EXPERIMENTAL DETERMINATION OF THE DYNAMIC CHARACTERISTICS OF A SYSTEM OF CONICAL SHELLS WITH ADDED MASSES

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A technique and results of experimental determination of vibrational stresses in a system of two truncated conical shells joined by three thin plates are presented. An analysis of the data obtained shows that the stress peaks near the added point mass at resonance

Keywords: system of shells, added masses, natural vibrations, impact force, strains, stresses

Introduction. The state of the art in the experimental investigation of the dynamics of shells of revolution is reviewed in [2, 7–11, 12]. Nonstationary loads on plates and shells are studied theoretically and experimentally in [1, 6, 7–14]. These studies dealt with structural members that deform elastically. The cited reviews indicate that the stress–strain state of a system of shells with added point masses is understood inadequately. Theoretical methods are expedient to combine with experimental ones to identify the validity limits of or to justify approaches used. Dynamic strains in shells with added point masses have been studied inadequately because of their local nature. The reviews also reveal the inadequate understanding of the synchronization of vibrations in harmonically excited systems of shells.

The objective of the present paper is to study the effect of vibrations of one shell in a system consisting of two or three shells mounted on a common massive elastic foundation on the vibrations and deformations of the other shells. Here conventional measurements of dynamic strains are supplemented by stress–strain analysis of a system consisting of truncated conical shells with an added point mass.

1. Experimental Determination of Natural Frequencies and Vibration Modes. The natural frequencies and vibration modes of a system consisting of two truncated conical shells joined by three plates were measured by the resonance method using a setup schematized in Fig. 1. This system models standard structures frequently used in shipbuilding, aircraft construction, and other branches. Shells No. 1 (*3* in Fig. 1) and No. 2 (*7* in Fig. 1) have the following parameters: the radius of the major base $R = 23.5$ cm, the radius of the minor base $r = 14.25$ cm, the height $L = 28.5$ cm, the wall thickness $h_0 = 0.03$ cm. The dimensions of the plates are 8×4×0.03 cm. Bands 2.5 cm wide and 0.03 cm thick were soldered round the shells at the major and minor bases. The shells, bands, and plates are made of OT4-1 titanium alloy. The shells were mounted vertically. Shell No. 1 was tested first. The minor end of the shell was attached to massive foundation *1* using conical disk *2*; the upper end remained free. The vibration modes of the free end were identified visually. The natural frequencies and vibration modes of shell No. 1 were then determined. After that, two shells (Nos. 1 and 2) joined by three plates *4* were subjected to the same tests. Vibrations were excited with a VÉDS-10A electrodynamic shaker through pin *9* connecting the moving table of electrodynamic vibrator *10* with the shell wall. The results obtained are given in Fig. 2 and Table 1.

Figure 2 shows the experimental natural frequency $f_{\exp}$ as a function of the number of circumferential waves n in shell No. 1 without added masses. Table 1 summarizes experimental data for shell No. 1 with added masses. There were no added point masses in test 1. In test 2, three masses were placed at regular intervals along the middle circumference of shell No. 1. One of the masses was fixed at the point of attachment of the vibrator coil. In test 3, the coil was between two added point masses. The compound shell was tested without added masses. The minimum natural frequency of the compound shell was $f_{\exp}^{\min} = 27$ Hz for $n = 3$ at the upper free end of shell No. 2.

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TABLE 1

Test number	M	f_{exp}^{min}	n
1	0	25	3
2	3 masses 0.345 kg each	18 Hz	3
3	3 masses 0.345 kg each	20 Hz	3

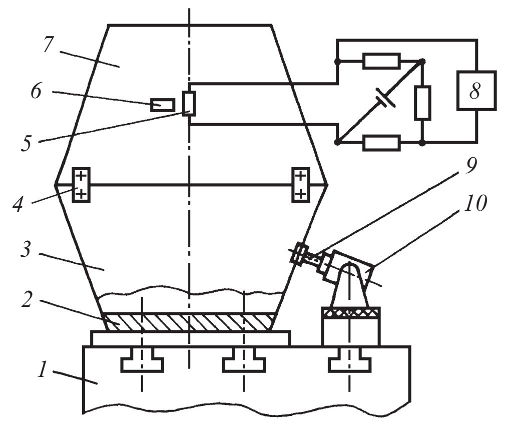

Fig. 1

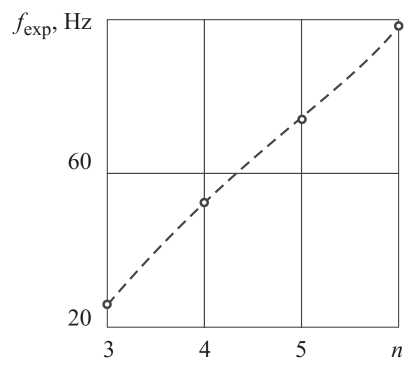

Fig. 2

Table 1 demonstrates that the added point masses have the strongest effect on the minimum natural frequencies when a point mass is fixed at the point of attachment of the vibrator. Connecting shells Nos. 1 and 2 by plates changes the boundary condition at the upper end and, hence, the flexural stiffness on the midsection of the compound shell, resulting in the minimum natural frequency of the new structure with new boundary conditions at the ends. Thus, both shells of the system have the same frequency, which, according to the synchronization law, is the minimum natural frequency of this system of shells.

2. Experimental Determination of the Stress–Strain State of Dynamically Excited System of Shells. The strains induced by the flexural vibrations of shell No. 1 excited by the vibrator moving at 40 m/sec^2 were measured in shell No. 2 (*7* in Fig. 1) using a setup schematized in Fig. 1. The vibration acceleration of the moving part of the vibrator was measured with a D14 transducer (not shown in Fig. 1). Its axis was parallel to the moving part of the vibrator. Two tests were conducted. There were no point masses attached in one test. In the other test, a 9.5 cm long cantilever with a point mass $M = 0.173$ kg at the end was attached near the strain gages. The strains were measured with four KTD7A strain gages bonded with BF-2 adhesive in the midsection of the shell, on its outer and inner surfaces. The circumferential (*6* in Fig. 1) and meridional (*5* in Fig. 1) strain gages were alternately connected into a bridge circuit consisting of three MSR-63 resistor banks. The offset voltage of the bridge was measured with a V3-56 millivoltmeter (*8* in Fig. 1) to determine the strains in the circumferential and meridional directions on both outer and inner surfaces. Measurements were conducted at the minimum natural frequency and constant level of excitation. The strains were calculated by the following formula [4]:

$$\varepsilon = \frac{4\Delta U}{kU}, \qquad (1)$$

where U is the bridge supply voltage; k is the strain-gauge factor; and ΔU is the bridge offset voltage.

The strains found were used to determine the membrane and bending stresses by the formulas from [3]. The results are summarized in Table 2.

TABLE 2

M, kg	$\sigma_{1tens}/10^5$, Pa	$\sigma_{1bend}/10^5$, Pa	$\sigma_{2tens}/10^5$, Pa	$\sigma_{2bend}/10^5$, Pa
0	53.0	16.0	58.5	10.5
0.173	57.5	14.5	100.5	7.5

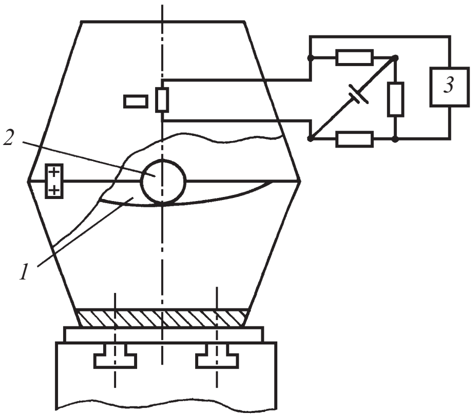

Fig. 3

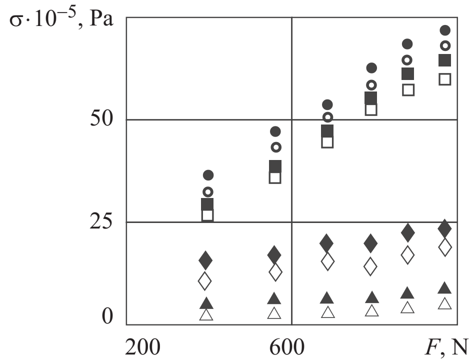

Fig. 4

It was established that the point mass ($M = 0.173$ kg, not shown in Fig. 1) attached near the strain gages increases the strains and the tensile (σ_{1tens} increased by 8.5% and σ_{2tens} by 71.8%) and bending (σ_{1bend} decreased by 9.3% and σ_{2bend} by 28.6%) stresses, the tensile stresses being much larger than the bending stresses.

3. Experimental Determination of the Stress–Strain State of a System of Shells under Impulsive Loading. The experimental setup is shown in Fig. 3. An impulsive load was generated by dropping solid *2* onto diaphragm *1*. The diaphragm is an OT4-1 titanium-alloy spherical cap with base radius $R_c = 23$ cm, deflection $A = 5$ cm, and thickness $h_c = 0.03$ cm. The diaphragm was attached, convex downward, with three screws at the points of fixation of shell No. 2 in the midsection of the compound shell. The solid is a steel ball 6 cm in diameter and 0.8 kg in mass. It was dropped from different heights $h = 5, 10, 15, 20, 25, 30$ cm. How the ball is dropped is not shown in Fig. 3. The strains were measured with the same strain gages as those used in the previous experiments and shown in Fig. 1. The bridge offset voltage was measured with a V3-56 millivoltmeter (*3* in Fig. 3). It was concluded in [5] that the magnitude of the impact force is constant throughout the impact and is given by

$$F_{imp} = \frac{m\sqrt{2gh}}{\tau}, \tag{2}$$

where m is the mass of the ball; g is the acceleration of gravity; τ is the impact duration (according to [6], this duration is $\tau = 2$ msec, vibrorecords made with a S8-17 oscilloscope with memory).

The stresses in longitudinal and transverse sections of shell No. 2 were determined from the measured strains by the formulas from [3]. Figure 4 shows these stresses as functions of the impact force F_{imp}. The circles and triangles stand for the stresses σ_{1tens} and σ_{1bend} in longitudinal sections, and squares and diamonds for the stresses σ_{2tens} and σ_{2bend} in transverse sections (obviously, the bending stresses were determined at points on the shell surface). The open symbols refer to the shells without added point mass, and the full symbols to the shell with such a mass ($M = 0.173$ kg).

An analysis of the experimental data shows that the vibrational stresses and the impact force are in a nearly linear relationship and that the tensile stresses are much larger than the bending stresses.

We also may conclude that the technique developed and the experimental data obtained can further be used to determine strains and stresses at various points of compound shells with attached solids under dynamic loading.

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