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EFFECT OF ACOUSTIC RADIATION ON A SPHERICAL DROP OF LIQUID

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The effect of radiation force on a drop of liquid in an acoustic field is examined. It is established that the force depends on the ratio of the densities of the liquid and the drop and on their adiabatic elastic moduli

Keywords: acoustic wave, plane wave, sound pressure, liquid sphere, radiation force

Introduction. Various issues of motion and interaction of liquid drops, gas bubbles, and cavities were addressed in [2, 3, 8, 13, etc.]. The motion of drops in an acoustic field under radiation forces is of certain technological interest. A radiation force acting on an obstacle in an acoustic field comes about from a change, within some volume, in the time-average momentum carried by a wave and is determined by the integral of the time-average sound (radiation) pressure over the surface of the obstacle. Note that the exact values of radiation pressure in Lagrangian and Eulerian coordinate systems are generally different [1]. In the former case, it is defined as the time-average sound pressure on the surface oscillating in an acoustic field. In the latter case, it is defined as a convolution of the momentum flux density tensor and the normal unit vector to the surface.

To determine the radiation force (independent of time) acting on an obstacle in an acoustic field, the liquid pressure is calculated up to the second-order terms, which are due to the nonlinearity of the acoustic field and, hence, do not vanish after averaging over time. The obstacle changes the acoustic field by generating a reflected wave. Therefore, it is necessary to solve a diffraction problem. If the obstacle is a gas bubble, the oscillatory processes excited by the incident wave in the bubble may affect the wave scattering significantly: if the frequency of the wave and the natural frequency of the bubble are close, then the effective scattering diameter will be many times that of the bubble [4, 7]. As a result, the reflected wave noticeably contributes to the radiation force.

The radiation force can be determined in several steps. The first step is to identify the wave scattered by the obstacle. Since the liquid pressure can be determined up to second-order terms in linear approximation [10], the reflected wave can be identified by solving a linear diffraction problem. The solution of this problem is used at the second step to determine the resultant force exerted by the liquid on the obstacle. At the third step, this force is averaged over time to filter out the constant (radiation) force and to study the behavior of the obstacle under the radiation force.

Since the wave reflected from the obstacle is determined by solving the linear diffraction problem, the interaction of the incident and reflected waves is neglected. In what follows, we will use the above approach to examine the case where the obstacle in a liquid is a drop of liquid of other kind. The cases of single solid particles and systems of such particles were discussed in [9].

1. Diffraction Problem. Let us first identify the wave reflected from a liquid drop. If the drop is a sphere of radius R, then we will solve a linear diffraction problem for a liquid sphere. Assume that the space is filled with a perfect liquid with density ρ_0 and speed of sound a_0. The liquid of the sphere has density $\overline{\rho}_0$ and speed of sound $\overline{a}_0$. Hereafter, overbar will refer to the liquid sphere.

We choose a Cartesian coordinate system *Oxyz* so that its origin *O* is at the center of the liquid sphere. Consider a plane acoustic wave propagating through the liquid space and described by the potential

$$\Phi_i = A\exp[i(kx-\omega t)], \tag{1.1}$$

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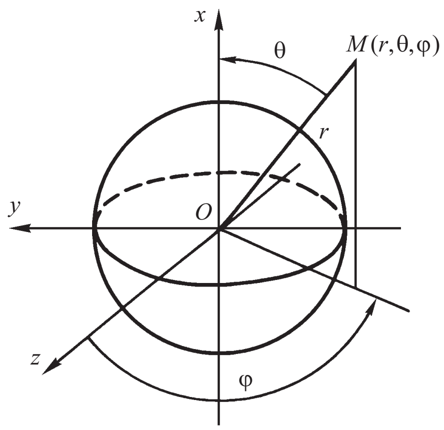

Fig. 1

where A is the amplitude, $k = \omega / a_0$ is the wave number, and ω is the angular frequency.

The incident wave causes the liquid sphere to contract and expand periodically. The effect of surface tension is neglected. Mathematically, the diffraction problem (1.1) is to determine the velocity potential Φ_s of the reflected wave, i.e., to solve the linear wave equation (its solution decays at infinity)

$$\Delta\Phi - \frac{1}{a_0^2}\frac{\partial^2\Phi}{\partial t^2} = 0, \tag{1.2}$$

with boundary conditions such that the pressure and normal velocity are continuous on the sphere surface. To describe the liquid sphere, we choose a spherical coordinate system (r, θ, φ) where the angle θ is reckoned from the Ox-axis (Fig. 1). Then the boundary conditions read

$$\left(p_i + p_s\right)_{r=R} = \bar{p}\big|_{r=R}, \tag{1.3}$$

$$\left(v_r^i + v_r^s\right)_{r=R} = \bar{v}_r\big|_{r=R}. \tag{1.4}$$

This approach is widely used to solve various problems in hydromechanics [11, 12].

In (1.3) and (1.4): p_i and v_r^i are the pressure and radial velocity in the surrounding liquid induced by the incident wave; p_s and v_r^s are the analogous quantities for the reflected wave; and $\bar{p}$ and $\bar{v}_r$ are the pressure and radial velocity in the liquid sphere. These quantities are determined in terms of the velocity potentials, which are solutions of Eq. (1.2). Given velocity potential, the pressure and radial velocity can be calculated in a linear approximation:

$$p = -\rho_0\frac{\partial\Phi}{\partial t}, \qquad v_r = \frac{\partial\Phi}{\partial r}. \tag{1.5}$$

It is assumed that the sphere surface oscillates with low amplitude and, thus, $R = \text{const}$. This assumption is reasonable for the liquid sphere.

To solve Eq. (1.2), we will use the method of separating variables in spherical coordinates. Then the velocity potentials of the reflected wave and for the wave inside the sphere can be expanded into generalized Fourier series in terms of spherical wave functions:

$$\Phi_s = \sum_{n=0}^{\infty} A_n h_n^{(1)}(kr) P_n(\cos\theta)\exp(-i\omega t), \tag{1.6}$$

$$\bar{\Phi} = \sum_{n=0}^{\infty} \bar{A}_n j_n\left(\bar{k}r\right) P_n(\cos\theta)\exp(-i\omega t), \tag{1.7}$$

where $h_n^{(1)}(w)$ are spherical Hankel functions of the first kind; $j_n(w)$ are spherical Bessel functions; and $P_n(\cos\theta)$ are Legendre polynomials. The coefficients A_n and $\overline{A}_n$ are determined from the boundary conditions (1.3) and (1.4) on the sphere surface $(r = R)$. Expression (1.6) satisfies the conditions at infinity.

Let us expand the potential (1.1) into a series in terms of spherical wave functions:

$$\Phi_\mathrm{i} = \sum_{n=0}^{\infty} A\left(2n+1\right) i^n j_n\left(kr\right) P_n\left(\cos\theta\right) \exp\left(-i\omega t\right). \tag{1.8}$$

Substituting the velocity potentials (1.6) and (1.8) into (1.5), we obtain the following formulas for the sound pressure $p = p_\mathrm{i} + p_\mathrm{s}$ and radial velocity $v_\mathrm{r} = v_\mathrm{r}^\mathrm{i} + v_\mathrm{r}^\mathrm{s}$ in the surrounding liquid:

$$p = i\omega\rho_0 \sum_{n=0}^{\infty} \left[A(2n+1) i^n j_n\left(kr\right) + A_n h_n^{(1)}\left(kr\right)\right] P_n\left(\cos\theta\right) \exp\left(-i\omega t\right), \tag{1.9}$$

$$v_\mathrm{r} = \sum_{n=0}^{\infty} \left[A\left(2n+1\right) i^n \frac{dj_n\left(kr\right)}{dr} + A_n \frac{dh_n^{(1)}\left(kr\right)}{dr}\right] P_n\left(\cos\theta\right) \exp\left(-i\omega t\right). \tag{1.10}$$

Dealing with the potential $\overline{\Phi}$ in a similar way, we obtain formulas for the sound pressure and radial velocity in the liquid sphere:

$$\overline{p} = i\omega\rho_0 \sum_{n=0}^{\infty} \overline{A}_n j_n\left(\overline{k}r\right) P_n\left(\cos\theta\right) \exp\left(-i\omega t\right), \tag{1.11}$$

$$\overline{v}_r = \overline{k} \sum_{n=0}^{\infty} \overline{A}_n \frac{dj_n(\overline{k}r)}{dr} P_n\left(\cos\theta\right) \exp\left(-i\omega t\right). \tag{1.12}$$

Conditions (1.3) and (1.4) and expressions (1.9)–(1.12) lead to an infinite system of algebraic equations ($n = 0, 1, \ldots$):

$$h_n^{(1)}(\alpha) A_n - \frac{\overline{\rho}_0 \overline{k} \overline{a}_0}{\rho_0 k a_0} j_n(\overline{\alpha}) \overline{A}_n = -A\left(2n+1\right) i^n j_n(\alpha),$$

$$\frac{dh_n^{(1)}(\alpha)}{dr} A_n - \frac{\overline{k}}{k} \frac{dj_n(\overline{\alpha})}{dr} \overline{A}_n = -A\left(2n+1\right) i^n \frac{dj_n(\alpha)}{dr} \qquad (\alpha = kR, \overline{\alpha} = \overline{k}R), \tag{1.13}$$

whence we obtain formulas for the unknown coefficients A_n and $\overline{A}_n$:

$$A_n = A\left(2n+1\right) i^n \frac{j_n(\alpha) \dfrac{dj_n(\overline{\alpha})}{dr} - \dfrac{\overline{\rho}_0 \overline{a}_0}{\rho_0 a_0} j_n(\overline{\alpha}) \dfrac{dj_n(\alpha)}{dr}}{\dfrac{\overline{\rho}_0 \overline{a}_0}{\rho_0 a_0} j_n(\overline{\alpha}) \dfrac{dh_n^{(1)}(\alpha)}{dr} - \dfrac{dj_n(\overline{\alpha})}{dr} h_n^{(1)}(\alpha)}, \tag{1.14}$$

$$\overline{A}_n = A\left(2n+1\right) i^{n+1} \frac{k}{\overline{k}\alpha^2} \frac{1}{\dfrac{\overline{\rho}_0 \overline{a}_0}{\rho_0 a_0} j_n(\overline{\alpha}) \dfrac{dh_n^{(1)}(\alpha)}{dr} - \dfrac{dj_n(\overline{\alpha})}{dr} h_n^{(1)}(\alpha)}. \tag{1.15}$$

Let us restrict ourselves to the case where the sphere radius is small compared with the wavelength. Then $\alpha \ll 1$. It can be shown that $\overline{\alpha} \ll 1$. Now we can simplify expressions (1.14) and (1.15) using the asymptotic representations of the functions $j_n(w)$ and $h_n^{(1)}(w)$ and their derivatives at small values of w [5, 7]:

$$j_n(w) \approx \frac{w^n}{\overline{n}(2n+1)}, \quad \overline{n} = 1\cdot 3\cdot 5\cdots(2n-1), \quad n(0) = 1,$$

$$\frac{d}{dw} j_n(w) = -D_n(w)\sin\delta_n(w), \quad D_n(w) \approx \frac{\overline{n}(n+1)}{w^{n+2}},$$

$$\delta_n(w) \approx -\frac{nw^{2n+1}}{\overline{n}^2(2n+1)(n+1)}, \quad n > 0,$$

$$h_n^{(1)}(w) = G_n(w)\exp[i\varepsilon(w)], \quad G_n(w) \approx \frac{\overline{n}}{w^{n+1}},$$

$$\varepsilon(w) \approx \frac{w^{2n+1}}{\overline{n}^2(2n+1)} - \frac{\pi}{2}, \quad \frac{d}{dw} h_n^{(1)}(w) = iD_n(w)\exp[i\delta(w)]. \tag{1.16}$$

Expressions (1.14)–(1.16) yield formulas for the coefficients A_n and $\overline{A}_n$ ($n = 0, 1, 2, \ldots$) when $\alpha, \overline{\alpha} << 1$. Here are the formulas for the first three coefficients A_n and $\overline{A}_n$:

$$A_0 = i\frac{1}{3}Ak^*\alpha^3, \quad k^* = \frac{3\overline{\kappa}(\kappa - \overline{\kappa})}{\overline{k}(3\overline{\kappa} - \kappa\alpha^2)}, \tag{1.17}$$

$$A_1 = A\beta^*\alpha^3, \quad \beta^* = \frac{\rho_0 - \overline{\rho}_0}{2\overline{\rho}_0 + \rho_0}, \tag{1.18}$$

$$A_2 = i\frac{2}{9}A\rho^*\alpha^5, \quad \rho^* = \frac{\rho_0 - \overline{\rho}_0}{2\rho_0 + 3\overline{\rho}_0}, \tag{1.19}$$

where $\kappa = \rho_0 a_0^2$ and $\overline{\kappa} = \overline{\rho}_0 \overline{a}_0^2$ are the adiabatic elastic moduli of the surrounding liquid and liquid sphere.

Since α, $\overline{\alpha} << 1$ by assumption, the scattered wave is mainly determined by the first terms in (1.6) because even A_2 (1.19), having the order of α^5, differs from A_0 (1.17) and A_1 (1.18) by a quantity of the order of α^2. Therefore, we will retain the first three terms in (1.6). Note that the zero term in (1.6) is responsible for the pulsations of the liquid sphere, and the second term for its oscillations. If the condition $\overline{\kappa} >> \kappa$ fails, then, according to (1.17), the pulsation amplitude will tend to peak as α tends to $\sqrt{3\overline{\kappa}/\kappa}$, which represents the pulsation resonance of the sphere [7]. The velocity potential $\overline{\Phi}$ in the liquid sphere will no longer be used here; therefore, the expressions for $\overline{A}_n$ are omitted. The solution of the diffraction problem is complete once the potentials Φ_s and $\overline{\Phi}$ are determined.

2. Radiation Force. The radiation force can be calculated by averaging over time the hydrodynamic force exerted by the liquid on the sphere in the acoustic field. Since the wave field is axially symmetric about the *Ox*-axis, the hydrodynamic force is parallel to this axis and is defined by

$$F_x = -2\pi R^2 \int_0^\pi p \sin\theta \cos\theta d\theta. \tag{2.1}$$

Since force (2.1) is to be further averaged over the period of the incident wave, the sound pressure p in the liquid can be determined in a quadratic approximation. To this end, we use the approach from [10], which suggests calculating the pressure p up to the second power of the Mach number:

$$p = -\rho_0 \frac{\partial\Phi}{\partial t} - \frac{1}{2}\rho_0(\nabla\Phi)^2 + \frac{1}{2}\frac{\rho_0}{a_0^2}\left(\frac{\partial\Phi}{\partial t}\right)^2. \tag{2.2}$$

Here, we can use the potential $\Phi = \Phi_i + \Phi_s$ determined in Sec. 1 by solving the linear diffraction problem [9, 10]. The structure of (2.2) suggests that the real part of the expression for the potential Φ should be used to determine the pressure p. In calculating the radiation force, it is necessary to take into account the change in the position of the oscillating sphere because the associated terms in (2.2) have the same order as that of the second and third terms. The reason is that the partial derivative $\partial\Phi/\partial t$ in (2.2) should be evaluated in the same fixed coordinate system that was used to determine the potential Φ. In the axisymmetric case, this derivative can be found in the spherical coordinate system fixed to the moving sphere as follows:

$$\frac{\partial\Phi}{\partial t} = \frac{d\Phi}{dt} - V_x \cos\theta \frac{\partial\Phi}{\partial r} + V_x \frac{\sin\theta}{r}\frac{\partial\Phi}{\partial\theta}, \tag{2.3}$$

where $d\Phi/dt$ is the total derivative with respect to time. With the accuracy adopted, relation (2.3) should only be used to determine the contribution of the first term in (2.2) to the hydrodynamic force (2.1). Moreover, only the second and third terms in (2.3) should be kept because the subsequent time averaging makes the total derivative of a bounded function vanish.

Since

$$\nabla\Phi = \frac{\partial\Phi}{\partial r} e_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta} e_\theta, \tag{2.4}$$

in the axisymmetric case (e_r and e_θ are the unit vectors of the spherical coordinate system), it follows from (1.6), (1.8), (2.1)–(2.4) that when $r = R$ it is sufficient to separate the real parts only in the expressions for Φ and $\partial\Phi/\partial r$.

With (1.6), (1.8), and (1.17)–(1.19), we get

$$\operatorname{Re}\Phi(r=R,\theta,t) = \sum_{n=0}^{2}\left(L_n \sin\omega t - K_n \cos\omega t\right) P_n(\cos\theta), \tag{2.5}$$

$$\operatorname{Re}\frac{\partial\Phi}{\partial r}(r=R,\theta,t) = \sum_{n=0}^{2}\left(M_n \cos\omega t + N_n \sin\omega t\right) P_n(\cos\theta), \tag{2.6}$$

where

$$K_0 = -A\left(1+\frac{1}{3}k^*\alpha^2\right), \quad L_0 = \frac{1}{3}Ak^*\alpha^3,$$

$$K_1 = -\frac{1}{3}A\beta^*\alpha^4, \quad L_1 = A\left(1-\beta^*\right)\alpha,$$

$$K_2 = \frac{1}{3}A\left(1-2\rho^*\right)\alpha^2, \quad L_2 = \frac{2}{135}A\rho^*\alpha^7,$$

$$M_0 = -\frac{1}{3}Ak\left(1+k^*\right)\alpha, \quad N_0 = -\frac{1}{9}Akk^*\alpha^4,$$

$$M_1 = \frac{1}{3}Ak\beta^*\alpha^3, \quad N_1 = Ak\left(1+2\beta^*\right),$$

$$M_2 = -2Ak\left(\frac{1}{3}+\rho^*\right)\alpha, \quad N_2 = \frac{4}{135}Ak\rho^*\alpha^6. \tag{2.7}$$

In what follows, we will omit for simplicity the symbol Re in the notation of the real part Re Φ of the potential Φ and its derivatives.

Now we can calculate the hydrodynamic force (2.1). Averaging it over the period of the incident wave yields the radiation force. Let us determine the contribution of each of the terms in (2.2) to (2.1).

To determine the contribution of the first term in (2.2) to (2.1), it is necessary, as justified above, to use expression (2.3) for the partial derivative $\partial\Phi / \partial t$, where V_x is the velocity of the liquid sphere as a whole along the Ox-axis. This velocity is found by evaluating $\partial\Phi / \partial r$ at $r = R$ and $\theta = 0$ or $\theta = \pi$. From (2.6), (2.7), we get

$$v_{\mathrm{r}}\left(r=R,\theta,t\right)=Ak\left(1-2\frac{\bar{\rho}_0-\rho_0}{2\bar{\rho}_0+\rho_0}\right)\cos\theta\sin\omega t+O(\alpha), \tag{2.8}$$

whence

$$V_x=v_{\mathrm{r}}\left(r=R,\theta=0,t\right)=Ak\left(1-2\frac{\bar{\rho}_0-\rho_0}{2\bar{\rho}_0+\rho_0}\right)\sin\omega t. \tag{2.9}$$

Considering relation (2.3) and following the above justification, we obtain a formula for the contribution of the first term in (2.2) to the hydrodynamic force (2.1):

$$F_x^{(1)}=2\pi\rho_0R^2V_x\int_0^{\pi}\left(-\frac{\partial\Phi}{\partial r}\cos\theta+\frac{1}{R}\frac{\partial\Phi}{\partial\theta}\sin\theta\right)\sin\theta\cos\theta d\theta. \tag{2.10}$$

Integrating (2.10) in view of (2.5), (2.6), (2.9), and

$$\int_{-1}^{1}P_n(\mu)P_m(\mu)d\mu=\begin{cases}\dfrac{2}{2n+1}, & m=n,\\ 0, & m\neq n,\end{cases}\qquad(\mu=\cos\theta), \tag{2.11}$$

$$\int_{-1}^{1}P_n(\mu)P_m(\mu)\mu d\mu=\begin{cases}\dfrac{2(n+1)}{(2n+1)(2n+3)}, & m=n+1,\\ 0, & m\neq n+1,\end{cases} \tag{2.12}$$

and averaging the result over the period of the incident wave, we obtain a formula for the contribution of the first term in (2.2) to the radiation force:

$$\langle F_x^{(1)}\rangle=-A\pi\rho_0kR^2\left(1-2\frac{\bar{\rho}_0-\rho_0}{2\bar{\rho}_0+\rho_0}\right)\left(\frac{2}{3}N_0+\frac{4}{15}N_2\right), \tag{2.13}$$

where the terms of the order of α^8 are omitted. Substituting (2.7) into (2.13), we get

$$\langle F_x^{(1)}\rangle=\frac{2}{9}A^2\pi\rho_0k^*\frac{\rho_0}{(2\bar{\rho}_0+\rho_0)}\alpha^6+O(\alpha^8). \tag{2.14}$$

Considering (2.4), we obtain a formula for the contribution of the second term in (2.2) to the hydrodynamic force (2.1):

$$F_x^{(2)}=\pi\rho_0R^2\int_0^{\pi}\left[\left(\frac{\partial\Phi}{\partial r}\right)^2+\frac{1}{R^2}\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right]\sin\theta\cos\theta d\theta. \tag{2.15}$$

Integrating (2.15) in view of (2.5), (2.6), (2.12), and

$$\int_{-1}^{1}\frac{dP_n(\mu)}{d\mu}\frac{dP_m(\mu)}{d\mu}\left(1-\mu^2\right)\mu d\mu=\begin{cases}\dfrac{2n(n+1)(n+2)}{(2n+1)(2n+3)}, & m=n+1,\\ 0, & m\neq n+1,\end{cases} \tag{2.16}$$

and averaging the result over the period of the incident wave, we obtain a formula for the contribution of the second term in (2.2) to the radiation force:

$$\langle F_x^{(2)}\rangle = 2\pi\rho_0 \sum_{n=0}^{1} \frac{n+1}{(2n+1)(2n+3)} \left[R^2 (M_n M_{n+1} + N_n N_{n+1}) + \frac{1}{\omega^2} n(n+2)(K_n K_{n+1} + L_n L_{n+1}) \right], \tag{2.17}$$

$$\langle F_x^{(2)}\rangle = \frac{2}{27} A^2 \pi \frac{\rho_0}{2\bar\rho_0 + \rho_0} \left[\bar\rho_0 - \rho_0 - k^* (\bar\rho_0 + 2\rho_0) \right] \alpha^6 + O(\alpha^8). \tag{2.18}$$

The contribution of the third terms in (2.2) to the hydrodynamic force (2.1) is described by the formula

$$F_x^{(3)} = -\frac{\pi\rho_0 R^2}{a_0^2} \int_0^\pi \left(\frac{\partial\Phi}{\partial t} \right)^2 \sin\theta \cos\theta d\theta. \tag{2.19}$$

Considering (2.5) and (2.12), integration (2.19), and averaging the result over the period of the incident wave, we get

$$\langle F_x^{(3)}\rangle = -\frac{\pi\rho_0 R^2}{a_0^2} \sum_{n=0}^{1} \frac{2(n+1)}{(2n+1)(2n+3)} (K_n K_{n+1} + L_n L_{n+1}). \tag{2.20}$$

Finally, substituting (2.7) into (2.20), we obtain a formula for the contribution of the third term in (2.2) to the radiation force:

$$\langle F_x^{(3)}\rangle = -\frac{2}{3} A^2 \pi\rho_0 \left[k^* \frac{\bar\rho_0}{(2\bar\rho_0 + \rho_0)} - \frac{\bar\rho_0 - \rho_0}{2\bar\rho_0 + \rho_0} \right] \alpha^6 + O(\alpha^8). \tag{2.21}$$

Summing (2.14), (2.18), and (2.21) and denoting $\eta = \rho_0 / \bar\rho_0$, we arrive at an expression for the radiation force exerted by the acoustic wave (1.1) on the liquid sphere:

$$\langle F_x\rangle = \frac{2}{27} A^2 \pi\rho_0 \frac{1}{2+\eta} \left[k^* (\eta - 10) + 4(1-\eta) \right] \alpha^6 + O(\alpha^8). \tag{2.22}$$

As indicated above, the condition $\alpha = \sqrt{3\bar\kappa / \kappa}$ corresponds to the pulsation resonance of the sphere [4, 7]. Therefore, according to (2.22), the radiation force increases as α tends to this resonant value. It should be noted that expression (2.22) has been derived assuming that α is different from $\sqrt{3\bar\kappa / \kappa}$. In this case, oscillations of the liquid sphere are dominant over pulsations [7].

When the liquid drop is similar in mechanical properties to the surrounding liquid ($\bar\rho_0 = \rho_0, \bar\kappa = \kappa$), the radiation force (2.22) is zero.

Within the framework of the approximation adopted here, we may conclude that the radiation force (2.22) acting on a liquid sphere, unlike a solid sphere, can be either in the same or opposite direction as the wave (for $\bar\rho_0 < \rho_0$). It is obvious that the effect of the radiation pressure on the liquid sphere is of the same nature as that on the interface between two liquids. In this case, according to [6], the interface always deflects toward the liquid with lower acoustic energy density, no matter in what direction the acoustic wave propagates.

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