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THERMOELASTOPLASTIC DEFORMATION OF NONCIRCULAR CYLINDRICAL SHELLS

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A method to determine the nonstationary temperature fields and the thermoelastoplastic stress–strain state of noncircular cylindrical shells is developed. It is assumed that the physical and mechanical properties are dependent on temperature. The heat-conduction problem is solved using an explicit difference scheme. The temperature variation throughout the thickness is described by a power polynomial. For the other two coordinates, finite differences are used. The thermoplastic problem is solved using the geometrically nonlinear theory of shells based on the Kirchhoff–Love hypotheses. The theory of simple processes with deformation history taken into account is used. Its equations are linearized by a modified method of elastic solutions. The governing system of partial differential equations is derived. Variables are separated in the case where the curvilinear edges are hinged. The partial case where the stress–strain state does not change along the generatrix is examined. The systems of ordinary differential equations obtained in all these cases are solved using Godunov's discrete orthogonalization. The temperature field in a shell with elliptical cross-section is studied. The stress–strain state found by numerical integration along the generatrix is compared with that obtained using trigonometric Fourier series. The effect of a Winkler foundation on the stress–strain state is analyzed

Keywords: thermoelastoplasticity, noncircular cylindrical shell, Kirchhoff–Love hypotheses, linearization method, explicit difference scheme, Godunov's discrete orthogonalization, cylindrical shell of elliptical cross-section

Introduction. Methods and elastic problems of designing noncircular cylindrical shells with arbitrary cross-section and arbitrary thickness are addressed in [3–6, 8]. These methods were further developed and some problems were solved in [14–21, 29]. The thermoelastoplastic stress–strain state (SSS) of this class of inelastic shells is analyzed below. To calculate thermal stresses, we will preliminarily solve the nonstationary heat-conduction problem for shells that transfer heat to the environment by convection.

1. Problem Formulation. Basic Equations. Let us determine the thermoelastoplastic SSS of a cylindrical shell with arbitrary cross-section and thickness varying in two directions. The shell can be coupled with an elastic foundation so that there can be no separation between them. At time zero, the shell, which is unstressed at temperature T_0, is subjected to mechanical and thermal loads that do not cause buckling. We will formulate a noncoupled quasistatic problem and use the geometrically nonlinear theory of shells to solve it. The meridian and thickness of the shell and the applied loads permit accepting the Kirchhoff–Love hypotheses. The physical and mechanical characteristics of the shell material are assumed temperature-dependent.

The position of points on the mid-surface of the shell is defined by the longitudinal coordinate s ($s_0 \le s \le s_N$) and the circumferential coordinate q ($q_0 \le q \le q_N$) (Fig. 1).

Let the mid-surface be bounded by the principal curvature lines s = const and q = const. The distance from an arbitrary point of the shell to the mid-surface is denoted by ζ ($-h/2 \le \zeta \le h/2$), where $h(s, q)$ is the thickness of the shell. The cross section

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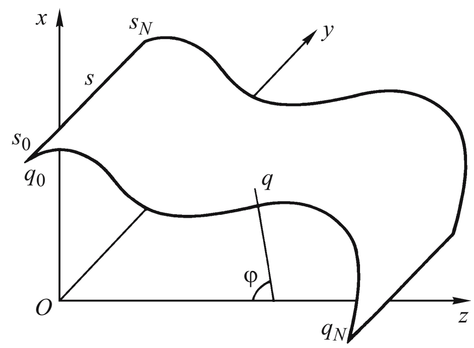

Fig. 1

lies in the plane xz of the Cartesian coordinate system $Oxyz$ with the Oy-axis being collinear with the generatrix s. The mid-surface in described by the following equations in Cartesian coordinates:

$$x = x(q), \qquad z = z(q), \qquad y = s. \tag{1.1}$$

The Lamé parameters $A_s = A_q = 1$; the principal curvatures $k_s = 0$, $k_q = d\varphi / dq$, where $\pi - \varphi$ is the angle between the normal to the mid-surface and the z-axis (Fig. 1).

The loading process is divided into rather small steps. Their number and length are selected so as to describe with adequate accuracy the deformation process in each element of the shell. The loading process is considered to be such that elastic unloading may occur in individual elements after active elastoplastic deformation. In turn, after elastic unloading, there may be reloading in either the initial or opposite direction. It is assumed that the strain paths of elements under initial loading slightly deviate from straight lines and the strain paths under unloading and reloading slightly deviate from the strain paths under initial loading. It is also assumed that while the shell deforms, stress, temperature, and time change within such limits that the rheological properties of the material can be neglected. The geometrically nonlinear theory of shells of the second order [9] assumes that strains and shears are small compared with unity and retains the terms with squared angles between the normal and the mid-surface in the kinematic equations and the nonlinear terms in the equilibrium equations. It is also assumed that the difference between the directions of unit vectors in the deformed and undeformed coordinate systems has a minor effect on the stress–strain relationship.

The displacement components of an arbitrary point of the shell are expressed in terms of the displacement components u, v, and w of the mid-surface and the angles between the normal and the mid-surface ϑ_s and ϑ_q as follows:

$$u_\zeta = u + \zeta\vartheta_s, \qquad v_\zeta = v + \zeta\vartheta_q, \qquad w_\zeta = w - \frac{1}{2}\zeta(\vartheta_s^2 + \vartheta_q^2). \tag{1.2}$$

The strain components at a point of the shell are expressed in terms of the strain components $\varepsilon_s, \varepsilon_q, \varepsilon_{sq}, \kappa_s, \kappa_q$, and κ_{sq} of the mid-surface (up to $k_q\zeta$ compared with unity) as follows:

$$\varepsilon_{ss}^{\varsigma} = \varepsilon_s + \zeta\kappa_s, \qquad \varepsilon_{qq}^{\varsigma} = \varepsilon_q + \zeta\kappa_q, \qquad \varepsilon_{sq}^{\varsigma} = \varepsilon_{sq6} + \zeta\kappa_{sq}. \tag{1.3}$$

Let us replace the components of the stress tensor by their integral characteristics—forces and moments. The differential equilibrium equations for the mid-surface's element bounded by arcs of the coordinate lines and contacting with the elastic foundation are the following [5]:

$$\frac{\partial N_s}{\partial s} + \frac{\partial \bar{S}}{\partial q} + q_s = 0, \qquad \frac{\partial \bar{S}}{\partial s} + \frac{\partial N_q}{\partial q} + k_q\left(Q_q + \frac{\partial H}{\partial s}\right) + q_q = 0,$$

$$\frac{\partial H}{\partial q} + \frac{\partial M_s}{\partial s} - Q_s - (N_s - k_q M_q)\vartheta_s - \bar{S}\vartheta_q + m_s = 0,$$

$$\frac{\partial M_q}{\partial q}+\frac{\partial H}{\partial s}-Q_q-N_q\vartheta_q-\bar{S}\vartheta_s+m_q=0, \tag{1.4}$$

where N_s, N_q, $\bar{S}$, Q_s, Q_q, M_s, M_q, and H are forces and moments; q_s, q_q, q_ζ, m_s, and m_q are the components of the distributed load in the mid-surface statically equivalent to mass and surface forces; μ_ζ is the Winkler modulus.

The nonlinear kinematic equations are the following [5]:

$$\varepsilon_s=\frac{\partial u}{\partial s}+\frac{1}{2}\vartheta_s^2,\quad \varepsilon_q=\frac{\partial v}{\partial q}+k_q w+\frac{1}{2}\vartheta_q^2,\quad \varepsilon_{sq}=\frac{\partial u}{\partial q}+\frac{\partial v}{\partial s}+\vartheta_s\vartheta_q,$$

$$\kappa_s=\frac{\partial\vartheta_s}{\partial s},\quad \kappa_q=\frac{\partial\vartheta_q}{\partial q}-\frac{1}{2}k_q\vartheta_q^2,\quad \kappa_{sq}=\frac{\partial\vartheta_q}{\partial s}+\frac{\partial\vartheta_s}{\partial q}+k_q\frac{\partial v}{\partial s},$$

$$\vartheta_s=-\frac{\partial w}{\partial s},\quad \vartheta_q=-\frac{\partial w}{\partial q}+k_q v. \tag{1.5}$$

The forces N_s, N_q, $\bar{S}$, the moments M_s, M_q, H, and the strains of the mid-surface are related as follows [10]:

$$N_s=D_N(\varepsilon_s+\nu_0\varepsilon_q+P_s),\quad N_q=D_N(\varepsilon_q+\nu_0\varepsilon_s+P_q),\quad \bar{S}=\frac{1}{2}D_N(1-\nu_0)(\varepsilon_{sq}+P),$$

$$M_s=D_M(\kappa_s+\nu_0\kappa_q+I_s),\quad M_q=D_M(\kappa_q+\nu_0\kappa_s+I_q),\quad H=\frac{1}{2}D_M(1-\nu_0)(\kappa_{sq}+I),$$

$$D_N=\frac{2G_0h}{1-\nu_0},\quad D_M=\frac{G_0h^3}{6(1-\nu_0)}. \tag{1.6}$$

Equations (1.6) have been written in the form of Hooke's law including additional integral terms P_s, P_q, P, I_s, I_q, and I to describe plastic and thermal strains and the temperature dependence of the mechanical properties of the material. In deriving Eq. (1.6), we considered the fact that the strains change linearly throughout the thickness and neglected $k_q\zeta$ compared with unity. The stress–strain relationship (hereafter the index ζ for strains is omitted) is written using the theory of simple deformation processes and linearized by the modified method of elastic solutions [22, 25]:

$$\sigma_{ss}=\frac{2G_0}{1-\nu_0}(\varepsilon_{ss}+\nu_0\varepsilon_{qq}+\beta_{ss0})\quad (s\leftrightarrow q),\quad \sigma_{sq}=2G_0(\varepsilon_{sq}+b_{sq0}), \tag{1.7}$$

$$\beta_{ss0}=(1-\omega)\beta_{ss}-\omega\varepsilon_{ss}-(\omega+\omega_2-\omega\omega_2)\nu_0\varepsilon_{qq}\quad (s\leftrightarrow q),\quad \beta_{sq0}=-\omega_1\varepsilon_{sq}-(1-\omega_1)\beta_{sq}, \tag{1.8}$$

$$\beta_{ss}=\frac{\nu^*\sigma_{\zeta\zeta}^{\mathrm{T}}-(1-\nu^*)\sigma_{ss}^{\mathrm{T}}}{2G^*}\quad (s\leftrightarrow q),\quad \beta_{sq}=-\frac{1}{2G^*}\sigma_{sq}^{\mathrm{T}}, \tag{1.9}$$

$$\sigma_{ss}^{\mathrm{T}}=K\varepsilon_{\mathrm{T}}\delta_{ss}+2G^*\varepsilon_{ss}^{1p}\quad (ss\leftrightarrow qq\leftrightarrow\zeta\zeta\leftrightarrow sq), \tag{1.10}$$

$$\varepsilon_{\zeta\zeta}=-\frac{\nu^*}{1-\nu^*}(\varepsilon_{ss}+\varepsilon_{qq})+\beta_{\zeta\zeta},\quad \beta_{\zeta\zeta}=\frac{1-2\nu^*}{2G^*(1-\nu^*)}\sigma_{\zeta\zeta}^{\mathrm{T}}, \tag{1.11}$$

$$\omega_1=1-\frac{G^*}{G_0},\quad \omega_2=1-\frac{\nu^*}{\nu_0},\quad \omega=1-\frac{(1-\omega_1)(1-\nu_0)}{1-\nu_0(1-\omega_2)}, \tag{1.12}$$

where $\varepsilon_{\mathrm{T}}=\alpha_{\mathrm{T}}(T-T_0)$ is the thermal strain; α_{T} is the coefficient of linear thermal expansion; T is the temperature at a point of the shell; T_0 is the initial temperature of the shell; K is the bulk modulus; G is the shear modulus; ν is Poisson's ratio; G_0, ν_0, K_0, and λ_0 are the respective quantities at the initial temperature T_0; δ_{ij} is the Kronecker delta; $2G^*=S/\Gamma$ is the secant modulus; ν^*

is Poisson's ratio; S is the tangential-stress intensity; Γ is the shear-strain intensity; σ_0 is the mean stress; and ε_0 is the mean deformation.

It is assumed that under initial loading the quantities S, Γ, and T are related by a function $S = F(\Gamma, T)$ that is independent of the stress mode and is determined from the thermomechanical surface $\sigma = f(\varepsilon, T)$ obtained in uniaxial tension tests for cylindrical specimens at different temperatures [11, 12]. Here σ and ε are the stress and strain in the specimen. The uniaxial and complex stress states are related by the formulas

$$S = \frac{\sigma}{\sqrt{3}}, \qquad \Gamma = \frac{1+\nu^*}{\sqrt{3}}\varepsilon, \qquad \nu^* = \frac{1}{2} - \frac{1-2\nu}{4G(1+\nu)}\frac{\sigma}{\varepsilon}. \tag{1.13}$$

Under elastic unloading, $G^* = G$. The quantities ε_{ss}^{1p} $(ss \leftrightarrow qq \leftrightarrow \zeta\zeta \leftrightarrow sq)$ are equal to zero under initial loading and to the components of the plastic-strain tensor at the time of unloading under unloading and reloading in the region of secondary plastic strains [13]. Under reloading in the region of secondary plastic strains, the function $S = F(\Gamma, T)$ accounts for the perfect Bauschinger effect. This makes it possible to use the function $S = F(\Gamma, T)$ obtained for initial loading instead of testing specimens under compression after tension to different levels of plastic strains. The relevant relations are presented in [13].

To ascertain how well the constitutive equations describe the deformation of the shell, it is necessary to draw strain paths in Il'yushin's three-dimensional subspace. The radii of curvature and torsions of these trajectories can be used to judge the reliability of the results [28, 26]. If the strain paths hardly deviate from straight lines, then it is justified to use the theory of deformation along straight-line paths. Otherwise, it would be necessary to use more complicated theories of plasticity [12].

Note that Eqs. (1.4) and (1.5) are based on the Kirchhoff–Love hypotheses and the condition $\varepsilon_{\zeta\zeta} = 0$. At the same time, the strain $\varepsilon_{\zeta\zeta}$ (1.11) in the plastic relations is determined from the condition $\sigma_{\zeta\zeta} = 0$.

2. Determining Nonstationary Temperature Fields. To determine the temperature fields, it is necessary to solve the nonstationary heat-conduction problem for cylindrical shells with arbitrary cross-section. It is assumed that there are no heat sources and that the thermal and physical characteristics of the material may depend on temperature. The initial temperature of the shells is given by the function T_0

$$T = T_0(s, q, \zeta) \quad \text{at} \quad t = 0. \tag{2.1}$$

The shell transfer heat to the environment by convection:

$$\lambda\frac{\partial T}{\partial \nu_1} = -\alpha_1(s,q)(T-\Theta_1) \ \text{ at } \ \zeta = \frac{h}{2}, \quad \lambda\frac{\partial T}{\partial \nu_2} = -\alpha_2(s,q)(T-\Theta_2) \ \text{ at } \ \zeta = -\frac{h}{2},$$

$$\lambda\frac{\partial T}{\partial s} = \alpha_3(q)(T-\Theta_3) \ \text{ at } \ s = s_0, \quad \lambda\frac{\partial T}{\partial s} = -\alpha_4(q)(T-\Theta_4) \ \text{ at } \ s = s_N,$$

$$\lambda\frac{\partial T}{\partial q} = \alpha_5(s)(T-\Theta_5) \ \text{ at } \ q = q_0, \quad \lambda\frac{\partial T}{\partial q} = -\alpha_6(s)(T-\Theta_6) \ \text{ at } \ q = q_N, \tag{2.2}$$

where t is time; Θ_1, Θ_2, Θ_3, Θ_4, Θ_5, and Θ_6 are the temperatures of the ambient media; $\alpha_1(T)$, $\alpha_2(T)$, $\alpha_3(T)$, $\alpha_4(T)$, $\alpha_5(T)$, and $\alpha_6(T)$ and $\lambda(T)$ are the temperature-dependent heat-transfer and thermal-conductivity coefficients; ν_1 and ν_2 are the outward normals to the boundary surfaces $\zeta = h(s, q)/2$ and $\zeta = -h(s, q)/2$.

To determine the nonstationary temperature field of an arbitrary cylindrical shell, it is necessary to integrate the equation

$$\frac{\partial}{\partial s}\left(\lambda\frac{\partial T}{\partial s}\right) + \frac{\partial}{\partial q}\left(\lambda\frac{\partial T}{\partial q}\right) + \frac{\partial}{\partial \zeta}\left(\lambda\frac{\partial T}{\partial \zeta}\right) + k_q\lambda\frac{\partial T}{\partial \zeta} = c\rho\frac{\partial T}{\partial t}, \tag{2.3}$$

where $c = c(T)$ is heat capacity; $\rho = \rho(T)$ is the density of the shell material. The differential equation (2.3) has been derived from the heat-conduction equation [11] written in arbitrary curvilinear orthogonal coordinates and including the Lamé coefficients with $k_q\zeta$ neglected as small compared with unity.

To simplify the solution of the nonstationary heat-conduction problem, we introduce a function Φ [24]:

$$\lambda\frac{\partial T}{\partial s}=\frac{\partial\Phi}{\partial s},\quad \lambda\frac{\partial T}{\partial q}=\frac{\partial\Phi}{\partial q},\quad \lambda\frac{\partial T}{\partial\zeta}=\frac{\partial\Phi}{\partial\zeta},\quad \lambda\frac{\partial T}{\partial t}=\frac{\partial\Phi}{\partial t}. \tag{2.4}$$

Then

$$\Phi=\int_0^T\lambda dT \tag{2.5}$$

and Eqs. (2.1)–(2.3) become

$$\frac{\partial^2\Phi}{\partial s^2}+\frac{\partial^2\Phi}{\partial q^2}+\frac{\partial^2\Phi}{\partial\zeta^2}+k_q\frac{\partial\Phi}{\partial\zeta}=\frac{1}{a}\frac{\partial\Phi}{\partial t}, \tag{2.6}$$

$$\frac{\partial\Phi}{\partial n_i}=-\alpha_i(T-\Theta_i),\quad i=1,\dots,6, \tag{2.7}$$

$$\Phi=\Phi_0=\int_0^{T_0}\lambda dT\quad\text{at}\quad t=0, \tag{2.8}$$

where $a=\lambda/(c\rho)$ is the thermal-diffusivity coefficient.

In solving Eq. (2.6), we approximate the function Φ by a power polynomial of order n in the thickness coordinate [1, 7, 23, 24], which makes it possible to reduce the solution of the problem to the determination of integral functions independent of the coordinate ζ:

$$\Phi(s,q,\zeta,t)=\sum_{p=0}^{n}\Phi_p(s,q,t)\zeta^p. \tag{2.9}$$

To determine the functions Φ_p appearing in (2.9), we multiply the differential equation (2.6) by ζ^p and integrate it over ζ $(-h/2\le\zeta\le h/2)$. Transforming the resulting integrals and introducing the function

$$v_p(s,q,t)=\int_{-h/2}^{h/2}\Phi(s,q,\zeta,t)\zeta^p d\zeta,\quad p=0,1,\dots,n, \tag{2.10}$$

we arrive at the heat-conduction equations written for the integral functions v_p:

$$\begin{aligned}\frac{1}{a}\frac{\partial v_p}{\partial t}&=\frac{\partial^2 v_p}{\partial s^2}+\frac{\partial^2 v_p}{\partial q^2}-pk_q v_{p-1}+p(p-1)v_{p-2}\\
&-\frac{1}{2^{p+1}}\left[\frac{\partial}{\partial s}\left(h^p\frac{\partial h}{\partial s}\Phi^+\right)+\frac{\partial}{\partial q}\left(h^p\frac{\partial h}{\partial q}\Phi^+\right)+\frac{\partial}{\partial s}\left(h^p\frac{\partial h}{\partial s}\Phi^-\right)+\frac{\partial}{\partial q}\left(h^p\frac{\partial h}{\partial q}\Phi^-\right)\right]\\
&+\left(\frac{h}{2}\right)^{p-1}\left[\left(\frac{hk_q}{2}-p\right)\Phi^+ +\left(\frac{hk_q}{2}+p\right)\Phi^-\right]\\
&-\left(\frac{h}{2}\right)^{p}\sqrt{1+\frac{1}{4}\left[\left(\frac{\partial h}{\partial s}\right)^2+\left(\frac{\partial h}{\partial q}\right)^2\right]}\left[\alpha_1(T^+-\Theta_1)-\alpha_2(T^--\Theta_2)\right],\end{aligned} \tag{2.11}$$

where

$$T^{+}(s,q,t)=T(s,q,h/2,t),\qquad T^{-}(s,q,t)=T(s,q,-h/2,t),$$

$$\Phi^{+}(s,q,t)=\Phi(s,q,h/2,t),\qquad \Phi^{-}(s,q,t)=\Phi(s,q,-h/2,t). \tag{2.12}$$

In deriving Eqs. (2.11), we assumed that the thermal-diffusivity coefficient a hardly varies throughout the thickness so that this variation can be neglected. We similarly transform conditions (2.7) and (2.8), also assuming that the heat-transfer coefficients α_3, α_4, α_5, and α_6 hardly vary throughout the thickness. As a result, we have

$$\frac{\partial v_p}{\partial s}=\alpha_3(\bar{v}_p-\Theta_{3p})+\frac{1}{2^{p+1}}h^p\frac{\partial h}{\partial s}(T^{+}-T^{-}),$$

$$\frac{\partial v_p}{\partial s}=-\alpha_4(\bar{v}_p-\Theta_{4p})+\frac{1}{2^{p+1}}h^p\frac{\partial h}{\partial s}(T^{+}-T^{-}), \tag{2.13}$$

$$\frac{\partial v_p}{\partial q}=\alpha_5(\bar{v}_p-\Theta_{5p})+\frac{1}{2^{p+1}}h^p\frac{\partial h}{\partial s}(T^{+}-T^{-}),$$

$$\frac{\partial v_p}{\partial q}=-\alpha_6(\bar{v}_p-\Theta_{6p})+\frac{1}{2^{p+1}}h^p\frac{\partial h}{\partial s}(T^{+}-T^{-}), \tag{2.14}$$

$$v_p=T_{0p}\quad \text{at}\quad t=0, \tag{2.15}$$

where

$$\Theta_{ip}=\int_{-h/2}^{h/2}\Theta_i\zeta^p\,d\zeta,\quad i=3\text{–}6,\ \ p=0,1,\ldots,n;\qquad \bar{v}_p=\int_{-h/2}^{h/2}T\zeta^p\,d\zeta,\qquad T_{0p}=\int_{-h/2}^{h/2}\Phi_0\zeta^p\,d\zeta. \tag{2.16}$$

Let us use an explicit difference scheme in time to solve system (2.11):

$$v_p(t+\Delta t)=v_p(t)+a\Delta t F_p(t),\qquad p=0,1,\ldots,n, \tag{2.17}$$

where Δt is the time step; $F_p(t)$ is the right-hand side of (2.11).

The derivatives with respect to s and q in the expression for $F_p(t)$ are approximated by finite-difference formulas [7, 15, 16]. To this end, the generatrix of the mid-surface is divided by K_s nodal points into $K_s - 1$ intervals of variable length Δs. The directrix of the mid-surface is divided by K_q nodal points into $K_q - 1$ intervals. We use the following finite-difference approximations in s:

$$\left[\frac{\partial v}{\partial s}\right]_i=\frac{v_{i+1}-v_{i-1}}{\Delta s_{i-1}+\Delta s_i},\qquad \left[\frac{\partial^2 v}{\partial^2 s}\right]_i=\frac{(v_{i+1}-v_i)\Delta s_{i-1}-(v_i-v_{i-1})\Delta s_i}{\Delta s_i\Delta s_{i-1}(\Delta s_{i-1}+\Delta s_i)}, \tag{2.18}$$

where i is the number of an arbitrary node in the s-direction ($1\le i\le K_s$).

The derivatives with respect to q can be represented in a similar manner. To exclude points beyond the boundary, we use the boundary conditions (2.13) and (2.14) where the derivatives with respect to s and q are also approximated as in (2.18). Then relations (2.17) can be used to determine the function v_p at the time $t + \Delta t$ from its values at the time t. Next, we find the function Φ_p. To this end, we substitute (2.9) into (2.10) and perform transformations to obtain

$$v_p=\sum_{l=0}^{n}\left[\left(\frac{h}{2}\right)^{l+p+1}\frac{1-(-1)^{l+p+1}}{l+p+1}\right]\Phi_l,\qquad p=0,1,\ldots,n. \tag{2.19}$$

The values of Φ_l found by solving the system of linear algebraic equations (2.19) are then used to determine the function Φ (2.9) and then the temperature T (2.5) at all nodal points.

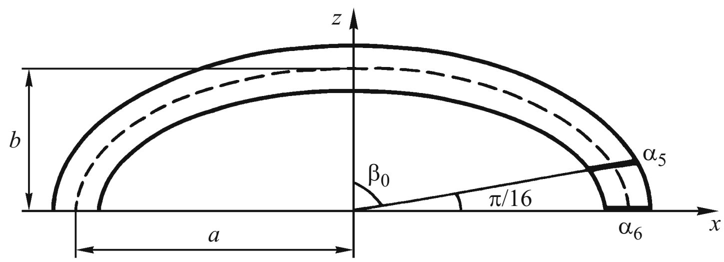

Fig. 2

3. Analyzing the Temperature Field in a Shell of Elliptic Cross-Section. As an illustrative example, let us determine the nonstationary temperature field of a cylindrical shell with infinite length and elliptic cross-section (Fig. 2).

The Cartesian coordinates x and z (1.1) defining the position of an arbitrary point on the mid-surface of the shell are specified parametrically using an angle β:

$$x = a \sin \beta, \quad z = b \sin \beta. \tag{3.1}$$

In the calculations, the angle β varies within the limits $\beta_0 \le \beta \le \pi/2$, $\beta_0 = 7\pi/16$ (see Fig. 2 for the sense of the angle). This choice of the limits is because the temperature remains constant in the circumferential direction as the angle changes from 0 to β_0. One of the ellipse semiaxes a = 5 m. The thickness and semiaxis b were not varied in the calculations. The initial temperature, the temperature of the environment, and the heat-transfer factors are the following:

$$T_0 = 293 \text{ K}, \ \Theta_1 = 373 \text{ K}, \ \Theta_2 = 873 \text{ K}, \ \alpha_1 = \alpha_2 = 500 \text{ W/(m}^2\cdot\text{deg)}, \ \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0. \tag{3.2}$$

The thermal-conductivity and thermal-diffusivity coefficients are $\lambda = 20$ W/(m^2·deg), $a = 4{\cdot}10^{-4}$ m/sec. The number of nodes partitioning the directrix $K_q = 33$, while coordinate β varies within $\beta_0 \le \beta \le \pi/2$. The number of nodes partitioning the thickness $K_\zeta = 9$. In the power series (2.9), $n = 7$. The step of integration over time $\Delta t = 0.01$ sec. Numerical experiments confirm that with this partitioning, the results do not depend on the discretization parameters. Some results calculated at $t = 10$ sec are presented in Tables 1 and 2. These are temperatures on the outside ($\zeta = h/2$), middle ($\zeta = 0$), and inside ($\zeta = -h/2$) surfaces calculated for different values of a and b and for $h = 0.01$ m (Table 1) and $h = 0.005$ m (Table 2).

The variation of the temperature throughout the thickness is essentially nonlinear. Of especial interest, however, is the considerable decrease in the circumferential temperature at high ellipticity in a small neighborhood of $\beta = \pi/2$, where the curvature of the ellipse is maximum. With $b = 5$ m, the ellipse turns into a circle, and, naturally, the circumferential temperature remains constant. At great values of b ($b = 1$; 0.5 m), the change in the temperature is insignificant. In the case of high ellipticity ($b = 0.2$; 0.1 m), the temperature can decrease by 25%.

4. Governing Equations to Determine the Stress–Strain State of Cylindrical Shells. The equilibrium equations (1.4), the kinematic equations (1.5), the thermoplastic equations (1.6), and the boundary conditions at the edges of the shell constitute a closed-form system of equations for solving the problem with external loads prescribed and the temperature fields found.

For the basic unknowns of the governing equations, we choose functions in terms of which the boundary conditions at the shell edges are expressed:

$$N_q, \bar{S}, \hat{Q}_q, M_q, u, v, w, \vartheta_q, \tag{4.1}$$

where $\hat{Q}_q = Q_q + \dfrac{\partial H}{\partial s}$ is the reduced transverse force.

Functions (4.1) and their linear combinations make it possible to formulate a wide range of boundary conditions for forces, moments, displacements, and their combinations. Using the equilibrium, kinematic, and thermoplastic equations, we obtain a system of partial differential equations for the vector of unknown functions:

TABLE 1

b, m	ζ	$7\pi/16$	$15\pi/32$	$61\pi/128$	$31\pi/64$	$63\pi/128$	$\pi/2$
5	$h/2$	83	83	83	83	83	83
	0	116	116	116	116	116	116
	$-h/2$	422	422	422	422	422	422
1	$h/2$	83	83	83	83	83	83
	0	115	114	114	114	114	114
	$-h/2$	421	420	420	420	419	419
0.5	$h/2$	83	83	83	83	83	83
	0	115	113	112	110	108	108
	$-h/2$	421	418	417	415	413	412
0.2	$h/2$	83	83	83	83	84	85
	0	116	113	110	103	92	87
	$-h/2$	421	418	414	405	386	374
0.1	$h/2$	86	83	83	83	85	87
	0	116	114	111	103	91	83
	$-h/2$	422	420	416	405	375	345

TABLE 2

b, m	ζ	$7\pi/16$	$15\pi/32$	$61\pi/128$	$31\pi/64$	$63\pi/128$	$\pi/2$
5	$h/2$	155	155	155	155	155	155
	0	256	256	256	256	256	256
	$-h/2$	433	433	433	433	433	433
1	$h/2$	154	154	154	154	154	154
	0	255	255	255	255	255	255
	$-h/2$	432	431	431	431	430	430
0.5	$h/2$	155	153	152	151	149	149
	0	255	252	250	247	245	244
	$-h/2$	432	429	428	426	424	424
0.2	$h/2$	155	153	150	145	136	131
	0	256	252	247	235	216	205
	$-h/2$	432	430	426	417	401	390
0.1	$h/2$	155	154	152	145	134	127
	0	256	254	249	236	208	189
	$-h/2$	433	431	428	417	390	363

$$\frac{\partial \vec{Y}}{\partial q} = A(s,q)\vec{Y} + \vec{F}(s,q), \tag{4.2}$$

where $\vec{Y}(s,q) = \{y_1, \ldots, y_8\}$ is the vector of unknown functions (4.1), $A(s, q) = \{a_{ij}(s, q)\}$ is a differential operator containing partial derivatives of the unknown functions with respect to the coordinate s; $\vec{F}(s,q) = \{f_i(s, q)\}$ is the vector of free terms; $i, j = 1$, 2,..., 8.

System (4.2) has the following expanded form:

$$\frac{\partial N_q}{\partial q} = -\frac{\partial \bar{S}}{\partial s} - k_q \hat{Q}_q - q_q, \quad \frac{\partial \bar{S}}{\partial q} = -\nu_0 \frac{\partial N_q}{\partial s} - D_N (1-\nu_0^2)\frac{\partial^2 u}{\partial s^2} - \frac{1}{2} D_N (1-\nu_0^2)\frac{\partial \vartheta_s^2}{\partial s}$$

$$-D_N \frac{\partial P_s}{\partial s} + \nu_0 D_N \frac{\partial P_q}{\partial s} - q_s, \quad \frac{\partial \hat{Q}_q}{\partial q} = k_q N_q - \nu_0 \frac{\partial^2 M_q}{\partial s^2} + \left[D_M (1-\nu_0^2)\frac{\partial^4}{\partial s^4} + \mu_\varsigma \right] w$$

$$-D_M \frac{\partial^2 I_s}{\partial s^2} + \nu_0 D_M \frac{\partial^2 I_q}{\partial s^2} + \frac{\partial}{\partial s}[(N_s - k_q M_q)\vartheta_s] + \frac{\partial}{\partial s}(\bar{S}\vartheta_q) - \frac{\partial m_s}{\partial s} - q_\zeta,$$

$$\frac{\partial M_q}{\partial q} = \hat{Q}_q - 4D_M (1-\nu_0)\frac{\partial^2 \vartheta_q}{\partial s^2} - 2D_M (1-\nu_0)\frac{\partial I}{\partial s} + N_q \vartheta_q + \bar{S}\vartheta_s - m_q,$$

$$\frac{\partial u}{\partial q} = \frac{2\bar{S}}{D_N (1-\nu_0)} - \frac{\partial v}{\partial s} - P - \vartheta_s \vartheta_q, \quad \frac{\partial v}{\partial q} = \frac{N_q}{D_N} - \nu_0 \frac{\partial u}{\partial s} - k_q w - P_q - \frac{1}{2}\vartheta_q^2 - \frac{\nu_0}{2}\vartheta_s^2,$$

$$\frac{\partial w}{\partial q} = k_q v - \vartheta_q, \quad \frac{\partial \vartheta_q}{\partial q} = \frac{M_q}{D_M} + \nu_0 \frac{\partial^2 w}{\partial s^2} - I_q - \frac{1}{2} k_q \vartheta_q^2. \tag{4.3}$$

The operator A (s, q) is independent of the SSS. The terms that describe plastic and thermal strains, geometrical nonlinearity, and the temperature dependence of mechanical properties appear only in the free terms $\vec{F}(s,q)$. These specific features of the system of equations, which are owing to the linearization methods used, highly reduces the amount of computation. At each step of loading, the problem is solved by the method of successive approximations. The vector of free terms $\vec{F}(s,q)$ is known from the previous iteration, and the system of equations (4.3) is linear at each iteration of each step.

Boundary conditions should be prescribed at the edges $s = s_0$, $s = s_N$, $q = q_0$, $q = q_N$. For example, if the edges $s = s_0$ and $s = s_N$ are hinged, then

$$N_s = M_s = v = w = 0. \tag{4.4}$$

For open shells, boundary conditions at the rectilinear edges $q = q_0$ and $q = q_N$ are formulated similarly. For example, the periodicity conditions are

$$\bar{S} = \hat{Q}_q = v = \vartheta_q = 0. \tag{4.5}$$

If a closed shell has a plane of symmetry (for the geometry and loads) coming through the Oy-axis, then it is enough to compute half the shell, i.e., an open shell with the periodicity conditions (4.5) set at the edges $q = q_0$ and $q = q_N$.

5. Governing Equations with Some Specific Boundary Conditions. The dimension of the problem can further be reduced by various methods [3–6, 8]. We will dwell on one of the boundary conditions—hinged edges (4.4). Satisfying these boundary conditions, we expand the functions entering them as follows:

$$\{N_s(q,s), M_s(q,s), u(q,s), w(q,s)\} = \sum_{n=1}^{N} \{N_{sn}(q), M_{sn}(q), u_n(q), w_n(q)\}\sin[\lambda_n (s - s_0)],$$

$$\lambda_n = \frac{n\pi}{(s_N - s_0)}. \tag{5.1}$$

Expanding all the given and unknown functions in a similar manner, we can separate variables in (4.2) and obtain a system of ordinary differential equations of the eighth order for each number $n = 1, 2, \ldots, N$:

$$\frac{d\vec{Y}_n(q)}{dq} = A_n(q)\vec{Y}_n(q) + \vec{F}_n(q), \tag{5.2}$$

where $\vec{Y}_n = \{y_{1n}, \ldots, y_{8n}\}$ are the amplitudes of the unknown functions (4.1); the elements of the operator matrix $A_n(q)$ and of the vector $\vec{F}_n(q)$ depend on the harmonic number n. The nonzero elements of the matrix $A_n(q)$ and of the vector $\vec{F}_n(q)$ are given by

$$a_{12} = -a_{56} = \lambda_n, \quad a_{13} = -a_{31} = a_{67} = -a_{76} = -k_q, \quad a_{21} = -a_{66} = -\nu_0\lambda_n, \quad a_{25} = D_N(1-\nu_0^2)\lambda_n^2,$$

$$a_{34} = -a_{88} = \nu_0\lambda_n^2, \quad a_{37} = D_M(1-\nu_0^2)\lambda_n^4 + \mu_\zeta, \quad a_{43} = a_{78} = 1, \quad a_{48} = 4D_M(1-\nu_0)\lambda_n^2,$$

$$a_{52} = 2[D_N(1-\nu_0)]^{-1}, \quad a_{61} = D_N^{-1}, \quad a_{84} = D_M^{-1},$$

$$f_{1n} = -q_{qn}, \quad f_{2n} = -q_{sn} - D_N\lambda_n P_{sn} + \nu_0 D_N\lambda_n P_{qn} - \frac{1}{2}D_N(1-\nu_0^2)\lambda_n q_{2n},$$

$$f_{3n} = q_{\zeta n} + \lambda_n m_{sn} + D_M\lambda_n^2 I_{sn} - \nu_0 D_M\lambda_n^2 I_{qn} - q_{3n},$$

$$f_{4n} = -m_{qn} + 2D_M(1-\nu_0)I_{sn} + q_{4n}, \quad f_{5n} = -P_n + q_{5n},$$

$$f_{6n} = -P_{qn} + q_{6n}, \quad f_{8n} = -I_{qn} + q_{8n}, \tag{5.3}$$

where P_{sn}, P_{qn}, P_n, I_{sn}, I_{qn}, and I_n are the amplitudes of the additional terms P_s, P_q, P, I_s, I_q, I, and g_{jn} ($j = 1, \ldots, 8$) are the amplitudes of the terms g_j. The nonzero terms g_j are given by

$$g_2 = \vartheta_s^2, \quad g_3 = (N_s - k_q M_q)\vartheta_s + \bar{S}\vartheta_q, \quad g_4 = N_q\vartheta_q + \bar{S}\vartheta_s,$$

$$g_5 = -\vartheta_s\vartheta_q, \quad g_6 = -\frac{1}{2}\vartheta_q^2 - \frac{\nu_0}{2}\vartheta_s^2, \quad g_8 = \frac{1}{2}k_q\vartheta_q^2. \tag{5.4}$$

The problem is reduced to the integration of independent equations of the eighth order for the amplitudes of unknown functions at each iteration of each step N.

In the specific case of an infinitely long shell (its SSS does not vary in the s-direction), it is sufficient to choose six unknown functions N_q, Q_q, M_q, v, w, and ϑ_q. Then the governing equations become

$$\frac{dN_q}{dq} = -k_q Q_q - q_q, \quad \frac{dQ_q}{dq} = k_q N_q + \mu_\zeta w - q_\zeta, \quad \frac{dM_q}{dq} = Q_q - m_q + N_q\vartheta_q,$$

$$\frac{dv}{dq} = \frac{1}{D_N}N_q - k_q w - P_q - \frac{1}{2}\vartheta_q^2, \quad \frac{dw}{dq} = k_q v - \vartheta_q, \quad \frac{d\vartheta_q}{dq} = \frac{1}{D_M}M_q - I_q + \frac{1}{2}k_q\vartheta_q^2. \tag{5.5}$$

The following boundary conditions should be satisfied for the unknown functions or their linear combinations:

$$D_1\vec{Y} = \vec{b}_1 \quad \text{at} \quad q = q_0, \tag{5.6}$$

$$D_2\vec{Y} = \vec{b}_2 \quad \text{at} \quad q = q_N, \tag{5.7}$$

where D_1 and D_2 are the matrices of boundary conditions; $\vec{b}_1$ and $\vec{b}_2$ are the vectors of free terms of boundary conditions.

TABLE 3

$q = 0$	σ_{ss}	σ_{qq}	Γ
$\zeta = -h/2$	–2524	–6793	0.003759
$\zeta = h/2$	2496	6759	0.003672

To determine the thermoelastoplastic SSS numerically, we divide the generatrix, directrix, and thickness of the shell into K_s, K_q, and K_ζ intervals, respectively. The shell appears covered with a three-dimensional mesh. A surface load is prescribed at the nodes of the mid-surface. The elements of the vectors of unknown functions are determined at the same nodes. The temperature is set or calculated and the strains, stresses, and other quantities needed for the subsequent approximation are calculated at the nodes of the mesh.

System (5.2) is solved by the Runge–Kutta method with intermediate orthogonalization and normalization of partial solutions [2]. The solution of the boundary-value problem (5.2), (5.6), (5.7) is sought in the following form [28]:

$$Z = Z^0 + Z^* = \sum_{i=1}^{4} C_i Z_i + Z^*, \tag{5.8}$$

where Z_i are the partial solutions of the Cauchy problems for the homogeneous system of equations (5.2) ($\vec{F}_n(q) = 0$) with the initial conditions at the edge $q = q_0$ equal to zero for the given unknown functions and equal to the respective columns of a unit matrix for the other functions; Z^* is the solution of the Cauchy problem for the inhomogeneous system of equations (5.2) with the initial conditions at the edge $q = q_0$ coinciding with the boundary conditions (5.6) for the given unknown functions and equal to zero for the other functions; C_i are the constants of integration determined from the boundary conditions (5.7). Since $A_n(q)$ is independent of the SSS, this operator and Z^0 can be calculated only in the first approximation of the first step. Thus, five Cauchy problems are solved at the first iteration of the first step, and only one Cauchy problem is solved (to find Z^*) at the subsequent iterations.

The process of successive approximations at each step is considered to converge once the tangential-stress intensity found from the calculated stresses and the tangential-stress intensity obtained from the thermomechanical surface have differed by some preset amount δ_1. Simultaneously, the absolute values of the displacement vector found in two successive approximations must differ by less than a small amount δ_2. These criteria ensure convergence in both physical and geometrical nonlinearities.

6. Assessing the Accuracy of the Method. To test the method and to assess its accuracy, we will consider an infinitely long circular cylindrical shell without geometrical nonlinearity and compare the solution obtained by numerical integration over the circumferential coordinate and the solution found using trigonometric Fourier series in powers of the same coordinate [28]. The radius and thickness of the shell are $R = 0.1$ m and $h = 0.005$ m. The shell is made of ÉI-395 alloy; its Young's modulus and Poisson's ratio are $E = 1.95$ MPa and $\nu_0 = 0.3$; its tensile stress–strain curve is given in [27]. The shell is subjected to loads $q_\zeta = 10\cos(2q/R)$. Since the load is symmetric, we can consider half the shell ($0 \le q/R \le \pi$). The periodicity conditions $Q_q = v = \vartheta_q = 0$ are prescribed at the edges $q = 0$ and $q = \pi R$. The number of intervals of integration $K_q = 2001$ along the circumference and $K_\zeta = 21$ across the thickness. In addition to the second harmonic corresponding to the load, the following higher harmonics were retained in the Fourier series: m = 6, 10, 14, 18, 22, 26, 30. The plastic problem has been solved with accuracy $\delta_1 = 0.001$. The results obtained are practically independent of the above discretization parameters.

Table 3 collects the values of the stresses σ_{ss} and σ_{qq} and the shear-strain intensity Γ at $q = 0$. As is seen, both methods give identical values, which means high accuracy of our method. Note that the lower harmonics in the Fourier series used to solve the elastoplastic problem always generate an infinite decaying spectrum of higher harmonics. The contribution of these harmonics to the maximum values of shear-strain intensity exceeds 10%.

7. Effect of the Winkler Foundation on the SSS. Let us consider the shell described in Sec. 6. It is subjected to load $q_\zeta = 10\cos(mq/R),\ m = 0, 1, 2.$

TABLE 4

μ_ζ	$q=0$	$q_\zeta = 10$		$q_\zeta = 10\cos(q/R)$		$q_\zeta = 10\cos(2q/R)$	
		σ_{ss}	σ_{qq}	σ_{ss}	σ_{qq}	σ_{ss}	σ_{qq}
0	$\zeta = -h/2$	60	200	58	195	–2524	–6793
	$\zeta = h/2$	60	200	61	205	2496	6759
10^3	$\zeta = -h/2$	55	183	53	178	–48	–159
	$\zeta = h/2$	55	183	56	187	47	156

Table 4 summarizes the stresses σ_{ss} and σ_{qq} for $q = 0$, two values of the Winkler modulus μ_ζ, and three first harmonics. The amplitudes of the load being equal, the stress of the second harmonic is much higher than the stresses due to the zero and first harmonics. Comparing the results for different μ_ζ reveals a strong effect of this parameter for the second harmonic. For the zero and first harmonics, the effect the Winkler foundation is much weaker.

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