International Applied Mechanics, Vol. 42, No. 9, 2006

CHAOTIC FRICTIONAL VIBRATIONS EXCITED BY A QUASIPERIODIC LOAD

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UDC 531.39

A Duffing oscillator frictionally interacting with a moving belt under a quasiperiodic load is studied. The multiple-scales method is used to derive a system of two nonautonomous equations with small parameters, which describes the modulation of vibrations. It is shown that the system of modulation equations has a heteroclinic structure. Melnikov functions are used to analyze the domain of heteroclinic chaos

Keywords: frictional vibrations, quasiperiodic load, Duffing oscillator, chaotic vibrations, modulation, heteroclinic Melnikov function, numerical modeling

Introduction. Much effort went into research of nonlinear vibrations in the presence of dry friction. Some of the publications on the subject are noteworthy. Den-Hartog [6] studied the vibrations of a string excited by a moving bow. Andronov, Vitt, Khaikin [4] and Kauderer [7] were one of the first to study self-excited frictional vibrations. In [4], the descending branch of the kinetic characteristics of friction is considered to be the cause of self-excited vibrations. A frictional self-oscillation model is used in [7] to explain acoustic phenomena. The frictional self-excited vibrations of a mass on a moving belt are detailed in [3, 8] using the averaging method and considering that the belt is moved by an energy source of limited power. Many examples of using frictional self-excited vibrations in engineering are given in the book [9], which also reports on results from extensive experiments to determine the kinetic characteristic of friction. The paper [1] addresses the bifurcations in the steady-state motion of a Duffing oscillator on a moving belt. Results from studies on nonlinear frictional vibrations are reviewed in [14]. Compound vibrations and nonlinear vibrations due to dry friction are addressed in [15, 17], and the dynamic processes associated with vibration damping are examined in [13, 18].

The present paper is concerned with a quasiperiodically excited Duffing oscillator frictionally interacting with a moving belt. We will use the multiple-scales method to derive a system of nonautonomous equations to describe modulation of oscillations. We will show that the system of modulation equations has a heteroclinic structure that causes chaotic vibrations. The heteroclinic Melnikov function will be used to identify the domain of chaotic modulation of vibrations. Analytic and numerical results will be compared.

A great many publications use heteroclinic Melnikov functions to analyze domains of chaotic vobrations. Such studies are reviewed in [5, 12]. The previous publications used homoclinic Melnikov functions to analyze Lagrange equations of the second kind. The novelty of the present paper is in applying homoclinic Melnikov functions to the modulation equations. This makes it possible to identify the domain of chaotic modulation in a Duffing oscillator subjected to quasiperiodic excitation and interacting with a moving belt.

1. Problem Formulation. Consider a one-degree-of-freedom system (Fig. 1). The vibrations of the mass are described by a generalized coordinate x. The belt moves with a constant velocity v_* and interacts with the oscillator through dry friction (force $f(v_R)$, where v_R is the relative velocity of the rubbing surfaces). The nonlinear spring is described by a restoring force $R = cx + c_3 x^3$. The vibrations of the system are excited by a quasiperiodic load $p(t) = \Gamma_1 \cos\omega_1 t + \Gamma_2 \cos\omega_2 t$.

The differential equation of motion of this system is given by

National Technical University (Kharkov Polytechnic Institute), Kharkov, Ukraine. Translated from Prikladnaya Mekhanika, Vol. 42, No. 9, pp. 127–133, September 2006. Original article submitted September 2, 2005.

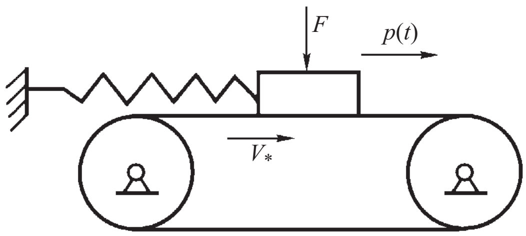


Fig. 1

$$m\ddot{x}+cx+c_3x^3=\Gamma_1\cos\omega_1 t+\Gamma_2\cos\omega_2 t-f(\dot{x}-v_*), \tag{1}$$

$$f(\dot{x}-v_\bullet)=\theta_0\,\mathrm{sign}(\dot{x}-v_\bullet)-A(\dot{x}-v_\bullet)+B(\dot{x}-v_\bullet)^3. \tag{2}$$

Let us introduce dimensionless variables and parameters:

$$\varepsilon\mu\gamma_2=\frac{\Gamma_2}{cx_\bullet},\quad \alpha=\frac{A\omega_0 x_\bullet}{\theta_0},\quad \beta=\frac{B\omega_0^3x_\bullet^3}{\theta_0},\quad x=x_\bullet\xi(t),\quad \tau=\omega_0 t,$$

$$\Omega_1=\frac{\omega_1}{\omega_0},\quad \Omega_2=\frac{\omega_2}{\omega_0},\quad \varepsilon\lambda=\frac{c_3x_\bullet^2}{c},\quad \varepsilon\mu\tilde{\theta}=\frac{\theta_0}{cx_\bullet},\quad \varepsilon\gamma_1=\frac{\Gamma_1}{cx_\bullet}, \tag{3}$$

where μ and ε are two independent small parameters $(0<\varepsilon<<\mu<<1)$. The mechanical system (1) can now be rearranged into a dimensionless form:

$$\xi''+\xi=\varepsilon\left\{-\lambda\xi^3+\gamma_1\cos\Omega_1\tau+\mu\left[\gamma_2\cos\Omega_2\tau-\tilde{\theta}P(\xi'-v_B)\right]\right\},$$

$$P(\xi'-v_B)=\mathrm{sign}(\xi'-v_B)-\alpha(\xi'-v_B)+\beta(\xi'-v_B)^3. \tag{4}$$

The second small parameter μ indicates that the force of friction is much less than the nonlinear component of the restoring force and that the amplitude γ_1 is much greater than the amplitude $\mu\gamma_2$. Let us examine the near-resonant vibrations

$$\Omega_1=1+\varepsilon\sigma,\qquad \Omega_2=\Omega_1+\varepsilon\Delta, \tag{5}$$

where σ and Δ are two independent mismatch parameters. Note that the load remains quasiperiodic at resonance (5). Using the standard procedure of the multiple-scales method [2, 11], we obtain a system of modulation equations:

$$\rho'=\sqrt{\rho}\frac{\gamma_1}{\sqrt{2}}\sin\theta+\mu\left\{\rho\tilde{\theta}(\alpha-3\beta v_B^2)-\tilde{\theta}\alpha_1\sqrt{2\rho}-\frac{3}{2}\tilde{\theta}\beta\rho^2+\sqrt{\rho}\frac{\gamma_2}{\sqrt{2}}\sin\theta\cos\Delta T_1+\sqrt{\rho}\frac{\gamma_2}{\sqrt{2}}\cos\theta\sin\Delta T_1\right\}, \tag{6}$$

$$\theta'=\sigma-\frac{3\lambda}{4}\rho+\frac{\gamma_1}{2\sqrt{2\rho}}\cos\theta+\mu\frac{\gamma_2}{2\sqrt{2\rho}}(\cos\theta\cos\Delta T_1-\sin\theta\sin\Delta T_1), \tag{7}$$

$$\alpha_1(\rho)=\begin{cases}0, & v_B>\sqrt{2\rho},\\ \dfrac{2}{\pi}\sqrt{1-\dfrac{v_B^2}{2\rho}}, & v_B<\sqrt{2\rho},\end{cases}$$

where $(\cdot)'=\dfrac{d(\cdot)}{dT_1}$, $T_1=\varepsilon\tau$, and μ is a small parameter.

The coordinate ξ in (4) and the modulation variables (ρ,θ) are related by

$$\xi=\sqrt{2\rho}\cos(\Omega_1\tau-\theta)+O(\varepsilon). \tag{8}$$

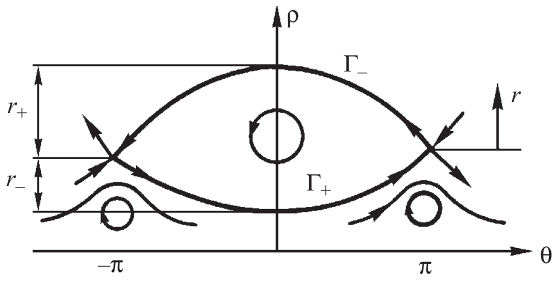


Fig. 2

2. Analysis of the Generating System of Modulation Equations. The generating system of equations (6), (7) with $\mu = 0$ is conservative and has the following Hamiltonian:

$$H = -\sqrt{2\rho}\,\frac{\gamma_1}{2}\cos\theta + \frac{3\lambda}{8}\rho^2 - \sigma\rho. \tag{9}$$

The generating system of equations has two groups of fixed points (θ_1, ρ_1) and (θ_2, ρ_2) that satisfy the cubic equation

$$\sigma - \frac{3\lambda}{4}\rho_{1,2} \pm \frac{\gamma_1}{2\sqrt{2\rho_{1,2}}} = 0, \quad \theta_1 = 0, \quad \theta_2 = \pm\pi. \tag{10}$$

These fixed points are shown in Fig. 2. They are stable fixed points (centers) and unstable fixed points (saddles). The saddles are connected by heteroclinic orbits (separatrices) denoted as $\Gamma_-(\rho_-(T_1), \theta_-(T_1))$ and $\Gamma_+(\rho_+(T_1), \theta_+(T_1))$ (Fig. 2). The saddles and separatrices correspond to one constant value of the Hamiltonian and can easily be obtained from (9). The trajectories Γ_- and Γ_+ are the upper and the lower heteroclinic orbits, respectively. With $\mu = 0$, Eq. (7) yields

$$\theta_\pm(\tau) = \arcsin\left(\frac{\sqrt{2}\rho'_\pm}{\gamma_1\sqrt{\rho_\pm}}\right).$$

Substituting it into Eq. (6) and assuming that $\mu = 0$, we get

$$\rho'^2 = \left(\frac{\gamma_1\sqrt{\rho}}{\sqrt{2}} + \sigma\rho - \frac{3\lambda}{8}\rho^2 + H_s\right)\left(\frac{\gamma_1\sqrt{\rho}}{\sqrt{2}} - \sigma\rho + \frac{3\lambda}{8}\rho^2 - H_s\right). \tag{11}$$

Replacing variables $\rho(T_1) = \rho_2^{(1)} + r(T_1)$ and using the initial conditions $\theta_\pm(0) = 0$, $\rho_\pm(0) = \rho_2^{(1)} + \tilde{r}_\pm$, $\tilde{r}_\pm = 2(k \pm \sqrt{2k\rho_2^{(1)}})$, $k = \frac{4\sigma}{3\lambda} - \rho_2^{(1)}$, we integrate Eq. (11) to obtain

$$\rho_\pm(T_1) = \rho_2^{(1)} \pm \frac{2\tilde{r}_-\tilde{r}_+}{(\tilde{r}_+ - \tilde{r}_-)\cosh(\tilde{a}T_1) \pm (\tilde{r}_+ + \tilde{r}_-)}, \tag{12}$$

where $\tilde{a} = \frac{3\lambda}{8}\sqrt{-\tilde{r}_-\tilde{r}_+}$, $\rho_2^{(1)}$ is the coordinate ρ of a saddle point.

3. Analytic Study of the Beginning of Chaos. There are intersections of invariant manifolds (heteroclinic structure) in the perturbed system (6), (7). A theory of such intersections is outlined in the monograph [5]. The heteroclinic structure gives rise to Smale horseshoes, which are an elementary mathematical model of chaotic vibrations. To determine the domain of heteroclinic structure, Melnikov functions are used [10]. A method for treating these functions is outlined in [5]. We will use this approach to determine the domain of existence of a heteroclinic structure in the system of modulation equations (6), (7). The heteroclinic Melnikov function of the dynamic system (6), (7) is defined by

$$\widetilde{M}=\int_{-\infty}^{\infty}\left\{-\frac{\gamma_1\gamma_2}{4}\sin\theta\cos(\Delta t-\Delta t_0+\theta)+\frac{\gamma_2}{\sqrt{2}}\sqrt{\rho}\sin(\Delta t-\Delta t_0+\theta)\right.$$

$$\left.\times\left(\sigma-\frac{3}{4}\lambda\rho+\frac{\gamma_1}{2\sqrt{2\rho}}\cos\theta\right)\right\}dt+\int_{-\infty}^{\infty}\mathrm{P}(\rho)\left(\sigma-\frac{3}{4}\lambda\rho+\frac{\gamma_1}{2\sqrt{2\rho}}\cos\theta\right)dt, \tag{13}$$

where $\mathrm{P}(\rho)=-\widetilde{\theta}\alpha_1\sqrt{2\rho}+\rho\widetilde{\theta}(\alpha-3\beta v_B^2)-\frac{3}{2}\widetilde{\theta}\beta\rho^2$. The integrals in (13) are defined along the heteroclinic trajectories of system (6), (7) at μ = 0. Therefore, the integration involves the following relations: $\rho(T_1)=\rho(-T_1)$ and $\theta(T_1)=-\theta(-T_1)$. After transformations, the Melnikov function takes the form

$$\widetilde{M}=\frac{\gamma_2}{2}\sin(\Delta t_0)\left\{\int_{-\infty}^{\infty}\left(-\sigma\sqrt{2\rho}\cos\theta+\frac{3\lambda}{4}\rho\sqrt{2\rho}\cos\theta-\frac{\gamma_1}{2}\right)\cos(\Delta t)dt+\sigma\int_{-\infty}^{\infty}\sqrt{2\rho}\sin\theta\sin(\Delta t)dt\right.$$

$$\left.-\frac{3\lambda}{4}\int_{-\infty}^{\infty}\rho\sqrt{2\rho}\sin\theta\sin(\Delta t)dt\right\}+\int_{-\infty}^{\infty}\mathrm{P}(\rho)\left(\sigma-\frac{3}{4}\lambda\rho+\frac{\gamma_1}{2\sqrt{2\rho}}\cos\theta\right)dt. \tag{14}$$

Finally,

$$\widetilde{M}_{\pm}(t_0)=\frac{\gamma_2}{2}A_{\pm}\sin(\Delta t_0)+D_{\pm}, \tag{15}$$

where

$$D_{\pm}=\mp\left(3\beta v_B^2-\alpha\right)\widetilde{\theta}\,\frac{16}{3\lambda}\left(\sigma\theta_0^{\pm}\mp\frac{9\lambda}{16}\widetilde{\rho}\right)+\widetilde{\theta}\beta\left\{\frac{14\widetilde{\rho}\sigma}{\lambda}\mp\theta_0^{\pm}\left[\left(9\rho_2^{(1)}-\frac{4\sigma}{\lambda}\right)\left(\frac{4\sigma}{\lambda}-2\rho_2^{(1)}\right)+\frac{7\sigma(\widetilde{r}_{+}+\widetilde{r}_{-})}{\lambda}\right]\right\}$$

$$+\widetilde{\theta}\frac{9\lambda}{16}J_2^{(\pm)}+\widetilde{\theta}\left(\frac{9}{8}\lambda\rho_2^{(1)}-\frac{\sigma}{2}\right)J_1^{(\pm)},\quad \theta_0^{+}=\theta_0,\quad \theta_0^{-}=\pi-\theta_0,\quad \theta_0=\arccos\left(\frac{\widetilde{r}_{+}+\widetilde{r}_{-}}{\widetilde{r}_{+}-\widetilde{r}_{-}}\right),\quad \widetilde{\rho}=\sqrt{-\widetilde{r}_{+}\widetilde{r}_{-}}.$$

The parameters $J_2^{(\pm)}$ and $J_1^{(\pm)}$ are omitted for brevity, and $A_{\pm}$ is defined by

$$A_{\pm}(\Delta,\lambda,\gamma_1,\sigma)=\frac{9\lambda^2}{16\gamma_1}K_3^{\pm}-\frac{9\lambda}{8\gamma_1}\left(2\sigma-\frac{3}{2}\rho_2^{(1)}\lambda\right)K_2^{\pm}$$

$$-\frac{1}{2\gamma_1}\left[\frac{9}{4}\gamma_1\lambda\sqrt{2\rho_2^{(1)}}-\sigma(4\sigma-3\lambda\rho_2^{(1)})\right]K_1^{\pm}-\frac{3\lambda}{2\gamma_1}L_1^{\pm}+\left(\frac{2\sigma}{\gamma_1}-\frac{3\lambda}{2\gamma_1}\rho_2^{(1)}\right)L_0^{\pm}, \tag{16}$$

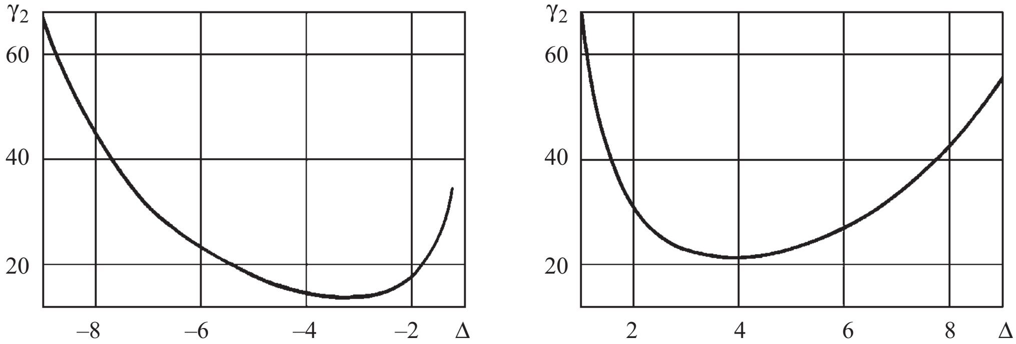
where

$$K_n^{\pm}=\int_{-\infty}^{\infty}r_{\pm}^{n}(t)\cos(\Delta t)dt,\quad L_n^{\pm}=\int_{-\infty}^{\infty}r_{\pm}^{n}(t)\dot{r}_{\pm}(t)\sin(\Delta t)dt,\quad n=1,2,3. \tag{17}$$

The values of these integrals satisfy the relations

$$L_0=-\Delta K_1,\quad K_2=-\frac{2}{\Delta}L_1. \tag{18}$$

The following values of the integrals have been determined with the help of residue theory:



a *b*

Fig. 3

$$K_1^{\pm} = \mp \frac{16\pi \sinh(\Delta'\theta_0^{\pm})}{3\lambda \sinh(\Delta'\pi)}, \qquad K_2^{\pm} = \frac{16\pi\tilde{\rho}}{3\lambda}\left[\Delta' \frac{\cosh(\Delta'\theta_0^{\pm})}{\sinh(\Delta'\pi)} \mp \cot\theta_0 \frac{\sinh(\Delta'\theta_0^{\pm})}{\sinh(\Delta'\pi)}\right],$$

$$K_3^{\pm} = \mp \frac{8\pi\tilde{\rho}^2}{3\lambda \sinh(\Delta'\pi)}\left\{\sinh(\Delta'\theta_0^{\pm})(1+3\cot^2\theta_0 + \Delta'^2) \mp 3\Delta'\cot\theta_0 \cosh(\Delta'\theta_0^{\pm})\right\},$$

$$L_0^{\pm} = \pm \frac{16\Delta\pi \sinh(\Delta'\theta_0^{\pm})}{3\lambda \sinh(\Delta'\pi)}, \qquad L_1^{\pm} = -\frac{8\Delta\pi\tilde{\rho}}{3\lambda}\left[\Delta' \frac{\cosh(\Delta'\theta_0^{\pm})}{\sinh(\Delta'\pi)} \mp \cot\theta_0 \frac{\sinh(\Delta'\theta_0^{\pm})}{\sinh(\Delta'\pi)}\right],$$

where $\Delta' = \frac{8\Delta}{3\tilde{\rho}\lambda}$. Then, $A_{\pm}(\Delta,\lambda,\gamma_1,\sigma)$ can finally be represented as

$$A_{\pm}(\Delta,\lambda,\gamma_1,\sigma) = \operatorname{cosech}\left(\frac{8\Delta\pi}{3\tilde{\rho}\lambda}\right)\left\{\mp\left[\frac{8\pi\Delta}{3\lambda\gamma_1}\left(\frac{3}{2}\lambda\tilde{\rho}\cot\theta_0 - 2\left[2\sigma - \frac{3}{2}\lambda\rho_2^{(1)}\right]\right) + \frac{2\pi}{3\lambda\gamma_1}\right.\right.$$
$$\times\left\{\frac{27}{4}\lambda^2\tilde{\rho}^2\cot^2\theta_0 - 9\lambda\tilde{\rho}\left(2\sigma - \frac{3}{2}\rho_2^{(1)}\lambda\right)\cot\theta_0 - 9\gamma_1\lambda\sqrt{2\rho_2^{(1)}} + 8\sigma\left(2\sigma - \frac{3}{2}\rho_2^{(1)}\lambda\right) + \frac{9}{4}\lambda^2\tilde{\rho}^2\right\}$$
$$\left.\left. + \frac{\pi\Delta^2 32}{3\gamma_1\lambda}\right]\sinh(\Delta'\theta_0^{\pm}) + \left[\frac{\Delta^2\pi 32}{3\gamma_1\lambda} + \frac{8\pi\Delta}{\lambda\gamma_1}\left\{\frac{3}{2}\tilde{\rho}\lambda\cot\theta_0 - 2(2\sigma - \frac{3}{2}\rho_2^{(1)}\lambda)\right\}\right]\cosh(\Delta'\theta_0^{\pm})\right\}. \quad (19)$$

It was shown in [5] that the intersections of invariant manifolds are described by the simple roots of the equation $\tilde{M}_{\pm}(t_0) = 0$. A homoclinic structure is observed in the following domain:

$$\left|D_{\pm} A_{\pm}^{-1}\right| < 0.5\gamma_2^{(\pm)}. \quad (20)$$

The domain of chaotic vibrations (20) has been analyzed numerically for the following parameters of the mechanical system (1), (2) taken from [8]: $m = 0.981$ kg, $c = 9.81\cdot10^3$ N/m, $c_3 = 1.67\cdot10^3$ N/m^3, $\Gamma_1 = 100$ N, $\theta_0 = 4.9$ N, $A = 0.2$ kg/sec, $B = 3\cdot10^{-6}$ (kg·sec)/m^2.

Then the dimensionless parameters (3) have the following values: $\varepsilon = 0.01$, $\mu = 0.1$, $\lambda = 17$, $\tilde{\theta} = 0.5$, $\gamma_1 = 1.02$, $\alpha = 4.08$, $\beta = 0.61$, $\sigma = 10$, $\nu_B = 4$.

The calculated boundaries $\gamma_2^{(+)}(\Delta)$ and $\gamma_2^{(-)}(\Delta)$ of the domain of chaotic vibrations are shown in Fig. 3*a*, *b*. The heteroclinic structure of (6), (7) is observed above these curves.

4. Numerical Modeling of Steady-State Vibrations. Here, we will discuss the numerical results on steady states in the system of modulation equations (6), (7) and compare them with the analytic results. The steady states have been determined by varying γ_2 stepwise at fixed $\Delta = 1.944$. For each values of γ_2, the integrals were evaluated by the fourth-order Runge–Kutta method.

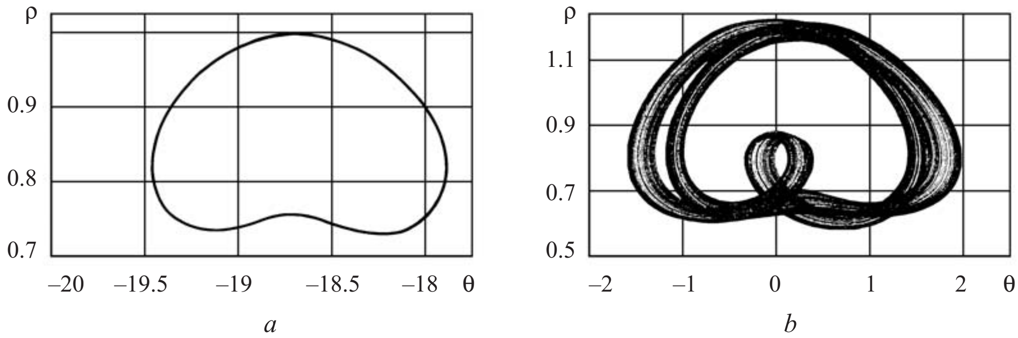


Fig. 4

The analysis has revealed the following behavior of the dynamic system (6), (7). Periodic vibrations with period $T_2 = 2\pi\Delta^{-1}$ are observed for $\gamma_2 \in [10; 43]$. As an example, periodic vibrations at $\gamma_2 = 43$ are shown in Fig. 4*a* on the plane (ρ, θ). A period-doubling bifurcation occurs in the interval $\gamma_2 \in [43; 44]$. Periodic motions with double period are observed for $\gamma_2 \in [46; 74]$. Another period-doubling bifurcation occurs in the interval $\gamma_2 \in [74; 75.5]$. This bifurcation gives rise to a fourth-order limit cycle. As the parameter γ_2 increases, chaotic vibrations occur. Chaos at $\gamma_2 = 77.3$ is shown in Fig. 4*b* as an example.

Thus, the chaotic behavior predicted based on the homoclinic Melnikov function has been confirmed by numerical modeling.

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