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# DEFORMATION OF A LAMINATED COMPOSITE WITH A PHYSICALLY NONLINEAR REINFORCEMENT AND MICRODAMAGEABLE MATRIX

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**The structural theory of short-term microdamage is generalized to a laminated composite with a microdamageable matrix and physically nonlinear reinforcement. The basis for the generalization is the stochastic elasticity equations of a laminated composite with a porous matrix. Microvolumes in the matrix material meet the Huber–Mises failure criterion. The damaged-microvolume balance equation for the matrix is derived. This equation and the equations relating macrostresses and macrostrains of a laminated composite with porous matrix and physically nonlinear reinforcement constitute a closed-form system of equations. This system describes the coupled processes of physically nonlinear deformation and microdamage occurring in different composite components. Algorithms for computing the microdamage–macrostrain relationships and deformation diagrams are developed. Uniaxial tension curves are plotted for a laminated composite with linearly hardening reinforcement**

**Keywords:** laminated composite, microdamageable matrix, physically nonlinear reinforcement, coupled process of physically nonlinear deformation and microdamage

The structural model of damage [6, 8–21] is one of the new trends in damage theory. The model is based on the stochastic equations of the mechanics of microinhomogeneous bodies. Dispersed microdamages are modeled by a system of quasispherical micropores [6], and the accumulation of microdamages is modeled by increased porosity. A microvolume is damaged, producing a micropore, according to the Huber–Mises or Schleicher–Nadai failure criterion, where the ultimate strength is a random function of coordinates. The effective properties and stress–strain state of a macrovolume of a porous composite material are determined from the stochastic differential equations of microinhomogeneous elastic medium. The general properties of the distribution function for the statistically homogeneous random field of ultimate microstrength are used to derive the equation of balance of damaged microvolumes or porosity. This equation relates porosity and microstress invariants. The simultaneous equations relating the effective elastic constants and porosity and the porosity balance equation describe the coupled processes of deformation and microdamage and lead to nonlinear deformation.

We will study the deformation of a two-component laminated composite with physically nonlinear reinforcement and microdamageable matrix. To this end, we will use the stochastic elasticity equations for a laminated composite with a porous matrix and nonlinearly elastic reinforcement. The damage of microvolumes is described by the Huber–Mises failure criterion, where the ultimate strength is a random function of coordinates with power or Weibull distribution. The deformation of the laminated material will be described and its effective deformation properties will be determined using the stochastic elasticity equations for a laminated composite with a porous matrix and physically nonlinear reinforcement. Using the properties of the distribution function of the statistically homogeneous random field of ultimate microstrength, we will derive a balance equation for damaged microvolumes, which will be nonlinear with respect to porosity. This equation and the equations relating macrostresses and macrostrains in a laminated composite with physically nonlinear reinforcement and microdamaged matrix constitute a closed-form system of equations. This system describes the coupled processes of physically nonlinear deformation and microdamage in different components of the composite. We will propose algorithms for computing the

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microdamage–macrostrain relationships and deformation curves. Also we will plot uniaxial tension curves for a laminated composite with a linearly hardening reinforcement.

**1.** Let us consider a two-component laminated composite with physically nonlinear reinforcement and matrix acquiring microdamages during loading. The microdamages are simulated by randomly dispersed quasispherical micropores appearing in microvolumes where the stresses exceed the ultimate microstrength. Let the matrix have porosity $p_2$. The physically nonlinear deformation of the reinforcement is described by the dependence of its elastic moduli on strains. Denote the bulk and shear moduli of the reinforcement and matrix by $K_1, \mu_1$, and $K_2, \mu_2$, respectively, and the volume fractions of the reinforcement and porous matrix by $c_1$ and $c_2$, respectively. The moduli $K_1$ and $\mu_1$ depend on strains, and the moduli $K_{2p}$ and $\mu_{2p}$ are defined in terms of $K_2$ and $\mu_2$ [6, 7],

$$K_{2p} = \frac{4K_2\mu_2(1-p_2)^2}{3K_2 p_2 + 4\mu_2(1-p_2)}, \quad \mu_{2p} = \frac{(9K_2 + 8\mu_2)\mu_2(1-p_2)^2}{3K_2(3-p_2) + 4\mu_2(2+p_2)}, \quad \lambda_{2p} = K_{2p} - \frac{2}{3}\mu_{2p}, \tag{1.1}$$

and depend on the porosity $p_2$. Then the macrostresses $\langle\sigma_{ij}\rangle$ and macrostrains $\langle\varepsilon_{ij}\rangle$ are related by

$$\langle\sigma_{jk}\rangle = \left(\lambda_{11}^* - \lambda_{12}^*\right)\langle\varepsilon_{jk}\rangle + \left(\lambda_{12}^*\langle\varepsilon_{rr}\rangle + \lambda_{13}^*\langle\varepsilon_{33}\rangle\right)\delta_{jk},$$

$$\langle\sigma_{33}\rangle = \lambda_{13}^*\langle\varepsilon_{rr}\rangle + \lambda_{33}^*\langle\varepsilon_{33}\rangle, \quad \langle\sigma_{j3}\rangle = 2\lambda_{44}^*\langle\varepsilon_{j3}\rangle \quad (j,k=1,2), \tag{1.2}$$

and the effective elastic moduli $\lambda_{11}^*, \lambda_{12}^*, \lambda_{13}^*, \lambda_{33}^*$, and $\lambda_{44}^*$ are functions of the porosity $p_2$ and macrostrains $\langle\varepsilon_{jk}\rangle$.

The effective elastic moduli of a laminated composite with physically nonlinear reinforcement and porous matrix are determined by the following iterative algorithm. In the *n*th-order approximation, the effective moduli $\lambda_{11}^{*(n)}, \lambda_{12}^{*(n)}, \lambda_{13}^{*(n)}, \lambda_{33}^{*(n)}$, and $\lambda_{44}^{*(n)}$ of the composite are determined [4, 5, 7] in terms of the corresponding *n*th-order moduli of the reinforcement, $\lambda_1^{(n)} = \lambda_1(\langle\varepsilon_{jk}^1\rangle^{(n)})$ and $\mu_1^{(n)} = \mu_1(\langle\varepsilon_{jk}^1\rangle^{(n)})$ $(\lambda_1^{(n)} = K_1^{(n)}(-2/3)\mu_1^{(n)})$, and of the matrix, $\lambda_{2p}$ and $\mu_{2p}$:

$$\lambda_{11}^{*(n)} = \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^{-1} \left\langle \frac{\lambda_p^{(n)}}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^2 + \left\langle \frac{\mu_p^{(n)}(\lambda_p^{(n)} + \mu_p^{(n)})}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle,$$

$$\lambda_{12}^{*(n)} = \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^{-1} \left\langle \frac{\lambda_p^{(n)}}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^2 + 2\left\langle \frac{\lambda_p^{(n)}\mu_p^{(n)}}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle, \quad \lambda_{44}^{*(n)} = \left\langle \frac{1}{\mu_p^{(n)}} \right\rangle^{-1},$$

$$\lambda_{13}^{*(n)} = \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^{-1} \left\langle \frac{\lambda_p^{(n)}}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle, \quad \lambda_{33}^{*(n)} = \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^{-1}. \tag{1.3}$$

Here $\lambda_p^{(n)}$ and $\mu_p^{(n)}$ take the values $\lambda_1^{(n)}, \mu_1^{(n)}$, and $\lambda_{2p}, \mu_{2p}$ in the nonlinear reinforcement and porous matrix, respectively; and an arbitrary function $\varphi$ is defined by

$$\langle\varphi_p^{(n)}\rangle = c_1\varphi_1\left(\langle\varepsilon_{jk}^1\rangle^{(n)}\right) + c_2\varphi_{2p}, \tag{1.4}$$

where $\langle\varepsilon_{jk}^1\rangle^{(n)}$ are the mean *n*th-order strains in the reinforcement. They are determined in terms of the macrostrains $\langle\varepsilon_{jk}\rangle$ by the formulas

$$\langle\varepsilon_{jk}^1\rangle^{(n)} = \langle\varepsilon_{jk}\rangle, \quad \langle\varepsilon_{j3}^1\rangle^{(n)} = \frac{1}{\mu_1^{(n)}} \left\langle \frac{1}{\mu_p^{(n)}} \right\rangle^{-1} \langle\varepsilon_{j3}\rangle,$$

$$\langle \varepsilon_{33}^{1} \rangle^{(n)} = \frac{1}{\lambda_1^{(n)} + 2\mu_1^{(n)}} \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle^{-1} \left[ \left\langle \frac{\lambda_p^{(n)}}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle - \lambda_1^{(n)} \left\langle \frac{1}{\lambda_p^{(n)} + 2\mu_p^{(n)}} \right\rangle \right] \langle \varepsilon_{rr} \rangle + \langle \varepsilon_{33} \rangle . \tag{1.5}$$

In the zero-order approximation, the reinforcement is considered linearly elastic:

$$\lambda_1 \left( \langle \varepsilon_{jk}^{1} \rangle^{(0)} \right) = \lambda_1(0), \qquad \mu_1 \left( \langle \varepsilon_{jk}^{1} \rangle^{(0)} \right) = \mu_1(0). \tag{1.6}$$

Given the macrostrains $\langle \varepsilon_{jk} \rangle$, the effective moduli are evaluated as the following limits:

$$\lambda_{lm}^{*} = \lim_{n \to \infty} \lambda_{lm}^{*(n)} . \tag{1.7}$$

The Huber–Mises failure criterion defines a condition for the formation of a single microdamage in a microvolume of the undamaged portion of the matrix:

$$I_{\sigma}^{12} = k_2 , \tag{1.8}$$

where $I_{\sigma}^{12} = (\langle \sigma_{jk}^{12} \rangle' \langle \sigma_{jk}^{12} \rangle')^{1/2}$ is the second invariant of the mean stress deviator $\langle \sigma_{jk}^{12} \rangle'$ for the undamaged portion of the matrix, and $k_2$ is the ultimate microstrength, which is a random function of coordinates.

The one-point distribution function $F_2(k_2)$ may have the form of a power law on some interval

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{20} , \\ \left( \dfrac{k_2 - k_{20}}{k_{12} - k_{20}} \right)^{n_2} , & k_{20} \le k_2 \le k_{21} , \\ 1, & k_2 > k_{21} \end{cases} \tag{1.9}$$

or an exponential–power law (Weibull distribution)

$$F_2(k_2) = \begin{cases} 0, & k_2 < k_{20} , \\ 1 - \exp\left[ -m_2 (k_2 - k_{20})^{n_2} \right], & k_2 \ge k_{20} , \end{cases} \tag{1.10}$$

where $k_{20}$ is the minimum value of the ultimate microstrength of the matrix; and $k_{21}$, $m_2$, and $n_2$ are deterministic constants describing a specific distribution function, which are determined by approximating experimental microstrength spread curves or deformation curves.

The random field of the ultimate microstrength $k_2$ of the matrix is statistically homogeneous for real materials. Its scale of correlation and the dimensions of single microdamages and distances between them are assumed negligible compared with a macrovolume of the material. Then the random field $k_2$ and the distribution of microstresses in the matrix under homogeneous loading satisfy the property of ergodicity, and the distribution function $F_2(k_2)$ determines the relative fraction of the undamaged material in the matrix in which the ultimate strength is less than $k_2$. Therefore, if $\langle \sigma_{jk}^{12} \rangle \neq 0$, then the function $F_2(I_{\sigma}^{12})$, according to (1.8)–(1.10), determines the relative fraction of damaged microvolumes in the matrix. Since damaged microvolumes are modeled by pores, we can write the balance equation for the damaged microvolumes in the matrix or for its porosity:

$$p_2 = p_{20} + (1 - p_{20}) F_2(I_{\sigma}^{12}), \tag{1.11}$$

where the mean stresses $\langle \sigma_{jk}^{12} \rangle$ are related to the macrostrains $\langle \varepsilon_{jk} \rangle$ by the following formulas [7]:

$$\langle \sigma_{jk}^{12} \rangle = \frac{1}{1 - p_2} \left[ 2\mu_{2p} \langle \varepsilon_{jk} \rangle + \frac{\lambda_{2p}}{\lambda_{2p} + 2\mu_{2p}} \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left( \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle + 2\mu_{2p} \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle \right) \langle \varepsilon_{rr} \rangle \delta_{jk} \right.$$

$$+\frac{\lambda_{2p}}{\lambda_{2p}+2\mu_{2p}}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}\langle\varepsilon_{33}\rangle\delta_{jk}\Bigg],$$

$$\langle\sigma_{33}^{12}\rangle=\frac{1}{1-p_2}\left[\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}\left(\left\langle\frac{\lambda_p}{\lambda_p+2\mu_p}\right\rangle\langle\varepsilon_{rr}\rangle+\langle\varepsilon_{33}\rangle\right)\right],$$

$$\langle\sigma_{j3}^{12}\rangle=\frac{2}{1-p_2}\left\langle\frac{1}{\mu_p}\right\rangle^{-1}\langle\varepsilon_{j3}\rangle \qquad (j,k,r=1,2) \tag{1.12}$$

and effective moduli $\lambda_{2p}$ and $\mu_{2p}$ of the matrix are defined by (1.1).

Equations (1.2), (1.11), and (1.12) constitute a closed-form system describing the coupled processes of statistically homogeneous, physically nonlinear deformation and damage of a two-component laminated composite. The physical nonlinearity of its reinforcement influences the formation of microdamages in (i.e., the porosity of) the matrix under loading, which is reflected on the deformation curve of the composite. Therefore, the resulting deformation curve of the laminated composite includes the physical nonlinearity of the reinforcement and the nonlinearity due to increasing porosity of the matrix.

The deformation of a laminated composite with physically nonlinear reinforcement and damaged matrix at given macrostrains are described by determining the macrostrain-dependent effective elastic moduli of the laminated material with porous matrix and physically nonlinear reinforcement by the iterative algorithm (1.1), (1.3)–(1.7) and determining the porosity of the matrix from Eqs. (1.11) and (1.12), also using a certain iterative method. Let us represent Eq. (1.11) for the $n$th step of the iterative process (1.1), (1.3)–(1.7) in the form

$$f_2^{(n)}\equiv p_2-p_{20}-(1-p_{20})\,F_2\left(I_\sigma^{12(n)}\right), \tag{1.13}$$

where

$$I_\sigma^{12(n)}=\left(\langle\sigma_{jk}^{12(n)}\rangle'\langle\sigma_{jk}^{12(n)}\rangle'\right)^{1/2},$$

$$\langle\sigma_{jk}^{12(n)}\rangle=\frac{1}{1-p_2}\left[2\mu_{2p}^{(n)}\langle\varepsilon_{jk}\rangle+\frac{\lambda_{2p}^{(n)}}{\lambda_{2p}^{(n)}+2\mu_{2p}^{(n)}}\left\langle\frac{1}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle^{-1}\left(\left\langle\frac{\lambda_p^{(n)}}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle+2\mu_{2p}^{(n)}\left\langle\frac{1}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle\right)\right.$$

$$\left.\times\langle\varepsilon_{rr}\rangle\delta_{jk}+\frac{\lambda_{2p}^{(n)}}{\lambda_{2p}^{(n)}+2\mu_{2p}^{(n)}}\left\langle\frac{1}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle^{-1}\langle\varepsilon_{33}\rangle\delta_{jk}\right],$$

$$\langle\sigma_{33}^{12(n)}\rangle=\frac{1}{1-p_2}\left[\left\langle\frac{1}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle^{-1}\left(\left\langle\frac{\lambda_p^{(n)}}{\lambda_p^{(n)}+2\mu_p^{(n)}}\right\rangle\langle\varepsilon_{rr}\rangle+\langle\varepsilon_{33}\rangle\right)\right],$$

$$\langle\sigma_{j3}^{12(n)}\rangle=\frac{2}{1-p_2}\left\langle\frac{1}{\mu_p^{(n)}}\right\rangle^{-1}\langle\varepsilon_{j3}\rangle \qquad (j,k,r=1,2). \tag{1.14}$$

Then the root $p_2$ of Eq. (1.13), (1.14) at the $m$th step of some iterative process can be found by the formula

$$p_2^{(m,n)}=A_2 f_2^{(n)}\left(p_2^{(m-1)}\right), \tag{1.15}$$

where $A_2$ is an operator acting on the function $f_2^{(n)}(p_2)$. Its expression will be given below.

The desired root is determined as follows:

$$p_2 = \lim_{\substack{m\to\infty \\ n\to\infty}} p_2^{(m,n)}. \tag{1.16}$$

Relations (1.1)–(1.7), (1.13)–(1.16) give a solution to the problem posed, i.e., they produce macrodeformation curves ($\langle\sigma_{jk}\rangle$ versus $\langle\varepsilon_{jk}\rangle$) and microdamage curves ($p_2$ versus $\langle\varepsilon_{jk}\rangle$) for the laminated composite under consideration.

**2.** As an example, let us analyze the coupled processes of nonlinear deformation and microdamage of a laminated composite with nonlinearly elastic reinforcement. Let the bulk strains of the reinforcement be linear, and the shear strains be described by a linear hardening curve, i.e., the following relations hold within a microvolume in the reinforcement:

$$\sigma_{rr}^1 = K_1 \varepsilon_{rr}^1, \quad \sigma_{ij}'^1 = 2\mu_1(S_1)\varepsilon_{ij}'^1. \tag{2.1}$$

Here the bulk modulus $K_1$ does not depend on strains, and the shear modulus $\mu_1(S_1)$ is described by the function

$$\mu_1(S_1) = \begin{cases} \mu_{10}, & T_1 \le T_{10}, \\ \mu_1' + \left(1 - \dfrac{\mu_1'}{\mu_{10}}\right)\dfrac{T_{10}}{2S_1}, & T_1 \ge T_{10}, \end{cases} \tag{2.2}$$

and

$$S_1 = (\varepsilon_{jk}'^1 \varepsilon_{jk}'^1)^{1/2}, \quad T_1 = (\sigma_{jk}'^1 \sigma_{jk}'^1)^{1/2}, \quad T_{10} = \sqrt{\frac{2}{3}}\sigma_{10}, \tag{2.3}$$

where $\varepsilon_{ij}'^1$ and $\sigma_{ij}'^1$ are the deviators of the strain and stress tensors, respectively, $\sigma_{10}$ is the coordinate-independent tensile elastic limit, and $\mu_{10}$ and $\mu_1'$ are the material constants of the reinforcement.

We will use the secant method [1] to find the root $p_2$ of Eq. (1.13), (1.14). Since the root $p_2$ falls into the interval $[p_{20}, 1]$, which follows from the inequalities

$$f_2^{(n)}(p_{20}) \le 0, \quad f_2^{(n)}(1) \ge 0, \tag{2.4}$$

the zero-order approximation of $p_2^{(0,n)}$ is determined, according to the secant method, from the formula

$$p_2^{(0,n)} = \frac{a_2^{(0)} f_2^{(n)}(b_2^{(0)}) - b_2^{(0)} f_2^{(n)}(a_2^{(0)})}{f_2^{(n)}(b_2^{(0)}) - f_2^{(n)}(a_2^{(0)})}, \tag{2.5}$$

where $a_2^{(0)} = p_{20}$ and $b_2^{(0)} = 1$.

The subsequent approximations are determined in the iterative process

$$p_2^{(m,n)} = A_2 f_2^{(n)}(p_2^{(m-1,n)}) \equiv \frac{a_2^{(m)} f_2^{(n)}(b_2^{(m)}) - b_2^{(m)} f_2^{(n)}(a_2^{(m)})}{f_2^{(n)}(b_2^{(m)}) - f_2^{(n)}(a_2^{(m)})},$$

$$a_2^{(m)} = a_2^{(m-1)}, \quad b_2^{(m)} = p_2^{(m-1,n)} \quad \text{for} \quad f_2^{(n)}(a_2^{(m-1)})\, f_2^{(n)}(p_2^{(m-1,n)}) \le 0,$$

$$a_2^{(m)} = p_2^{(m-1,n)}, \quad b_2^{(m)} = b_2^{(m-1)} \quad \text{for} \quad f_2^{(n)}(a_2^{(m-1)})\, f_2^{(n)}(p_2^{(m-1,n)}) \ge 0$$

$$(m = 1, 2, \ldots), \tag{2.6}$$

which proceeds until

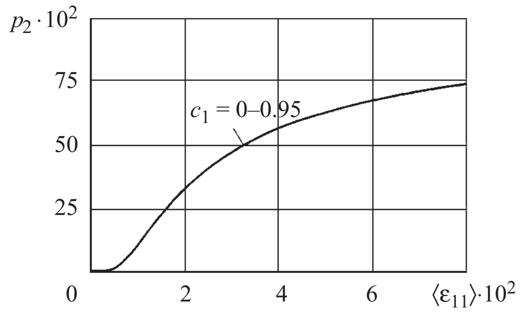


Fig. 1

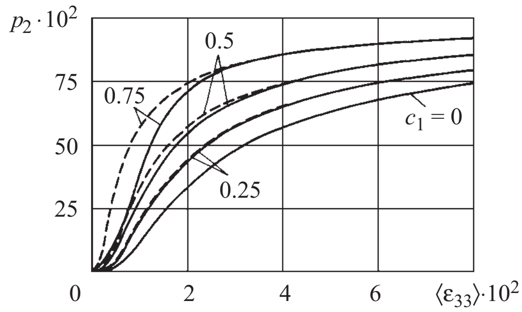


Fig. 2

$$\left|f_2^{(n)}\left(p_2^{(m,n)}\right)\right| < \delta, \tag{2.7}$$

where $\delta$ is the accuracy of computing the root.

Based on the theory stated above, we have studied the coupled processes of nonlinear deformation and microdamage of a laminated composite with Weibull-distributed microstrength in its matrix. The components of the composite are aluminoborosilicate-glass reinforcement described by the linear-hardening curve (2.1), (2.2) with the following parameters [3]:

$$K_1 = 38.89 \text{ GPa}, \quad \mu_1 = 29.17 \text{ GPa}, \quad \mu_1' = 0.334 \text{ GPa} \tag{2.8}$$

and linearly elastic epoxy matrix with the following characteristics [2, 3]:

$$K_2 = 3.33 \text{ GPa}, \quad \mu_2 = 1.11 \text{ GPa}. \tag{2.9}$$

The limits of proportionality of the reinforcement and the minimum tensile microstrength of the matrix $\sigma_{2p} = \sqrt{\frac{3}{2}} k_{20}$ have the following values:

$$\sigma_{10} = 0.01 \text{ GPa}, \quad \sigma_{2p} = 0.007 \text{ GPa}, \tag{2.10}$$

$$\sigma_{10} = 0.07 \text{ GPa}, \quad \sigma_{2p} = 0.007 \text{ GPa}. \tag{2.11}$$

When

$$\langle\varepsilon_{11}\rangle \neq 0, \quad \langle\sigma_{22}\rangle = \langle\sigma_{33}\rangle = 0 \tag{2.12}$$

according to (1.2), the macrostresses $\langle\sigma_{11}\rangle$ are related to the macrostrain $\langle\varepsilon_{11}\rangle$ by

$$\langle\sigma_{11}\rangle = \frac{\lambda_{11}^* - \lambda_{12}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2} \left[\left(\lambda_{11}^* + \lambda_{12}^*\right)\lambda_{33}^* - 2\left(\lambda_{13}^*\right)^2\right] \langle\varepsilon_{11}\rangle . \tag{2.13}$$

And

$$\langle\varepsilon_{22}\rangle = \frac{(\lambda_{13}^*)^2 - \lambda_{12}^*\lambda_{33}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2} \langle\varepsilon_{11}\rangle, \quad \langle\varepsilon_{33}\rangle = \frac{(\lambda_{12}^* - \lambda_{11}^*)\lambda_{13}^*}{\lambda_{11}^*\lambda_{33}^* - (\lambda_{13}^*)^2} \langle\varepsilon_{11}\rangle \tag{2.14}$$

in the porosity balance equation (1.11). When

$$\langle\varepsilon_{33}\rangle \neq 0, \quad \langle\sigma_{11}\rangle = \langle\sigma_{22}\rangle = 0 \tag{2.15}$$

according to (1.2), the macrostresses $\langle\sigma_{33}\rangle$ are related to the macrostrain $\langle\varepsilon_{33}\rangle$ by

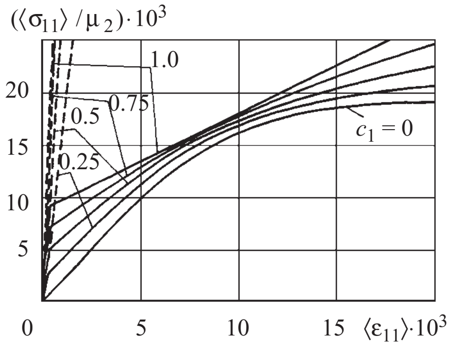


Fig. 3

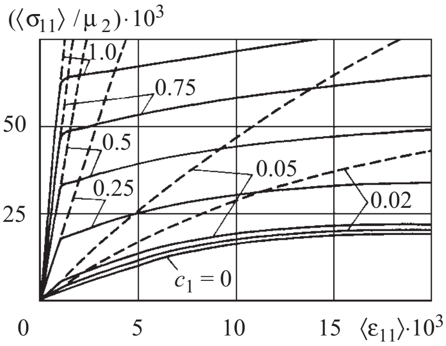


Fig. 4

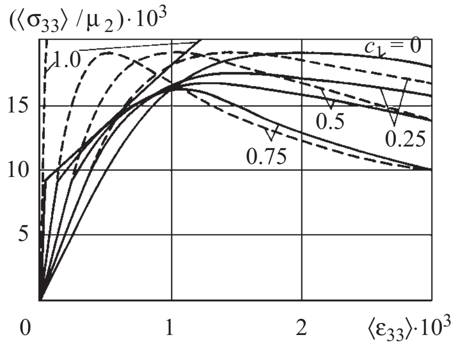


Fig. 5

$$\langle\sigma_{33}\rangle = \frac{1}{\lambda_{11}^* + \lambda_{12}^*}\left[\left(\lambda_{11}^* + \lambda_{12}^*\right)\lambda_{33}^* - 2\left(\lambda_{13}^*\right)^2\right]\langle\varepsilon_{33}\rangle. \tag{2.16}$$

And

$$\langle\varepsilon_{11}\rangle = \langle\varepsilon_{22}\rangle = -\frac{\lambda_{13}^*}{\lambda_{11}^* + \lambda_{12}^*}\langle\varepsilon_{11}\rangle \tag{2.17}$$

in the porosity balance equation (1.11).

Figures 1 and 2 show by solid lines the porosity $p_2$ of the matrix versus the macrostrains $\langle\varepsilon_{11}\rangle$ and $\langle\varepsilon_{33}\rangle$, respectively, for a laminated composite with a linearly hardening reinforcement with the limit of proportionality (2.10) and a matrix with the minimum microstrength (2.10). For reference, similar curves (dashed) for a laminated composite with a linear reinforcement have been plotted in the same figures. From Fig. 1 it is seen that the physical nonlinearity and volume fraction $c_1$ of the reinforcement do not affect the microdamage of the matrix. According to Fig. 2, the physical nonlinearity of the reinforcement has a strong effect on the microdamage of the matrix for all, especially large ($c_1 \geq 0.5$), values of $c_1 > 0$. With nonlinear reinforcement, microdamage is less intensive at the initial stage (for a fixed value of $\langle\varepsilon_{33}\rangle$, the porosity $p_2$ of the matrix is less).

Figures 3 and 4 show by solid lines the macrostress $\langle\sigma_{11}\rangle$ versus the macrostrain $\langle\varepsilon_{11}\rangle$ for (2.10) and (2.11), respectively. Figure 5 shows the macrostress $\langle\sigma_{33}\rangle$ versus the macrostrain $\langle\varepsilon_{33}\rangle$ for a laminated composite with a linearly hardening reinforcement with the limit of proportionality (2.10) and a matrix with the minimum microstrength (2.10). For reference, similar curves (dashed) for a laminated composite with a linear reinforcement have been plotted in the same figures. The figures demonstrate that the physical nonlinearity of the reinforcement has a significant effect on the deformation curves for all values of $c_1 > 0$. Comparing the results for linear and nonlinear reinforcements, we see that the level of macrostresses in the material with linearly hardening reinforcement is higher than in the material with linear reinforcement, especially for the dependence of $\langle\sigma_{11}\rangle$ on $\langle\varepsilon_{11}\rangle$.

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