ROTATION OF RIGID AND ELASTIC CYLINDRICAL SHELLS ELASTICALLY COUPLED WITH A PLATFORM

I. L. Solov'ev

UDC 539.3

The critical states in simple and compound rotation of thin cylindrical shells elastically coupled with a platform are modeled theoretically. The technique developed has been implemented in a software system intended to analyze the mechanical phenomena associated with the critical states and to establish general conditions for such phenomena to occur. The results obtained may be used to model the dynamic behavior of turbine rotors in aircraft and ship engines

Keywords: thin-walled rotor, simple and compound rotation, critical states, computer simulation

Introduction. Current trends in design of turbines rotors for aircraft and ship engines, which are distinguished by high specific power and high intensity of engine processes, add significance to the problems of their stability and vibration strength. Rotors are usually made from several thin-walled shells of various shapes. While in service, rotors are subject to nonstationary loads due to aerodynamic forces and high-temperature fields. The highest operational loads occur in reorientations of the rotor axis in space such as in maneuvering of an airplane or rocking of a ship, when structural members undergo compound rotation [6–12, etc.]. The dynamic behavior of the rotor in such states depends substantially on the stiffness of its elastic coupling with the base. In this connection, it is of interest to study the conditions for critical states to occur during simple and compound rotation of thin-walled rigid and elastic bodies and to examine the effect of the stiffness of their elastic coupling on these processes. We will do such studies for a cylindrical shell.

1. Critical States of a Rigid Cylindrical Shell. Let us first analyze the critical states of rigid cylindrical shells hinged to and elastically coupled with a rotating platform. We choose an inertial coordinate frame $OX^*Y^*Z^*$ (Fig. 1). The shell is coupled by its lower end with a plane that is hinged or elastically connected to a platform rotating with a constant angular velocity ω about the OZ^*-axis. Let a coordinate frame $Oxyz$ be fixed to the platform so that the Oz-axis is aligned with the OZ^*-axis. Suppose that the lower end plane can turn about the Ox- and Oy-axes through small angles $\vec{\alpha}$ and $\vec{\beta}$ and cannot turn about the Oz-axis. In the initial state, the axis of circular symmetry of the shell is aligned with the Oz-axis and $\vec{\alpha}=0$, $\vec{\beta}=0$.

Let us study small free vibrations described by the angles α and β and the stability of equilibrium of the shell in the coordinate frame $Oxyz$. Suppose that after rotation through these angles, the end of the shell is subjected to a restoring elastic torque $\vec{M}^{\text{el}}=-k\alpha\vec{i}+-k\beta\vec{j}$, where k is the stiffness. When $k = 0$, the shell is hinged to the platform.

In addition to this torque, the body is acted upon by the moment of inertial forces $\vec{M}^{\text{in}}=M_x^{\text{in}}\vec{i}+M_y^{\text{in}}\vec{j}$. D'Alembert's principle suggests that

$$\vec{M}^{\text{in}}+\vec{M}^{\text{el}}=0. \tag{1}$$

Under the above assumptions, the shell acts as a rigid gyroscope whose stability and oscillation are governed by its geometrical and inertial parameters [4, 5].

National University of Transport, Kyiv, Ukraine. Translated from Prikladnaya Mekhanika, Vol. 42, No. 7, pp. 107–115, July 2006. Original article submitted April 18, 2005.

 1063-7095/06/4207-0818

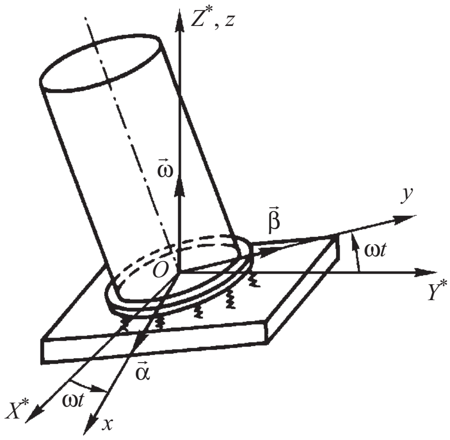

Fig. 1

To evaluate $\vec{M}^{\,\text{in}}$, we choose a small element of mass Δm in the shell and determine the inertial force $\Delta\vec{p} = -\Delta m \cdot \vec{a}$ that acts on it, where $\vec{a}$ is the absolute acceleration of the element in $Oxyz$, $\vec{a} = \vec{a}^{\,\text{c}} + \vec{a}^{\,\text{r}} + \vec{a}^{\,\text{C}}$ according to the Coriolis theorem.

The centrifugal ($\vec{a}^{\,\text{c}}$), relative ($\vec{a}^{\,\text{r}}$), and Coriolis ($\vec{a}^{\,\text{C}}$) accelerations are defined as follows [3]:

$$\vec{a}^{\,\text{c}} = \vec{\omega} \times (\vec{\omega} \times \vec{\rho}), \qquad \vec{a}^{\,\text{r}} = d^2\vec{\rho} / dt^2, \qquad \vec{a}^{\,\text{C}} = 2\vec{\omega} \times \vec{v}^{\,\text{r}}, \tag{2}$$

where $\vec{v}^{\,\text{r}} = d\vec{\rho} / dt$ is the relative velocity, and $\vec{\rho} = (x + z\beta)\vec{i} + (y - z\alpha)\vec{j} + (z + y\alpha - x\beta)\vec{k}$ is the position vector of the element in the disturbed state at small α and β.

Performing the necessary operations in (2), we obtain

$$\vec{v}^{\,\text{r}} = z\dot{\beta}\vec{i} - z\dot{\alpha}\vec{j} + (y\dot{\alpha} - x\dot{\beta})\vec{k}, \quad \vec{a}^{\,\text{c}} = -\omega^2 (x + z\beta)\vec{i} - \omega^2 (y - z\alpha)\vec{j},$$

$$\vec{a}^{\,\text{r}} = z\ddot{\beta}\vec{i} - z\ddot{\alpha}\vec{j} + (y\ddot{\alpha} - x\ddot{\beta})\vec{k}, \quad \vec{a}^{\,\text{C}} = 2\omega z\dot{\alpha}\vec{i} + 2\omega z\dot{\beta}\vec{j}. \tag{3}$$

Integrating these equations over the volume of the shell, we find the components of the moment of inertial forces:

$$M_x^{\text{in}} = \left(-\ddot{\alpha} + 2\omega\dot{\beta} + \omega^2\alpha\right)\left(I_0 - I_z\right) - \left(\ddot{\alpha} + \omega^2\alpha\right)\left(I_0 - I_x\right),$$

$$M_y^{\text{in}} = \left(-\ddot{\beta} - 2\omega\dot{\alpha} + \omega^2\beta\right)\left(I_0 - I_z\right) - \left(\ddot{\beta} + \omega^2\beta\right)\left(I_0 - I_x\right), \tag{4}$$

where I_x and I_z are the axial moments of inertia of the shell; and $I_0 = I_x + I_z / 2$ is the polar moment of inertia in the frame $Oxyz$ in the strain-free state.

Equations (1) and (4) yield equations describing the free vibrations of the shell in the rotating coordinate frame:

$$I_x\ddot{\alpha} - 2\omega\left(I_0 - I_z\right)\dot{\beta} + \left[\omega^2\left(I_z - I_x\right) + k\right]\alpha = 0,$$

$$I_x\ddot{\beta} + 2\omega\left(I_0 - I_z\right)\dot{\alpha} + \left[\omega^2\left(I_z - I_x\right) + k\right]\beta = 0. \tag{5}$$

These equations allow us to analyze the state of dynamic equilibrium $\alpha = \beta = 0$ for stability and to find frequencies and modes of free vibrations. When $\dot{\alpha} = \dot{\beta} = 0$ and $\ddot{\alpha} = \ddot{\beta} = 0$, the system of equations (5) is uncoupled and the inequality $[\omega^2 (I_z - I_x) + k] > 0$ is the condition of stability.

Then the critical angular velocity is defined by

$$\omega_{\text{cr}} = \pm\sqrt{-k / (I_z - I_x)}. \tag{6}$$

Equation (6) indicates that when $I_z - I_x > 0$, the rigid shell regarded as a gyroscope does not lose stability. For the shell to lose stability at some ω, it is necessary that $I_z - I_x < 0$. It is significant that if $k = 0$, then the gyroscope is stable when $I_z > I_x$ and is unstable when $I_z < I_x$, whatever the value of ω. The case $I_z = I_x$ is limiting.

Equations (5), which describe free vibrations, are distinguished by having gyroscopic terms. Therefore, they permit precession motions with frequency c:

$$\alpha = A \sin ct, \quad \beta = B \cos ct. \tag{7}$$

Let us first examine the case $A = B$. Each of Eqs. (5) is reduced to the characteristic equation

$$-I_x c^2 + 2(I_0 - I_z)c\omega + (I_z - I_x)\omega^2 + k = 0, \tag{8}$$

which defines the relationship between c and ω in the form of hyperbolic curves

$$c_{1,2} = \omega(I_0 - I_z)/I_x \pm \sqrt{\omega^2 (I_0 - I_x)^2 / I_x^2 + k/I_x} \tag{9}$$

with the asymptotes

$$c_1 = \omega, \quad c_2 = \omega(I_x - I_z)/I_x. \tag{10}$$

The angles between these lines and the ordinate axis are $\psi_1 = \pi/4$ and $\psi_2 = \arctan[(I_x - I_z)/I_x]$.

If $c_1 > 0$ and $c_2 > 0$ in (9), then, according to (7), at both frequencies, the gyroscope axis describes a cone in the rotating coordinate frame by moving in the opposite direction to self-rotation (regular retrograde precession). When one of the frequencies is negative, the corresponding motion (7) will be regular progressive precession. If $k = 0$, then c_1 and c_2 are defined by (10), which follows from Eq. (9).

The case $A = -B$ is also represented by the hyperbolic curves $c_i(\omega)$, but with the asymptotes $c_1 = -\omega$, $c_2 = -\omega(I_x - I_z)/I_x$. Again, retrograde precession occurs at both frequencies when $I_x > I_z$ and progressive precession at the frequency c_2 when $I_x < I_z$.

It should be emphasized that $c_2 = 0$ is the limiting (in the sense of static stability) case $I_x = I_z$.

Let us now examine compound rotation. Suppose that the Oz-axis of the rotating coordinate frame $Oxyz$ to which the body is fixed is additionally forced to turn with a constant angular velocity $\omega_0 \ll \omega$ in the plane X^*OZ^*. Let us introduce a turning coordinate frame $OXYZ$: the OY-axis is fixed and aligned with the OY^*-axis, and the OZ-axis is turning and aligned with the Oz-axis. The interaction of two rotary motions generates a gyroscopic moment $\vec{M}^{\text{gyr}} = I_z \vec{\omega} \times \vec{\omega}_0$ that remains constant in the frame $OXYZ$ [3] and rotates with angular velocity $-\omega$ in the frame $Oxyz$. Therefore, this moment has the following components in this frame:

$$M_x^{\text{gyr}} = -I_z \omega\omega_0 \cos \omega t, \quad M_y^{\text{gyr}} = I_z \omega\omega_0 \sin \omega t, \quad M_z^{\text{gyr}} = 0. \tag{11}$$

Substituting M_x^{gyr} and M_y^{gyr} into the right-hand sides of Eqs. (5), we obtain

$$I_x \ddot{\alpha} - 2\omega(I_0 - I_z)\dot{\beta} + [\omega^2(I_z - I_x) + k]\,\alpha = -I_z \omega\omega_0 \cos \omega t,$$

$$I_x \ddot{\beta} + 2\omega(I_0 - I_z)\dot{\alpha} + [\omega^2(I_z - I_x) + k]\,\beta = I_z \omega\omega_0 \sin \omega t. \tag{12}$$

The solution of this system of equations has the form

$$\alpha = (-I_z \omega\omega_0 / k)\cos \omega t, \quad \beta = (I_z \omega\omega_0 / k)\sin \omega t. \tag{13}$$

It indicates that during compound rotation, the body undergoes retrograde precession with angular velocity ω in the frame $Oxyz$; but since this frame itself rotates with velocity ω, the motion of the body in the turning frame $OXYZ$ is a stationary state in which the Oz-axis remains turned through an angle $I_z\omega\omega_0/k$ in the plane YOZ. Critical equilibrium states in the frame

OXYZ or resonant precession oscillations (13) in the frame *Oxyz* do not occur, because the circular frequency ω of the moments (11) never becomes equal to the natural frequencies (9).

2. Critical States of an Elastic Cylindrical Shell. A technique for elastic strain analysis of thin rotating shells without possible rotations through angles α and β at the elastic supports was discussed in [1, 2, 6–12]. It is based on the general equations of motion of shells in a curvilinear orthogonal coordinate frame $ox^1x^2x^3$ with the basis vectors $\vec{e}_\alpha$ on its mid-surface:

$$\nabla_\alpha \vec{T}^\alpha + \vec{p} = 0, \qquad \nabla_\alpha \vec{M}^\alpha + (e_\alpha \times \vec{T}^\alpha)\sqrt{a_{11}a_{22}} = 0 \quad (\alpha = 1, 2), \tag{14}$$

where the x^1-axis lies in the plane of the generating section of the shell; the x^2-axis is directed in the circumferential direction; the x^3-axis is directed along the inward normal to the mid-surface; $\vec{T}^\alpha$ is the vector of internal forces; $\vec{M}^\alpha$ is the vector of internal moments; a_{11} and a_{22} are the coefficients of the first quadratic form of the mid-surface; and $\vec{p}$ is the vector of intensity of the external distributed load.

Here the load $\vec{p}$ is the inertial forces of compound motion defined by the formula $\vec{p} = -\gamma h \vec{a}$, where γ is the density of the shell material; *h* is the thickness of the shell; and $\vec{a}$ is the absolute acceleration of the element.

In the general case, the acceleration $\vec{a}$ is calculated from formulas (2) and (3) in which, however, the vectors $\vec{\omega}$, $\vec{v}^{\,r}$, and $\vec{\rho}$ should account for all elastic motions of the shell and its turn through the angles α and β. To this end, we transform the accelerations $\vec{a}^{\,c}$, $\vec{a}^{\,r}$, and $\vec{a}^{\,C}$ defined in the basis $\vec{i}, \vec{j}, \vec{k}$ of the rigid shell to the local basis $\vec{e}_1, \vec{e}_2, \vec{e}_3$ of the elastic shell:

$$a^1 = \big[(-z\sin x^2 \sin\varphi + r\sin x^2 \cos\varphi)\ddot{\alpha} + (z\cos x^2 \sin\varphi - r\cos x^2 \cos\varphi)\ddot{\beta} + 2\omega z\cos x^2 \sin\varphi\dot{\alpha}$$

$$+ 2\omega z\sin x^2 \sin\varphi\dot{\beta} + \omega^2 z\sin x^2 \sin\varphi\alpha - \omega^2 z\cos x^2 \sin\varphi\beta\big] \Big/ \sqrt{a_{11}},$$

$$a^2 = \left(-z\cos x^2\ddot{\alpha} - z\sin x^2\ddot{\beta} - 2\omega z\sin x^2\dot{\alpha} + 2\omega z\cos x^2\dot{\beta} + \omega^2 z\cos x^2\alpha + \omega^2 z\sin x^2\beta\right) \Big/ \sqrt{a_{22}},$$

$$a^3 = (z\sin x^2 \cos\varphi + r\sin x^2 \sin\varphi)\ddot{\alpha} + (-z\cos x^2 \cos\varphi - r\cos x^2 \sin\varphi)\ddot{\beta} - 2\omega z\cos x^2 \cos\varphi\dot{\alpha}$$

$$-2\omega z\sin x^2 \cos\varphi\dot{\beta} - \omega^2 z\sin x^2 \cos\varphi\alpha + \omega^2 z\cos x^2 \cos\varphi\beta \tag{15}$$

and sum them with the corresponding contravariant components of the accelerations of the elastic shell undergoing compound rotation [1, 2, 6–12].

To examine the critical states of simple rotation and the precession resonances of compound rotation, we linearize Eqs. (14) about the state of simple rotation with angular velocity ω, assuming that the shell is prestressed by membrane forces T^{11} and T^{22}. The radius *r*, the angle φ, and the parameters of the second quadratic form b_{ii} increase by Δ*r*, $\Delta\varphi = \Delta\vartheta_1$, and Δb_{ii}, respectively, and the strains ε_{ij} are defined by nonlinear relations [1, 2, 6–12].

We finally obtain a linearized system of differential equations of dynamic equilibrium for forces:

$$\partial\Delta T^{11}/\partial x^1 + \partial\Delta T^{12}/\partial x^2 + (2\Gamma^1_{11} + \Gamma^2_{21})\Delta T^{11} + \Gamma^1_{22}\Delta T^{22} - b^1_1\Delta T^{13}$$

$$-\gamma h\left[-\omega^2 \sin\varphi\Delta r/\sqrt{a_{11}} - \omega^2 r\cos\varphi\Delta\vartheta^*_1/\sqrt{a_{11}} - 2\omega\sin\varphi\Delta\dot{u}_2/\sqrt{a_{11}a_{22}} + \Delta\ddot{u}_1/a_{11}\right]$$

$$-\gamma h\big[(-z\sin x^2 \sin\varphi + r\sin x^2 \cos\varphi)\ddot{\alpha} + (z\cos x^2 \sin\varphi - r\cos x^2 \cos\varphi)\ddot{\beta} + 2\omega z\cos x^2 \sin\varphi\dot{\alpha}$$

$$+2\omega z\sin x^2 \sin\varphi]\dot{\beta} + \omega^2 \sin x^2 (z\sin\varphi + r\cos\varphi)\alpha - \omega^2 \cos x^2 (z\sin\varphi + r\cos\varphi)\beta\big] \Big/ \sqrt{a_{11}}$$

$$= 2\gamma h\omega_0\omega r\sin(\omega t + x^2)\cos\varphi \Big/ \sqrt{a_{11}},$$

$$\partial\Delta T^{12}/\partial x^1 + \partial\Delta T^{22}/\partial x^2 + (3\Gamma^2_{12} + \Gamma^1_{11})\Delta T^{12} - b^2_2\Delta T^{23}$$

$$-\gamma h\left[-\omega^2 r\cos\varphi\Delta\vartheta^*_2/\sqrt{a_{22}} + 2\omega\sin\varphi\Delta\dot{u}_1/\sqrt{a_{11}a_{22}} - 2\omega\cos\varphi\Delta\dot{u}_3/\sqrt{a_{22}} + \Delta\ddot{u}_2/a_{22} - \omega^2\Delta u_2/a_{22}\right]$$

$$-\gamma h\left[-z\cos x^2\ddot{\alpha}-z\sin x^2\ddot{\beta}-2\omega z\sin x^2\dot{\alpha}+2\omega z\cos x^2\dot{\beta}+\omega^2 z\cos x^2\alpha+\omega^2 z\sin x^2\beta\right]\Big/\sqrt{a_{22}}=0,$$

$$\partial\Delta T^{13}/\partial x^1+\partial\Delta T^{23}/\partial x^2+(\Gamma_{12}^2+\Gamma_{11}^1)\Delta T^{13}+b_{11}\Delta T^{11}+\Delta b_{11}T^{11}+b_{22}\Delta T^{22}+\Delta b_{22}T^{22}$$

$$-\gamma h\left[\omega^2\cos\varphi\Delta r-\omega^2 r\sin\varphi\Delta\vartheta_1^*+2\omega\cos\varphi\Delta\dot{u}_2/\sqrt{a_{22}}+\Delta\ddot{u}_3\right]$$

$$-\gamma h\left[(z\sin x^2\cos\varphi+r\sin x^2\sin\varphi)\ddot{\alpha}+(-z\cos x^2\cos\varphi-r\cos x^2\sin\varphi)\ddot{\beta}\right.$$

$$-2\omega z\cos x^2\cos\varphi\dot{\alpha}-2\omega z\sin x^2\cos\varphi\dot{\beta}-\omega^2\sin x^2(z\cos\varphi-r\sin\varphi)\alpha$$

$$\left.+\omega^2\cos x^2(z\cos\varphi-r\sin\varphi)\beta\right]=2\gamma h\omega_0\omega r\sin\varphi\sin(\omega t+x^2),\quad(16)$$

where Γ_{jk}^i are the Christoffel symbols, and Δ denotes an increment.

The system of equations (16) is supplemented with the linearized equations for moments following from (14). What distinguishes this system of equations is that its unknown functions are Δu_1, Δu_2, and Δu_3 dependent on x^1, x^2, and t and are α and β dependent on t. Therefore, the governing equations of shell theory should be supplemented with conditions at the end $x^1=0$:

$$\vec{M}_{\text{sh}}^{\text{el}}+\vec{M}^{\text{el}}=0\quad\text{or}\quad\vec{M}_{\text{sh}}^{\text{el}}-k\left(\alpha\vec{i}+\beta\vec{j}\right)=0,\quad(17)$$

where $\vec{M}_{\text{sh}}^{\text{el}}$ is the resultant moment of internal elastic forces at the end $x^1=0$.

The governing system of equations is a hybrid one because it includes both partial and ordinary derivatives with respect to the unknown variables. This circumstance imposes some restrictions on the solution technique.

3. Technique. For analyzing the stability of equilibrium of a rotating elastic shell, we suppose that its bifurcation buckling occurs at $\alpha\neq 0$ and $\beta=0$ in the first harmonic of the coordinate x^2 for the distributed unknown variables. Then, assuming that the odd and even functions describing the strain state of the shell are proportional to $\sin x^2$ and $\cos x^2$, respectively, we reduce the governing system of equations to the form

$$\frac{d\vec{y}}{dx}=A(x,\omega)\vec{y}+\vec{g}(x,\omega)\alpha,\quad\vec{b}(\omega)\vec{y}(0)-k\alpha=0,\quad(18)$$

where $x\equiv x^1$ is an independent variable, $0\le x\le L$; $\vec{y}(x)$ is an eight-component vector of unknown variables; $A(x,\omega)$ is an 8×8 matrix of coefficients; $\vec{g}(x,\omega)$ is a known vector function; and $\vec{b}(\omega)$ is a known vector.

We add the boundary equations

$$D\vec{y}(0)=0,\quad F\vec{y}(L)=0,\quad(19)$$

where D and F are constant 4×8 matrices.

We seek a solution of the system of ordinary differential equations (18) with an unknown parameter α in the form $\vec{y}(x)=C_i\cdot\vec{y}_i(x)+\vec{y}_\alpha(x)\cdot\alpha$. The partial solutions $\vec{y}_i$ and $\vec{y}_\alpha$ are found by the Runge–Kutta method, and the constants C_i and α are calculated from the system of equations (19) and the second equation of (18). The states in which the determinant of the matrix of coefficients vanishes are bifurcational.

If the subject of study is the natural vibrations of a rotating shell, then the odd and even unknown functions will have the multipliers $\sin(x^2+ct)$ and $\cos(x^2+ct)$, respectively. In this case, it is necessary to solve the eigenvalue problem for the homogeneous system of equations

$$\frac{d\vec{y}}{dx}=A(x,\omega)\vec{y}+c^2G(x,\omega)\vec{y}+cH(x,\omega)\vec{y}+\vec{g}(x,\omega)\alpha,\quad\vec{b}(\omega)\vec{y}(0)-k\alpha=0.$$

TABLE 1

$k, \frac{\text{N}\cdot\text{m}}{\text{rad}}$	Rigid structures				Elastic structures			
	Bar Thin-walled pipe				Beam Shell			
	l = 0.1 m h = 1 mm	l = 0.25 m h = 0.5 mm	l = 0.25 m h = 1 mm	l = 1 m h = 1 mm	l = 0.1 m h = 1 mm	l = 0.25 m h = 0.5 mm	l = 0.25 m h = 1 mm	l = 1 m h = 1 mm
1	49.483 51.979	17.703 17.838	12.518 12.613	1.565 1.566	49.483 51.998	17.703 17.845	12.518 12.634	1.565 1.605
10	156.48 164.37	55.983 56.408	39.586 39.886	4.958 4.951	156.48 164.43	55.980 56.428	39.585 39.953	4.948 5.076
10^3	1564.78 1643.73	559.83 564.08	395.86 398.86	49.483 49.506	1662.99 1640.74	556.66 560.72	394.74 398.27	48.920 50.205
10^5	15647.8 16437.3	5598.33 5640.79	3958.62 3988.64	494.83 495.06	14110.2 13710.9	3808.17 3718.56	3151.78 3112.43	241.01 278.85

When the elastic shell undergoes compound rotation, $\sin(x^2+\omega t)$ and $\cos(x^2+\omega t)$ are used as basis functions and the equations describing forced vibrations constitute an inhomogeneous system of equations:

$$\frac{d\vec{y}}{dx}=A(x,\omega)\vec{y}+\vec{g}(x,\omega)\alpha+\vec{f}(x)\omega\omega_0, \quad \vec{b}(\omega)\vec{y}(0)-k\alpha=0.$$

Given ω and ω_0, it is solved by the method of initial parameters, and the partial solutions are found by the Runge–Kutta method.

4. Results. To establish the conditions for critical states to occur in elastic cylindrical shells elastically coupled with a rotating platform, we analyzed for stability of perfectly rigid and elastic structures with equivalent geometrical and inertial parameters. In these two cases, the shell was modeled by a thin-walled cylindrical pipe and a bar with appropriate length and mass per unit length. The results of the analysis are summarized in Table 1. The critical angular velocities for the rigid thin-walled pipes have been determined by formula (6). The moments I_x and I_z have been calculated for pipe lengths from 0.1 m to 1 m, mid-surface radius r = 0.025 m, wall thicknesses h = 0.0005 m and h = 0.001 m, and stiffnesses k from 1 to 10^5 (N·m/rad). For the equivalent rigid bar, we have $I_z=0$; therefore, $\omega_{\text{cr}}=\sqrt{k/I_x}$, where $I_x=M\cdot l^2/3$, M is the mass of the bar.

Each cell of the table includes two values of ω_{cr}: for the bar/beam (upper) and pipe/shell (lower). In both cases: $E=2.1\cdot10^{11}$ Pa, ν = 0.3, $\gamma=7.8\cdot10^3$ kg/m^3.

Analyzing the data in the table, we conclude that the difference between the values of ω_{cr} for the rigid and elastic structures is negligible when k is low ($k\le1\cdot10^3$ N·m/rad). This indicates that with low k all the structures behave as equivalent rigid bodies. For $k=1\cdot10^5$ N·m/rad, however, this difference becomes significant, with ω_{cr} being less for the elastic structures apparently because of their additional elastic compliance.

Note that with low k, the value of ω_{cr} for the elastic shells is somewhat greater than the value of ω_{cr} for the elastic beams. For $k=1\cdot10^5$ N·m/rad, the situation is opposite.

We also examined how the stiffness of the elastic coupling between the shell and the platform affects the natural frequencies. Figure 2 shows the multiple natural frequencies $c_1^{\pm}$ and $c_2^{\pm}$ as functions of the angular velocity ω for a shell with l = 1 m, r = 0.25 m, and h = 0.02 m rigidly fixed to the rotating platform. It is seen that the curves $c_1^{+}(\omega)$ and $c_2^{+}(\omega)$ smoothly join, crossing neither the bisector of the right quadrant nor the ordinate axis. This is indicative of no critical states at these frequencies,

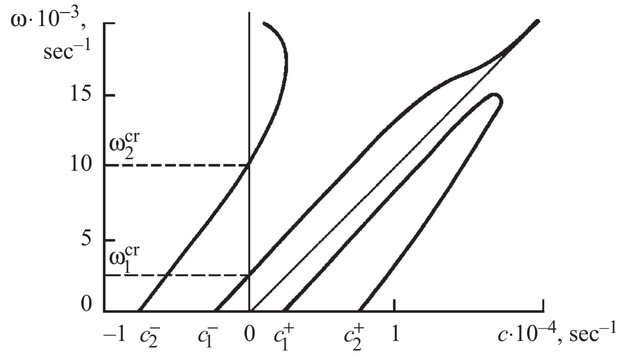

Fig. 2

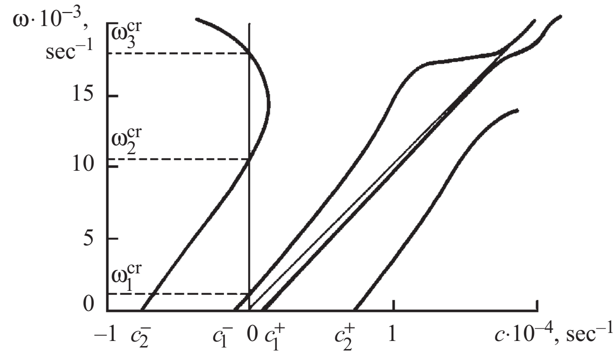

Fig. 3

whether the rotation is simple or compound [1, 2, 6–8]. The curves $c_1^-(\omega)$ and $c_2^-(\omega)$ cross the ordinate axis at $\omega_1^{cr} = 2487.1$ Hz and $\omega_2^{cr} = 10154.0$ Hz, which are angular velocities of simple rotation that cause the shell to lose stability.

Figure 3 shows the same curves for the shell connected by an elastic coupling with $k = 1 \cdot 10^8$ N·m/rad to the platform. It is seen that the curves $c_1^+(\omega)$ and $c_2^+(\omega)$ do not join, and the curves $c_1^-(\omega)$ and $c_2^-(\omega)$ cross the ordinate axis at smaller values of ω. This indicates that reducing the stiffness of the elastic coupling causes the first critical angular velocity to decrease and produces additional critical velocities ω_3^{cr}.

REFERENCES

1. M. A. Belova, V. I. Gulyaev, and I. L. Solov'ev, "Static and dynamic critical states of parabolic shells undergoing simple and compound rotation," *Izv. RAN*, *Mekh. Tverd. Tela*, No. 3, 152–163 (2004).
2. V. I. Gulyaev, P. Z. Lugovoi, and I. L. Solov'ev, "Elastic vibrations of a single-support thin-walled rotor (compound shell) during complex rotation," *Int. Appl. Mech*., **39**, No. 8, 969–975 (2003).
3. A. I. Lurie, *Analytical Mechanics*, Springer, Berlin (2001).
4. D. R. Merkin, *An Introduction to the Theory of Stability of Motion* [in Russian], Nauka, Moscow (1976).
5. H. Ziegler, *Principles of Structural Stability*, Blaisdell Publ. Comp., Waltham, MA (1968).
6. V. I. Gulyaev and S. N. Khudolii, "Vibrations of curved and twisted blades during complex rotation," *Int. Appl. Mech*., **41**, No. 4, 449–454 (2005).
7. V. I. Gulyayev and M. Nabil, "Resonant interaction of a beam and an elastic foundation during the motion of a periodic system of concentrated loads," *Int. Appl. Mech*., **41**, No. 5, 560–565 (2005).
8. V. I. Gulyayev, I. L. Solovjov, and M. A. Belova, "Critical states of thin ellipsoidal shells in simple and compound rotations," *J. Sound Vibr*., **270**, 323–339 (2004).
9. V. I. Gulyayev, I. L. Solovjov, and M. A. Belova, "Interconnection of critical states of parabolic shells in simple and compound rotations with values of their natural precession vibration frequencies," *Int. J. Solids Struct*., **41**, 3565–3583 (2004).
10. V. I. Gulyayev, I. L. Solovjov, and P. Z. Lugovoy, "Analysis of precession vibrations of thin-wall elastic shells in compound rotation," *J. Sound Vibr*., **246**, No. 3, 491–504 (2001).
11. V. I. Gulyaev, I. L. Solov'ev, and M. A. Belova, "Bifurcations of a single-support elastic thin-walled rotor," *Int. Appl. Mech*., **41**, No. 3, 330–335 (2005).
12. P. Z. Lugovoi, V. F. Meish, and S. E. Shtantsel', "Forced nonstationary vibrations of a sandwich cylindrical shell with cross-ribbed core," *Int. Appl. Mech*., **41**, No. 2, 161–167 (2005).