International Applied Mechanics, Vol. 42, No. 7, 2006

LAMINATED SHELLS WITH DEBONDING BETWEEN LAMINAS IN TEMPERATURE FIELD

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UDC 539.3

A heat-conduction problem is formulated for laminated plates and shells with a heat-conducting layer and debonding between laminas. The approach consists in analyzing how the layer thickness changes in the process of debonding of laminas and deformation of plates and shells. The three-dimensional thermoelastic and heat-conduction equations are expanded into polynomial Legendre series in thickness. The first-order, Timoshenko's, and Kirchhoff–Love equations are examined. A numerical example of laminated shells with a heat-conducting layer is considered

Keywords: laminated plates and shells, heat conductivity, heat-conducting layer, debonding of laminas, polynomial Legendre series, numerical example

1. 3-D Formulation. Let an elastic homogeneous anisotropic laminated shell of arbitrary geometry consist of Q layers with $2h^q$ thickness. Here and henceforth, all the parameters related to layers are marked with superscripts in brackets. The same parameters related to the whole shell have no subscripts in brackets.

We consider the possibility of debonding between laminas. There is a heat-conducting medium in the gap $h_0(\boldsymbol{x})$ between laminas in the debonding area. The medium does not resist the deformation of laminas, and the heat exchange between laminas is due to the thermal conductivity of the medium. We assume that h_0 is commensurable with the displacements of laminas and that these displacements are small.

The thermodynamic state of the system, including the laminas and the heat-conducting medium, is defined by the following parameters: $h_0, \varepsilon_{ij}^{(q)}(\boldsymbol{x})$, and $u_i^{(q)}(\boldsymbol{x})$ are the components of the stress and strain tensors and displacement vector, and $\theta^{(q)}(\boldsymbol{x})$, $\chi^{(q)}(\boldsymbol{x})$, $\theta^*(\boldsymbol{x})$, and $\chi^*(\boldsymbol{x})$ are the temperature and specific strength of internal heat sources in the bodies and the medium, respectively. In this case, the following relations hold [1, 4]:

$$\partial_j \sigma_{ij}^{(q)} + b_j^{(q)} = 0, \qquad \varepsilon_{ij}^{(q)} = \frac{1}{2}(\partial_i u_j^{(q)} + \partial_j u_i^{(q)}), \qquad \sigma_{ij}^{(q)} = c_{ijkl}^{(q)} \sigma_{kl}^{(q)} + \beta_{ij}^{(q)} \theta^{(q)}, \tag{1}$$

where $\partial_i = \partial / \partial x_i$ are partial derivatives with respect to the space variables x_i; and $c_{ijkl}^{(q)}$ and $\beta_{ij}^{(q)}$ are the elastic modulus and the coefficients of linear thermal expansion. In the isotropic case,

$$c_{ijkl}^{(q)} = \lambda^{(q)} \delta_{ij} \delta_{kl} + \mu^{(q)} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{kj}), \qquad \beta_{ij}^{(q)} = (\mu^{(q)} + 3\lambda^{(q)}) \alpha^{(q)} \delta_{ij},$$

where λ and μ are the Lamé constants, $\alpha^{(q)}$ are the coefficients of linear thermal expansion, and the summation convention for repeated indices is adopted.

The differential equilibrium equations for the displacement components may be presented in the form

Centro de Investigacion Cientifica de Yucatan, Mexico. Published in Prikladnaya Mekhanika, Vol. 42, No. 7, pp. 135–141, July 2006. Original article submitted September 2, 2005.

$$A_{ij}^{(q)} u_j^{(q)} + A_i^{(q)} \theta^{(q)} + b_i^{(q)} = 0, \tag{2}$$

with $A_{ij}^{(q)} = c_{ijkl}^{(q)} \partial_k \partial_l$, $A_i^{(q)} = \beta_{ij}^{(q)} \partial_j$ and $A_{ij}^{(q)} = \mu^2 \delta_{ij} \partial_k \partial_k + (\lambda^{(q)} + \mu^{(q)}) \partial_i \partial_j$, $A_i^{(q)} = (\mu^{(q)} + 3\lambda^{(q)}) \alpha^{(q)} \partial_i$ in the anisotropic and isotropic cases, respectively.

Boundary conditions for displacements and traction on the parts $\partial V_p^{(q)}$ and $\partial V_u^{(q)}$ have the form

$$p_i^{(q)} = \sigma_{ij}^{(q)} n_j = \psi_i^{(q)}, \quad \forall \boldsymbol{x} \in \partial V_p^{(q)}, \quad u_i^{(q)} = \varphi_i^{(q)}, \quad \forall \boldsymbol{x} \in \partial V_u^{(q)}, \quad \forall \boldsymbol{x} \in V^{(q)}. \tag{3}$$

In the debonding areas $\partial V_e^{(q)}$, the boundary conditions at the crack edges have the form of inequalities [2, 3, 6]:

$$\Delta u_n^{(q)} = u_n^{(q)} - u_n^{(q+1)} \geq h_0^{(q)}, \quad q_n^{(q)} \geq 0, \quad \left(\Delta u_n^{(q)} - h_0^{(q)}\right) q_n^{(q)} = 0,$$

$$\left|\boldsymbol{q}_\tau^{(q)}\right| < k_\tau^{(q)} q_n^{(q)} \to \partial_t \boldsymbol{u}_\tau^{(q)} = 0, \quad \left|\boldsymbol{q}_\tau^{(q)}\right| = k_\tau^{(q)} q_n^{(q)} \to \partial_t \boldsymbol{u}_\tau^{(q)} = -\lambda_\tau^{(q)} \boldsymbol{p}_\tau^{(q)},$$

$$\boldsymbol{p}^{(q)} = -\boldsymbol{p}^{(q+1)} = \boldsymbol{q}^{(q)}, \quad \boldsymbol{n}^{(q)} = -\boldsymbol{n}^{(q+1)}, \quad \forall \boldsymbol{x} \in \partial V_e, \quad \partial V_e = \partial V_e^{(q)} \cap \partial V_e^{(q+1)}, \tag{4}$$

where $q_n^{(q)}$, $\Delta u_n^{(q)} = u_n^{(q)} - u_n^{(q+1)}$, $\boldsymbol{q}_\tau^{(q)}$, and $\Delta \boldsymbol{u}_\tau^{(q)} = \boldsymbol{u}_\tau^{(q)} - \boldsymbol{u}_\tau^{(q+1)}$ are the normal and tangential components of the contact force vector and the displacement discontinuity vector, respectively; and $k_\tau^{(q)}$ and $\lambda_\tau^{(q)}$ are coefficients that depend upon the properties of the contact surfaces.

The linear heat-conduction equations have the form

$$\lambda_{ij}^{(q)} \partial_i \partial_j \theta^{(q)} - \chi^{(q)} = 0, \quad \forall\, x \in V^{(q)}, \tag{5}$$

where $\lambda_{ij}^{(q)}$ are the coefficients of thermal conductivity. In the isotropic case, we have $\lambda_{ij}^{(q)} = \delta_{ij} \lambda_T^{(q)}$.

The boundary conditions for temperature and heat flux on the parts $\partial V_\theta^{(q)}$ and $\partial V_q^{(q)}$ have the form

$$\theta^{(q)} = \theta_b^{(q)}, \quad \forall \boldsymbol{x} \in \partial V_0, \quad \boldsymbol{q}^{(q)} = \boldsymbol{q}_b^{(q)}, \quad \forall \boldsymbol{x} \in V^{(q)}. \tag{6}$$

The temperature distribution within the heat-conducting medium is described by the heat-conduction equations

$$\lambda_{ij}^* \partial_i \partial_j \theta_* - \chi_* = 0, \quad \forall \boldsymbol{x} \in V^*. \tag{7}$$

The boundary conditions on the lateral sides of the heat-conducting medium have the form

$$\lambda_{ij} \partial_n \theta^{(q)} + \beta_{ij} \left(\theta^{(q)} - \theta_b^{(q)}\right) = 0. \tag{8}$$

The heat-conduction conditions for the heat-conducting medium have the form

$$\theta_* = \theta^{(q)}, \quad \lambda_{ij}^* \partial_n \theta_* = \lambda_{ij}^\alpha \partial_n \theta^{(q)}, \quad \forall \boldsymbol{x} \in \partial V_e^{(q)}. \tag{9}$$

In the area of close mechanical contact, the thermal conditions transform into the form

$$q_\theta = \alpha_e \left(\theta^{(q)} - \theta_b^{(q)}\right), \quad \forall \boldsymbol{x} \in \partial V_e^{(q)}, \tag{10}$$

where q_θ is the heat flux across the close mechanical contact area, and α_e is the contact heat conductivity.

The analysis of the problem encounters mathematical difficulties caused by the dimension and non-linearity of the problem. The problem can be partially simplified by considering thin bodies. In this case, we can reduce the dimension of the problem.

2. 2-D Formulation. Let the component parameters, which describe the stress–strain state of each lamina as a three-dimensional body, be sufficiently smooth functions of x_3 expandable into Legendre series. Using the approach developed in [5, 8], we can express them as

$$u_i^{(q)}(\boldsymbol{x})=\sum_{k=0}^{\infty} u_i^{(q)k}(\boldsymbol{x}_\alpha)P_k(\omega),\quad u_i^{(q)k}(\boldsymbol{x}_\alpha)=\frac{2k+1}{2h^{(q)}}\int_{-h^{(q)}}^{h^{(q)}} u_i^{(q)}(\boldsymbol{x}_\alpha,x_3)P_k(\omega)dx_3,$$

$$\sigma_{ij}^{(q)}(\boldsymbol{x})=\sum_{k=0}^{\infty} \sigma_{ij}^{(q)k}(\boldsymbol{x}_\alpha)P_k(\omega),\quad \sigma_{ij}^{(q)k}(\boldsymbol{x}_\alpha)=\frac{2k+1}{2h^{(q)}}\int_{-h^{(q)}}^{h^{(q)}} \sigma_{ij}^{(q)}(\boldsymbol{x}_\alpha,x_3)P_k(\omega)dx_3,$$

$$\varepsilon_{ij}^{(q)}(\boldsymbol{x})=\sum_{k=0}^{\infty} \varepsilon_{ij}^{(q)k}(\boldsymbol{x}_\alpha)P_k(\omega),\quad \varepsilon_{ij}^{(q)k}(\boldsymbol{x}_\alpha)=\frac{2k+1}{2h^{(q)}}\int_{-h^{(q)}}^{h^{(q)}} \varepsilon_{ij}^{(q)}(\boldsymbol{x}_\alpha,x_3)P_k(\omega)dx_3,$$

$$\theta^{(q)}(\boldsymbol{x})=\sum_{k=0}^{\infty} \theta^{(q)k}(\boldsymbol{x}_\alpha)P_k(\omega),\quad \theta^{(q)k}(\boldsymbol{x}_\alpha)=\frac{2k+1}{2h^{(q)}}\int_{-h^{(q)}}^{h^{(q)}} \theta^{(q)}(\boldsymbol{x}_\alpha,x_3)P_k(\omega)dx_3,$$

$$\theta_*^{(q)}(\boldsymbol{x})=\sum_{n=0}^{\infty} \theta_*^{(q)k}(\boldsymbol{x}_\alpha)P_k(\omega),\quad \theta_*^{(q)k}(\boldsymbol{x}_\alpha)=\frac{2k+1}{2h^{(q)}}\int_{-h^{(q)}}^{h^{(q)}} \theta_*^{(q)}(\boldsymbol{x}_\alpha,x_3)P_k(\omega)dx_3, \tag{11}$$

where $\omega=x_3/h^{(q)}$ is a dimensionless coordinate.

Equations (1) and (2) may be easily rewritten for the coefficients of the Legendre series. The boundary conditions (3), (5), (8)–(10) may also be easily rewritten for the coefficients of the Legendre series. The corresponding 2-D equations for the k coefficient in the Legendre polynomial will have the same form, but the boundary conditions in the debonding area (9) become

$$\sum_{k=0}^{\infty}\Delta u_n^{(q)k}P_k(\omega)\geq h_0^{(q)},\quad \sum_{k=0}^{\infty} q_n^{(q)k}P_k(\omega)\geq 0,\quad \left(\sum_{k=0}^{\infty}\Delta u_n^{(q)k}P_k(\omega)-h_0^{(q)}\right)\sum_{k=0}^{\infty} q_n^{(q)k}P_k(\omega)=0,$$

$$\left|\sum_{n=0}^{\infty}\boldsymbol{q}_\tau^{(q)n}P_n(\omega)\right|<k_\tau^{(q)}\sum_{n=0}^{\infty}q_n^{(q)n}P_n(\omega)\to\partial_t\sum_{n=0}^{\infty}\Delta\boldsymbol{u}_\tau^{(q)n}P_n(\omega)=0,$$

$$\left|\sum_{n=0}^{\infty}\boldsymbol{q}_\tau^{(q)n}P_n(\omega)\right|=k_\tau^{(q)}\sum_{n=0}^{\infty}q_n^{(q)n}P_n(\omega)\to\partial_t\sum_{n=0}^{\infty}\Delta\boldsymbol{u}_\tau^{(q)n}P_n(\omega)=-\lambda_\tau^{(q)}\sum_{n=0}^{\infty}\boldsymbol{q}_\tau^{(q)n}P_n(\omega). \tag{12}$$

Now instead of one 3-D boundary-value problem, we have an infinite set of 2-D boundary-value problems for the coefficients of the Legendre series. In order to construct an approximate theory, we have to keep only a finite number of terms in (11).

3. First-Order Equations. In the first approximation, shell theory retains only the first two terms in the Legendre series [8, 10]. In this case, the thermodynamic parameters, which describe the state of the laminated shell, can be represented in the form

$$\sigma_{ij}^{(q)}(\boldsymbol{x})=\sigma_{ij}^{(q)0}(\boldsymbol{x}_\nu)P_0(\omega)+\sigma_{ij}^{(q)1}(\boldsymbol{x}_\nu)P_1(\omega),\quad u_i^{(q)}(\boldsymbol{x})=u_i^{(q)0}(\boldsymbol{x}_\nu)+u_i^{(q)1}(\boldsymbol{x}_\nu)\frac{x_3}{h},$$

$$\varepsilon_{ij}^{(q)}(\boldsymbol{x})=\varepsilon_{ij}^{(q)0}(\boldsymbol{x}_\nu)P_0(\omega)+\varepsilon_{ij}^{(q)1}(\boldsymbol{x}_\nu)P_1(\omega),\quad \theta^{(q)}(\boldsymbol{x})=\theta^{(q)0}(\boldsymbol{x}_\nu)+\theta^{(q)1}(\boldsymbol{x}_\nu)\frac{x_3}{h},$$

$$\theta_*^{(q)}(\boldsymbol{x}) = \theta_*^{(q)0}(\boldsymbol{x}_\nu) + \theta_*^{(q)1}(\boldsymbol{x}_\nu)\frac{x_3}{h}. \quad (13)$$

Then the 2-D thermoelastic equations (2) have the form

$$L_{ij}^{00} u_j^{(q)0} + L_{ij}^{01} u_j^{(q)1} + L_i^0 \left(\theta^{(q)0} - \theta_0^{(q)0}\right) + b_i^{(q)0} = 0,$$

$$L_{ij}^{10} u_j^{(q)0} + L_{ij}^{11} u_j^{(q)1} + L_i^1 \left(\theta^{(q)1} - \theta_0^{(q)1}\right) + b_i^{(q)1} = 0 \quad (14)$$

and the 2-D heat-conduction equations (5) have the form

$$\Delta_0 \theta^{(q)0} + \frac{1}{2h}\left(Q_3^{(q)+} - Q_3^{(q)-}\right) + \left(k_1 + k_2\right) Q_3^{(q)0} + \frac{\chi^{(q)0}}{\lambda_{(q)0}} = 0,$$

$$\Delta_0 \theta^{(q)1} + \frac{3}{2h}\left(Q_3^{(q)+} + Q_3^{(q)-}\right) + \left(k_1 + k_2\right) Q_3^{(q)1} + \frac{\chi^{(q)1}}{\lambda_{(q)0}} = 0. \quad (15)$$

We will keep only one term in the Legendre series for θ_*. Then the 2-D heat-conduction equations (7) for the layer become

$$\Delta_0 \theta_*^0 + \Delta h \theta_*^0 + \left(\nabla^* h \cdot \nabla \theta_*^0\right) - \frac{1}{2}\left(\theta_*^+ \Delta_* h^+ + \theta_*^- \Delta_* h_*\right) + \left(\nabla^* h \cdot Q_2^0\right)$$

$$+ \frac{1}{2h}(Q_3^{*+} - Q_3^{*-}) + \left(k_1 + k_2\right) Q_3^{*0} + \frac{\chi^0}{\lambda_*} = 0. \quad (16)$$

The unknown parameters in Eqs. (14) have the form

$$Q_3^{(q)+} - Q_3^{(q)-} = \frac{3}{4h}\left(\theta^+ + T_k\right) + \frac{3\theta^{(q)0}}{2h}, \quad Q_3^{(q)0} = \frac{1}{2h}\left(\theta^+ - T_k\right),$$

$$Q_3^{(q)+} + Q_3^{(q)-} = \frac{3}{2h}\left(\theta^+ - T_k\right) - \frac{5\theta^{(q)1}}{2h}, \quad Q_3^1 = \frac{3}{2h}\left(\theta^+ + T_k\right) - \frac{3\theta^1}{2h}, \quad (17)$$

where

$$T_k^{(q)} = \frac{\left[9\lambda_0^2\left(h_0 - \Delta u_3^{(q)}\right) + \lambda_0 \lambda_* h^{(q+1)}\right]\left(3\theta^+ + 6\theta^{(q)0} - 10\theta^{(q)1}\right) + \lambda_0 \lambda_* h^{(q)}\left(3\theta^- + 6\theta^{(q+1)0} - 10\theta^{(q+1)1}\right)}{9\left[9\lambda_0^2\left(h_0 - \Delta u_3^{(q)}\right) + \lambda_0 \lambda_* \left(h^{(q)} + h^{(q+1)}\right)\right]},$$

$$T_k^{(q+1)} = \frac{\left[9\lambda_0^2\left(h_0 - \Delta u_3^{(q)}\right) + \lambda_0 \lambda_* h^{(q)}\right]\left(3\theta^+ + 6\theta^{(q+1)0} - 10\theta^{(q+1)1}\right) + \lambda_0 \lambda_* h^{(q+1)}\left(3\theta^+ + 6\theta^{(q)0} - 10\theta^{(q)1}\right)}{9\left[9\lambda_0^2\left(h_0 - \Delta u_3^{(q)}\right) + \lambda_0 \lambda_* \left(h^{(q)} + h^{(q+1)}\right)\right]}. \quad (18)$$

See [7] for details.

4. Timoshenko's Shell Equations. Timoshenko's theory of shells is based on the following assumptions concerning the nature of the stress–strain state of the shell: $\sigma_{33} = 0$ and $\varepsilon_{33} = 0$. In this theory, the thermodynamic state of shells is determined by quantities specified on the middle surface. The stress state is characterized by the normal ($n_{\alpha\alpha}$), tangential ($n_{\alpha\beta}$ ($\alpha \neq \beta$)), and shear ($n_{\alpha 3}$) forces, as well as the bending ($m_{\alpha\alpha}$) and twisting ($m_{\alpha\beta}$ ($\alpha \neq \beta$)) moments. The components of the stress tensor are given by the equations

$$\sigma_{\alpha\beta}(\boldsymbol{x}) = \frac{n_{\alpha\beta}(\boldsymbol{x}_\nu)}{2h} + \frac{3m_{\alpha\beta}(\boldsymbol{x}_\nu) x_3}{h^3}, \quad \sigma_{\alpha 3}(\boldsymbol{x}) = \frac{n_{\alpha 3}(\boldsymbol{x}_\nu)}{2h}, \quad \sigma_{33}(\boldsymbol{x}) = 0.$$

The components of the stress tensor are

$$\varepsilon_{\alpha\beta}(\boldsymbol{x})=e_{\alpha\beta}(\boldsymbol{x}_\nu)+\kappa_{\alpha\beta}(\boldsymbol{x}_\nu)x_3, \quad \varepsilon_{\alpha 3}(\boldsymbol{x})=e_{\alpha 3}(\boldsymbol{x}_\nu), \quad \varepsilon_{33}(\boldsymbol{x})=0,$$

where $e_{\alpha i}$ characterize deformation that is uniform throughout the thickness of the shell and is associated with the tension and compression of the middle surface and the displacement in perpendicular planes, while $\kappa_{\alpha\beta}$ is associated with the bending and twisting of the middle surface [7, 10]. In these and further equations, we omit the index (q).

The components of the displacement vector are given by the equations

$$u_\alpha(\boldsymbol{x})=v_\alpha(\boldsymbol{x}_\nu)+\gamma_\alpha(\boldsymbol{x}_\nu)x_3, \quad u_3(\boldsymbol{x})=v_3(\boldsymbol{x}_\nu),$$

where v_i is the displacement of the middle surface and γ_α is the angle of rotation of the middle surface.

According to Timoshenko's theory, the differential thermoelastic equations for shells have the form

$$L_{ij}^{u}v_j+L_{i\beta}^{u\gamma}\gamma_\beta+L_i^0\left(\theta^0-\theta_0^0\right)+\bar{b}_i=0,$$

$$L_{ij}^{\gamma u}v_j+L_{\alpha\beta}^{\gamma}\gamma_\beta+L_\alpha^1\left(\theta^1-\theta_0^1\right)+\bar{m}_\alpha=0, \tag{19}$$

where $\bar{b}_i$ and $\bar{m}_\alpha$ are the external loads acting on Ω^+ and Ω^- and reduced to the middle surface; L_{ij}^u, $L_{i\beta}^{u\gamma}$, $L_{\alpha j}^{\gamma u}$, and $L_{\alpha\beta}^{\gamma}$ are second-order differential operators; L_i^0 and L_α^1 are first-order differential operators. Their expressions are given in [9] for some shell geometries.

The differential heat-conduction equations for the shells and the layer do not change and have the form of (14) and (16).

5. Kirchhoff–Love Shell Equations. In addition to the assumptions of Timoshenko's theory, the classical Kirchhoff–Love theory of shells assumes that $\varepsilon_{\alpha 3}=0$ and that the angles of rotation of the normal to the middle surface are dependent and are given by the equations

$$\gamma_\alpha(\boldsymbol{x}_\nu)=-\frac{1}{A_\alpha(\boldsymbol{x}_\nu)}\partial_\alpha v_3(\boldsymbol{x}_\nu)+k_\alpha(\boldsymbol{x}_\nu)\,v_\alpha(\boldsymbol{x}_\nu).$$

The inconsistencies of the Kirchhoff–Love theory of shells resulting from these hypotheses are well known [7, 10]. Nevertheless, the differential thermoelastic equations for shells have a simple form in this case:

$$L_{ij}v_j+\sum_{k=0}^{1}L_i^k\left(\theta^k-\theta_0^k\right)+b_i=0. \tag{20}$$

The differential heat-conduction equations for the shells and the heat-conducting layer do not change too and have the form of (14) and (16).

6. Example. Let us consider an axisymmetric cylindrical shell in adhesive contact with a foundation in a uniform temperature field. There is a debonding area with gap $h_0(x)$ between the shell and the foundation. Let us study the temperature field and stress–strain state using the approach presented above.

The differential thermoelastic and heat-conduction equations in the Kirchhoff–Love theory for an axisymmetric cylindrical shell have the form

$$\frac{d^4w}{dx^4}+4\beta^4w-\beta_0\theta^0-\beta_1\frac{d^2\theta^1}{dx^2}=\frac{1}{D}\left(p-q\right), \quad \beta^4=\frac{3(1-\nu^2)}{4h^2r^2},$$

$$\frac{d^2\theta^0}{dx^2}-\varepsilon_0^2\theta^0+F_0=0, \quad \frac{d^2\theta^1}{dx^2}-\varepsilon_1^2\theta^1+F_1=0,$$

$$F_0=0.5\varepsilon_0\left(\theta^-+T_k\right)+\frac{1}{2hr}\left(T_k-\theta^-\right), \quad \varepsilon_0=\frac{3}{h^2}, \quad \varepsilon_1=\frac{15}{h^2},$$

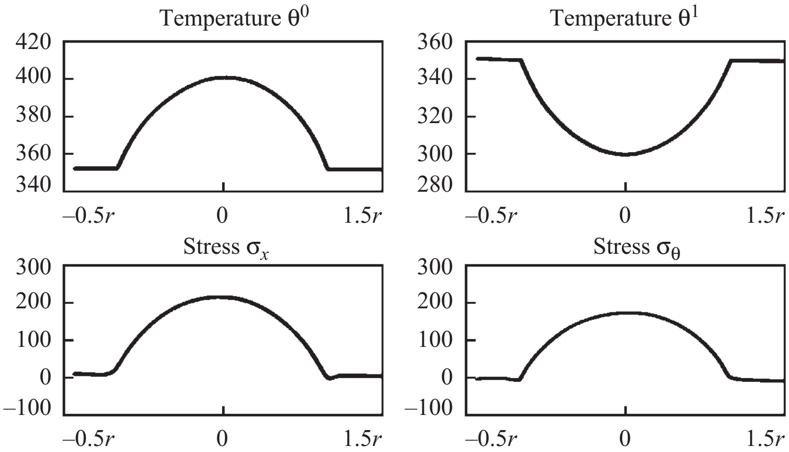

Fig. 1

$$F_1 = 0.5\varepsilon_1\left(T_k - \theta^-\right) + \frac{3}{2hr}\left(T_k + \theta^-\right) - \frac{3}{hr}\theta^1, \qquad D = \frac{2Eh^3}{3(1-\nu^2)},$$

$$T_k = \frac{\lambda_0\left(h_0 - u_3\right)\left(3\theta^+ + 6\theta^0 - 10\theta^1\right) + \lambda_* h\theta^-}{9\lambda_0\left(h_0 - u_3\right) + \lambda_* h}. \tag{21}$$

Then the boundary-value problem (21) can be transformed into Hammerstein-type integral equations:

$$\int_l G_\alpha(x,y)F_\alpha(y)dy = \theta^\alpha, \qquad \int_l W(x,y)\left\{\frac{1}{D}\left[p(y) - q(y)\right] - \beta_0 F_3(y)\right\}dy = w. \tag{22}$$

The kernels in these integral equations are Green's functions

$$G_i(x,y) = \exp\left(-\varepsilon_i\left|x-y\right|\right)/2\varepsilon_i, \qquad i = 0, 1,$$

$$W(x,y) = \frac{1}{8\beta^3 D}\exp\left(-\beta\left|x-y\right|\right)\left[\cos\left(\beta\left|x-y\right|\right) + \sin\left(\beta\left|x-y\right|\right)\right],$$

$$F_3 = \beta_1\left(F_1 + \varepsilon_1^2\theta^1\right) - \beta_0\theta^0, \qquad \beta_0 = \frac{3(1-\nu)\alpha_\tau}{h^2 r}, \qquad \beta_1 = \frac{(1+\nu)\alpha_\tau}{h}. \tag{23}$$

An algorithm for solving the problem has been elaborated in [5].

Calculations have been performed for the following data: temperature: $h_0(x) = h_m \sin \pi x / l$, $\theta_0^+ = 700$ °C, $\theta^- = 0$ °C, geometrical parameters $r = 0.7$ m, $h = 0.01$ m, $h_0(x) = h_m \sin \pi x / l$, $h_m = 0.0025$ m, $l_d = 2r$, material properties: $E = 2.5 \cdot 10^5$ MPa, $\nu = 0.25$, $\alpha_\tau = 2.5 \cdot 10^{-5}$ °C^{-1}, $\lambda_1 = 20$ V/(m·°C), $\lambda_* = 10$ V/(m·°C).

The temperature field and the stresses σ_x and σ_θ are presented in Fig. 1. It is assumed that the shell is in unstressed state in a homogeneous temperature field. The results show that even in this model problem, debonding changes the thermal conditions and affects the temperature field and stress distribution.

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