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NONLINEAR ONE-DIMENSIONAL SEISMODYNAMIC MODEL OF A SOLID WITH SHOCK ABSORBING SUPPORTS

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UDC 531.38

A generalized method for constructing a nonlinear one-dimensional mathematical model of a solid on massive shock-absorbing supports of five different types is justified. The periods of small oscillations of the solid about the equilibrium position are determined. The equations of motion of the solid–support system with broken frictional bond are set up

Keywords: mathematical model, solid, seismic damper, period of oscillations, variable-structure support, frictional bond

Introduction. Seismic isolation of structures is an effective way of protection against earthquakes or mitigation of disastrous effects of earthquakes. A design model of a building with a seismic-isolation mechanism is a solid connected to a moving base (foundation) by compliant joints, which may be seismic dampers. This allows us to analyze the inertial loads on a structure with seismic dampers within the framework of solid dynamics. For low structures, seismic-isolation mechanisms based on pendulum suspensions, roller, tiltable, or fixed seismic dampers may be an effective means of seismic protection [1–9, 11–13]. The mechanisms with roller or tilting supports have not been studied adequately, since there is no limit to the number of their designs. How efficient a seismic-isolation mechanism is depends on the ratio between the natural periods of the structure as a solid and the predominant period of the ground during a seismic event. The more this ratio, the better the performance of the seismic-isolation mechanism.

The present paper proposes a generalized method for constructing a mathematical dynamic one-dimensional model of a solid on shock absorbing supports of five different types. We will determine the periods of small one-dimensional oscillations about the equilibrium position of the solid and set up the equations of motion of the solid–support system with broken frictional bonds.

1. The Method of Instantaneous Radius of Rotation for One-Dimensional Dynamics of a Solid with Seismic Dampers as Supports. There are elastic, sliding, roller, or tilting seismoisolating supports. Roller supports are shown in Figs. 1–3 and tilting supports in Figs. 4 and 5. We will call them supports *1–5*. Supports *1–3* and *5* are rolling systems [10]. All the supports are solids of revolution. Supports *1* and *2* are bodies defined by cutting a segment of thickness NN_1 out of a solid sphere; the equatorial plane of the segment is its plane of symmetry. Support *3* is a solid sphere, support *4* is a circular cylinder, and support *5* is a torus with diameters $2R$ and $2R + b$. These supports, which may be more than three in number, contact with a flat rigid foundation plate. Thus, the problem of determining the axial load on the supports is statically indeterminate and omitted here.

We will consider one-dimensional translational motion, as a special case of plane-parallel motion. Let us set up equations to describe such a motion.

Let a generalized coordinate representing the configuration of the mechanism in an arbitrary position be the angle between the vertical and the diameter connecting the points of contact with the solid and the foundation plate in the static equilibrium position. The differential equation of motion of a solid supported by seismic dampers is a Lagrange equation of the second kind:

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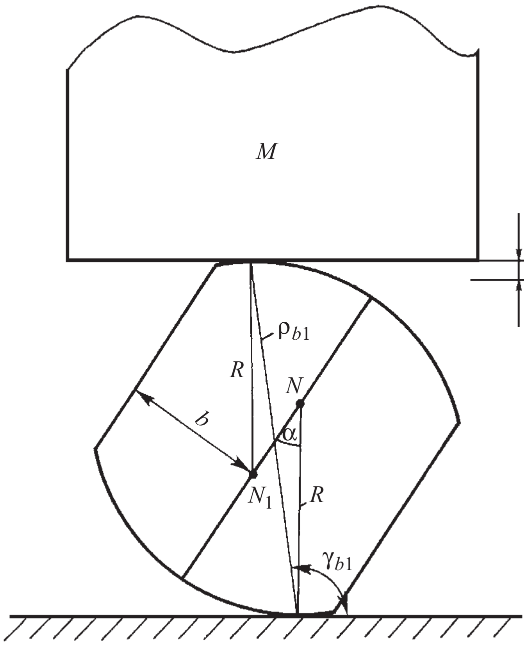

Fig. 1

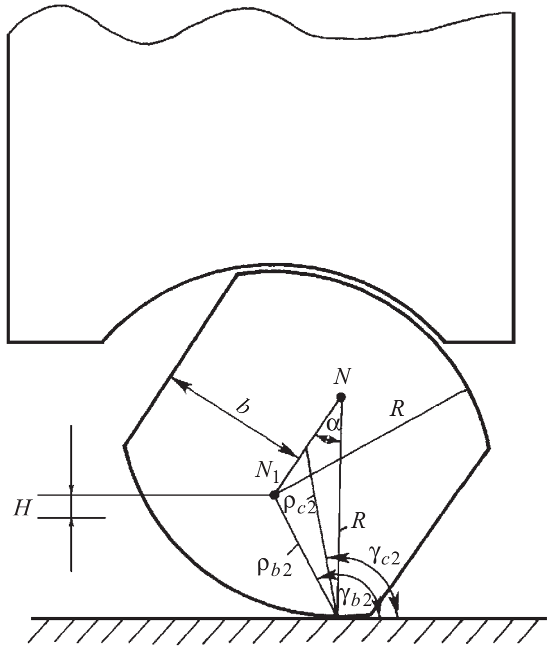

Fig. 2

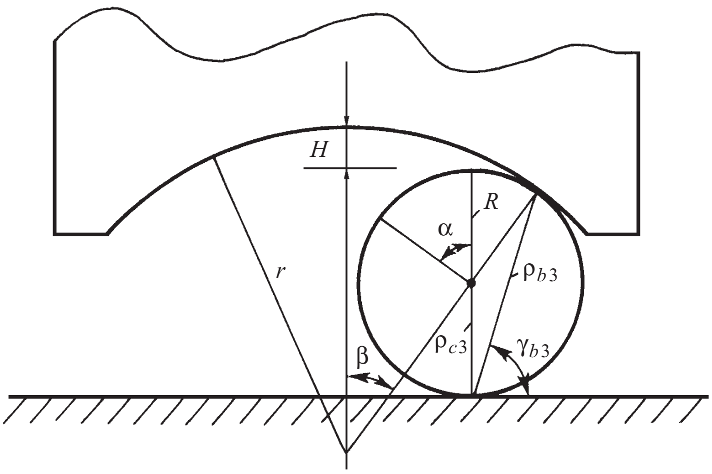

Fig. 3

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = -\frac{\partial \Pi}{\partial \alpha} + Q(t,\alpha) - D(\alpha,\dot{\alpha}), \tag{1}$$

where T and Π are the kinetic and potential energies of the system, respectively; and $Q(t,\alpha)$ and $D(\alpha,\dot{\alpha})$ are nonpotential generalized forces (inertial disturbing moments and moments of friction).

For all the supports, the kinetic energy of relative motion can be expressed as

$$T = \frac{1}{2}M\dot{\alpha}^2 \left(\rho_b^2(\alpha) + \mu\rho_s^2(\alpha)\right) = \frac{1}{2}M\dot{\alpha}^2\rho^2(\alpha,\mu), \tag{2}$$

where $\rho_b(\alpha)$ is the instantaneous kinematic radius of rotation that connects the point of rolling of the support over the plate with the solid's point that is either fixed or a unique point of contact with the support; $\rho_s(\alpha)$ is the instantaneous dynamic radius of

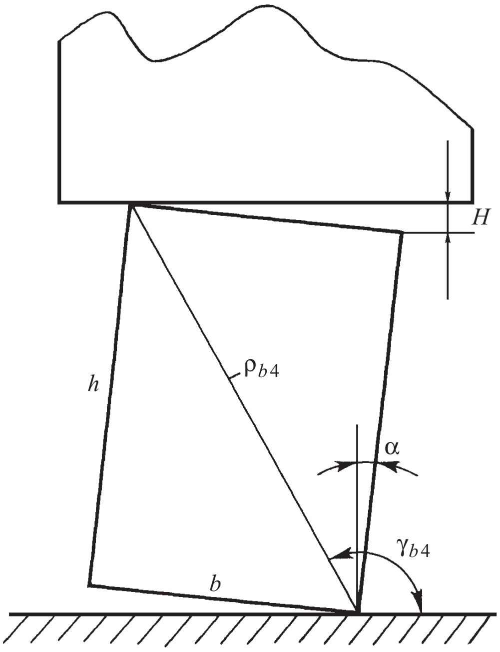

Fig. 4

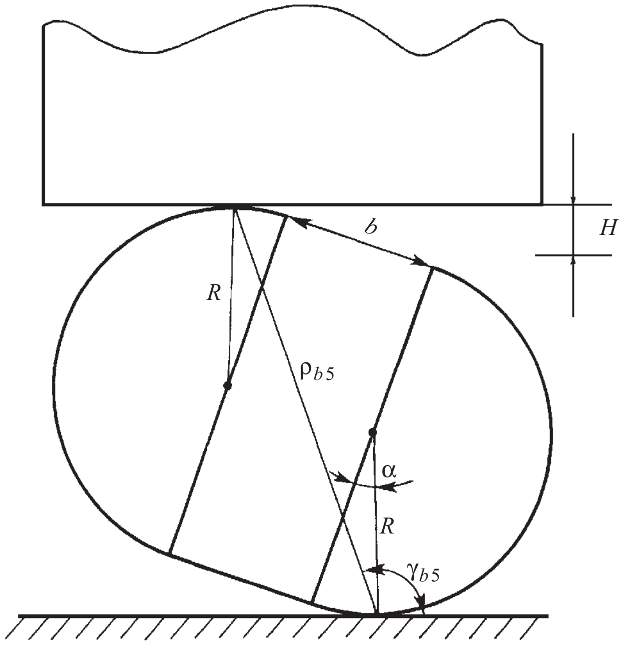

Fig. 5

gyration of the support about the axis running through the point of contact with the foundation plate; $\mu = m/M$, M is the mass of the solid per one support; and m is the mass of the support.

The potential energy of the solid and support is

$$\Pi = Mg(H_b(\alpha) + \mu H_s(\alpha)) \equiv MgH(\alpha, \mu), \tag{3}$$

where $H_b(\alpha)$ and $H_s(\alpha)$ are the lifting height of the solid and the height of the support's center of mass when the support axis is deflected from the vertical by an angle α.

According to Steiner's theorem, for the ith support we have $\rho_{si}^2(\alpha) = R_{ci}^2 + \rho_{ci}^2(\alpha)$, where R_{ci} = const is the central radius of gyration of the support about the axis lying in its equatorial plane; and $\rho_{ci}(\alpha)$ is the distance from the support's center of mass to the point of contact with the foundation plate. Table 1 presents the expressions of the functions $\rho_{bi}(\alpha)$, $\rho_{si}(\alpha)$, and $H_i(\alpha, \mu)$ for the five supports (r) shown in Figs. 1–5.

The functions $\rho_{bi}(\alpha)$ and $H_i(\alpha)$ must be even in α for all the supports. This is why the factor sign α appears in the expressions for $\rho_{bi}(\alpha)$ and $H_i(\alpha)$ for supports *4* and *5* in Table 1. In evaluating the partial derivatives $(\partial H)/(\partial \alpha)$ and $(\partial \rho)/(\partial \alpha)$ that include $\sin\alpha \cdot \operatorname{sign}\alpha$, we will neglect $\delta(\alpha)\cdot\sin\alpha$ (where $\delta(\alpha)$ is the Dirac distribution) as vanishing when integrated over an infinite interval:

$$\int_{-\infty}^{\infty} \delta(\alpha) \sin\alpha \, d\alpha = 0.$$

Let us now calculate the moments of inertial forces due to the translational horizontal motion of the foundation plate, regardless of the type of support:

$$Q(t, \alpha) = -M\ddot{\xi}(t)[\rho_b(\alpha)\sin\gamma_b(\alpha) + \mu\rho_c(\alpha)\sin\gamma_c(\alpha)], \tag{4}$$

where $\ddot{\xi}(t)$ is the translational horizontal acceleration of the plate; and $\gamma_b(\alpha)$ or $\gamma_c(\alpha)$ is the angle between the horizon and the instantaneous radius $\rho_b(\alpha)$ or $\rho_c(\alpha)$.

Using Figs. 1–5 and the formulas for the angles of the triangles with $\rho_{bi}(\alpha)$ and $\rho_{ci}(\alpha)$ as sides, we obtain

TABLE 1

N	$\rho_{bi}(\alpha)$	$\rho_{si}(\alpha)$	$H_i(\alpha,\mu)$
1	$R(4+\chi^2-4\chi\cos\alpha)^{1/2}$ $\chi=NN_1/R$	$\left(R_{c1}^2+1/4\rho_{b1}^2(\alpha)\right)^{1/2}$	$NN_1(1-\cos\alpha)(1+\mu/2)$
2	$R(1+\chi^2-2\chi\cos\alpha)^{1/2}$	$\left(R_{c1}^2+1/4\rho_{b1}^2(\alpha)\right)^{1/2}$	$NN_1(1-\cos\alpha)(1+\mu/2)$
3	$R(2+2\cos\lambda\alpha)^{1/2}$ $\lambda=R/(r-R)$	$R\sqrt{7/5}$	$(r-R)(1-\cos\lambda\alpha)$
4	$\left(h^2+b^2\right)^{1/2}$	$\left(5b^2/16+h^2/3\right)^{1/2}$	$\left[h(\cos\alpha-1)+b\sin\alpha\operatorname{sign}\alpha\right]\times$ $\times(1+\mu/2)$
5	$\dfrac{(2R+b\sin\alpha\operatorname{sign}\alpha)}{\cos\beta(\alpha)}$, $\beta(\alpha)=\beta_0\left(1-\dfrac{2}{\pi}\alpha\operatorname{sign}\alpha\right)$, $\beta_0=\arctan b/2R$	$\left(R_{c5}^2+1/4\rho_{b5}^2(\alpha)\right)^{1/2}$	$b\sin\alpha\operatorname{sign}\alpha\,(1+\mu/2)$

$$\gamma_{bi}(\alpha)=\frac{\pi}{2}+\beta_{bi}(\alpha),\qquad \gamma_{ci}(\alpha)=\frac{\pi}{2}+\beta_{ci}(\alpha)\quad (i=\overline{1,5}). \tag{5}$$

Table 2 presents the formulas for the angles $\beta_{bi}(\alpha)$ and $\beta_{ci}(\alpha)$ $(i=\overline{1,5})$.

For $i = 4, 5$, the formulas are valid when α is negative, because they include signature factors, which provide oddness in α. For supports 1, 4, and 5, we obviously have $\gamma_{bi}(\alpha)=\gamma_{ci}(\alpha)$.

The subscript i in the formulas of Table 1 and 2 will sometimes be omitted.

Let us now calculate the moment of dry friction. For the mechanisms with rolling (supports *1*, *3*, and *5*), this moment is equal to the sum of the moments of rolling friction at the points of contact between the solid and the support and between the support and the foundation plate:

$$D(\alpha,\dot\alpha)=Mgf_r+(M+m)gf_r=f_rM(2+\mu)g\operatorname{sign}\dot\alpha,$$

where f_r is the coefficient of rolling friction.

For support *2* with sliding friction in the joint and rolling friction on the plate, we have

$$D(\alpha,\dot\alpha)=[Mgf_sR+(m+M)gf_r]\operatorname{sign}\dot\alpha,$$

where f_s is the coefficient of sliding friction.

For support *4* with friction at the fixed points of contact between the solid and the support and between the support and the plate, which is torsional friction characterized by the coefficient f_c, we have

$$D(\alpha,\dot\alpha)=Mf_c\cdot(2+\mu)\,g\operatorname{sign}\dot\alpha.$$

In the case of supports *4* and *5*, which have area contact with the foundation and the solid, we deal with dry (Coulomb) friction. Assume that the coefficients of friction between the solid and the support and between the support and the plate are equal. If the coefficients of dry friction are greater than some critical value, then the angle α will change during the motion of the solid over the supports. Otherwise, the supports will not tilt. The critical value of the coefficient of dry sliding friction follows

TABLE 2

N	$\beta_{bi}(\alpha)$	$\beta_{ci}(\alpha)$	$v_i(\alpha^*)$
1	$\arcsin\left(\dfrac{NN_1}{\rho_{b1}(\alpha)}\sin\alpha\right)$	$\beta_{b1}(\alpha)$	$\sin\alpha^* - \dfrac{b}{R}$
2	$\arcsin\left(\dfrac{NN_1}{\rho_{b2}(\alpha)}\sin\alpha\right)$	$\beta_{b1}(\alpha)$	$\sin\alpha^* - \dfrac{b}{R}$
3	$-\dfrac{\alpha}{2}(1+\lambda)$	0	$\tan\dfrac{\lambda\alpha^*}{2} - f_s$
4	$\arctan\dfrac{b}{h}\operatorname{sign}\alpha - \alpha$	$\beta_{b4}(\alpha)$	$\tan\alpha^* - \dfrac{b}{h}$
5	$\pi\operatorname{sign}\alpha - \alpha -$ $-\arcsin\left(\dfrac{2R\sin^2\alpha}{\rho_{b5}(\alpha)\sin\alpha - b\operatorname{sign}\alpha}\right)$	$\beta_{b5}(\alpha)$	$\alpha^* - \dfrac{\pi}{2}$

from the condition that the torque due to the horizontal friction force is greater than the counterbalance moment due to the weight of the solid and support: $Mghf_s > M(1+\mu/2)gb$. From here

$$f_s > b(1+\mu/2)/h = f_{\text{cr}}.$$

Obviously, for supports *4* and *5*, the probability of oscillation (sliding) onset increases if $bh^{-1}, b(2R)^{-1} << 1$ (>> 1).

Thus, if $f_s > f_{\text{cr}}$, then, according to (1)–(3), we have the following equations for all the five supports:

$$\ddot\alpha + \dot\alpha^2 \frac{\partial}{\partial\alpha}\ln\rho(\alpha,\mu) + \frac{g}{\rho^2(\alpha,\mu)}\frac{\partial H(\alpha,\mu)}{\partial\alpha}$$

$$= -\frac{\ddot\xi(t)}{\rho(\alpha,\mu)}\big(u(\alpha)\sin\gamma_b(\alpha) + \mu w(\alpha)\sin\gamma_c(\alpha)\big) - D(\alpha,\dot\alpha)\big(M\rho^2(\alpha,\mu)\big)^{-1}, \quad |\alpha| < \alpha^*, \tag{6}$$

where $\rho^2(\alpha,\mu) = \rho_b^2(\alpha) + \mu\rho_s^2(\alpha)$, $u(\alpha) = \rho_b(\alpha)/\rho(\alpha,\mu)$, $w(\alpha) = \rho_c(\alpha)/\rho(\alpha,\mu)$, α^* is the boundary angle of the definition domain of Eq. (6), $v_i(\alpha^*) = 0$ $(i = \overline{1,5})$.

The expressions for the function $v_i(\alpha^*)$ are given in the last column of Table 2. The value of α^* for support *3* is taken from [8], considering that $\lambda\alpha = \beta$ (Fig. 3).

If there are no friction and inertial disturbing moments, then Eq. (6) has the energy integral

$$\frac{1}{2}\rho^2(\alpha,\mu)\dot\alpha^2 + gH(\alpha,\mu) = C = \text{const}, \tag{7}$$

where C is an arbitrary constant determined from the initial conditions. If $\alpha(0) = \alpha_0$ and $\dot\alpha(0) = 0$ at $t = 0$, then $C = gH(\alpha_0,\mu)$. Separating variables in (10), we obtain a formula for a quarter the period of sustained oscillations of the solid:

$$\frac{1}{\sqrt{2g}}\int_0^{\alpha_0}\frac{\rho(\alpha,\mu)\,d\alpha}{\sqrt{H(\alpha_0,\mu) - H(\alpha,\mu)}} = \int_0^{T/4} dt = \frac{T}{4}. \tag{8}$$

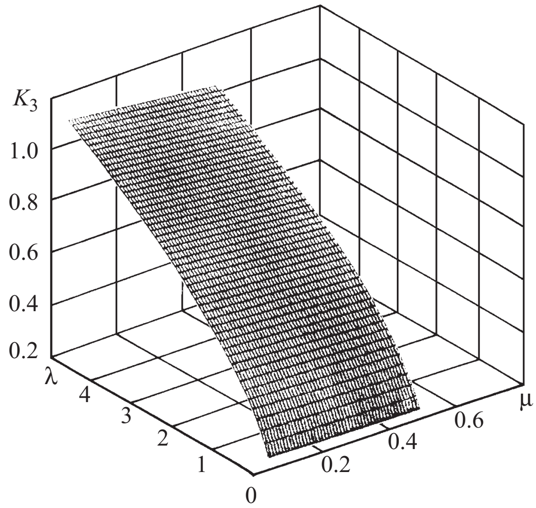

Fig. 6

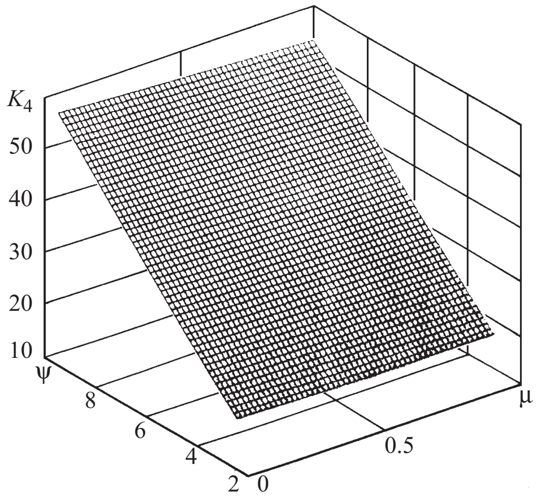

Fig. 7

Generally, the integral on the left-hand side should be evaluated numerically. If the amplitude of oscillations is small, then quantities proportional to α^n $(n \geq 2)$ can be neglected and the integral of the left-hand side of Eq. (8) can be evaluated in closed form.

For supports *1–3*, the linearized equations yield a general formula for the constant periods T_i $(i=\overline{1,3})$ of small oscillations of the solid:

$$T_i = 2\pi / \omega_i, \quad \omega_i = \sqrt{\frac{g}{R}} K_i(\mu,\chi,\lambda,\nu),$$

where the dimensionless coefficients $K_i(\dots)$ are defined by

$$K_1 = \frac{1}{2-\chi}\sqrt{\frac{\chi(1+\mu/2)}{1+\mu/4+\mu(2-\chi)^{-2}\nu^2}}, \qquad \nu = \frac{R_c}{R},$$

$$K_2 = \sqrt{\frac{\chi(1+\mu/2)}{(1-\chi)^2+\mu[(1-\chi/2)^2+\nu^2]}}, \qquad K_3 = \frac{1}{2}\sqrt{\frac{\lambda}{1+0.35\mu}}. \tag{9}$$

The coefficients K_1 and K_2 depend on three structural parameters, and K_3 on two. Therefore, the dependence $K_3(\lambda, \mu)$ is a surface over the plane of parameters λ and μ. It is shown in Fig. 6 over a rectangular domain $(0.05 \leq \mu \leq 0.5,\ 0.2 \leq \lambda \leq 5)$.

According to the last formula in (9) and Table 1, as the radius of the recess decreases, $\lambda \to \infty$ and, hence, $\omega_3 \to \sqrt{\infty}$.

For multistory buildings, μ is of the second order of smallness ($M >> m$) and, thus, can be neglected. In this case, formulas (9) yield the expression $\omega_1 = [1/(2-\chi)]\sqrt{g\chi/R}$, which coincides with the formula for the frequency of small oscillations of buildings with Nazin's supports, as those in the city of Sevastopol [4].

For supports *4* and *5* with nonlinearized models, the period of small oscillations depends on the amplitude, i.e., on the initial deflection. We will calculate these dependences using the formulas of Table 1 with small α:

$$\rho_4(\alpha,\mu) = \bar{\rho}_0 = \sqrt{h^2(1+\mu/3)+b^2(1+5\mu/16)} = \text{const}, \quad H(\alpha, M) = \bar{b}\alpha \operatorname{sign}\alpha, \quad \bar{b} = b(1+\mu/2).$$

The integral (8) for positive α is

$$T_4 = \frac{4\bar{\rho}_0}{\sqrt{2\bar{b}g}} \int_0^{\alpha_0} (\alpha_0 - \alpha)^{-1/2} \, d\alpha.$$

The integration yields $T_4 = T_4(\alpha_0) = \sqrt{\alpha_0}\, 8\bar{\rho}_0 / \sqrt{2\bar{b}g}$. Substituting the expression of $\bar{\rho}_0$ into this formula and denoting $\psi = h/b > 1$, we obtain

$$T_4 = \sqrt{\alpha_0} \sqrt{\frac{b}{g}} K_4(\mu, \psi), \tag{10}$$

where $K_4(\mu,\psi) = 4\sqrt{2}\sqrt{\dfrac{\psi^2(1+\mu/3)+1+5\mu/16}{1+\mu/2}}$.

This dimensionless coefficient depends on two parameters and is represented by a surface shown in Fig. 7 for $0.05 \le \mu \le 1$ and $3 \le \psi \le 10$. According to formula (10), as $\alpha_0 \to 0$, the frequency $\omega_4 = 2\pi / T_4$ of the solid on support *4* tends to infinity.

For support *5*, from Table 1 for small α, we get $H_5(\alpha,\mu) = \bar{b}\alpha \operatorname{sign}\alpha$,

$$\rho_5(\alpha,\mu) = \sqrt{A^2 + 2BA\alpha \operatorname{sign}\alpha} \cdot \sqrt{1+\mu\left(\frac{1}{4} + \frac{R_{c5}^2}{A^2 + 2BA\alpha \operatorname{sign}\alpha}\right)} + O(\alpha^2),$$

where $A = 2R / \cos\beta_0$ and $B = b\left(1 - \dfrac{2\beta_0}{\pi}\right) \Big/ \cos\beta_0$. If $\mu << 1$, then

$$\rho_5(\alpha,\mu) = P + Q\alpha \operatorname{sign}\alpha + O(\alpha^2, \mu^2), \qquad P = A(1+\mu/8+\mu R_{c5}^2/(2A^2)),$$

$$Q = B(1+\mu/8-\mu R_{c5}^2/(2A^2)).$$

Evaluating the quadrature for positive α using formula (8), we obtain

$$T_5 = T_5(\alpha_0) = \frac{8\sqrt{\alpha_0}}{\sqrt{2\bar{b}g}}\left(P + \frac{2Q}{3}\alpha_0\right) + O(\alpha_0^{5/2}). \tag{11}$$

When α_0 is small, formulas (10) and (11) suggest that the natural frequency is inversely proportional to $\sqrt{\alpha_0}$. What this means is that depending on the initial deflection, the solid on supports *4* or *5* may come into resonance for a wide range of external disturbances.

2. Mathematical Model of Sliding. If the coefficient of Coulomb sliding friction $f_s < f_{\text{cr}}$, then supports *4* and *5* do not tilt during the motion of the solid, i.e., $\alpha = 0$.

Let us mathematically describe the motion of three interacting bodies with broken frictional bonds. Denote the coefficients of sliding friction between the solid and the support and between the support and the plate by f_1 and f_2 and the Cartesian coordinates of the solid and support in the coordinate system fixed to the moving foundation by x_1 and x_2. When $f_1 < f_{\text{cr}}$, the solid may slide over the fixed or moving (sliding) support. Suppose the masses $M = 1$ and $m = \mu$ are subjected to dry friction forces and inertial forces due to the horizontal translational motion of the plate. Using the D'Alembert principle, we obtain the following two nonlinear coupled differential equations:

$$\ddot{x}_1 + gf_1 \operatorname{sign}(\dot{x}_1 - \dot{x}_2) = -\ddot{\xi}(t), \tag{12}$$

$$\mu\ddot{x}_2 + (1+\mu) f_1 g \operatorname{sign}(\dot{x}_2) - f_1 g \operatorname{sign}(\dot{x}_1 - \dot{x}_2) = -\mu\ddot{\xi}(t). \tag{13}$$

We assume that the sliding friction force depends on the normal pressure on the surfaces over which the solid slides and that the initial conditions are zero: $x_i(0) = 0, \dot{x}_i(0) = 0, i = 1, 2$.

Summing Eqs. (12) and (13), we obtain

$$(\ddot{x}_1 + \mu\ddot{x}_2) + (\mu+1) f_2 g \,\mathrm{sign}\, \dot{x}_2 = -(1+\mu)\ddot{\xi}(t). \tag{14}$$

It follows herefrom that when $f_2 g < |\ddot{\xi}(t)|$, Eq. (14) may have the solution

$$x_1(t) = x_2(t) \neq 0. \tag{15}$$

This solution represents a system with the masses M and m stuck together. This system is mathematically described by the equation

$$\ddot{x}_2 + f_2 g \,\mathrm{sign}\, \dot{x}_2 = -\ddot{\xi}(t), \qquad x_2(0) = \dot{x}_2(0) = 0.$$

If $f_2 g > |\ddot{\xi}(t)|$, then $x_1(t) = x_2(t) \equiv 0$ in (15).

This solution is valid under the condition $f_1 g > |\ddot{\xi}(t)|$, which prevent the solid from moving over the supports.

Let us consider the condition for the following mode of motion:

$$x_1(t) \neq 0, \quad x_2(t) \equiv 0 \quad \text{for} \quad \ddot{\xi}(t) \neq 0. \tag{16}$$

This motion occurs when the static friction force between the solid and the support satisfies the condition

$$f_1 g < \max_t |\ddot{\xi}(t)|, \tag{17}$$

and the friction force on the plate surface is greater than the sum of the inertial force due to the translational motion of the plate and the sliding friction force on the body surface: $(1+\mu) f_2 g > |\mu\ddot{\xi}(t)| + |f_1 g \,\mathrm{sign}\, \dot{x}_1|$.

Assuming that $|\mathrm{sign}\, \dot{x}| = 1$, we get

$$g(f_2 - f_1) + \mu f_2 g > |\mu\ddot{\xi}(t)|. \tag{18}$$

Inequality (18) holds if $f_2 > f_1$ and $f_2 g > \max_t |\ddot{\xi}(t)|$.

This mode of motion is described by the equation

$$\ddot{x}_1 + g f_1 \mathrm{sign}\, x = -\ddot{\xi}(t), \qquad x_1(0) = \dot{x}_1(0) = 0.$$

When $f_1 = f_2$, condition (18) contradicts condition (17). Thus, motion (16) is impossible.

The mechanism with a boll rolling in a recess and over a plate is the most complicated in dynamic behavior when frictional bond is broken. One of the possible modes of motion in this case is sliding of the solid over the fixed ball. Such a motion was analyzed in [13].

Equation (1) allows us to study the purely torsional oscillations of the solid if the supports are arranged along a circle of the coplanar plane of the plate. The center of mass of the body must be on the axis coming through the center of this circle. However, this position of the center of mass is unlikely in real systems. Therefore, translational oscillations would necessarily be accompanied by torsional oscillations and vice versa. A mathematical model of a solid on three or four imponderable supports *1–3* uniformly arranged along a circle was addressed in [6].

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