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DYNAMICS OF A PRESTRESSED INCOMPRESSIBLE LAYERED HALF-SPACE UNDER MOVING LOAD

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The paper addresses a plane problem: a concentrated force acts on a plate resting on an elastic half-space with homogeneous prestrain. The equations of motion of the plate incorporate shear and rotary inertia. The half-space is assumed to be incompressible and isotropic in the natural state. The elastic potential is given in general form and is only specified for numerical purposes. The dependence of the critical velocity of the load and the stress–strain state on the prestresses is analyzed for different ratios between the stiffnesses of the layer and half-space and different contact conditions. The calculations are carried out for a half-space with Bartenev–Khazanovich potential

Keywords: initial stresses, load moving with constant velocity, two-layer half-space, incompressible material, Bartenev–Khazanovich potential

Introduction. Three-dimensional linearized theories of stability of deformable bodies and elastic waves in prestressed bodies were analyzed from a contemporary standpoint in [7, 8], respectively. The results of [7, 8] were used in modern analysis of inverse problems for elastic waves in prestressed bodies in [19], contact interaction of elastic prestressed bodies in [5, 6], stability of mine workings in the case of an inhomogeneous subcritical state in [9], and exact solutions of mixed plane problems in the case of prestresses in [18]. The motion of cracks in elastic prestressed bodies was studied in [10–13] for homogeneous bodies and in [14–17] for piecewise-homogeneous bodies. Static contact problems for elastic bodies were solved with and without regard to prestresses in [20–22, 24, etc.].

The present paper addresses a plane problem for a plate subject to a concentrated mechanical load and lying on an elastic half-space with homogeneous prestrain. The equations of motion of the plate incorporate shear and rotary inertia. The half-space is incompressible and isotropic in the natural state. The elastic potential has a general form, which will be specified for numerical purposes. We will analyze the dependence of the critical velocity of the load and the stress–strain state on the prestresses for different ratios between the stiffnesses of the layer and half-space and different contact conditions. The calculations will be conducted for a half-space with Bartenev–Khazanovich potential.

1. Consider a plate of thickness $2h$ on an elastic half-space whose initial strain state is determined by the following components of the generalized stress tensor:

$$\sigma_{11}^0 \neq 0, \quad \sigma_{22}^0 \neq 0, \quad \sigma_{33}^0 \neq 0. \tag{1.1}$$

The plate and the half-space are described by Cartesian coordinates (ξ_1, ξ_2, ξ_3) introduced in the initial strain state and related to Lagrangian coordinates (x_1, x_2, x_3) introduced in the natural state by the formula $\xi_i = \lambda_i x_i$, where λ_i are tensile strains elongations (i = 1, 2, 3).

The coordinate plane $\xi_1 O \xi_3$ coincides with the mid-surface of the plate. The half-space occupies the domain $|\xi_1| < \infty$, $|\xi_3| < \infty$, $\xi_2 + h \le 0$. Assume that the load moves over the surface of the plate ($\xi_2 = h$) with constant velocity v for a long time;

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therefore, the plane strain state in the coordinate system fixed to this load is steady-state. The coordinates of the moving coordinate system are determined from the relations $y_1 = \xi_1 - vt$ and $y_2 = \xi_2$.

The equations of motion of the plate with shear and rotary inertia have the following form in the moving coordinate system [4]:

$$2h\left(\frac{2G_1}{1-\nu_1} - \rho_1 v^2\right)\frac{\partial^2 u}{\partial y_1^2} - \tau = P_1,$$

$$2h\left(\kappa G_1 - \rho_1 v^2\right)\frac{\partial^2 w}{\partial y_1^2} - 2\kappa G_1 h\frac{\partial \phi}{\partial y_1} - q = P_2,$$

$$\frac{2h^2}{3}\left(\frac{2G_1}{1-\nu_1} - \rho_1 v^2 \delta_0\right)\frac{\partial^2 \phi}{\partial y_1^2} + 2\kappa G_1\left(\frac{\partial w}{\partial y_1} - \phi\right) - \tau = 0, \tag{1.2}$$

where ρ_1, G_1, and ν_1 are the density, shear modulus, and Poisson's ratio of the plate material, respectively; u and w are the displacements of the mid-surface ($y_2 = 0$); ϕ is the angle of rotation of the cross-section; κ is Timoshenko's shear coefficient; q, τ and P_2, P_1 are the normal and tangential stresses at the interface between the plate and the half-space and on the free surface of the plate; δ_0 is equal to 1 or 0.

The bending moment in the plate in the moving coordinate system is defined by the formula

$$M = \frac{4}{3}\frac{G_1 h^3}{1-\nu_1}\frac{d\phi}{dy_1}. \tag{1.3}$$

Using formulas from [1], we rearrange the linearized equations of motion of the half-space during plane deformation in the coordinates (y_1, y_2) in terms of the function χ (y_1, y_2):

$$\left(\eta_1^2\frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\left(\eta_2^2\frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\chi^{(j)} = 0, \quad j = 1, 2, \tag{1.4}$$

where the roots η_1 and η_2 are found from the equation

$$\eta^4 + 2A\eta^2 + A_1 = 0, \tag{1.5}$$

and the coefficients A and A_1 are determined from the relations

$$2A\tilde{q}_{22}^2\tilde{\kappa}_{2112} = \tilde{q}_{11}^2\tilde{\kappa}_{2222} + \tilde{q}_{22}^2\left(\tilde{\kappa}_{1111} - \tilde{\rho}v^2\right) - 2\tilde{q}_{11}\tilde{q}_{22}\left(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212}\right),$$

$$2A_1\tilde{q}_{22}^2\tilde{\kappa}_{2112} = \tilde{q}_{11}^2\left(\tilde{\kappa}_{1221} - \tilde{\rho}v^2\right), \quad \tilde{q}_{ij} = \delta_{ij}\lambda_i q_i, \quad \tilde{\rho} = \rho, \tag{1.6}$$

ρ is the density of the half-space in the natural state.

Let us consider two cases of contact between the plate and the half-space at $y_2 = -h$:

tight contact:

$$\tilde{Q}_{21} = \tau, \quad \tilde{Q}_{22} = q, \quad u_2 = w, \quad u_1 = u + h\phi, \tag{1.7}$$

nontight contact:

$$\tilde{Q}_{21} = 0, \quad \tau = 0, \quad \tilde{Q}_{22} = q, \quad u_2 = w. \tag{1.8}$$

Thus, the perturbations in the layered medium are determined by solving Eqs. (1.2) and (1.4) with the boundary conditions (1.7) or (1.8).

Let us consider the cases of unequal and equal roots of Eq. (1.5).

Unequal Roots. Let $\eta_1 \neq \eta_2$. Suppose

$$\chi = \tilde{q}_{11}^{-1}\chi^{(1)}, \quad \chi^{(2)} = 0. \tag{1.9}$$

Considering (1.9), we obtain a solution in the form

$$u_1 = -\frac{\partial^2 \chi}{\partial y_1 \partial y_2}, \quad u_2 = \tilde{q}_{11}\tilde{q}_{22}\frac{\partial^2 \chi}{\partial y_1^2},$$

$$p = \tilde{q}_{11}^{-1}\left\{\left[\tilde{\kappa}_{1111} - \tilde{q}_{11}\tilde{q}_{22}^{-1}(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212}) - \tilde{\rho}v^2\right]\frac{\partial^2}{\partial y_1^2} + \tilde{\kappa}_{2112}\frac{\partial^2}{\partial y_2^2}\right\}\frac{\partial \chi}{\partial y_2}. \tag{1.10}$$

When the roots of Eq. (1.5) are unequal, it is possible to represent the solution in terms of functions Φ and Ψ given by

$$\Phi = -\frac{\partial \chi^{(1)}}{\partial y_2}, \quad \Psi = \frac{\partial \chi^{(2)}}{\partial y_1}, \quad \chi = \chi^{(1)} + \chi^{(2)}. \tag{1.11}$$

Substituting (1.10) into (1.3), we obtain two wave equations for the half-space:

$$\left(\eta_1^2 \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\Phi = 0, \quad \left(\eta_2^2 \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}\right)\Psi = 0. \tag{1.12}$$

If there are no prestresses, these equations coincide with the wave equations in the classical theory of elasticity, while the functions Φ and Ψ coincide with the longitudinal and transverse potentials for an incompressible body.

Solutions (1.10) with (1.11) can be represented in the form

$$u_1 = \frac{\partial \Phi}{\partial y_1} - \frac{\partial \Psi}{\partial y_2}, \quad u_2 = \alpha_1 \frac{\partial \Phi}{\partial y_2} + \alpha_2 \frac{\partial \Psi}{\partial y_1},$$

$$p = \tilde{q}_{11}^{-1}\left\{-\left[\tilde{\kappa}_{1111} - \tilde{\rho}v^2 - \eta_1^2\tilde{\kappa}_{2112} - \tilde{q}_{11}\tilde{q}_{22}^{-1}(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212})\right]\frac{\partial^2 \Phi}{\partial y_1^2}\right.$$

$$\left. + \left[\tilde{\kappa}_{1111} - \tilde{\rho}v^2 - \eta_2^2\tilde{\kappa}_{2112} - \tilde{q}_{11}\tilde{q}_{22}^{-1}(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212})\right]\frac{\partial^2 \Psi}{\partial y_1 \partial y_2}\right\} \tag{1.13}$$

$$(\alpha_1 = \tilde{q}_{11}\tilde{q}_{22}^{-1}\eta_1^{-2}, \quad \alpha_2 = \tilde{q}_{11}\tilde{q}_{22}^{-1}). \tag{1.14}$$

If there are no prestresses, the solutions (1.13) are identical with the Lamé solution for an incompressible body.

Thus, Eqs. (1.12) with (1.6) and solution (1.13) are a generalization of the wave equations of motion and the Lamé solution to the case of prestressed incompressible bodies. The functions Φ and Ψ are the longitudinal and transverse potentials for these bodies.

The stresses and displacement rates in the incompressible half-space can be expressed in terms of the function $\chi(y_1, y_2)$ and in view of (1.9) as

$$\tilde{Q}_{ii} = \alpha_{ii}^{(1)}\frac{\partial^3 \chi}{\partial y_1^2 \partial y_2} + \alpha_{ii}^{(2)}\frac{\partial^3 \chi}{\partial y_2^3}, \quad \tilde{Q}_{ij} = \alpha_{ij}^{(1)}\frac{\partial^3 \chi}{\partial y_1^3} + \alpha_{ij}^{(2)}\frac{\partial^3 \chi}{\partial y_1 \partial y_2^2},$$

$$\dot{u}_1 = v\frac{\partial^3 \chi}{\partial y_1^2 \partial y_2}, \quad \dot{u}_2 = -v\left[\beta_1 \frac{\partial^2}{\partial y_1^2} + \beta_2 \frac{\partial^2}{\partial y_2^2}\right]\frac{\partial \chi}{\partial y_1}, \quad i, j = 1, 2; \quad i \neq j, \tag{1.15}$$

$$\alpha_{ii}^{(1)} = \tilde{q}_{ii}\tilde{q}_{11}^{-1}\left[\tilde{\kappa}_{1111} - \tilde{\rho}v^2 - \tilde{q}_{11}\tilde{q}_{22}^{-1}\left(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212}\right)\right] - \tilde{\kappa}_{ii11} + \tilde{\kappa}_{ii22}\tilde{q}_{11}\tilde{q}_{22}^{-1},$$

$$\alpha_{ii}^{(2)} = \tilde{q}_{ii}\tilde{q}_{11}^{-1}\tilde{\kappa}_{2112}, \quad \alpha_{ij}^{(2)} = \tilde{q}_{11}\tilde{q}_{22}^{-1}\tilde{\kappa}_{ij21}, \quad \alpha_{ij}^{(2)} = -\tilde{\kappa}_{ij12}, \quad i \neq j, \quad \beta_1 = \tilde{q}_{11}\tilde{q}_{22}^{-1}, \quad \beta_2 \equiv 0. \tag{1.16}$$

The stresses and displacement rates in the incompressible half-space can also be expressed in terms of the potentials Φ and Ψ:

$$\tilde{Q}_{ii} = \left(\eta_1^2\alpha_{ii}^{(2)} - \alpha_{ii}^{(1)}\right)\frac{\partial^2\Phi}{\partial y_1^2} + \left(\alpha_{ii}^{(1)} - \eta_2^2\alpha_{ii}^{(1)}\right)\frac{\partial^2\Psi}{\partial y_1 \partial y_2},$$

$$\tilde{Q}_{ij} = \left(\eta_1^{-2}\alpha_{ij}^{(1)} - \alpha_{ij}^{(2)}\right)\frac{\partial^2\Phi}{\partial y_1 \partial y_2} + \left(\alpha_{ij}^{(1)} - \eta_2^2\alpha_{ij}^{(1)}\right)\frac{\partial^2\Psi}{\partial y_1^2}, \quad i, j = 1, 2; \quad i \neq j,$$

$$\dot{u}_1 = -v\left(\frac{\partial^2\Phi}{\partial y_1^2} - \frac{\partial^2\Psi}{\partial y_1 \partial y_2}\right), \quad \dot{u}_2 = -v\left(\alpha_1\frac{\partial^2\Phi}{\partial y_1 \partial y_2} + \alpha_2\frac{\partial^2\Psi}{\partial y_1^2}\right). \tag{1.17}$$

The boundary conditions (1.7) and (1.8) for tight and nontight contact can be written as

$$-\theta_1\frac{\partial^4\chi}{\partial y_1^3 \partial y_2} - 2\kappa G_1\left(\beta_1\frac{\partial^2}{\partial y_1^2} + \beta_2\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial}{\partial y_1}\chi - \theta_4\frac{\partial^2\phi}{\partial y_1^2} + 2\kappa G_1\phi = P_1,$$

$$\theta_3\left(\beta_1\frac{\partial^2}{\partial y_1^2} + \beta_2\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial^2}{\partial y_1^2}\chi - \left(\alpha_{22}^{(1)}\frac{\partial^2}{\partial y_1^2} + \alpha_{22}^{(2)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial}{\partial y_2}\chi - 2\kappa h G_1\frac{\partial\phi}{\partial y_1} = P_2,$$

$$2\kappa G_1\left(\beta_1\frac{\partial^2}{\partial y_1^2} + \beta_2\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial}{\partial y_1}\chi - \left(\alpha_{21}^{(1)}\frac{\partial^2}{\partial y_1^2} + \alpha_{21}^{(2)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial}{\partial y_1}\chi + \theta_2\frac{\partial^2\phi}{\partial y_1^2} - 2\kappa G_1\phi = 0, \tag{1.18}$$

$$\alpha_{21}^{(1)}\frac{\partial^3\chi}{\partial y_1^3} + \alpha_{21}^{(2)}\frac{\partial^3\chi}{\partial y_1 \partial y_2^2} = 0,$$

$$\theta_3\left(\beta_1\frac{\partial^2}{\partial y_1^2} + \beta_2\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial^2}{\partial y_1^2}\chi - \left(\alpha_{22}^{(1)}\frac{\partial^2}{\partial y_1^2} + \alpha_{22}^{(2)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial}{\partial y_2}\chi - 2\kappa h G_1\frac{\partial\phi}{\partial y_1} = P_2,$$

$$2\kappa G_1\left(\beta_1\frac{\partial^3\chi}{\partial y_1^3} + \beta_2\frac{\partial^3\chi}{\partial y_1 \partial y_2^2}\right) + \theta_2\frac{\partial^2\phi}{\partial y_1^2} - 2\kappa G_1\phi = 0, \tag{1.19}$$

$$\theta_1 = 2h\left(\frac{2G_1}{1-\nu_1} - \rho_1 v^2\right), \quad \theta_2 = \frac{2h^2}{3}\left(\frac{2G_1}{1-\nu_1} - \delta_0\rho_1 v^2\right), \quad \theta_3 = 2h\left(\kappa G_1 - \rho_1 v^2\right). \tag{1.20}$$

Similar relations with the potentials Φ and Ψ have the form:

$$\left(\theta_1\frac{\partial^2}{\partial y_1^2} - 2\kappa G_1\alpha_1\frac{\partial}{\partial y_2}\right)\frac{\partial\Phi}{\partial y_1} - \left(\theta_1\frac{\partial}{\partial y_2} + 2\kappa G_1\alpha_2\right)\frac{\partial^2\Psi}{\partial y_1^2} - \theta_4\frac{\partial^2\phi}{\partial y_1^2} + 2\kappa G_1\phi = P_1,$$

$$\left(\alpha_1\theta_3\frac{\partial}{\partial y_2} + \alpha_{22}^{(1)} - \eta_1^2\alpha_{22}^{(2)}\right)\frac{\partial^2\Phi}{\partial y_1^2} + \left[\alpha_2\theta_3\frac{\partial^2}{\partial y_1^2} - \left(\alpha_{22}^{(1)} - \eta_2^2\alpha_{22}^{(2)}\right)\frac{\partial}{\partial y_2}\right]\frac{\partial\Psi}{\partial y_1} - 2\kappa h G_1\frac{\partial\phi}{\partial y_1} = P_2,$$

$$\left(2\kappa G_1\alpha_1-\eta_1^{-2}\alpha_{21}^{(1)}+\alpha_{21}^{(2)}\right)\frac{\partial^2\Phi}{\partial y_1\partial y_2}+\left(2\kappa G_1\alpha_2-\alpha_{21}^{(1)}+\eta_2^2\alpha_{21}^{(2)}\right)\frac{\partial^2\Psi}{\partial y_1^2}+\theta_2\frac{\partial^2\phi}{\partial y_1^2}-2\kappa G_1\phi=0, \tag{1.21}$$

$$\left(\eta_1^{-2}\alpha_{21}^{(1)}-\alpha_{21}^{(2)}\right)\frac{\partial^2\Phi}{\partial y_1\partial y_2}+\left(\alpha_{21}^{(1)}-\eta_2^2\alpha_{21}^{(2)}\right)\frac{\partial^2\Psi}{\partial y_1^2}=0,$$

$$\left(\alpha_1\theta_3\frac{\partial}{\partial y_2}+\alpha_{22}^{(1)}-\eta_1^2\alpha_{22}^{(2)}\right)\frac{\partial^2\Phi}{\partial y_1^2}+\left[\alpha_2\theta_3\frac{\partial^2}{\partial y_1^2}-\left(\alpha_{22}^{(1)}-\eta_2^2\alpha_{22}^{(2)}\right)\frac{\partial}{\partial y_2}\right]\frac{\partial\Psi}{\partial y_1}-2\kappa hG_1\frac{\partial\phi}{\partial y_1}=P_2,$$

$$2\kappa G_1\alpha_1\frac{\partial^2\Phi}{\partial y_1\partial y_2}+2\kappa G_1\alpha_2\frac{\partial^2\Psi}{\partial y_1^2}+\theta_2\frac{\partial^2\phi}{\partial y_1^2}-2\kappa G_1\phi=0. \tag{1.22}$$

Thus, in the case of unequal roots, the problem for a prestressed incompressible two-layer half-space steadily moving under the action of a load moving with constant velocity is reduced to the determination of the functions χ and ϕ from the boundary conditions (1.18) (tight contact) or (1.19) (nontight contact). When the roots of Eq. (1.5) are unequal, the solution can be represented in terms of longitudinal (Φ) and transverse (Ψ) potentials, which follow from Eqs. (1.22) or (1.21), depending on the chosen contact conditions between the plate and the half-space.

Equal Roots. If the roots of Eq. (1.5) are equal ($\eta_1=\eta_2=\eta$), we represent the solution as

$$u_1=\lambda_1^{-1}q_1^{-1}\left(-\frac{\partial^2}{\partial y_1\partial y_2}\chi^{(1)}+\frac{\partial^2}{\partial y_2^2}\chi^{(2)}\right),\quad u_2=\lambda_2^{-1}q_2^{-1}\left(\frac{\partial^2}{\partial y_1^2}\chi^{(1)}-\frac{\partial^2}{\partial y_1 y_2}\chi^{(2)}\right),$$

$$p=\lambda_1^{-2}q_1^{-2}\left\{\left[\tilde\kappa_{1111}-\tilde\rho v^2-\lambda_1q_1\lambda_2^{-1}q_2^{-1}\left(\tilde\kappa_{1122}+\tilde\kappa_{1212}\right)\right]\frac{\partial^2}{\partial y_1^2}+\tilde\kappa_{2112}\frac{\partial^2}{\partial y_2^2}\right\}\frac{\partial}{\partial y_2}\chi^{(1)}$$

$$+\lambda_2^{-2}q_2^{-2}\left\{\left(\tilde\kappa_{1221}-\tilde\rho v^2\right)\frac{\partial^2}{\partial y_1^2}+\left[\tilde\kappa_{2222}-\lambda_2q_2\lambda_1^{-1}q_1^{-1}\left(\tilde\kappa_{1122}+\tilde\kappa_{1212}\right)\right]\frac{\partial^2}{\partial y_2^2}\right\}\frac{\partial}{\partial y_1}\chi^{(2)}. \tag{1.23}$$

With (1.23), the boundary conditions (1.7) and (1.8) can be expressed as

$$\left[\theta_1\left(\beta_{11}^{(2)}\frac{\partial^2}{\partial y_1^2}+\beta_{12}^{(2)}\frac{\partial^2}{\partial y_2^2}\right)+2\kappa G_1\beta_{21}^{(2)}\right]\frac{\partial^2\chi^{(2)}}{\partial y_1^2}$$

$$-\left[\theta_1\beta_{11}^{(1)}\frac{\partial^3}{\partial y_1^2\partial y_2}+2\kappa G_1\left(\beta_{21}^{(1)}\frac{\partial^2}{\partial y_1^2}+\beta_{22}^{(1)}\frac{\partial^2}{\partial y_2^2}\right)\right]\frac{\partial\chi^{(1)}}{\partial y_1}-\theta_4\frac{\partial^2\phi}{\partial y_1^2}+2\kappa G_1\phi=P_1,$$

$$\left[\theta_3\left(\beta_{21}^{(1)}\frac{\partial^2}{\partial y_1^2}+\beta_{22}^{(1)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial^2}{\partial y_1^2}-\alpha_{22}^{(11)}\frac{\partial^3}{\partial y_1^2\partial y_2}-\alpha_{22}^{(21)}\frac{\partial^3}{\partial y_2^3}\right]\chi^{(1)}$$

$$-\left[\theta_3\beta_{21}^{(2)}\frac{\partial^3}{\partial y_1^2\partial y_2}+\alpha_{22}^{(12)}\frac{\partial^2}{\partial y_1^2}+\alpha_{22}^{(22)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(2)}}{\partial y_1}-2\kappa hG_1\frac{\partial\phi}{\partial y_1}=P_2, \tag{1.24}$$

$$\left[\left(2\kappa G_1\beta_{21}^{(1)}-\alpha_{21}^{(11)}\right)\frac{\partial^2}{\partial y_1^2}+\left(2\kappa G_1\beta_{22}^{(1)}-\alpha_{21}^{(21)}\right)\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(1)}}{\partial y_1}$$

$$-\left[\left(2\kappa G_1\beta_{21}^{(2)}+\alpha_{21}^{(12)}\right)\frac{\partial^2}{\partial y_1^2}+\alpha_{21}^{(22)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(2)}}{\partial y_2}+\theta_2\frac{\partial^2\phi}{\partial y_1^2}-2\kappa G_1\phi=0,$$

$$\left[\alpha_{21}^{(12)}\frac{\partial^2}{\partial y_1^2}+\alpha_{21}^{(22)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(2)}}{\partial y_2}+\left[\alpha_{21}^{(11)}\frac{\partial^2}{\partial y_1^2}+\alpha_{21}^{(21)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(1)}}{\partial y_1}=0,$$

$$\left[\theta_3\left(\beta_{21}^{(1)}\frac{\partial^2}{\partial y_1^2}+\beta_{22}^{(1)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial^2}{\partial y_1^2}-\alpha_{22}^{(11)}\frac{\partial^3}{\partial y_1^2\partial y_2}-\alpha_{22}^{(21)}\frac{\partial^3}{\partial y_2^3}\right]\chi^{(1)}$$

$$-\left[\theta_3\beta_{21}^{(2)}\frac{\partial^3}{\partial y_1^2\partial y_2}+\alpha_{22}^{(12)}\frac{\partial^2}{\partial y_1^2}+\alpha_{22}^{(22)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(2)}}{\partial y_1}-2\kappa hG_1\frac{\partial\phi}{\partial y_1}=P_2, \tag{1.25}$$

$$-2\kappa G_1\beta_{21}^{(2)}\frac{\partial^3\chi^{(2)}}{\partial y_1^2\partial y_2}+2\kappa G_1\left[\beta_{21}^{(1)}\frac{\partial^2}{\partial y_1^2}+\beta_{22}^{(1)}\frac{\partial^2}{\partial y_2^2}\right]\frac{\partial\chi^{(1)}}{\partial y_1}+\theta_2\frac{\partial^2\phi}{\partial y_1^2}-2\kappa G_1\phi=0,$$

$$\alpha_{ii}^{(kn)}=\tilde{q}_{ii}\tilde{q}_{nn}^{-2}\left(\tilde{\kappa}_{knnk}-\delta_{n2}\tilde{\rho}v^2\right),$$

$$\alpha_{ii}^{(kk)}=\tilde{\kappa}_{iinn}\tilde{q}_{nn}^{-1}-\tilde{\kappa}_{iikk}\tilde{q}_{kk}^{-1}+\tilde{q}_{ii}\tilde{q}_{kk}^{-2}\left[\tilde{\kappa}_{kkkk}-\tilde{\rho}v^2-\tilde{q}_{kk}\tilde{q}_{nn}^{-1}\left(\tilde{\kappa}_{1122}+\tilde{\kappa}_{1212}\right)\right],$$

$$\alpha_{ij}^{(kn)}=-\tilde{\kappa}_{ijnk}\tilde{q}_{nn}^{-1},\quad \alpha_{ij}^{(kk)}=\tilde{\kappa}_{ijnk}\tilde{q}_{nn}^{-1},\quad i,j,n,k=1,2,\quad i\neq j,\quad n\neq k,$$

$$\beta_{12}^{(2)}=\beta_{11}^{(1)}=\tilde{q}_{11}^{-1},\quad \beta_{21}^{(2)}=\beta_{21}^{(1)}=\tilde{q}_{22}^{-1},\quad \beta_{11}^{(2)}=\beta_{22}^{(1)}=0. \tag{1.26}$$

The stresses and displacement rates in the half-space are expressed in terms of the functions $\chi^{(j)}$ $(j=1,2)$ in view of (1.26) as

$$\tilde{Q}_{ii}=\left(\alpha_{ii}^{(12)}\frac{\partial^2}{\partial y_1^2}+\alpha_{ii}^{(22)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(2)}}{\partial y_1}+\left(\alpha_{ii}^{(11)}\frac{\partial^2}{\partial y_1^2}+\alpha_{ii}^{(21)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(1)}}{\partial y_2},$$

$$\tilde{Q}_{ij}=\left(\alpha_{ij}^{(12)}\frac{\partial^2}{\partial y_1^2}+\alpha_{ij}^{(22)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(2)}}{\partial y_2}+\left(\alpha_{ij}^{(11)}\frac{\partial^2}{\partial y_1^2}+\alpha_{ij}^{(21)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(1)}}{\partial y_1},$$

$$\dot{u}_1=v\beta_{11}^{(1)}\frac{\partial^3\chi^{(1)}}{\partial y_1^2\partial y_2}-v\left(\beta_{11}^{(2)}\frac{\partial^2}{\partial y_1^2}+\beta_{12}^{(2)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(2)}}{\partial y_1},$$

$$\dot{u}_2=v\beta_{21}^{(2)}\frac{\partial^3\chi^{(2)}}{\partial y_1^2\partial y_2}-v\left(\beta_{21}^{(1)}\frac{\partial^2}{\partial y_1^2}+\beta_{22}^{(1)}\frac{\partial^2}{\partial y_2^2}\right)\frac{\partial\chi^{(1)}}{\partial y_1},\quad i,j=1,2;\quad i\neq j. \tag{1.27}$$

Thus, in the case of equal roots, the dynamic problem for a prestressed incompressible two-layer half-space subject to a moving load is reduced to the determination of the functions $\chi^{(j)}$ and ϕ from the boundary conditions (1.24) or (1.25) (depending on the contact conditions between the plate and the half-space). The stresses and displacement rates in the half-space and the bending moment in the plate are defined by formulas (1.27) and (1.3).

2. Let us examine the effect of the velocity of the load on the roots of Eq. (1.5). We represent the solution of Eq. (1.5) in the form

TABLE 1

Velocity of load	η_1^2	η_2^2
$0 < v \le v_1$	+	+
$v_1 < v < v_2$	к	к
$v_2 \le v \le c_1$	+	+
$v > c_1$	+	–

$$\eta_i^2(v) = -A \pm \sqrt{D}, \tag{2.1}$$

where $D = A^2 - A_1$.

Denote

$$\tilde{\rho} d = \tilde{q}_{11}^2 \tilde{\kappa}_{2222} + \tilde{q}_{22}^2 \tilde{\kappa}_{1111} - 2\tilde{q}_{11}\tilde{q}_{22}(\tilde{\kappa}_{1122} + \tilde{\kappa}_{1212}), \quad \tilde{\rho} c_i^2 = \tilde{\kappa}_{ijji}, \quad i, j = 1, 2; \quad i \ne j. \tag{2.2}$$

With (2.2), formulas (1.6) become

$$2A\tilde{q}_{22}^2 c_2^2 = d - v^2 \tilde{q}_{22}^2, \quad A_1 \tilde{q}_{22}^2 c_2^2 = \tilde{q}_{11}^2 (c_1^2 - v^2), \tag{2.3}$$

where c_1 and c_2 are the velocities of transverse waves along the Oy_1- and Oy_2-axes in an unbounded prestressed incompressible body [3].

As in the case of a compressible body, the functions $\eta_i^2(v)$ take on real values when $D \ge 0$ and complex values when $D < 0$.

If $D = 0$, then the roots of Eq. (1.5) can be equal. The roots of the biquadratic equation $D = 0$ are given by

$$v_i^2 = \tilde{q}_{22}^{-2}\left[d - 2\tilde{q}_{11}^2 c_2^2 + (-1)^i \sqrt{4\tilde{q}_{11}^2 c_2^2 (\tilde{q}_{11}^2 c_2^2 + \tilde{q}_{22}^2 c_1^2 - d)}\right]. \tag{2.4}$$

Hence, the equation $D = 0$ has real roots if

$$\tilde{q}_{11}^2 c_2^2 + \tilde{q}_{22}^2 c_1^2 \ge d. \tag{2.5}$$

Note that $v_i < c_1$.

Thus, if inequality (2.5) does not hold, the roots of Eq. (1.5) for this incompressible material and any velocity v of the load will not be equal ($\eta_1 \ne \eta_2$). Inequality (2.5) is the necessary condition for the roots of Eq. (1.5) to be equal.

If $\operatorname{Im} v_i^2 = 0$ and $\operatorname{Re} v_i^2 \ge 0$ (i = 1, 2), then the functions η_1^2 and η_2^2 take on complex values when $v_1 < v < v_2$ and real, positive, and equal values ($\eta_1 = \eta_2$) when $v = v_1$ or $v = v_2$. Table 1 demonstrates what sign the functions $\eta_i^2(v)$ (i = 1, 2) have for different velocities of the load on an incompressible body. Note that $\eta_2 = 0$ if $v = c_1$.

3. The problem is solved by applying the Fourier transform with respect to the variable y_1

$$f^{\mathrm{F}}(k) = \int_{-\infty}^{+\infty} f(y_1) e^{-iky_1} dy_1 \tag{3.1}$$

and the inverse transform

$$f(y_1)=\frac{1}{2\pi}\int_{-\infty+i\gamma}^{+\infty+i\gamma} f^{\mathrm{F}}(k)e^{iky_1}\,dk, \qquad \gamma>0. \tag{3.2}$$

Taking the Fourier transform of Eqs. (1.2), we obtain

$$\left(\frac{d^2}{dy_2^2}-k^2\eta_1^2\right)\left(\frac{d^2}{dy_2^2}-k^2\eta_2^2\right)\chi^{(j)\mathrm{F}}=0, \qquad j=1,2. \tag{3.3}$$

Let us find the solution in the cases of unequal and equal roots and for different contact conditions between the layer and the half-space. The solution will be represented in the general form for any velocity of load (subsonic, transonic, or supersonic).

Unequal Roots. Taking the Fourier transform of the boundary conditions (1.7) and (1.8) yields

$$ik^3\theta_1\frac{d\chi^{\mathrm{F}}}{dy_2}-2ik\kappa\left(\beta_2\frac{d^2}{dy_2^2}-k^2\beta_1\right)\chi^{\mathrm{F}}+(k^2\theta_4+2\kappa)\phi^{\mathrm{F}}=P_1^{\mathrm{F}},$$

$$k^2\theta_3\left(k^2\beta_1-\beta_2\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}}+\left(k^2\alpha_{22}^{(1)}-\alpha_{22}^{(2)}\frac{d^2}{dy_2^2}\right)\frac{d\chi^{\mathrm{F}}}{dy_2}-2ik\kappa h\phi^{\mathrm{F}}=P_2^{\mathrm{F}},$$

$$2ik\kappa\left(-k^2\beta_1+\beta_2\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}}+ik\left(k^2\alpha_{21}^{(1)}-\alpha_{21}^{(2)}\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}}-(k^2\theta_2+2\kappa)\phi^{\mathrm{F}}=0, \tag{3.4}$$

$$-ik^3\alpha_{21}^{(1)}\chi^{\mathrm{F}}+ik\alpha_{21}^{(2)}\frac{d^2\chi^{\mathrm{F}}}{dy_2^2}=0,$$

$$k^2\theta_3\left(k^2\beta_1-\beta_2\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}}+\left(k^2\alpha_{22}^{(1)}-\alpha_{22}^{(2)}\frac{d^2}{dy_2^2}\right)\frac{d\chi^{\mathrm{F}}}{dy_2}-2ik\kappa h\phi^{\mathrm{F}}=P_2^{\mathrm{F}},$$

$$2ik\kappa\left(-k^2\beta_1+\beta_2\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}}-(k^2\theta_2+2\kappa)\phi^{\mathrm{F}}=0. \tag{3.5}$$

The equation of motion and the boundary conditions written in terms of Φ and Ψ can be transformed similarly:

$$\left(\frac{d^2}{dy_2^2}-k^2\eta_1^2\right)\Phi^{\mathrm{F}}=0, \qquad \left(\frac{d^2}{dy_2^2}-k^2\eta_2^2\right)\Psi^{\mathrm{F}}=0, \tag{3.6}$$

$$-ik\left(k^2\theta_1+2\kappa\alpha_1\frac{d}{dy_2}\right)\Phi^{\mathrm{F}}+k^2\left(\theta_1\frac{d}{dy_2}+2\kappa\alpha_2\right)\Psi^{\mathrm{F}}+(k^2\theta_4+2\kappa)\phi^{\mathrm{F}}=P_1^{\mathrm{F}},$$

$$-k^2\left(\alpha_1\theta_3\frac{d}{dy_2}+\alpha_{22}^{(1)}-\eta_1^2\alpha_{22}^{(2)}\right)\Phi^{\mathrm{F}}-ik\left[\alpha_2\theta_3k^2+(\alpha_{22}^{(1)}-\eta_2^2\alpha_{22}^{(2)})\frac{d}{dy_2}\right]\Psi^{\mathrm{F}}-2ik\kappa h\phi^{\mathrm{F}}=P_2^{\mathrm{F}},$$

$$ik(2\kappa\alpha_1-\eta_1^{-2}\alpha_{21}^{(1)}+\alpha_{21}^{(2)})\frac{d\Phi^{\mathrm{F}}}{dy_2}-k^2(2\kappa\alpha_2-\alpha_{21}^{(1)}+\eta_2^2\alpha_{21}^{(2)})\Psi^{\mathrm{F}}-(k^2\theta_2+2\kappa)\phi^{\mathrm{F}}=0 \tag{3.7}$$

for tight contact and

$$ik(\eta_1^{-2}\alpha_{21}^{(1)}-\alpha_{21}^{(2)})\frac{d\Phi^{\mathrm{F}}}{dy_2}-k^2(\alpha_{21}^{(1)}-\eta_2^2\alpha_{21}^{(2)})\Psi^{\mathrm{F}}=0,$$

$$-k^2\left(\alpha_1\theta_3\frac{d}{dy_2}+\alpha_{22}^{(1)}-\eta_1^2\alpha_{22}^{(2)}\right)\Phi^{\mathrm F}-ik\left[\alpha_2\theta_3k^2+(\alpha_{22}^{(1)}-\eta_2^2\alpha_{22}^{(2)})\frac{d}{dy_2}\right]\Psi^{\mathrm F}-2ik\kappa h\phi^{\mathrm F}=P_2^{\mathrm F},$$

$$2ik\kappa\alpha_1\frac{d\Phi^{\mathrm F}}{dy_2}-2k^2\kappa\alpha_2\Psi^{\mathrm F}-(k^2\theta_2+2\kappa)\phi^{\mathrm F}=0 \tag{3.8}$$

for nontight contact.

The solution of Eq. (3.3) is sought in the following form considering decay at infinity:

$$\chi^{\mathrm F}=Ae^{k_1k\eta_1y_2}+Be^{k_2k\eta_2y_2}, \tag{3.9}$$

where A and B are constants of integration; $k_j\equiv\sigma=|k|/k$ if $\eta_j^2>0$, and $k_j=i$ if $\eta_j^2<0$.

If η_i takes complex values, then it is necessity to set $k_j=1$ and $\eta_j=\sigma\operatorname{Re}\eta_j-(-1)^j i\operatorname{Im}\eta_j$ (j = 1, 2) in (3.9).

The solutions for $\Phi^{\mathrm F}$ and $\Psi^{\mathrm F}$ are represented in the form

$$\Phi^{\mathrm F}=A_0e^{k_1k\eta_1y_2},\quad \Psi^{\mathrm F}=B_0e^{k_2k\eta_2y_2}, \tag{3.10}$$

where A_0 and B_0 are constants of integration.

Note that for the values of $\chi^{\mathrm F}$, $\Phi^{\mathrm F}$, and $\Psi^{\mathrm F}$ to be finite, it is necessary that $\operatorname{Re}\eta_j>0$.

Note also that the function $\phi^{\mathrm F}$ appears linearly in the systems of equations (3.4), (3.5), (3.7), and (3.8). Substituting (3.9) into (3.4) or (3.5) depending on the boundary condition, we obtain a system of algebraic equations for A, B, and $\phi^{\mathrm F}$. The solution of this system can be written in the form

$$A=\frac{iP_1^{\mathrm F}U_1^{(2)}+P_2^{\mathrm F}U_2^{(2)}}{k^3\Delta(k)}e^{k_1k\eta_1h},\quad B=-\frac{iP_1^{\mathrm F}U_1^{(1)}+P_2^{\mathrm F}U_2^{(1)}}{k^3\Delta(k)}e^{k_2k\eta_2h},$$

$$\phi^{\mathrm F}=\frac{P_1^{\mathrm F}U_1+iP_2^{\mathrm F}U_2}{\Delta(k)}, \tag{3.11}$$

where

$$\begin{aligned}\Delta(k)=&-k^4\theta_1\theta_2\theta_3(k_1\gamma_1^{(1)}\gamma_2^{(2)}-k_2\gamma_1^{(2)}\gamma_2^{(1)})+k^3[\theta_3\theta_4(\gamma_2^{(1)}\gamma_{21}^{(2)}-\gamma_2^{(2)}\gamma_{21}^{(1)})\\&-k_1k_2\theta_1\theta_2(\gamma_1^{(1)}\gamma_{22}^{(2)}-\gamma_1^{(2)}\gamma_{22}^{(1)})]+k^2[2\kappa h\theta_1(k_2\gamma_1^{(2)}\gamma_{21}^{(1)}-k_1\gamma_1^{(1)}\gamma_{21}^{(2)}\\&+k_2\gamma_2^{(1)}\gamma_{22}^{(2)}-k_1\gamma_2^{(2)}\gamma_{22}^{(1)})-2\kappa\theta_1(\theta_3-2\kappa h)(k_1\gamma_1^{(1)}\gamma_2^{(2)}-k_2\gamma_1^{(2)}\gamma_2^{(1)})\\&+\theta_4(k_1\gamma_{22}^{(1)}\gamma_{21}^{(2)}-k_2\gamma_{21}^{(1)}\gamma_{22}^{(2)})]+2k\kappa\Big[(\theta_3-2\kappa h)(\gamma_2^{(1)}\gamma_{21}^{(2)}-\gamma_2^{(2)}\gamma_{21}^{(1)})\\&-k_1k_2\theta_1(\gamma_1^{(1)}\gamma_{22}^{(2)}-\gamma_1^{(2)}\gamma_{22}^{(1)})\Big]+2\kappa(k_1\gamma_{22}^{(1)}\gamma_{21}^{(2)}-k_2\gamma_{22}^{(2)}\gamma_{21}^{(1)}),\end{aligned} \tag{3.12}$$

$$U_1^{(j)}=k^3\theta_2\theta_3\gamma_2^{(j)}+k_jk^2\theta_2\gamma_{22}^{(j)}+2k\kappa[\gamma_2^{(j)}(\theta_3-2\kappa h)+h\gamma_{21}^{(j)}]+2k_j\kappa\gamma_{22}^{(j)},$$

$$U_2^{(j)}=k_jk^3\theta_1\theta_2\gamma_1^{(j)}+k^2(\theta_4\gamma_{21}^{(j)}-2\kappa h\theta_1\gamma_2^{(j)})+2k_jk\kappa\theta_1\gamma_1^{(j)}+2\kappa\gamma_{21}^{(j)},\quad j=1,2,$$

$$U_1=k\theta_3(\gamma_2^{(1)}\gamma_{21}^{(2)}-\gamma_2^{(2)}\gamma_{21}^{(1)})+k_1\gamma_{22}^{(1)}\gamma_{21}^{(2)}-k_2\gamma_{22}^{(2)}\gamma_{21}^{(1)}-2\kappa(k_1\gamma_{22}^{(1)}\gamma_2^{(2)}-k_2\gamma_{22}^{(2)}\gamma_2^{(1)}),$$

$$U_2=-k\theta_1[k_1\gamma_1^{(1)}\gamma_{21}^{(2)}-k_2\gamma_1^{(2)}\gamma_{21}^{(1)}-2\kappa(k_1\gamma_1^{(1)}\gamma_2^{(2)}-k_2\gamma_1^{(2)}\gamma_2^{(1)})]-2\kappa(\gamma_2^{(1)}\gamma_{21}^{(2)}-\gamma_2^{(2)}\gamma_{21}^{(1)}) \tag{3.13}$$

in the case of tight contact and

$$\Delta(k)=k^3\theta_2\theta_3(\gamma_2^{(2)}\gamma_{21}^{(1)}-\gamma_2^{(1)}\gamma_{21}^{(2)})+k^2\theta_2(k_2\gamma_{21}^{(1)}\gamma_{22}^{(2)}-k_1\gamma_{21}^{(2)}\gamma_{22}^{(1)})$$

$$+2k\kappa(\theta_3-2\kappa h)(\gamma_2^{(2)}\gamma_{21}^{(1)}-\gamma_2^{(1)}\gamma_{21}^{(2)})+2\kappa(k_2\gamma_{22}^{(2)}\gamma_{21}^{(1)}-k_1\gamma_{22}^{(1)}\gamma_{21}^{(2)}), \tag{3.14}$$

$$P_1^F\equiv 0,\quad U_1^{(j)}=0,\quad U_2^{(j)}=-\gamma_{21}^{(j)}(k^2\theta_2+2\kappa),\qquad j=1,2,$$

$$U_1=0,\quad U_2=-2\kappa(\gamma_2^{(2)}\gamma_{21}^{(1)}-\gamma_2^{(1)}\gamma_{21}^{(2)}) \tag{3.15}$$

in the case of nontight contact.

Here

$$\gamma_{in}^{(j)}=\alpha_{in}^{(1)}-k_j^2\eta_j^2\alpha_{in}^{(2)},\quad \gamma_{ii}^{(j)}=\eta_j(\alpha_{ii}^{(1)}-k_j^2\eta_j^2\alpha_{ii}^{(2)}),$$

$$\gamma_1^{(j)}=\eta_j,\quad \gamma_2^{(j)}=\beta_1-k_j^2\eta_j^2\beta_2,\quad i,j,n=1,2;\quad i\neq n. \tag{3.16}$$

Similarly, we use solution (3.10) and the boundary conditions (3.7) or (3.8) to determine A_0, B_0, and ϕ^{F}:

$$A_0=\frac{iP_1^{\mathrm{F}}U_{10}^{(2)}+P_2^{\mathrm{F}}U_{20}^{(2)}}{k^2\Delta_0(k)}e^{k_1k\eta_1h},\quad B_0=-\frac{iP_1^{\mathrm{F}}U_{10}^{(1)}+P_2^{\mathrm{F}}U_{20}^{(1)}}{k^2\Delta_0(k)}e^{k_2k\eta_2h},$$

$$\phi^{\mathrm{F}}=\frac{P_1^{\mathrm{F}}U_{10}+iP_2^{\mathrm{F}}U_{20}}{\Delta_0(k)}, \tag{3.17}$$

$$\Delta_0(k)=-\frac{\Delta(k)}{k_1\eta_1},\quad U_{j0}^{(2)}=U_j^{(2)},\quad U_{j0}^{(1)}=-\frac{iU_j^{(1)}}{k_1\eta_1},\quad U_{j0}=-\frac{U_j}{k_1\eta_1},\quad j=1,2. \tag{3.18}$$

Taking the Fourier transform of (1.15) and (1.3), we get

$$\tilde{Q}_{ii}^{\mathrm{F}}=-k^2\alpha_{ii}^{(1)}\frac{d\chi^{\mathrm{F}}}{dy_2}+\alpha_{ii}^{(2)}\frac{d^3\chi^{\mathrm{F}}}{dy_2^3},\quad \tilde{Q}_{ij}^{\mathrm{F}}=-ik^3\alpha_{ij}^{(1)}\chi^{\mathrm{F}}+ik\alpha_{ij}^{(2)}\frac{d^2\chi^{\mathrm{F}}}{dy_2^2},$$

$$\dot{u}_1^{\mathrm{F}}=-vk^2\frac{d\chi^{\mathrm{F}}}{dy_2},\quad \dot{u}_2^{\mathrm{F}}=ikv\left(k^2\beta_1\frac{\partial^2}{\partial y_1^2}-\beta_2\frac{d^2}{dy_2^2}\right)\chi^{\mathrm{F}},$$

$$M^{\mathrm{F}}=\frac{4}{3}\frac{ikG_1h^3}{1-\nu_1}\phi^{\mathrm{F}},\quad i,j=1,2;\quad i\neq j. \tag{3.19}$$

With (3.9) and (3.11), expressions (3.19) become

$$\tilde{Q}_{jj}^{\mathrm{F}}=\frac{1}{\Delta(k)}(iP_1^{\mathrm{F}}\Gamma_{jj}^{(1)}+P_2^{\mathrm{F}}\Gamma_{jj}^{(2)}),\quad \tilde{Q}_{nj}^{\mathrm{F}}=\frac{1}{\Delta(k)}(-P_1^{\mathrm{F}}\Gamma_{nj}^{(1)}+iP_2^{\mathrm{F}}\Gamma_{nj}^{(2)}),$$

$$\dot{u}_1^{\mathrm{F}}=\frac{1}{\Delta(k)}(iP_1^{\mathrm{F}}\Gamma_1^{(1)}+P_2^{\mathrm{F}}\Gamma_1^{(2)}),\quad \dot{u}_2^{\mathrm{F}}=\frac{1}{\Delta(k)}(-P_1^{\mathrm{F}}\Gamma_2^{(1)}+iP_2^{\mathrm{F}}\Gamma_2^{(2)}),$$

$$M^{\mathrm{F}}=\frac{1}{\Delta(k)}(iP_1^{\mathrm{F}}\Gamma_\varphi^{(1)}-P_2^{\mathrm{F}}\Gamma_\varphi^{(2)}),\quad n,j=1,2;\quad n\neq j, \tag{3.20}$$

$$\Gamma_{jj}^{(t)} = k_2 \gamma_{jj}^{(2)} U_t^{(1)} e^{k_2 k \eta_2 (y_2 + h)} - k_1 \gamma_{jj}^{(1)} U_t^{(2)} e^{k_1 k \eta_1 (y_2 + h)},$$

$$\Gamma_{ij}^{(t)} = \gamma_{ij}^{(2)} U_t^{(1)} e^{k_2 k \eta_2 (y_2 + h)} - \gamma_{ij}^{(1)} U_t^{(2)} e^{k_1 k \eta_1 (y_2 + h)},$$

$$\Gamma_1^{(t)} = v \left(k_2 \gamma_1^{(2)} U_t^{(1)} e^{k_2 k \eta_2 (y_2 + h)} - k_1 \gamma_1^{(1)} U_t^{(2)} e^{k_1 k \eta_1 (y_2 + h)} \right),$$

$$\Gamma_2^{(t)} = v \left(\gamma_2^{(2)} U_t^{(1)} e^{k_2 k \eta_2 (y_2 + h)} - \gamma_2^{(1)} U_t^{(2)} e^{k_1 k \eta_1 (y_2 + h)} \right),$$

$$\Gamma_\phi^t = \frac{4}{3} \frac{k G_1 h^3 U_t}{1 - \nu_1}, \quad i, j, t = 1, 2; \quad i \neq j. \tag{3.21}$$

The Fourier-transformed stresses and displacement rates in the half-space and the bending moment in the plate can be expressed in a similar way, using formulas (1.17), (1.3), (3.1), (3.10), and (3.17).

Equal Roots. Consider the case $\eta_1 = \eta_2 = \eta$. Taking the Fourier transform of (1.24) and (1.25) yields

$$-k^2 \left[\theta_1 \left(-k^2 \beta_{11}^{(2)} + \beta_{12}^{(2)} \frac{d^2}{dy_2^2} \right) + 2\kappa \beta_{21}^{(2)} \right] \chi^{(2)\mathrm{F}} - ik \left[-k^2 \theta_1 \beta_{11}^{(1)} \frac{d}{dy_2} \right.$$

$$\left. + 2\kappa \left(-k^2 \beta_{21}^{(1)} + \beta_{22}^{(1)} \frac{d^2}{dy_2^2} \right) \right] \chi^{(1)\mathrm{F}} - (k^2 \theta_4 + 2\kappa) \phi^{\mathrm{F}} = P_1^{\mathrm{F}},$$

$$\left[-k^2 \theta_3 \left(-k^2 \beta_{21}^{(1)} + \beta_{22}^{(1)} \frac{d^2}{dy_2^2} \right) + \left(k^2 \alpha_{22}^{(11)} - \alpha_{22}^{(21)} \frac{d^2}{dy_2^2} \right) \frac{d}{dy_2} \right] \chi^{(1)\mathrm{F}}$$

$$-ik \left(-k^2 \theta_3 \beta_{21}^{(2)} \frac{d}{dy_2} - k^2 \alpha_{22}^{(12)} + \alpha_{22}^{(22)} \frac{d^2}{dy_2^2} \right) \chi^{(2)\mathrm{F}} - 2ik\kappa h \phi^{\mathrm{F}} = P_2^{\mathrm{F}},$$

$$ik \left[-(2\kappa \beta_{21}^{(1)} - \alpha_{21}^{(11)}) k^2 + (2\kappa \beta_{22}^{(1)} - \alpha_{21}^{(21)}) \frac{d^2}{dy_2^2} \right] \chi^{(1)\mathrm{F}}$$

$$-\left[-k^2 (2\kappa \beta_{21}^{(2)} + \alpha_{21}^{(12)}) + \alpha_{21}^{(22)} \frac{d^2}{dy_2^2} \right] \frac{d}{dy_2} \chi^{(2)\mathrm{F}} - (k^2 \theta_2 + 2\kappa) \phi^{\mathrm{F}} = 0 \tag{3.22}$$

for tight contact and

$$\left(-k^2 \alpha_{21}^{(12)} + \alpha_{21}^{(22)} \frac{d^2}{dy_2^2} \right) \frac{d}{dy_2} \chi^{(2)\mathrm{F}} + ik \left(-k^2 \alpha_{21}^{(11)} + \alpha_{21}^{(21)} \frac{d^2}{dy_2^2} \right) \chi^{(1)\mathrm{F}} = 0,$$

$$\left[-k^2 \theta_3 \left(-k^2 \beta_{21}^{(1)} + \beta_{22}^{(1)} \frac{d^2}{dy_2^2} \right) + \left(k^2 \alpha_{22}^{(11)} - \alpha_{22}^{(21)} \frac{d^2}{dy_2^2} \right) \frac{d}{dy_2} \right] \chi^{(1)\mathrm{F}}$$

$$-ik \left(-k^2 \theta_3 \beta_{21}^{(2)} \frac{d}{dy_2} - k^2 \alpha_{22}^{(12)} + \alpha_{22}^{(22)} \frac{d^2}{dy_2^2} \right) \chi^{(2)\mathrm{F}} - 2ik\kappa h \phi^{\mathrm{F}} = P_2^{\mathrm{F}},$$

$$2k^2\kappa\beta_{21}^{(2)}\frac{d}{dy_2}\chi^{(2)\mathrm{F}}+2ik\kappa\left(-k^2\beta_{21}^{(1)}+\beta_{22}^{(1)}\frac{d^2}{dy_2^2}\right)\chi^{(1)\mathrm{F}}-(k^2\theta_2+2\kappa)\phi^{\mathrm{F}}=0 \tag{3.23}$$

for nontight contact.

The solution of Eqs. (3.3) is sought in the form

$$\chi^{(j)\mathrm{F}}=[A^{(j)}+|k|\eta(y_2+h)B^{(j)}]e^{|k|\eta y_2}, \tag{3.24}$$

where $A^{(j)}$ and $B^{(j)}$ are constants of integration. Let

$$A^{(1)}=\eta A,\quad B^{(1)}=\eta B,\quad A^{(2)}=\sigma iA,\quad B^{(2)}=\sigma iB. \tag{3.25}$$

The constants A and B and the function ϕ^{F} are determined as in the case of unequal roots of Eq. (1.5). As a result, we obtain expressions similar to (3.11). The components of formulas (3.11) are determined from either (3.12) and (3.13) or (3.14) and (3.15), depending on the contact conditions. The following notation should be used:

$$\gamma_{ii}^{(1)}=-\alpha_{ii}^{(12)}+\eta^2(\alpha_{ii}^{(22)}+\alpha_{ii}^{(11)}-\eta^2\alpha_{ii}^{(21)}),\quad \gamma_{ii}^{(2)}=\eta^2(2\alpha_{ii}^{(22)}+\alpha_{ii}^{(11)}-3\eta^2\alpha_{ii}^{(21)}),$$

$$\gamma_{ij}^{(1)}=\eta\left[\alpha_{ij}^{(12)}+\alpha_{ij}^{(11)}-\eta^2(\alpha_{ij}^{(22)}+\alpha_{ij}^{(21)})\right],\quad \gamma_{ij}^{(2)}=\eta\left[\alpha_{ij}^{(12)}-\eta^2(3\alpha_{ij}^{(22)}+2\alpha_{ij}^{(21)})\right],$$

$$\gamma_2^{(1)}=\eta(-\beta_{21}^{(2)}+\beta_{21}^{(1)}-\eta^2\beta_{22}^{(1)}),\quad \gamma_2^{(2)}=-\eta(2\eta^2\beta_{22}^{(1)}+\beta_{21}^{(2)}),\quad i,j=1,2;\quad i\neq j,$$

$$\gamma_1^{(1)}=\beta_{11}^{(2)}+\eta^2(\beta_{11}^{(1)}-\beta_{12}^{(2)}),\quad \gamma_1^{(2)}=\eta^2(\beta_{11}^{(1)}-\beta_{12}^{(2)}). \tag{3.26}$$

Formulas (1.3), (1.27), (3.1), and (3.11) can be used to represent the transformed stresses and displacement rates in the half-space and the bending moment in the plate in the form (3.20), where the functions Γ_{ij}^t, Γ_i^t, and Γ_ϕ^t are defined by formulas (3.21) with (3.26).

Thus, the Fourier-transformed solution for a prestressed two-layer elastic half-space steadily moving under the action of a moving load has the form (3.9) or (3.10) in the case of unequal roots and (3.24) in the case of equal roots. The components of formulas (3.9), (3.10), and (3.24) are determined from (3.11)–(3.13) for tight contact and from (3.11), (3.14), and (3.15) for nontight contact. In (3.12)–(3.15), notation (3.16) is used for $\eta_1\neq\eta_2$ and notation (3.26) for $\eta_1=\eta_2$.

The Fourier-transformed stress/strain characteristics are determined from (3.20) using the roots of the characteristic equation (3.16) or (3.26) and the boundary conditions (3.12), (3.13) or (3.14), (3.15). It follows from (3.20) that the stress/strain characteristics of the half-space increase without limit as $\Delta(k)\to 0$. The expression for $\Delta(k)=0$ is a quartic polynomial (in k) for (3.12) and a cubic polynomial for (3.14). If the equation $\Delta(k)=0$ has real positive multiple roots, resonance is possible [4]. The associated velocity of the load is called critical.

To recover the original functions in (3.20), it is necessary to use the inverse Fourier transform (3.2).

4. Let us examine the effect of prestresses, the mechanical characteristics of the plate and half-space, and their contact conditions on the critical velocities of the load in the case where the half-space is incompressible and described by the Bartenev–Khazanovich potential [2]:

$$\Phi=2\mu S_1. \tag{4.1}$$

We will use the method outlined in [4] to determine the critical velocities.

The theory of finite (large) prestrains gives the following expressions for $A_{i\beta}$, μ_{ij}, $S_0^{\beta\beta}$, and q_j:

$$A_{i\beta}=-2\delta_{i\beta}\lambda_\beta^{-4}(\mu\lambda_\beta+p^0),\quad S_0^{\beta\beta}=(2\mu\lambda_\beta+p^0)\lambda_\beta^{-2},$$

$$\mu_{ij}=-[2\mu\lambda_i\lambda_j+p^0(\lambda_i+\lambda_j)]\lambda_i^{-2}\lambda_j^{-2}(\lambda_i+\lambda_j)^{-1},\quad q_j=\lambda_j^{-1}. \tag{4.2}$$

TABLE 2

λ_1	v_2		
	$\mu / G_1 = 0.1$	$\mu / G_1 = 0.5$	$\mu / G_1 = 0.8$
1	0	0	0
1.1	0.372	0.831	1.052
1.2	0.490	1.096	1.387
1.3	0.562	1.256	1.589
1.4	0.609	1.361	1.722

Assume that the initial strain state is plane ($\lambda_3 = 1$, $\lambda_1 = \lambda_2^{-1}$) and the surface load is zero ($S_0^{22} = 0$). Then the components of the tensor $\tilde{\kappa}$ are expressed as

$$\tilde{\kappa}_{1111} = \tilde{\kappa}_{2222} = 2\mu\lambda_1^{-1}, \quad \tilde{\kappa}_{2112} = 2\mu\lambda_1^{-1}(\lambda_1^2+1)^{-1}, \quad \tilde{\kappa}_{1221} = 2\mu\lambda_1^3(\lambda_1^2+1)^{-1},$$

$$\tilde{\kappa}_{1212} = \tilde{\kappa}_{2121} = 2\mu\lambda_1^{-1}(\lambda_1^2+1)^{-1}, \quad \tilde{\kappa}_{1122} = \tilde{\kappa}_{2211} = 0. \tag{4.3}$$

Using (2.2)–(2.4), (4.2), and (4.3), we obtain

$$v_1^2 = 0, \quad v_2^2 = 8\mu(\lambda_1^2 - 1)\lambda_1^{-1}(\lambda_1^2+1)^{-1}. \tag{4.4}$$

It follows from (4.4) that for $\lambda_1 > 1$ there exists a velocity of the load such that Eq. (1.5) has equal roots ($\eta_1 = \eta_2$). Thus, when $0 < v < v_2$ and $\lambda_1 > 1$, the roots η_j ($j = 1, 2$) are complex (Table 1).

Substituting (4.2) and (4.3) into (1.6), (1.16), (1.26), (3.16) or (3.26) and then into (3.12) or (3.14), we find the roots of the equation $\Delta(k) = 0$ in the cases of tight and nontight contact, respectively.

For numerical purposes, we set $\nu_1 = 0.25$ (Poisson's ratio), $\kappa = 0.845$ (shear coefficient), and $\rho / \rho_1 = 0.5$.

Table 2 summarizes the values of v_2 for different values of μ / G_1 and λ_1.

Table 3 collects the values of the critical velocity v_1^* (minimum of the dispersion curve) and v_2^* (maximum of the dispersion curve) and the corresponding values of *kh*, the velocity of Rayleigh waves v_R, and the velocity c_1 for different values of μ / G_1 and λ_1 and different contact conditions (tight (a) and nontight (b)).

Note that the velocities in Tables 2 and 3 are divided by the velocity c_s of transverse waves in the plate material.

Figure 1 shows the dispersion curves with the parameters corresponding to $\mu / G_1 = 0.8$. The dispersion curve and dashed horizontal lines (c_1^2 / c_s^2) corresponding to the same value of λ_1 have the same number $n = 10\lambda_1 - 5$.

We will restrict ourselves to the case $kh < \pi$ because the motion of the layer is described using the theory of plates and the Timoshenko hypothesis, which is valid for waves longer than the thickness of the layer.

An analysis of the above results leads to the following conclusions. The effect of prestresses on the critical velocity of the load is stronger for relatively soft plates and nontight contact. With nontight contact, the resonance occurs at a lower velocity of the load than with tight contact. The softer the plate than the half-space, the less the critical velocities of the load than the velocity v_R of Rayleigh waves. Note that in the range of λ_1 being considered, the velocity v_R decreases compared with c_1 under compression and tends to c_1 under tension. In the case of tight contact for $\mu / G_1 = 0.8$ (Fig. 1*a*), the dispersion curve for $\lambda_1 > 1$ (curves *5–9*) have two extrema. In the other cases, there is only one or no critical velocity.

The critical velocities for various contact conditions between the half-space and the plate are close to the velocity v_R under compression and less than v_R under tension.

The rate of change in the critical velocities increases under compression and decreases under tension.

TABLE 3

μ / G_1	λ_1		0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.1	c_1		0.252	0.303	0.353	0.401	0.447	0.491	0.532	0.572	0.609
	v_R		0.101	0.232	0.310	0.373	0.427	0.475	0.511	0.556	0.561
	$\frac{v_1^*}{kh}$	a	—	—	—	—	0.428	0.457	0.483	0.486	0.488
							0.199	0.253	0.291	0.300	0.308
		b	0.094	0.219	0.291	0.346	0.394	0.432	0.457	0.467	0.473
			0.059	0.138	0.186	0.226	0.262	0.294	0.314	0.323	0.329
	$\frac{v_2^*}{kh}$	a	—	—	—	—	0.435	0.478	0.522	0.560	0.566
							0.048	0.030	0.020	0.017	0.013
0.5	c_1		0.564	0.679	0.790	0.898	1.000	1.098	1.190	1.278	1.362
	v_R		0.220	0.518	0.694	0.834	0.955	1.063	1.162	1.251	1.339
	$\frac{v_1^*}{kh}$	a	—	—	—	0.822	0.844	0.878	0.914	0.931	0.941
						0.441	0.667	0.853	1.093	1.269	1.363
		b	0.207	0.460	0.597	0.694	0.723	0.823	0.880	0.911	0.927
			0.132	0.317	0.452	0.578	0.702	0.810	1.000	1.138	1.232
0.8	c_1		0.713	0.858	0.999	1.135	1.265	1.388	1.505	1.617	1.722
	v_R		0.285	0.655	0.878	1.055	1.208	1.345	1.470	1.586	1.710
	$\frac{v_1^*}{kh}$	a	—	—	0.861	—	—	—	—	—	—
					0.286						
		b	0.262	0.565	0.721	0.827	0.901	—	—	—	—
			0.168	0.418	0.620	0.835	1.088				

In the case of tight contact for $\lambda_1 < 1$, there exists a value λ_1^* at which there are no critical velocities. The same is observed in relatively soft plates under tension ($\lambda_1 > 1$) for both tight and nontight contacts.

Thus, the values and number of critical velocities of the load are strongly dependent on the prestresses in the half-space, the mechanical characteristics of the components of the two-layer medium, and the contact conditions.

5. Let us analyze, as an example, the stress–strain state of a two-layer medium: a layer (plate) on an incompressible half-space with Bartenev–Khazanovich elastic potential (4.1). Assume that the conditions formulated in Sec. 4 are satisfied and the two-layer medium is subject to a linear load with components given by $P_1 = P\delta(y_1)\cos\alpha$ and $P_2 = P\delta(y_1)\sin\alpha$, $P = G_1$, where α is the angle between the load direction and the Oy_1-axis.

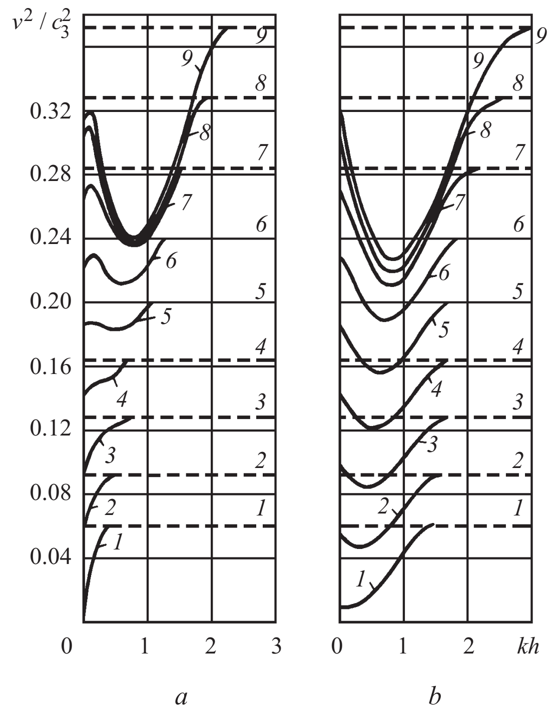


Fig. 1

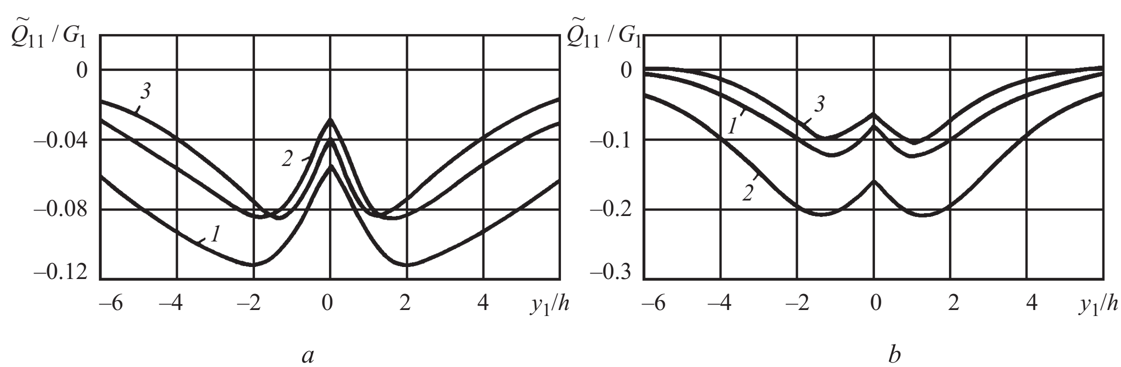


Fig. 2

To calculate the components of stresses and displacement rates in the half-space and the bending moment in the plate, we will use formulas (3.20), (3.2), and (4.3). The integrals in the inverse transform will be evaluated as in [23].

Let us examine the dependence of the stress/strain characteristics on the prestresses in the half-space for different velocities of the load (subsonic, transonic, and supersonic) and different contact conditions between the plate and the half-space.

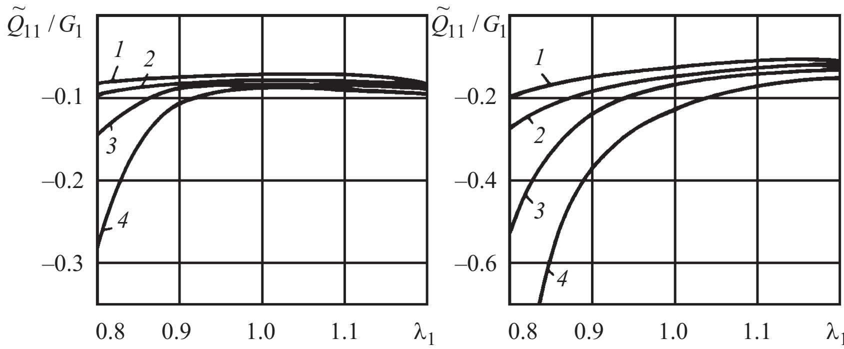
Let $\kappa = 0.845$, $\mu / G_1 = 0.5$, $\rho / \rho_1 = 0.5$, $\nu_1 = 0.25$, $\alpha = \pi/2$. With such values, $c_1 = c_s$ if $\lambda_1 = 1$.

When $v < c_1$, we consider only subcritical velocities of the load.

Let us examine the case $v < v^* < c_1$. Table 3 summarizes the critical velocities of the load for different λ_1 and contact conditions.

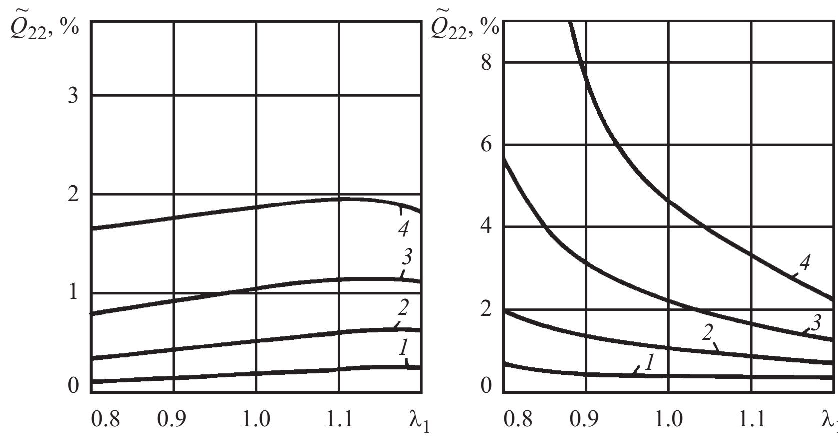
Figure 2 shows the distribution of the stress $\tilde{Q}_{11}$ for $y_2 = -2h / \lambda_2$ and $v^2 = 0.1c_s^2$ in the cases of tight contact (*a*) and nontight contact (*b*). Curves *1*, *2*, and *3* correspond to $\lambda_1 = 0.8$, $\lambda_1 = 1$, and $\lambda_1 = 1.2$. With such velocities of the load, the diagrams of stress/strain characteristics are symmetric about the point of load application.

Figure 3 shows the dependence of the stress/strain characteristics on the prestresses in the half-space and the velocity of the load at the point $y_1 = -\lambda_1 h$, $y_2 = -h / 2$ of the plate and at the point ($y_1 = -\lambda_1 h$, $y_2 = -2h\lambda_1$) of the half-space in the cases of tight contact (*a*) and nontight (*b*) contact. Curves *1*, *2*, *3*, and *4* correspond to $v^2 = 0.1c_s^2$, $0.2c_s^2$, $0.3c_s^2$, and $0.4c_s^2$.



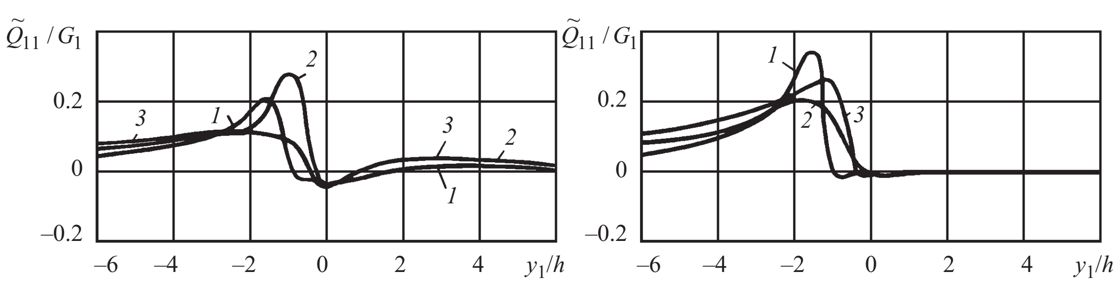
a *b*

Fig. 3



a *b*

Fig. 4



a *b*

Fig. 5

Figure 4 shows the effect of rotary inertia at different velocities of the load and prestrains on $\widetilde{Q}_{22}$ at the point ($y_1 = -\lambda_1 h$, $y_2 = -2h\lambda_1$) in the cases of tight (*a*) and nontight (*b*) contact. The notation in Fig. 4 is the same as in Fig. 3.

An analysis of the numerical results for subcritical velocities of the load leads to the following conclusions. The stress amplitude, displacement rates in the half-space, and the bending moment in the plate are less with tight contact than with nontight. At λ_1, the rate of their increase under compression is greater than under tension. Decrease with distance from the point of load application is slower under compression than under tension. The stress/strain characteristics and their dependence on the prestresses are determined by the coordinates of the point of interest.

As the velocity of the load increases, the effect of prestresses becomes stronger. The increase is more intensive under compression. With tight contact, the effects of the velocity and prestresses are weaker than with nontight contact.

Figure 4 demonstrates that rotary inertia is significant with nontight contact and high velocities of the load. With tight contact, the effect of the prestresses on the value of the inertia term is weak (Fig. 4*a*), whereas with nontight contact this value is strongly dependent on not only the velocity of the load, but also the prestresses (Fig. 4*b*).

Consider the case $v > c_1$. Figure 5 shows the stress $\tilde{Q}_{11}$ as a function of the distance to the point of load application in the cases of tight (Fig. 5*a*) and nontight (Fig. 5*b*) contact for $v^2 = 2c_s^2$ and $y_2 = -2h\lambda_1$ for the half-space and $y_2 = -h/2$ for the plate. The notation in Fig. 5 is the same as in Fig. 2. With supersonic velocity of the load, the curves are asymmetric about the point of load application, as in the case of a compressible half-space. The forward wave decays much quicker than the backward wave, but does not disappear completely because of the presence of the plate. An analysis of the results shows that the stresses and displacement rates in the half-space and the bending moment in the plate are strongly dependent on the prestresses in the half-space when $v > c_1$. The form of this relationship is determined by the position of a point of the layered medium relative to the point of load application.

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