SHORT-TERM MICRODAMAGE OF A PHYSICALLY NONLINEAR FIBROUS MATERIAL UNDER SIMULTANEOUS NORMAL AND TANGENTIAL LOADS

L. P. Khoroshun and E. N. Shikula

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The structural theory of short-term damage is generalized to the case where the undamaged isotropic matrix of a fibrous composite with transversely isotropic reinforcement deforms nonlinearly under loads that induce a combined stress state, microdamages occurring in the matrix alone. The basis for this generalization is the stochastic elasticity equations for a fibrous composite with porous matrix whose skeleton deforms nonlinearly. The Huber–Mises failure criterion is used to describe the damage of microvolumes in the matrix. The damaged microvolume balance equation is derived for the physically nonlinear material of the matrix based on the properties of the distribution function for the statistically homogeneous random field of ultimate microstrength. Together with the macrostress–macrostrain relationship, they constitute a closed-form system of equations. This system describes the coupled processes of physically nonlinear deformation and microdamage. Algorithms for calculating the dependences of macrostresses and microdamages on macrostrains are proposed. Stress–strain curves for a composite with a linearly hardened matrix under simultaneous normal and tangential loads are plotted. The effect of the volume fraction of reinforcement and tangential load on the curves is examined

Keywords: fibrous composite, microdamage of matrix, physically nonlinear matrix, coupled processes of physically nonlinear deformation and microdamage, combined stress state, combined effect of normal and tangent loads

Introduction. The structural theory of short-term microdamage for homogeneous and composite materials proposed in [6, 7, 9–26] is based on the mechanics of microinhomogeneous bodies of stochastic structure and on modeling dispersed microdamages by randomly arranged quasispherical micropores [6]. The accumulation of microdamages during deformation is modeled as increased porosity.

Here we generalize the structural theory of short-term damage to the case where the undamaged isotropic matrix of a fibrous composite with transversely isotropic fibers deforms nonlinearly under loads that induce a combined stress state, with microdamages occurring in the matrix alone. The basis for this generalization is the stochastic elasticity equations for a fibrous composite with a nonlinear elastic porous matrix. Damage in microvolumes of the matrix is described by the Huber–Mises failure criterion where the ultimate strength is a random function of coordinates with power or Weibull one-point distribution. The damaged microvolume balance equation, which is nonlinear with respect to the porosity of the matrix, is derived based on the properties of the distribution function for the statistically homogeneous random field of ultimate microstrength. Together with the macrostress–macrostrain relationship, they constitute a closed-form system of equations. This system describes the coupled processes of physically nonlinear deformation and microdamage. We will outline algorithms for calculating the dependences of macrostresses and microdamages on macrostrains. Also, we will plot stress–strain curves for a fibrous composite with linearly hardened matrix and examine the effect of the volume fraction of reinforcement and tangential and normal loads on the curves.

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1. Consider a fibrous composite material with unidirectional transversely isotropic fibers and isotropic matrix whose nonlinear deformation is described as a dependence of the bulk, K_2, and shear, μ_2, moduli on strains and accompanied by microdamage of the matrix. We will model microdamages of the matrix by randomly dispersed quasispherical micropores appearing in microvolumes where stresses exceed the ultimate microstrength. The fibers are normal to the plane of isotropy x_1x_2. Denote the elastic moduli of the reinforcement by $\lambda_{11}^1, \lambda_{12}^1, \lambda_{13}^1, \lambda_{33}^1$, and λ_{44}^1, the bulk and shear moduli and porosity of the skeleton of the matrix by K_2, μ_2, and p_2, respectively, and the volume fractions of the reinforcement and porous matrix by c_1 and c_2, respectively. Then the macrostresses $\langle\sigma_{ij}\rangle$ and macrostrains $\langle\varepsilon_{ij}\rangle$ are related by

$$\langle\sigma_{ij}\rangle = \left(\lambda_{11}^* - \lambda_{12}^*\right)\langle\varepsilon_{ij}\rangle + \left(\lambda_{12}^*\langle\varepsilon_{rr}\rangle + \lambda_{13}^*\langle\varepsilon_{33}\rangle\right)\delta_{ij},$$

$$\langle\sigma_{33}\rangle = \lambda_{13}^*\langle\varepsilon_{rr}\rangle + \lambda_{33}^*\langle\varepsilon_{33}\rangle, \quad \langle\sigma_{i3}\rangle = 2\lambda_{44}^*\langle\varepsilon_{i3}\rangle \qquad (i, j, r = 1, 2), \tag{1.1}$$

where the effective elastic moduli $\lambda_{11}^*, \lambda_{12}^*, \lambda_{13}^*, \lambda_{33}^*$, and λ_{44}^* are functions of the porosity p_2 and macrostrains $\langle\varepsilon_{ij}\rangle$.

The effective elastic moduli of a nonlinear elastic fibrous composite with porous matrix are determined by the following iterative algorithm. At the nth iteration, the effective moduli, $\lambda_{11}^{*(n)}, \lambda_{12}^{*(n)}, \lambda_{13}^{*(n)}, \lambda_{33}^{*(n)}$, and $\lambda_{44}^{*(n)}$, of the composite are determined [4, 5, 7] in terms of the corresponding moduli of the fibers, $\lambda_{11}^1, \lambda_{12}^1, \lambda_{13}^1, \lambda_{33}^1$, and λ_{44}^1, and matrix, $\lambda_{2p}^{(n)}$ and $\mu_{2p}^{(n)}$ ($\lambda_{2p}^{(n)} = K_{2p}^{(n)} - 2/3\mu_{2p}^{(n)}$):

$$\lambda_{11}^{*(n)} + \lambda_{12}^{*(n)} = c_1(\lambda_{11}^1 + \lambda_{12}^1) + 2c_2(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)}) - \frac{c_1c_2(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)})^2}{2c_1(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m^{(n)}},$$

$$\lambda_{11}^{*(n)} - \lambda_{12}^{*(n)} = c_1(\lambda_{11}^1 - \lambda_{12}^1) + 2c_2\mu_{2p}^{(n)} - \frac{c_1c_2(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}^{(n)})^2}{2c_1\mu_{2p}^{(n)} + c_2(\lambda_{11}^1 - \lambda_{12}^1) + \dfrac{2m^{(n)}n^{(n)}}{n^{(n)} + 2m^{(n)}}},$$

$$\lambda_{13}^{*(n)} = c_1\lambda_{13}^1 + c_2\lambda_{2p}^{(n)} - \frac{c_1c_2(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)})(\lambda_{13}^1 - \lambda_{2p}^{(n)})}{2c_1(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m^{(n)}},$$

$$\lambda_{33}^{*(n)} = c_1\lambda_{33}^1 + c_2(\lambda_{2p}^{(n)} + 2\mu_{2p}^{(n)}) - \frac{2c_1c_2(\lambda_{13}^1 - \lambda_{2p}^{(n)})^2}{2c_1(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)}) + c_2(\lambda_{11}^1 + \lambda_{12}^1) + 2m^{(n)}},$$

$$\lambda_{44}^{*(n)} = c_1\lambda_{44}^1 + c_2\mu_{2p}^{(n)} - \frac{c_1c_2(\lambda_{44}^1 - \mu_{2p}^{(n)})^2}{c_1\mu_{2p}^{(n)} + c_2\lambda_{44}^1 + s^{(n)}}, \tag{1.2}$$

where

$$2m^{(n)} = c_1\left(\lambda_{11}^1 - \lambda_{12}^1\right) + 2c_2\mu_{2p}^{(n)},$$

$$2n^{(n)} = c_1\left(\lambda_{11}^1 + \lambda_{12}^1\right) + 2c_2\left(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)}\right), \quad s^{(n)} = c_1\lambda_{44}^1 + c_2\mu_{2p}^{(n)} \tag{1.3}$$

if the stiffness of the matrix is greater than that of the reinforcement and

$$2m^{(n)} = \left(\frac{c_1}{\lambda_{11}^1 - \lambda_{12}^1} + \frac{c_2}{2\mu_{2p}^{(n)}}\right)^{-1},$$

$$2n^{(n)} = \left(\frac{c_1}{\lambda_{11}^1 + \lambda_{12}^1} + \frac{c_2}{2(\lambda_{2p}^{(n)} + \mu_{2p}^{(n)})} \right)^{-1}, \quad s^{(n)} = \left(\frac{c_1}{\lambda_{44}^1} + \frac{c_2}{\mu_{2p}^{(n)}} \right)^{-1} \tag{1.4}$$

otherwise.

According to [6, 7], the effective moduli of the porous matrix are determined at the nth iteration as

$$K_{2p}^{(n)} = \frac{4K_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) \mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) (1-p_2)^2}{3K_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) p_2 + 4\mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) (1-p_2)},$$

$$\mu_{2p}^{(n)} = \frac{\left[9K_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) + 8\mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right)\right] \mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) (1-p_2)^2}{3K_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) (3-p_2) + 4\mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right) (2+p_2)}, \tag{1.5}$$

where $\langle \varepsilon_{ij}^{12} \rangle^{(n)}$ are the mean strains in the undamage portion of the matrix determined at the nth iteration. They are related to the mean strains $\langle \varepsilon_{ij}^{2} \rangle^{(n)}$ in the matrix at the same iteration by

$$\langle \varepsilon_{ij}^{12} \rangle^{(n)} = \frac{1}{(1-p_2)} \left\{ \frac{\mu_{2p}^{(n-1)}}{\mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right)} \langle \varepsilon_{ij}^{2} \rangle^{(n)} + \frac{1}{3} \left[\frac{K_{2p}^{(n-1)}}{K_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right)} - \frac{\mu_{2p}^{(n-1)}}{\mu_2\left(\langle \varepsilon_{ij}^{12} \rangle^{(n)}\right)} \right] \langle \varepsilon_{rr}^{2} \rangle^{(n)} \delta_{ij} \right\}. \tag{1.6}$$

The strains $\langle \varepsilon_{ij}^{2} \rangle^{(n)}$ are determined in terms of the macrostrains $\langle \varepsilon_{ij} \rangle$ as

$$\langle \varepsilon_{ij}^{2} \rangle^{(n)} = \frac{\lambda_{11}^{*(n)} - \lambda_{12}^{*(n)} - \lambda_{11}^1 + \lambda_{12}^1}{c_2 (2\mu_{2p}^{(n)} - \lambda_{11}^1 + \lambda_{12}^1)} \langle \varepsilon_{ij} \rangle$$

$$- \frac{1}{\Delta_2^{(n)}} \left\{ \left[\left(\lambda_{11}^{*(n)} - \lambda_{11}^1\right) a_1^{(n)} - \left(\lambda_{12}^{*(n)} - \lambda_{12}^1\right) a_2^{(n)} - \left(\lambda_{13}^{*(n)} - \lambda_{13}^1\right) a_3^{(n)} \right] \langle \varepsilon_{rr} \rangle \right.$$

$$\left. + \left[\left(\lambda_{13}^{*(n)} - \lambda_{13}^1\right) \left(a_1^{(n)} - a_2^{(n)}\right) - \left(\lambda_{33}^{*(n)} - \lambda_{33}^1\right) a_3^{(n)} \right] \langle \varepsilon_{33} \rangle \right\} \delta_{ij},$$

$$\langle \varepsilon_{33}^{2} \rangle^{(n)} = -\frac{1}{\Delta_2^{(n)}} \left\{ \left[\left(\lambda_{13}^{*(n)} - \lambda_{13}^1\right) a_4^{(n)} - \left(\lambda_{11}^{*(n)} + \lambda_{12}^{*(n)} - \lambda_{11}^1 - \lambda_{12}^1\right) a_3^{(n)} \right] \langle \varepsilon_{rr} \rangle \right.$$

$$\left. + \left[\left(\lambda_{33}^{*(n)} - \lambda_{33}^1\right) a_4^{(n)} - 2\left(\lambda_{13}^{*(n)} - \lambda_{13}^1\right) a_3^{(n)} \right] \langle \varepsilon_{rr} \rangle \right\},$$

$$\langle \varepsilon_{i3}^{2} \rangle^{(n)} = \frac{\lambda_{44}^{*(n)} - \lambda_{44}^1}{c_2 (\mu_{2p}^{(n)} - \lambda_{44}^1)} \langle \varepsilon_{i3} \rangle \quad (i, j, r = 1, 2), \tag{1.7}$$

where

$$\Delta_2^{(n)} = c_2\left(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}^{(n)}\right) \left[\left(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right) \left(\lambda_{33}^1 - \lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right) - 2\left(\lambda_{13}^1 - \lambda_{2p}^{(n)}\right)^2 \right],$$

$$a_1^{(n)} = \left(\lambda_{13}^1 - \lambda_{2p}^{(n)}\right)^2 - \left(\lambda_{12}^1 - \lambda_{2p}^{(n)}\right) \left(\lambda_{33}^1 - \lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right),$$

$$a_2^{(n)} = \left(\lambda_{13}^1 - \lambda_{2p}^{(n)}\right)^2 - \left(\lambda_{11}^1 - \lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right)\left(\lambda_{33}^1 - \lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right),$$

$$a_3^{(n)} = \left(\lambda_{13}^1 - \lambda_{2p}^{(n)}\right)\left(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}^{(n)}\right),$$

$$a_4^{(n)} = \left(\lambda_{11}^1 + \lambda_{12}^1 - 2\lambda_{2p}^{(n)} - 2\mu_{2p}^{(n)}\right)\left(\lambda_{11}^1 - \lambda_{12}^1 - 2\mu_{2p}^{(n)}\right), \tag{1.8}$$

and $K_{2p}^{(n)}$, $\lambda_{2p}^{(n)}$, and $\mu_{2p}^{(n)}$ are defined by (1.5).

Given the macrostrains $\langle \varepsilon_{ij} \rangle$, the effective moduli are evaluated as the following limit:

$$\lambda_{lm}^* = \lim_{n \to \infty} \lambda_{lm}^{*(n)}. \tag{1.9}$$

We use the Huber–Mises failure criterion as a condition whereby a single microdamage appears within an undamaged microvolume of the matrix:

$$I_\sigma^{12} = k_2, \tag{1.10}$$

where $I_\sigma^{12} = (\langle \sigma_{ij}^{12} \rangle' \langle \sigma_{ij}^{12} \rangle')^{1/2}$ is the second invariant of the mean stress deviator $\langle \sigma_{ij}^{12} \rangle'$ for the undamaged portion of the matrix; and k_2 is the ultimate microstrength, which is a random function of coordinates.

The one-point distribution function $F_2(k_2)$ may have the form of a power law on some interval

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{02}, \\ \left(\dfrac{k_2 - k_{02}}{k_{12} - k_{02}}\right)^{n_2}, & k_{02} \le k_2 \le k_{12}, \\ 1, & k_2 > k_{12} \end{cases} \tag{1.11}$$

or Weibull distribution

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{02}, \\ 1 - \exp\left[-m_2 (k_2 - k_{02})^{n_2}\right], & k_2 \ge k_{02}, \end{cases} \tag{1.12}$$

where k_{02} is the minimum value of the ultimate microstrength of the matrix; k_{12}, m_2, and n_2 are deterministic constants describing a specific distribution function, which are determined by fitting experimental microstrength scatter curves or stress–strain curves for the matrix.

Assume that before deformation the matrix had initial microdamage characterized by porosity p_{02}. Then the distribution function $F_2(k_2)$ determines the relative fraction of the undamaged material in which the ultimate strength is less than k_2. Therefore, if $\langle \sigma_{ij}^{12} \rangle$, then the function $F_2(I_\sigma^{12})$, according to (1.10)–(1.12), determines the relative fraction of damaged microvolumes in the skeleton of the matrix. Since damaged microvolumes are modeled by pores, we can write the balance equation for the damaged microvolumes in the matrix or, which is the same, for porosity:

$$p_2 = p_{02} + (1 - p_{02}) F_2(I_\sigma^{12}), \tag{1.13}$$

where the mean stresses $\langle \sigma_{ij}^{12} \rangle^{(n)}$ in the undamaged portion of the matrix are related to the macrostrains $\langle \varepsilon_{ij} \rangle$ by [7] (1.7), (1.8), and

$$\langle \sigma_{ij}^{12} \rangle^{(n)} = \frac{1}{1 - p_2}\left[\left(K_{2p}^{(n)} - \frac{2}{3}\mu_{2p}^{(n)}\right)\langle \varepsilon_{rr}^2 \rangle^{(n)} \delta_{ij} + 2\mu_{2p}^{(n)} \langle \varepsilon_{ij}^2 \rangle^{(n)}\right]. \tag{1.14}$$

Equations (1.1), (1.7), (1.8), (1.13), and (1.14) constitute a closed-form system describing the coupled processes of statistically homogeneous, nonlinear deformation and damage of a fibrous composite with unidirectional transversely isotropic fibers and physically nonlinear microdamagable isotropic matrix. The physical nonlinearity of the matrix influences its porosity under load, which is reflected on the stress–strain curve of the composite. Therefore, the resulting stress–strain curve includes the physical nonlinearity of the matrix and the nonlinearity due to increasing porosity under physically nonlinear deformation.

The coupled processes of physically nonlinear deformation and damage of the fibrous composite at given macrostrains are described by determining the macrostrain-dependent effective elastic moduli of the fibrous material with porous matrix by the iterative algorithm (1.2)–(1.9) and by determining the porosity from Eqs. (1.7), (1.8), (1.13), and (1.14), also using a certain iterative method. Let us represent Eq. (1.13) for the *n*th step of the iterative process (1.2)–(1.9) in the form

$$f_2^{(n)} \equiv p_2 - p_{02} - (1 - p_{02})\, F_2 (I_\sigma^{12(n)}), \tag{1.15}$$

where

$$I_\sigma^{12(n)} = \left(\langle \sigma_{jk}^{12(n)} \rangle' \langle \sigma_{jk}^{12(n)} \rangle' \right)^{1/2}. \tag{1.16}$$

Then the root p_2 of Eq. (1.15), (1.16) at the *m*th step of some iterative process can be found by the formula

$$p_2^{(m,n)} = A_2 f_2^{(n)} \left(p_2^{(m-1)} \right), \tag{1.17}$$

where A_2 is an operator acting on the function $f_2^{(n)}(p_2)$.

The desired root is determined as the following limit:

$$p_2 = \lim_{\substack{m \to \infty \\ n \to \infty}} p_2^{(m,n)}. \tag{1.18}$$

Relations (1.1)–(1.9), (1.15)–(1.18) give the solution to the problem posed, i.e., they produce macrodeformation curves ($\langle \sigma_{ij} \rangle$ versus $\langle \varepsilon_{pq} \rangle$) and microdamage curves (p_2 versus $\langle \varepsilon_{pq} \rangle$) for the fibrous composite with physically nonlinear matrix.

2. Let us analyze, as an example, the coupled processes of nonlinear deformation and microdamage in a fibrous composite with linearly hardened matrix. Assume that the bulk strains of the matrix are linear and the shear strains are described by a linear hardening curve, i.e., the following relations hold within a microvolume:

$$\sigma_{rr}^2 = K_2 \varepsilon_{rr}^2, \quad \sigma_{ij}'^2 = 2\mu_2 (S_2)\, \varepsilon_{ij}'^2. \tag{2.1}$$

Here the bulk modulus K_2 does not depend on strains and the shear modulus $\mu_2(S_2)$ is described by the function

$$\mu_2 (S_2) = \begin{cases} \mu_{02}, & T_2 \le T_{02}, \\ \mu_2' + \left(1 - \dfrac{\mu_2'}{\mu_{02}}\right) \dfrac{T_{02}}{2S_2}, & T_2 \ge T_{02}, \end{cases} \tag{2.2}$$

where

$$S_2 = (\varepsilon_{ij}'^2 \varepsilon_{ij}'^2)^{1/2}, \quad T_2 = (\sigma_{ij}'^2 \sigma_{ij}'^2)^{1/2}, \quad T_{02} = \sqrt{\frac{2}{3}}\, \sigma_{02}, \tag{2.3}$$

$\varepsilon_{ij}'^2$ and $\sigma_{ij}'^2$ are the deviators of the strain and stress tensors, respectively; σ_{02} is the coordinate-independent tensile elastic limit; and μ_{02} and μ_2' are material constants.

We will use the secant method [1] to find the root p_2 of Eqs. (1.7), (1.8), (1.15), (1.16). Since the root p_2 falls into the interval [p_{02}, 1], which follows from the inequalities

$$f_2^{(n)}(p_{20}) \le 0, \quad f_2^{(n)}(1) \ge 0, \tag{2.4}$$

the zero approximation $p_2^{(0,n)}$ is determined, according to the secant method, from the formula

$$p_2^{(0,n)} = \frac{a_2^{(0)} f_2^{(n)}(b_2^{(0)}) - b_2^{(0)} f_2^{(n)}(a_2^{(0)})}{f_2^{(n)}(b_2^{(0)}) - f_2^{(n)}(a_2^{(0)})}, \tag{2.5}$$

where $a_2^{(0)} = p_{02}$ and $b_2^{(0)} = 1$.

The subsequent approximations are determined in the iterative process

$$p_2^{(m,n)} = A_2 f_2^{(n)}\left(p_2^{(m-1,n)}\right) \equiv \frac{a_2^{(m)} f_2^{(n)}(b_2^{(m)}) - b_2^{(m)} f_2^{(n)}(a_2^{(m)})}{f_2^{(n)}(b_2^{(m)}) - f_2^{(n)}(a_2^{(m)})},$$

$$a_2^{(m)} = a_2^{(m-1)}, \quad b_2^{(m)} = p_2^{(m-1,n)} \quad \text{at} \quad f_2^{(n)}(a_2^{(m-1)}) f_2^{(n)}(p_2^{(m-1,n)}) \le 0,$$

$$a_2^{(m)} = p_2^{(m-1,n)}, \quad b_2^{(m)} = b_2^{(m-1)} \quad \text{at} \quad f_2^{(n)}(a_2^{(m-1)}) f_2^{(n)}(p_2^{(m-1,n)}) \ge 0$$

$$(m = 1, 2, \ldots), \tag{2.6}$$

which proceeds until

$$\left| f_2^{(n)}(p_2^{(m,n)}) \right| < \delta, \tag{2.7}$$

where δ is the error of computing the root.

Based on the theory stated above, we have studied the coupled processes of nonlinear deformation and microdamage of a fibrous composite with Weibull-distributed ultimate microstrength of the matrix under loading of various types. The composite has epoxy matrix described by the linear-hardening diagram (2.1), (2.2) with the following constants [2, 3]:

$$K_2 = 3.33 \text{ GPa}, \quad \mu_{02} = 1.11 \text{ GPa}, \quad \mu'_2 = 0.331 \text{ GPa}, \tag{2.8}$$

and the following elastic limits and minimum tensile microstrength $\sigma_p = \sqrt{3/2}\, k_{02}$:

$$\sigma_{20} = 0.015 \text{ GPa}, \quad \sigma_{2p} = 0.003 \text{ GPa}, \tag{2.9}$$

and high-modulus carbon fibers with the following characteristics [3]:

$$E_1^1 = 8 \text{ GPa}, \quad E_3^1 = 226 \text{ GPa}, \quad \nu_{12}^1 = 0.2, \quad \nu_{13}^1 = 0.3, \quad G_{12}^1 = 60 \text{ GPa}, \tag{2.10}$$

where E_1^1 and E_3^1, ν_{12}^1 and ν_{13}^1, G_{12}^1 and G_{13}^1 are, respectively, transverse and longitudinal Young's moduli, Poisson's ratios, and shear moduli of the fibers, which are related to the elastic moduli $\lambda_{11}^1, \lambda_{12}^1, \lambda_{13}^1, \lambda_{33}^1$, and λ_{44}^1 by the formulas

$$\lambda_{11}^1 + \lambda_{12}^1 = E_1^1 E_3^1 \left[E_3^1 \left(2 - \frac{E_1^1}{2G_{12}^1} \right) - 2E_1^1 (\nu_{13}^1)^2 \right]^{-1}, \quad \lambda_{11}^1 - \lambda_{12}^1 = 2G_{12}^1,$$

$$\lambda_{13}^1 = \nu_{13}^1 \left(\lambda_{11}^1 + \lambda_{12}^1 \right), \quad \lambda_{33}^1 = \left(\lambda_{11}^1 + \lambda_{12}^1 \right) \frac{E_3^1}{E_1^1} \left(2 - \frac{E_1^1}{2G_{12}^1} \right), \quad \lambda_{44}^1 = G_{13}^1. \tag{2.11}$$

If

$$\langle \sigma_{11} \rangle \ne 0, \quad \langle \sigma_{13} \rangle \ne 0, \quad \langle \sigma_{22} \rangle = \langle \sigma_{33} \rangle = 0 \tag{2.12}$$

or

$$\langle \sigma_{11} \rangle \ne 0, \quad \langle \sigma_{12} \rangle \ne 0, \quad \langle \sigma_{22} \rangle = \langle \sigma_{33} \rangle = 0, \tag{2.13}$$

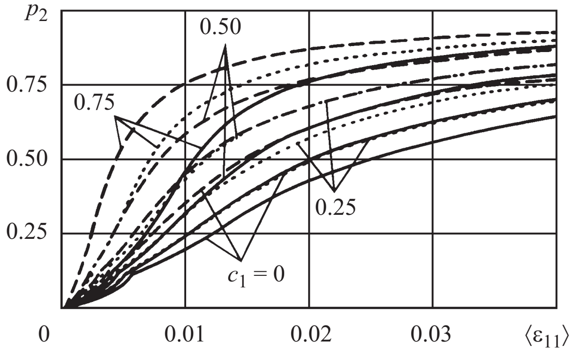

Fig. 1

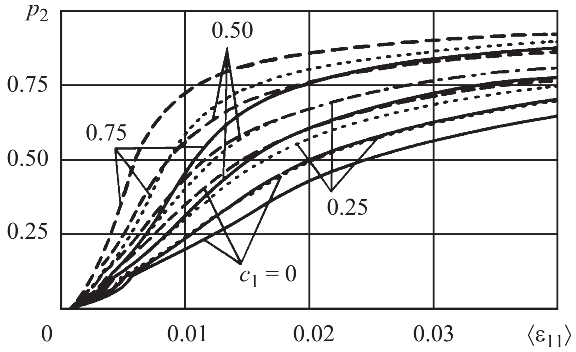

Fig. 2

then, according to (1.1), the macrostress $\langle\sigma_{11}\rangle$ is related to the macrostrain $\langle\varepsilon_{11}\rangle$ by

$$\langle\sigma_{11}\rangle = \frac{\lambda_{11}^* - \lambda_{12}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2}\left[(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2\right]\langle\varepsilon_{11}\rangle \tag{2.14}$$

and

$$\langle\varepsilon_{22}\rangle = \frac{(\lambda_{13}^*)^2 - \lambda_{12}^*\lambda_{33}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2}\langle\varepsilon_{11}\rangle, \quad \langle\varepsilon_{33}\rangle = \frac{(\lambda_{12}^* - \lambda_{11}^*)\lambda_{13}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2}\langle\varepsilon_{11}\rangle. \tag{2.15}$$

If

$$\langle\sigma_{33}\rangle \neq 0, \quad \langle\sigma_{13}\rangle \neq 0, \quad \langle\sigma_{11}\rangle = \langle\sigma_{22}\rangle = 0 \tag{2.16}$$

or

$$\langle\sigma_{33}\rangle \neq 0, \quad \langle\sigma_{12}\rangle \neq 0, \quad \langle\sigma_{11}\rangle = \langle\sigma_{22}\rangle = 0, \tag{2.17}$$

then, according to (1.1), the macrostress $\langle\sigma_{33}\rangle$ is related to the macrostrain $\langle\varepsilon_{33}\rangle$ by

$$\langle\sigma_{33}\rangle = \frac{1}{\lambda_{11}^* + \lambda_{12}^*}\left[(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2\right]\langle\varepsilon_{33}\rangle \tag{2.18}$$

and

$$\langle\varepsilon_{11}\rangle = \langle\varepsilon_{22}\rangle = \frac{-\lambda_{13}^*}{\lambda_{11}^* + \lambda_{12}^*}\langle\varepsilon_{11}\rangle. \tag{2.19}$$

Figures 1 and 2 show (for (2.12) and (2.13), respectively) the porosity p_2 as a function of the macrostrain $\langle\varepsilon_{11}\rangle$ in a fibrous composite with different volume fractions c_1 of reinforcement and different values of the macrostress $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$: $\langle\sigma_{12}\rangle = 0$ or $\langle\sigma_{13}\rangle = 0$ (solid line), $\langle\sigma_{12}\rangle = \langle\sigma_{11}\rangle$ or $\langle\sigma_{13}\rangle = \langle\sigma_{11}\rangle$ (dashed line), and $\langle\sigma_{12}\rangle = 1/2\langle\sigma_{11}\rangle$ or $\langle\sigma_{13}\rangle = 1/2\langle\sigma_{11}\rangle$ (dotted line). It is seen that as $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$ increases, microdamages begin to appear earlier and accumulate more intensively (higher porosity p_2 corresponds to fixed values of $\langle\varepsilon_{11}\rangle$). Note that the dependence of p_2 on $\langle\varepsilon_{11}\rangle$ is highly influenced by what is specified: $\langle\sigma_{12}\rangle \neq 0$ or $\langle\sigma_{13}\rangle \neq 0$. When $\langle\sigma_{13}\rangle \neq 0$, microdamages begin to appear earlier and accumulate more intensively than with $\langle\sigma_{12}\rangle \neq 0$.

Figures 3 and 4 show (for (2.12) and (2.13), respectively) the macrostress $\langle\sigma_{11}\rangle$ as a function of the macrostrain $\langle\varepsilon_{11}\rangle$ for different volume fractions c_1 of reinforcement and different values of the macrostress $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$. Note that here the dependence of $\langle\varepsilon_{11}\rangle$ on $\langle\sigma_{11}\rangle$, like the dependence of p_2 on $\langle\varepsilon_{11}\rangle$, is highly influenced by what is specified: $\langle\sigma_{12}\rangle \neq 0$ or $\langle\sigma_{13}\rangle \neq 0$. When $\langle\sigma_{13}\rangle \neq 0$, the maximum value of $\langle\sigma_{11}\rangle$ is less than that when $\langle\sigma_{12}\rangle \neq 0$.

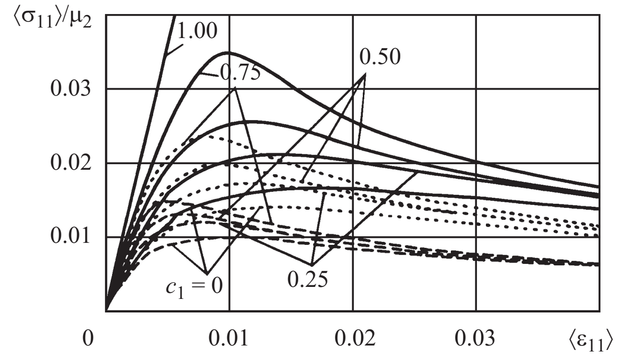

Fig. 3

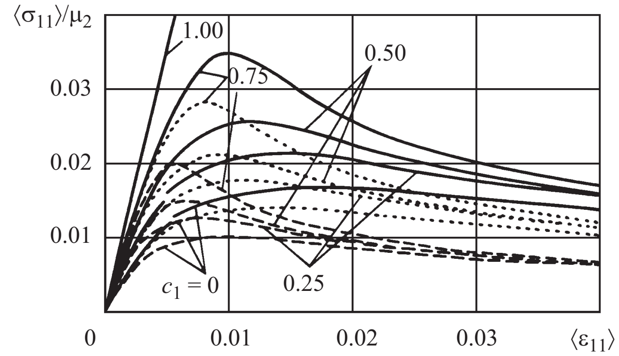

Fig. 4

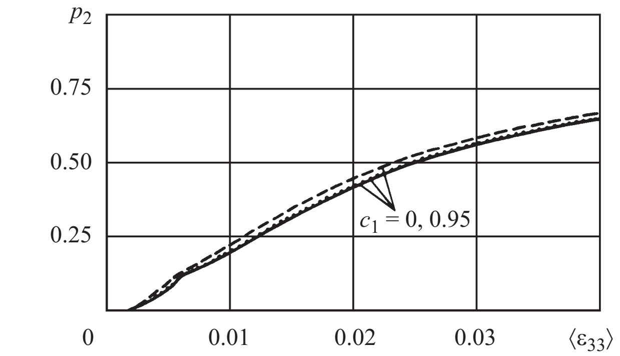

Fig. 5

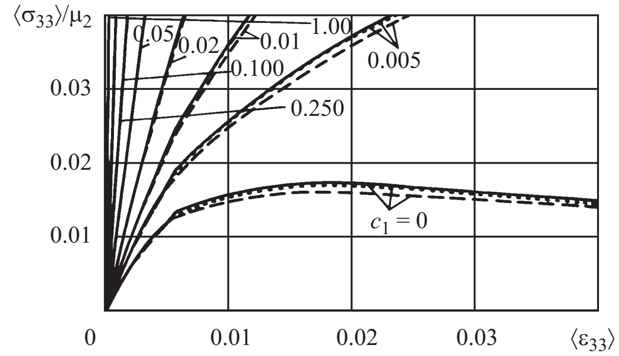

Fig. 6

Figure 5 shows (for (2.16) and (2.17)) the porosity p_2 as a function of the macrostrain $\langle\varepsilon_{33}\rangle$ for different values of c_1 and $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$. Note that this dependence is independent of what is specified: $\langle\sigma_{12}\rangle \neq 0$ or $\langle\sigma_{13}\rangle \neq 0$. It can be seen that as $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$ increases, microdamages begin to appear earlier and accumulate more intensively, as with conditions (2.12) and (2.13).

Figures 6 shows (for (2.16) and (2.17)) the macrostress $\langle\sigma_{33}\rangle$ as a function of the macrostrain $\langle\varepsilon_{33}\rangle$ for different values of c_1 and $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$. Here, as in Fig. 5, the dependence is independent of what is specified: $\langle\sigma_{12}\rangle \neq 0$ or $\langle\sigma_{13}\rangle \neq 0$. It can be seen that as $\langle\sigma_{12}\rangle$ or $\langle\sigma_{13}\rangle$ increases, the maximum value of $\langle\sigma_{33}\rangle$ decreases and fixed values of $\langle\varepsilon_{33}\rangle$ correspond to smaller values of $\langle\sigma_{33}\rangle$, as with conditions (2.12) and (2.13).

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