the properties of the passive and piezoactive materials, and their temperature sensitivity because of dissipative heating or heating by the environment. Special allowance should be made for thermal effects in calculating the vibratory characteristics of plates made of polymeric materials and their composites, which are rather sensitive to temperature variations [6]. Most studies on the subject involve thin-walled structural members and are based on the standard Kirchhoff–Love hypotheses generalized to electrothermomechanics and a plane stress state [5]. Some results of such studies can be found in [7, 8, 10, 12–14].

The objective of the present paper is to study, using a refined problem formulation and allowing for thermomechanical coupling, the forced vibrations and dissipative heating of a viscoelastic circular plate with piezoelectric actuators under monoharmonic electromechanical loading.

1. Problem Formulation. Consider a solid circular plate of thickness h and radius R described in a radial coordinate system with the origin at the center of the midsurface ($r = 0, z = 0$). Piezoelectric pads (actuators) of thickness δ and radius r_0 are rigidly attached to the outside surfaces $z = \pm h/2$ of the plate. The material of the passive layer is assumed viscoelastic and

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isotropic. The actuators consist of viscoelastic piezoceramic layers polarized throughout the thickness in opposite directions. We assume that polarization is characterized by a piezoelectric modulus d_{31} for $z > 0$ and by $-d_{31}$ for $z < 0$. Infinitely thin electrodes are on the outside surfaces of the actuators and between the actuators and the passive layer. The plate is subjected to a uniform surface pressure harmonically varying with time t as $P = P_0 \cos \omega t$ with a circular frequency ω close to the resonant one. An electric potential difference is applied to the external electrodes $\psi(h/2+\delta)-\psi(-h/2-\delta) = 2\,\mathrm{Re}\,(Ve^{i\omega t})$ and has the same frequency as the mechanical load. The internal electrodes are held at zero potential.

The general formulation of the electrothermoviscoelastic problem for a monoharmonically deformed body is given in the monograph [6]. According to [6], the viscoelastic behavior of the passive and piezoactive materials of the plate can be described using the concept of complex moduli dependent on the temperature of dissipative heating. Modeling the electromechanical vibrations of such a plate is based on Timoshenko's straight-line hypothesis [3] for the mechanical variables, which is valid for the whole layered structure. As far as the electric-field variables are concerned, it is assumed that the tangential components D_r and D_θ of electric-flux density in the piezolayers can be neglected compared with the normal component D_z, which is considered independent of the thickness coordinate [5, 6]. In this case, the electrostatic equations [5] hold identically, and the tangential components E_r and E_θ of electric-field intensity can be found from the three-dimensional equations of state of the polarized piezoceramics. Considering that the actuators are very thin compared with the passive layer and that the thermal contact is perfect, we assume that the temperature of dissipative heating is constant throughout the thickness.

The vibrations excited in the plate are purely flexural and axisymmetric due to the structural symmetry of the plate, the polarization of the actuators, and the behavior of the load. Based on the above hypotheses on the electromechanical variables, the three-dimensional equations of state for piezoceramics polarized in the z-direction [5, 6] used to describe the monoharmonic axisymmetric deformation of the piezolayers take the following form for complex amplitudes:

$$\sigma_r = c_{11}^E \varepsilon_r + c_{12}^E \varepsilon_\theta - b_{31} E_z, \quad \sigma_\theta = c_{12}^E \varepsilon_r + c_{11}^E \varepsilon_\theta - b_{31} E_z, \quad \sigma_{rz} = c_{55}^E \varepsilon_{rz}, \tag{1}$$

$$D_z = b_{31}(\varepsilon_r + \varepsilon_\theta) + b_{33} E_z, \quad E_z = -\frac{d\psi}{dz}, \quad E_s = -\frac{b_{15}}{b_{11}} \varepsilon_{sz}, \tag{2}$$

where

$$c_{11}^E = \frac{1}{s_{11}^E (1-\nu_E^2)}, \quad \nu_E = -\frac{s_{12}^E}{s_{11}^E}, \quad c_{12}^E = \nu_E c_{11}^E, \quad b_{11} = \varepsilon_{11}^T (1-k_{15}^2),$$

$$c_{55}^E = \frac{1}{s_{55}^E (1-k_{15}^2)}, \quad k_{15}^2 = \frac{d_{15}^2}{s_{55}^E \varepsilon_{11}^T}, \quad b_{15} = \frac{d_{15}}{s_{55}^E}, \quad b_{31} = \frac{d_{31}}{s_{11}^E (1-\nu_E)},$$

$$b_{33} = \varepsilon_{33}^T (1-k_p^2), \quad k_p^2 = \frac{2d_{31}^2}{s_{11}^E \varepsilon_{33}^T (1-\nu_E)}, \quad s_{mn}^E = s'_{mn}(1-i\delta_{mn}^s), \tag{3}$$

$d_{mn} = d'_{mn}(1-i\delta_{mn}^d)$, $\varepsilon_{mn}^T = \varepsilon'_{mn}(1-i\delta_{mn}^\varepsilon)$ are complex compliances, piezoelectric moduli, and permittivities whose components are functions of temperature; $\sigma = \sigma' + i\sigma''$ and $\varepsilon = \varepsilon' + i\varepsilon''$ are complex amplitudes of stresses and strains; primes and double primes mark real and imaginary components.

The isotropic material of the passive layer can be described by Eqs. (1) in which the coefficients c_{11}^E, c_{12}^E, c_{55}^E, and b_{31} should be replaced by

$$c_{11} = E/(1-\nu^2), \quad c_{12} = \nu c_{11}, \quad c_{55} = G_{13}, \quad b_{31} = 0, \tag{4}$$

where $E = E' + iE''$ is the temperature-dependent complex viscoelastic modulus; $\nu = \mathrm{const}$ is Poisson's ratio assumed to be constant and real; and $G_{13} = G = E/2(1+\nu)$ for an isotropic material.

Due to the straight-line hypothesis, the elastic parameters in (1) and (2) for the plate under pure bending can be expressed in terms of the deflection w and the angle of rotation ψ_r of the originally normal element:

$$\varepsilon_r = e_r + z\kappa_r, \quad \varepsilon_\theta = e_\theta + z\kappa_\theta, \quad e_r = e_\theta = 0,$$

$$\varepsilon_{rz} = \psi_r - \vartheta_r, \quad \kappa_r = \frac{d\psi_r}{dr}, \quad \kappa_\theta = \frac{\psi_r}{r}, \quad \vartheta_r = -\frac{dw}{dr}. \tag{5}$$

Integrating the first two equations in (2) over the thickness of the piezolayers using the assumptions for D_z and the electric boundary conditions, we obtain the electrostatic equations

$$D_z = -b_{33}\frac{V}{\delta} + \frac{1}{2}b_{31}(h+\delta)\kappa, \quad \kappa = \kappa_r + \kappa_\theta,$$

$$E_z = -\frac{V}{\delta} + \frac{b_{31}}{b_{33}}\left(\frac{h+\delta}{2} \pm z\right)\kappa \quad \left(z \le -\frac{h}{2},\ z \ge \frac{h}{2}\right). \tag{6}$$

Replacing the stresses by statically equivalent forces and moments in (1), (5), and (6), we obtain

$$M_r = D_{11}\kappa_r + D_{12}\kappa_\theta + M_E, \quad M_\theta = D_{12}\kappa_r + D_{11}\kappa_\theta + M_E, \quad Q_r = k_s C_{55}(\psi_r - \vartheta_r), \tag{7}$$

where $D_{11} = \frac{1}{12}(c_{11}h^3 + 2c_{11}^E\delta_*^3 + 2\gamma_{31}\delta^3)$, $\delta_*^3 = 4\delta^3 + 6\delta^2 h + 3\delta h^2$, $D_{12} = \frac{1}{12}(c_{12}h^3 + 2c_{12}^E\delta_*^3 + 2\gamma_{31}\delta^3)$, $C_{55} = G_{13}h + 2c_{55}^E\delta$, $\gamma_{31} = b_{31}^2 / b_{33}$, $M_E = b_{31}(h+\delta)V$, k_s is the shear coefficient, which can be determined using refined theories of layered elastic plates and shells [3]; M_r, M_θ, and Q_r are the complex amplitudes of bending moments and shearing force.

The flexural harmonic vibrations of the plate under axisymmetric loading are described by the equations

$$\frac{dQ_r}{dr} + \frac{1}{r}Q_r + \hat{\rho}\omega^2 w + P_0 = 0, \quad \frac{dM_r}{dr} + \frac{1}{r}(M_r - M_\theta) - Q_r + \hat{\rho}_*\omega^2\psi_r = 0, \tag{8}$$

where $\hat{\rho} = \rho h + 2\rho_*\delta$, $\hat{\rho}_* = (\rho h^3 + 2\rho_*\delta_*^3)/12$; ρ and ρ_* are the specific densities of the passive material and piezoceramics.

The viscoelastic equations (5)–(8) should be supplemented with an energy equation averaged over a cycle of vibrations and the thickness of the plate:

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{\alpha_0}{\lambda H}(T - T_c) + \frac{1}{\lambda H}\hat{W} \tag{9}$$

with the dissipation function

$$\begin{aligned}\hat{W} &= M_r''\kappa_r' - M_r'\kappa_r'' + M_\theta''\kappa_\theta' - M_\theta'\kappa_\theta'' + Q_{rz}''\varepsilon_{rz}' - Q_{rz}'\varepsilon_{rz}'' + 2(D_z''V' - D_z'V'')\\ &= D_{11}''(\kappa_r'^2 + \kappa_r''^2 + \kappa_\theta'^2 + \kappa_\theta''^2) + 2D_{12}''(\kappa_r'\kappa_\theta' + \kappa_r''\kappa_\theta'') + k_s C_{55}''(\varepsilon_{rz}'^2 + \varepsilon_{rz}''^2)\\ &\quad + 2(h+\delta)b_{31}''(\kappa'V' + \kappa''V'') + 2b_{33}''(V'^2 + V''^2)/\delta,\end{aligned} \tag{10}$$

and the boundary and initial conditions

$$\frac{\partial T}{\partial r} + \frac{\alpha_R}{\lambda}(T - T_c) = 0 (r = R), \quad T = T_0 (t = 0), \tag{11}$$

where λ and a are the thermal-conductivity and thermal-diffusivity coefficients; α_0 and α_R are the heat-transfer coefficients on the boundaries $z = \pm(h/2 + \delta)$ and $r = R$, respectively; $H = h + \delta$.

2. Constructing the Solution. To solve the electrothermoviscoelastic problem posed, we choose Q_r, ψ_r, w, and M_r to be unknown functions to integrate the electroelastic equations. After some transformations, Eqs. (5), (7), and (8) yield a system of ordinary differential equations in Cauchy's normal form:

$$\frac{dQ_r}{dr} = -\frac{1}{r}Q_r - \hat{\rho}\omega^2 w - P_0, \quad \frac{d\psi_r}{dr} = J_D M_r + \nu_D\frac{\psi_r}{r} - J_D M_E, \quad \frac{dw}{dr} = -\psi_r + \frac{1}{k_s C_{55}}Q_r,$$

$$\frac{dM_r}{dr} = Q_r + \left(\frac{D_{11}(1-\nu_D^2)}{r^2} - \hat{\rho} * \omega^2 \right) \psi_r - \frac{1+\nu_D}{r} M_r + \frac{1+\nu_D}{r} M_E, \tag{12}$$

where $J_D = 1/D_{11}$ and $\nu_D = -J_D D_{12}$.

System (12) is integrated using the following boundary conditions:

$$w = 0, \quad M_r = 0 \quad (r = R) \tag{13}$$

for hinged edge and

$$w = 0, \quad \psi_r = 0 \quad (r = R) \tag{14}$$

for clamped edge.

With the shear strain and rotary inertia neglected, system (12) goes over into the equations [15], which are based on the Kirchhoff–Love hypotheses generalized to the case of electroelasticity.

When the temperature dependence of the electric and mechanical characteristics of the piezoactive and passive materials is allowed for, the system of equations (12)–(14) together with the energy equations (9)–(11) defines the coupled nonlinear electrothermoelastic problem for the plate subject to harmonic deformation. To solve it, we use the time stepping method [6]. At each step, the boundary-value problem (12)–(14) is solved by the discrete-orthogonalization method using a standard program [1]. After calculating the dissipation function (10), the heat-conduction problem (9), (10) is solved by the explicit finite-difference method using the dimensionless space, $x = r/R$, and time, $\tau = at/R^2$, coordinates and the heat-transfer coefficients $\gamma_{0,R} = \alpha_{0,R} R/\lambda$. Note that the central point is singular in the case of a solid circular plate. Therefore, for numerical purposes, the plate is considered to have a small hole $r = \varepsilon$ with regularity conditions [4] for Eqs. (9) and thermal-insulation conditions for Eqs. (10) on its boundary:

$$Q_r = 0, \quad \vartheta = 0, \quad \frac{dT}{dr} = 0 \quad (r = \varepsilon). \tag{15}$$

To determine the optimal electric potential V_A that should be applied to the electrodes of the actuator to suppress the forced vibrations caused by surface pressure of amplitude P_0, the following linear relation is used:

$$V_A = -k_A P_0, \tag{16}$$

where k_A is the complex control coefficient. It is calculated by the following formula [7]:

$$k_A = \frac{R}{(h+\delta)} \int_0^1 wx dx \Big/ \int_0^{x_0} b_{31} \kappa x dx \quad (x_0 = r_0 / R), \tag{17}$$

which is valid at the resonant frequencies of mechanical vibrations of the plate. The minus sign in (16) means that the electric potential is applied in antiphase to balance the mechanical harmonic load.

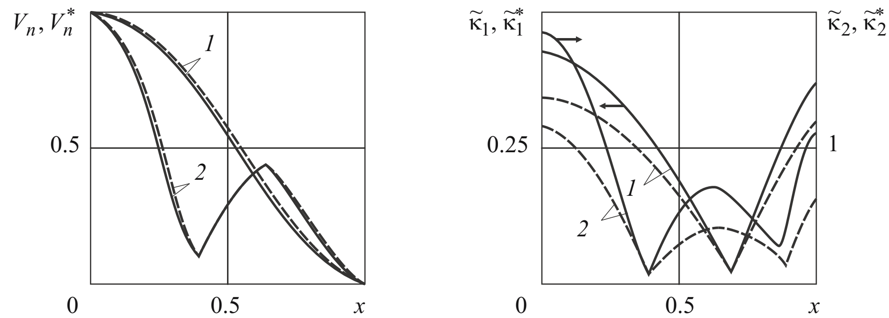
3. Calculated Results. The effect of shear strain and electromechanical coupling on the vibratory characteristics of the plate and damping characteristics of the actuator under mechanical and electric loads applied separately or together is examined for a polymer plate [11] with TsTStBS-2 piezoceramic actuators [2]. The temperature dependence of the complex shear modulus of the passive material (4) in the temperature range 20–80 °C is described by

$$G' = 968 - 8.69T \text{ MPa}, \quad G'' = 87.1 - 0.7T \text{ MPa}. \tag{18}$$

The other characteristics are the following: $\nu = 0.3636$, $\rho = 929$ kg/m^3, $\lambda = 0.47$ W/(m×deg), $\alpha_0 = \alpha_R = 2$ W/(m^2×deg).

The temperature approximations of the electromechanical characteristics of piezoceramics in (3) are the following ($20 \le T \le 160$ °C):

$$s'_{11} = s_{11}^0 \left(1 + 0.3077 \cdot 10^{-3}\, \overline{T}\right), \quad s'_{55} = s_{55}^0 \left(1 + 0.54575 \cdot 10^{-3}\, \overline{T}\right),$$



a *b*

Fig. 1

$$d'_{31} = d_{31}^0 \left(1 + 0.219 \cdot 10^{-2}\, \overline{T}\right), \qquad d'_{15} = d_{15}^0 \left(1 + 0.97221 \cdot 10^{-3}\, \overline{T}\right),$$

$$\varepsilon'_{11} = \varepsilon_{11}^0 (1 + 0.45045 \cdot 10^{-1}\, \overline{T}), \qquad \delta_{11} = \delta_{11}^0 (1 + 0.15 \cdot 10^{-1}\, \overline{T}),$$

$$\varepsilon'_{33} = \varepsilon_{33}^0 (1 + 0.111 \cdot 10^{-3}\, \overline{T} + 0.84256 \cdot 10^{-4}\, \overline{T}^2), \qquad \overline{T} = T - T_R,$$

$$\delta_{11}^s = \delta_{11}^0 (1 + 0.6155 \cdot 10^{-3}\, \overline{T} + 0.41575 \cdot 10^{-4}\, \overline{T}^2),$$

$$\delta_{55}^s = \delta_{55}^0 (1 + 0.8333 \cdot 10^{-2}\, \overline{T}) \cdots, \qquad \delta_{15}^d = \delta_{15}^0 (1 + 0.3571 \cdot 10^{-2}\, \overline{T}),$$

$$\delta_{31}^d = \delta_{31}^0 \left(1 + 1.198 \cdot 10^{-2}\, \overline{T} + 1.823 \cdot 10^{-4}\, \overline{T}^2\right),$$

$$\delta_{33} = \delta_{33}^0 \left(1 + 0.119 \cdot 10^{-1}\, \overline{T} + 0.119 \cdot 10^{-3}\, \overline{T}^2\right), \tag{19}$$

where $s_{11}^0 = 12.5 \cdot 10^{-12}$ m^2/N, $s_{12}^0 = -462 \cdot 10^{-2}$ m^2/N, $s_{55}^0 = 39.7 \cdot 10^{-12}$ m^2/N, $d_{31}^0 = -1.6 \cdot 10^{-10}$ C/m, $d_{15}^0 = 45 \cdot 10^{-10}$ C/m, $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m, $\delta_{11}^0 = 0.16 \cdot 10^{-2}$, $\delta_{55}^0 = 0.14 \cdot 10^{-2}$, $\delta_{31}^0 = 0.4 \cdot 10^{-2}$, $\delta_{15}^0 = 0.35 \cdot 10^{-2}$, $\varepsilon_{33}^0 = \varepsilon_0 \bar{\varepsilon}_{33}$, $\varepsilon_{11}^0 = \varepsilon_0 \bar{\varepsilon}_{11}$, $\bar{\varepsilon}_{33} = 21 \cdot 10^2$, $\bar{\varepsilon}_{11} = 18.5 \cdot 10^2$, $\delta_{33}^0 = 0.35 \cdot 10^{-2}$, $\delta_{11}^0 = 0.5 \cdot 10^{-2}$.

Moreover, the coefficient $\nu_E = -s_{12}^0 / s_{11}^0$ for the piezoceramics is considered constant and real; $\rho_* = 7520$ kg/m^3, $T_0 = T_c = T_R = 20$ °C. If the shear strain is allowed for, then it is assumed that $k_s = 5/6$ [3]. The radius of the plate $R = 0.2$ m, and $\varepsilon = 0.1 \cdot 10^{-6}$ m in (15).

Figure 1 shows the radial distribution of normalized amplitudes of deflections $(V_n, V_n^*) = |w_p| \cdot 10^{-2} / (A_n, A_n^*)$, $n = 1, 2$, (Fig. 1*a*) and total curvatures $(\widetilde{\kappa}_n, \widetilde{\kappa}_n^*) = |\kappa| \cdot 10^{-2} / (A_n, A_n^*)$ (Fig. 1*b*) for a clamped plate of thickness $h = 4$ cm with actuators of thickness $\delta = 0.1$ mm and relative radius $x_0 = 0.8$ under harmonic pressure $P_0 = 1$ Pa. The solid lines represent the results obtained using the Kirchhoff–Love hypotheses. The dashed lines represent the results (asterisked) obtained taking the shear strain into account. Curves *1* ($n = 1$) correspond to the first bending mode and $\omega_r = 5.32$ kHz, $A_1 = 0.2045 \cdot 10^{-7}$ m (solid lines), and curves *2* ($n = 2$) to the second mode and $\omega_r = 21.9$ kHz, $A_1 = 0.1016 \cdot 10^{-8}$ m (dashed lines). It can be seen that allowing for the shear strain in the calculation of the vibratory characteristics of the plate decreases the resonant frequencies ($\omega_r^* < \omega_r$), increases the maximum deflection ($A_{1,2}^* > A_{1,2}$), and has almost no effect on the vibration mode. The greatest effect of the shear strain is the redistribution of the normalized total curvature along the radius of the plate in the first and especially second vibration modes. This factor is decisive in assessing the effect of the shear strain on the control coefficient (17). As numerical calculations for thinner plates ($h / R \le 1/20$) show, the effect of these factors weakens with decreasing thickness. When the external edge is hinged (13), the shear strain shows a similar effect on the vibratory characteristics of the plate.

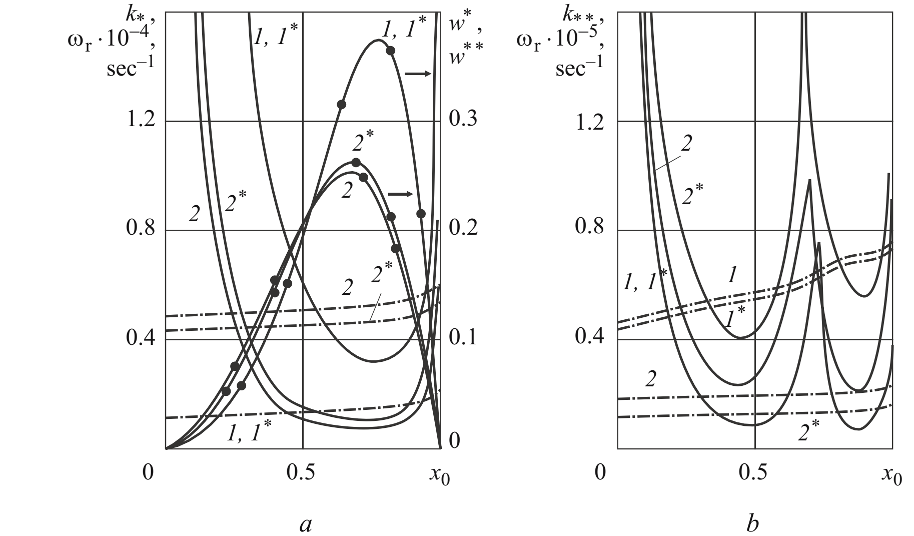


Fig. 2

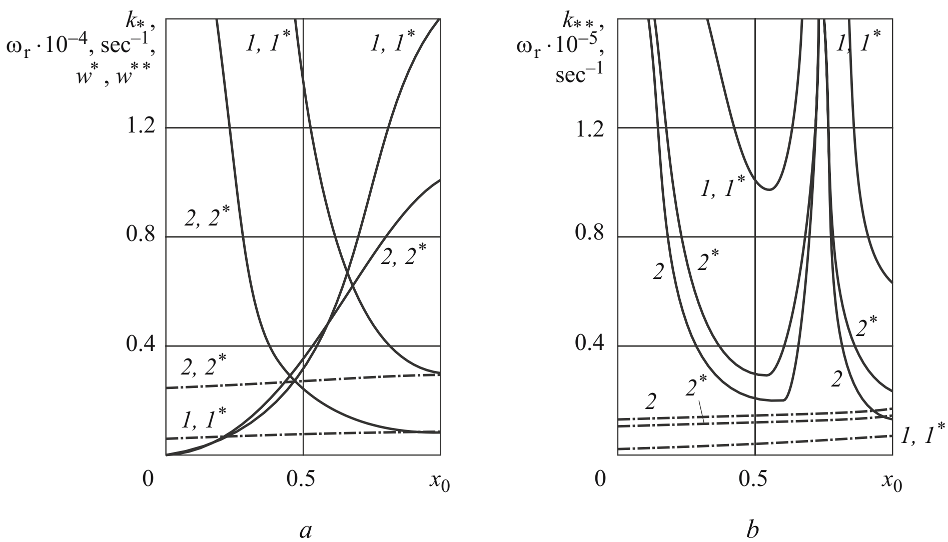


Fig. 3

Figure 2 shows the resonant frequency ω_r (dash-and-dot lines) and control coefficient $k_* = |k_A| \cdot 10$ V/Pa, $k_{**} = |k_A| \cdot 10^2$ V/Pa (solid lines) for damping of the first (Fig. 2*a*) and the second (Fig. 2*b*) modes of the forced flexural vibrations of a clamped plate under pressure P_0 = 1 Pa versus the dimensionless radius x_0 of the actuator of thickness $\delta = 0.1$ mm. Curves *1* and *2* represent the case of using the Kirchhoff–Love hypotheses and curves 1^* and 2^* the case of allowing for the shear strain. Curves *1* and 1^* correspond to a plate of thickness $h = 1$ cm, and curves *2* and 2^* to a plate of thickness $h = 4$ cm. Moreover, the solid curves with full circles in Fig. 2*a* show the maximum amplitude $w^* = |w_E(0)| \cdot 10^4$ m for a plate of thickness h = 1 cm (curves *1* and 1^*) and the maximum amplitude $w^{**} = |w_E(0)| \cdot 10^5$ m for a plate of thickness h = 4 cm (curves *2* and 2^*) electrically excited by a unit potential $V' = 1$ V, $V'' = 0$. Similar results for a hinged plate are presented in Fig. 3.

The calculations and the analysis of Figs. 2 and 3 reveal that an actuator with such dimensions that the deflection amplitude peaks under electric excitation (solid lines with full circles) is the most effective (optimal). The control coefficient is minimum (solid lines) in this case. The dimensions of such an actuator depend on the thickness of the passive layer. It can also be seen from Figs. 2 and 3 that the fixation conditions of the plate have qualitative and quantitative effects on the dimensions and control coefficient of the optimal actuator. For example, when a clamped plate is undergoing vibrations at the first resonant frequency, the maxima of the electrically excited amplitudes and control coefficient are ambiguous functions of the radius x_0 of the circular actuator, and the control coefficient reaches its minimum when the surfaces of the plate are incompletely covered

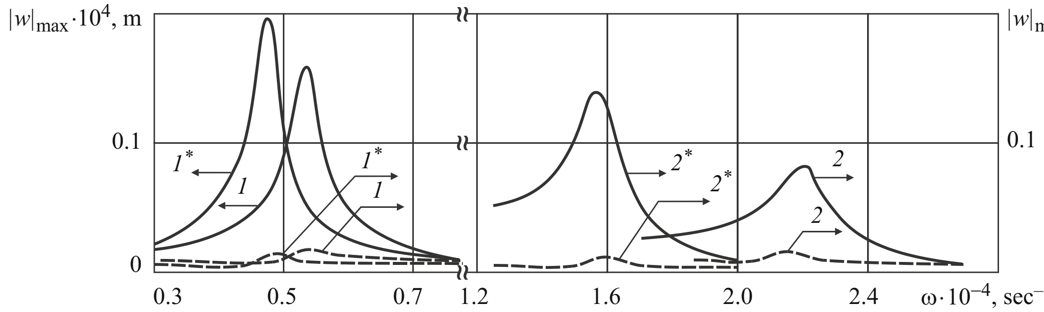


Fig. 4

with piezolayers ($0.6 \le x_0 \le 0.8$). If the plate is hinged (Fig. 3), these parameters increase monotonically (solid curves with circles) or decrease (solid curves) with increase in x_0, and the control coefficient $|k_A|$ of the optimal actuator reaches its minimum when the surfaces of the plate are completely coated ($x_0 = 1$). Allowing for the shear strain increases the control coefficient. This effect becomes stronger with increasing thickness of the plate and increasing frequency to the second resonance and higher. When the plate is undergoing forced vibrations at the second resonant frequency, the control coefficient is an ambiguous function of x_0 for both types of boundary conditions (Figs. 2*b* and 3*b*).

Figure 4 shows the amplitude–frequency characteristics (AFCs) for a clamped plate of thickness h = 4 cm with an actuator of relative radius x_0 = 0.8 and thickness $\delta = 0.1$ mm subject to harmonic excitation at the first (curves *1* and 1^*) and second (curves *2* and 2^*) resonant frequencies. Curves *1* and *2* represent the case of using the Kirchhoff–Love hypotheses, and curves 1^* and 2^* to the case of allowing for the shear strain. The solid lines correspond to the excitation of vibrations by external pressure $P_0 = 0.8$ kPa. The dashed lines represent the excitation by this mechanical load and an electric potential V_A of the same frequency applied in antiphase to the actuators and having the following active, V_A', and reactive, V_A'', amplitude components, according to (16): $V_A' = -6.923$ V, $V_A'' = -0.02856$ V (curves *1*); $V_A' = -8.848$ V, $V_A'' = -0.05365$ V (curves 1^*); $V_A' = -1.990$ V, $V_A'' = -0.03166$ V (curves *2*); $V_A' = -5.541$ V, $V_A'' = -0.3418$ V (curves 2^*). These results, as in the above examples, have been obtained for the isothermal ($T = T_R$) values of (18) and (19).

Comparing the curves (continuous and dashed) in Fig. 4, we conclude that it is possible to suppress mechanically excited resonant vibrations by using an actuator of optimal configuration and applying an electric potential with a frequency equal to that of mechanical resonance and an amplitude calculated by formula (16) to its electrodes in antiphase. The calculations show that allowing for the shear strain decreases the resonant frequencies and increases the amplitudes of deflections during forced vibrations at these frequencies.

Figure 4 and numerical experiments indicate that the same optimal actuator can be used to damp forced vibrations of the plate at either first or second resonant frequency. However, since the first mode is very intensive, it is damped by an electric potential of higher amplitude than that needed to damp the second mode. Moreover, numerical calculations establish that the reactive component V_A'' has almost no effect on the AFCs shown by dashed lines.

Figures 5 and 6 demonstrate the effect of the shear strain and the temperature dependence of the electromechanical properties (18) and (19), respectively, on the frequency dependence of the maximum amplitude of relative deflections $w_{max} = |w(0)|/h$ and dissipative heating temperature $T_{max} = T(0)$ of a plate undergoing forced vibrations excited by load P_0 = 1.4 kPa.

Figures 5*a* and 6*a* characterize the behavior of the system at the first resonant frequency, while Figs. 5*b* and 6*b* at the second resonant frequency for a clamped plate of radius R = 0.2 m with an actuator of relative radius x_0 = 0.8 and thickness δ = 0.1 mm for the following coefficients: $\alpha = 2$ W/(m^2·deg), $\lambda = 0.47$ W/(m·deg) and $T_0 = T_R = 20$ °C. The solid curves correspond to material properties independent of temperature, and the dashed curves to temperature-dependent properties. Curves *1* and *2* represent the case of using the Kirchhoff–Love hypotheses, and curves 1^* and 2^* the case of allowing for the shear strain. It can be seen that the temperature dependence of material properties (dashed lines) makes the amplitude– and temperature–frequency characteristics soft, which is a well-known effect [6], the resonant frequency, amplitude, and dissipative heating temperature decreasing. The contribution of the thermomechanical coupling to the isothermal case (solid lines) in the refined problem formulation is of the same order as in the classical problem formulation.

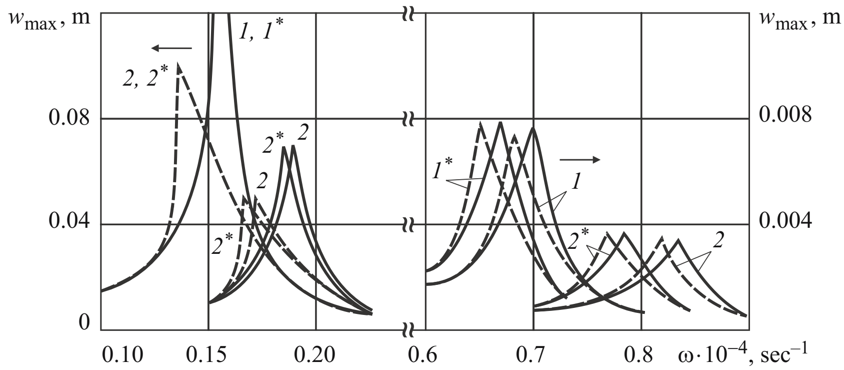


Fig. 5

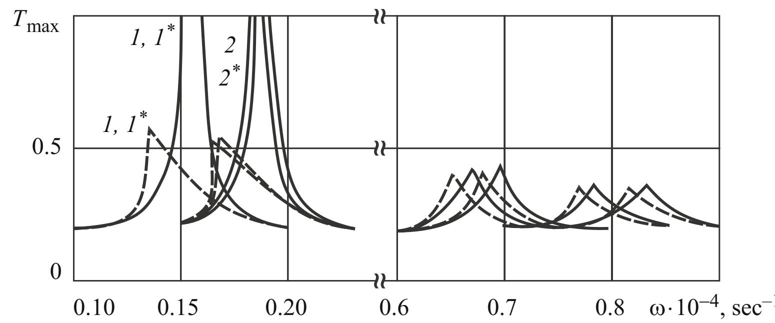


Fig. 6

TABLE 1

h / R	Kirchhoff–Love hypotheses			Timoshenko hypotheses		
	$\omega_r \cdot 10^{-4}$, Hz	$\lvert V_A \rvert$, V	$\lvert V_A^T \rvert$, V	$\omega_r \cdot 10^{-4}$, Hz	$\lvert V_A \rvert$, V	$\lvert V_A^T \rvert$, V
1/20	0.155	44.62	43.43	0.155	45.64	44.47
	0.700	16.64	17.90	0.670	18.95	20.56
1/16	0.190	36.18	35.27	0.185	37.32	36.27
	0.830	12.28	13.21	0.780	14.76	15.98

Table 1 characterizes the influence of the shear strain according to the Timoshenko hypotheses and the temperature dependence of material properties on the amplitude of electric potential $|V_A|$ applied to the actuators to balance the mechanical load $P_0 = 1.4$ kPa at the frequencies ω_r of the first two isothermal resonances of flexural vibrations. Here $|V_A|$ and $|V_A^T|$ have been calculated using the isothermal values of the electromechanical characteristics ($T = T_R$) and taking into account their dependence on the dissipative heating temperature (18), (19). It can be seen that the shear strain somewhat increases $|V_A|$. The thicker the plate and the higher the resonant frequency, the greater this increase. Allowing for the thermomechanical coupling reduces a little the parameter $|V_A^T|$ at the first resonant frequency and increases at the second resonant frequency.

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