



A High Capacity Preamble Sequence for Random Access in 5G IoT Networks: Design and Analysis

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Received: 23 June 2022 / Revised: 17 August 2022 / Accepted: 2 November 2022 / Published online: 11 December 2022
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Abstract

5G NR aims to enable the high density of Internet of Things (IoT), around one million (10^6) connections per square kilometer, through the Massive Machine Type Communication (mMTC). 5G NR employs a Random Access (RA) Procedure for uplink synchronization between User Equipment (UE) and Base Station (gNB). The Zadoff-Chu (ZC) preamble sequence is widely used as the preamble sequence for RA procedure. These ZC sequences have limitations in terms of the total number of unique preambles generated, forcing the reuse of preambles. An increase in the reuse of preambles increases the probability of collision of the preamble of a UE, resulting in the failure of uplink synchronization. This necessitates the study of alternate preamble sequences with higher preamble capacity. In this paper, we propose a preamble sequence called the *mALL* sequence using the concept of cover sequences to generate a large number of unique unambiguous preambles. We compare the performance of *mALL* sequence with the ZC sequence and other combination sequences proposed in the literature using the metrics namely, periodic correlation, detection probability, effects of diversity combining, Peak to Average Power Ratio (PAPR), Cubic Metric (CM), and the effects of Carrier Frequency Offset (CFO) on preamble detection. We show that the newly proposed *mALL* sequence achieves a much higher preamble capacity (10^4 times) compared to the legacy ZC sequence, without any deterioration in the correlation properties, detection performance. Further, *mALL* sequence exhibits a better performance in terms of Cubic Metric. Results also show that the detection of *mALL* sequence is unambiguous in presence of CFO implying a better detection performance in presence of non-idealities. Thus *mALL* sequence is a potential candidate to cater to the mMTC use case of 5G. This paper also presents the comparison between Zadoff-Chu, *m*-sequence and Alltop sequence and their detection performance in RA procedure for different number of receive antennas.

Keywords 5G IoT · Random access procedure · Preamble sequences · Cover sequences · Peak to average power ratio (PAPR) · Cubic metric

1 Introduction

The deployment of 5G NR is a step forward for providing Enhanced Mobile Broadband (eMBB), Ultra Reliable Low Latency Communications (URLLC) and Massive Machine Type Communications (mMTC). Enhanced Mobile Broadband (eMBB) communications have requirements of data

rates up to 10–20 Gbps with high mobility support of User Equipment (UE) [1]. Ultra Reliable Low Latency Communications (URLLC) require robust connectivity and very low end to end latency of 5 ms. Massive Machine Type Communications (mMTC) aims for density of One million (10^6) connections per square kilometer for IoT devices with low power requirements [1]. Today, the internet and telecommunications is responsible for 4–6% of total global power consumption. With the 5G and beyond 5G upgrades in the emerging markets, it is anticipated that the telecommunication sector triples their power consumption [2]. Hence, investigation and quantification of the energy efficiency of the 5G communication devices, signal transmitters and waveforms is a matter of importance [3].

For a UE to establish an uplink connection, Random Access (RA) procedure is initiated by the UE [4] after

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satisfying a few conditions. RA initiation requires the downlink connection to be established, where timing synchronisation between gNB and UE is essential. The downlink synchronisation is done with the help of Primary Synchronisation Signal (PSS) and Secondary Synchronisation Signals (SSS), using which the UE acquires information about Physical Cell Identity (PCI), Cell Specific Reference Signal (CSR) and resource allocations. Using this information, UE initiates RA procedure to establish an uplink connection with the gNB.

The main component for the initiation of RA procedure is the preamble. The preamble is a fixed length sequence that is primarily used for uplink synchronisation. In LTE and 5G NR, ZC sequences are used as preamble sequences, but the length of the sequence restricts the number of different sequences that can be generated. The UE chooses one of the preamble from the available set of preambles randomly and initiates the RA procedure for uplink synchronisation and transmission. RA procedure of a UE is successful if the preamble chosen by it is not transmitted in the same resource units by any other UE, otherwise its a failure due to preamble collision [4]. Especially for mMTC type communications, where there is a very high density of connection requests, the probability of collision of preambles becomes higher if ZC sequence is used, resulting in frequent failure of RA procedure [4].

The limited number of available preambles using ZC sequence create a need for other sequences to be studied that can provide more number of preambles. Higher number of unambiguous preambles can reduce the reuse of a preamble and hence reduce the probability of collision. Several sequences have been proposed for such use in the literature [5, 6]. A detailed discussion on several sequences proposed in the literature is done in Sect. 3. The improvement achieved in the preamble capacity of several recently proposed sequences is very nominal and cannot meet the requirement of mMTC use case of 5G. Hence, at this point of time in the evolution of new 5G use cases, the need for a high capacity preamble sequence is of high priority as the targeted device density is one million devices per square kilometer. The purpose of this work is to address the above mentioned research gap.

Aim of this paper is to propose a new preamble sequence and to carry out a detailed performance study of the different preamble sequences proposed in the literature for the random access procedure in 5G. The major contributions of our paper are as follows.

- We propose a new candidate preamble sequence called *mALL* and show that *mALL* has higher preamble capacity than other sequences proposed in the literature.

- We carry out a detailed performance study of the preamble sequences proposed in the literature for the random access procedure in 5G.
- We evaluate the performance of the preamble using metrics that include periodic correlation, false detection, mis-detection.
- We also explore other metrics related to preamble sequences such as PAPR and Cubic-Metric to better understand the energy requirements and inherent properties of such sequences.
- We observe the effect of number of antennas on detection performance of different preamble sequences, which provides us an insight on the effect of equal gain diversity combining on probability of detection of the preamble.

Rest of the paper is organised as follows. In Sect. 2 we discuss the background of preamble generation and some important definitions. In Sect. 3, we present the related work. We discuss the concept of cover sequences and compute the preamble capacity of existing cover sequences in Sect. 4. We propose a new candidate sequence for preamble generation in Sect. 5. In Sect. 6, we lay out the algorithm used for preamble detection. In Sect. 7 we present the simulation results comparing the Detection Probabilities, PAPR, Cubic Metric and effect of Carrier Frequency Offset for the different sequences which are being considered as preamble candidates. Finally in Sect. 8, we provide our conclusions regarding the features of preambles considered in the study.

2 Background

The preambles used for the random access procedure in 5G are generated on the principle of orthogonality, where the set of preamble sequences are orthogonal to each other making it easy to detect at the receiver. Constant Amplitude Zero Auto Correlation (CAZAC) Sequences are considered to be suitable candidates for preamble generation. These sequences maintain Constant Amplitude so that the Peak to Average Power Ratio (PAPR) is low, which is required to maintain the linearity of Power Amplifiers. CAZAC sequences have another property, i.e. any two different sequences in a CAZAC sequence set are orthogonal to each other. A type of CAZAC sequences, called ZC sequences have been used in LTE and 5G NR for preamble generation. The length of a preamble sequence, denoted by L_{RA} , is based on the numerology defined by 3GPP technical specification [7]. Before getting into the details of the preamble sequences, we introduce a few notations and definitions, that are used in the paper as follows.

2.1 Some Definitions

2.1.1 Cyclic Shift

Let $x_\mu[n]$ be a sequence of length L_{RA} with root index μ , then the cyclic shifted $x_\mu^{n_0}[n]$ is defined as:

$$x_\mu^{n_0}[n] = x_\mu[(n + n_0) \bmod L_{RA}] \tag{1}$$

where n_0 is the number of samples by which $x_\mu^{n_0}[n]$ is cyclically shifted from $x[n]$. Here, *mod* represents the modulo operation i.e., $a \bmod b$ gives the value of the remainder when b divides a .

2.1.2 Periodic Correlation

Periodic correlation of any two sequences $x[n]$ and $y[n]$ of length L is given by the following equation.

$$R_{x,y}[\tau] = \sum_{n=0}^{L_{RA}-1} x[n]y^*[(n + \tau) \bmod L_{RA}] \tag{2}$$

where $y^*[n]$ represents the conjugation of sequence $y[n]$

2.1.3 Zero/Low Correlation Zone (ZCZ) / (LCZ)

The ZCZ / LCZ is defined as the region where the value of $|R_{x,y}|$ is zero/very low as compared to the peak value of $|R_{x,y}|$.

2.1.4 Probability of Detection/Misdetection

$P(\text{detection})$ is defined as the conditional probability of correct detection of the preamble when the signal is present. $P(\text{miss})$ is complementary of $P(\text{detection})$. Standard (3GPP, TS 38.104 [8]) dictates that $P(\text{detection})$ should be equal to or exceed 99% [8].

2.1.5 Probability of False Alarm ($P(\text{false})$)

$P(\text{false})$ is defined as the conditional probability of erroneous detection of the preamble when input is noise only. $P(\text{false})$ should be less than 0.1% [8].

2.1.6 Preamble Capacity

Preamble Capacity can be defined as the total number of preambles from which a set of 64 preambles is generated for each cell. As the 5G NR aims for mMTC, there will be a need to increase the number of preambles per cell. We have a limited number of preamble sequences depending on the length of sequence and the allowed cyclic shift

between preamble sequences. The method of generating the preamble sequences is illustrated in TS 38.211 ([7], Sec. 6.3.3.1). In case of ZC sequences of length L_{RA} , we have $L_{RA} - 1$ different root sequences and for each root sequence we have $\lfloor \frac{L_{RA}}{N_{CS}} \rfloor$ different cyclically shifted sequences. Therefore preamble capacity for ZC sequence is given as:

$$PrCapacity^{ZC} = (L_{RA} - 1) \cdot \lfloor \frac{L_{RA}}{N_{CS}} \rfloor \tag{3}$$

From (3) we clearly see the dependence of preamble capacity on L_{RA} , the length of preamble sequence, and N_{CS} , the minimum cyclic shift between two sequences. As N_{CS} increases, the number of preambles available decreases. This may lead to the reuse of sequences among the UEs in the cell when the UEs randomly choose the available preambles during the RA procedure. This results in high probability of collision. If the number of unique preambles available is large, then the reuse of sequences will decrease. So, there is a need for the study of other sequences or the combination of some sequences such that higher preamble capacities are obtained. These sequences can then be used instead of ZC sequence to cater the needs of mMTC.

3 Related Work

ZC sequences and its properties have been extensively studied by Frank et al. [9] and Chu et al. [10]. The restricted length of preamble sequences, limit the number of different preambles generated for a ZC sequence as discussed in Sect. 2.1.6. In addition to the low preamble capacity, ZC sequences are found to be sensitive to frequency shift by Pitaval et al. [5]. Schreiber et al. [11] proposed a new candidate sequence for preamble generation called m-Sequence. The m-Sequences show better tolerance to frequency shifts. To increase the number of preambles available, several techniques have been proposed in the literature. Aggregation techniques are explored by Mostafa et al. [12] where the transmitted preamble is a weighted addition of two ZC sequences. Arana et al. [13] designed a new preamble by multiplying shifted versions of two different ZC sequences. The combination of ZC sequences with other CAZAC or near-CAZAC sequences has been suggested by Pitaval et al. [5]. Further, a concept of cover sequences has been employed to design new sequences namely *aZC* and *mZC* to achieve a better preamble capacity as discussed in Sect. 4.

The detection methods of *aZC* and *mZC* sequences have been proposed by Pitaval et al. in [5, 6] respectively. Moreover, the detection algorithm in both works assume a prior setting of threshold such that the false alarm can be mitigated as low as 10^{-3} . However the detection methods

in [5, 6] does not address the problem of adverse impact of Carrier frequency offsets(CFOs). Detection methods should also include analysis under non-zero CFOs because a non-zero CFO can actually lead to energy leakage of correlation peaks [14] and can even shift the peaks at wrong timing position under large CFOS. This degrades the detection performance as well. Yang et al. [15] and Wang et al. [16] uses the peak detector to analyze the detection performance but the adverse impact of non-zero CFOs is not taken in account. Whereas in [17] Generalised likelihood ratio test(GLRT) detector, clairvoyant detector is used to analyze the detection performance but with unknown value of CFOs these detectors are practically inapplicable. So in most of the above mentioned works, although the authors have adopted detectors that deal with multi-path environment, impact of CFO has been ignored.

Vukovic et al. [18] has systematically analysed the impact of AWGN channel on RACH throughput. A detection technique has been proposed by Pham et al. [19] for long sequences i.e., $L_{RA} = 839$ for rejecting false peaks. A preamble design and detection method for satellite random access is discussed by Zhen et al. [20]. Kim et al. [21], Kim [22] proposed more sophisticated detection schemes that make use of post-processing and reconstructions of received signal in order to achieve better detection but with increased complexity.

Although a number of preamble sequences have been proposed and studied in the literature, the effect of diversity combining on the detection performance has not been explored in detail for these preamble sequences. Furthermore, Cubic Metric (CM), which is an important parameter, has not been studied for these sequences. To meet the demand of increasing devices especially in mMTC, the preamble capacity should be as high as to serve 10^6 devices per km^2 . To fill in the said gaps in the literature, this paper aims to find the detection performance using diversity combining of the proposed sequences and their CM measurements, which can help to identify better overall sequences as candidates for preambles.

Unique contributions of our paper are as follows

- We propose a new candidate preamble sequence called *mALL* by combining the *m-sequence* and *Alltop* sequences. Further, we show that *mALL* has higher preamble capacity than other sequences proposed in the literature.
- Unlike the existing works in literature [5, 11], in addition to the PAPR performance, we also study Cubic-Metric performance of different preamble sequences for better understanding of the inherent properties of such sequences.
- Ours is the only work to present the effect of equal gain diversity combining on the detection performance of the

preamble sequences considered in [5, 11] and for the proposed *mALL* sequence.

- We capture the effect of carrier frequency offset on the correlation properties of the proposed preamble sequences $\{mALL\}$ and compare it with the legacy ZC sequences.

4 Existing Preamble Designs and their Preamble Capacity

4.1 Concept of Cover Sequences

From (3) we note that, we have a fixed number of preamble sequences that can be generated based on L_{RA} and N_{CS} . Let us denote this set of preambles available in ZC sequences as S^{ZC} . Consider another CAZAC sequence such as *m-sequence* [11] and *Alltop* [23] sequence of the same length L_{RA} . We call it a cover sequence and use it to generate more number of unique preambles. We denote the cover sequence as $c[n]$. Let N_c be the number of sequences that can be generated such that they are all orthogonal to $c[n]$ and non-ambiguous to one another. Here non-ambiguity implies that the cross-correlation between any two sequences in S^c is very low compared to the total signal energy, so that they can be identified and detected easily. We define this set of N_c cover sequences as S^c .

$$S^c = \{c_1[n], c_2[n], c_3[n], \dots, c_{N_c}[n]\} \quad (4)$$

We define a set of orthogonal signals as s_μ^{ZC} such that for a fixed μ and different ν , the generated sequences are orthogonal to each other. Therefore we have

$$s_\mu^{ZC} = \left\{ x_\mu^0, x_\mu^1, x_\mu^2, \dots, x_\mu^{\lfloor \frac{L_{RA}}{N_{CS}} \rfloor - 1} \right\} \quad (5)$$

Union of s_μ^{ZC} for all values of μ will generate a super set S^{ZC} such that $S^{ZC} = \bigcup_{\mu=1}^{L_{RA}-1} s_\mu^{ZC}$.

New combination sequences are generated when each sequence from S^c multiplies with each sequence from S^{ZC} element-wise as shown below.

$$\begin{aligned} y_l[n] &= c_l[n] \cdot * S^{ZC} \\ &= c_l[n] \cdot * \left\{ s_1^{ZC}, s_2^{ZC}, \dots, s_{L_{RA}-1}^{ZC} \right\} \quad l = 1, 2, \dots, N_c \end{aligned} \quad (6)$$

Here in (6), the notation ' $\cdot *$ ' represents element-wise multiplication. We define a set S_l^{ZC} , which is a set of sequences obtained from (6) for a fixed value of l which can be varied from 1 to N_c . The union of all such N_c sets will result into a superset S^{ZC} :

$$S^{cZC} = \bigcup_{l=1}^{N_c} S_l^{ZC} \tag{7}$$

Popovic in [24] defines such set in (7) as Quasi-Orthogonal Set.

4.2 *m*–Sequence and Combination with ZC Sequence(*mZC*)

m – sequences are binary sequences generated by using Linear Feedback Shift Registers (LFSR) [5, 11]. Generation of these sequences are defined by its primitive generator polynomials. A *m*th order polynomial is defined as follows.

$$g(P) = g_m P^m + g_{m-1} P^{m-1} + \dots + g_1 P^1 + g_0 \tag{8}$$

Where *g_m* is the polynomial coefficient which can take value of either 0 or 1.

The length of sequence is given by *N_m* = 2^{*m*} – 1. For example consider *m* = 7, we have a length of sequence as *N_m* = 127 and generator polynomial as *P*⁷ + *P*¹ + 1. Using these polynomial weights, LFSRs will generate a periodic binary bit streams *x_m[n]* of length 127. We can obtain a set of orthogonal sequences *S^m* from *x_m[n]* by cyclic shifting the sequences.

$$x_m^l[n] = x_m[(n + l) \bmod N_m] \tag{9}$$

$$S^m = \bigcup_{l=0}^{N_m-1} \{x_m^l[n]\} \tag{10}$$

The inherent issue with the *m*-sequences is that they are defined perfectly only for lengths of 2^{*m*} – 1 i.e., 3, 7, 15, 31, 63, 127, 255, 511 and so on. As defined by 3GPP technical specifications [7], the length for short preamble is defined as *L_{RA}* = 139. The value of *m* for which the length for *m*–sequence is closest to *L_{RA}* is *m* = 7, corresponding to 127 length *m*–sequence. At the defined lengths the cyclic shifted versions of *m* – sequences defined in (9) are orthogonal to each other. But the need for 139 length sequences as cover sequences raises the compatibility issue with *m* – sequences. To generate 139 length cover sequence, the first 12 samples of *m*–sequence are appended to itself. Although the new sequences generated lose their orthogonality, they maintain non-ambiguity since their cross-correlation is very low compared to the total energy of signal. Hence, they behave similar to near-CAZAC sequences.

As mentioned in Sect. 4.1, we can use the *m* – sequences as a cover sequence to obtain the new preamble sequence, *z_{l,μ,v}* and the Quasi-Orthogonal set *S^{mZC}* [5] as given below:

$$z_{l,\mu,v}[n] = (x_m^l[n]) \cdot * x_\mu^v[n] \tag{11}$$

$$S^{mZC} = \bigcup_{l,\mu,v} \{z_{l,\mu,v}[n]\} \tag{12}$$

To calculate the preamble capacity of *mZC* sequence, we see that we can use the set *S^{ZC}* as it is. Then we have *N_m* number of sequences in set *S^m*. Therefore multiplying each sequences in *S^m* with each sequence in *S^{ZC}* we will have *N_m(L_{RA} – 1)* $\left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor$ sequences along with the original ZC set. Therefore we have

$$\begin{aligned} PrCapacity^{mZC} &= N_m(L_{RA} - 1) \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \\ &+ (L_{RA} - 1) \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \\ &= (N_m + 1)(L_{RA} - 1) \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \end{aligned} \tag{13}$$

The maximum value for *N_m* can be attained using a cyclic shift of each sample which gives us *N_m^{max}* = *L_{RA}*. Substituting this value in (13), we have

$$PrCapacity^{mZC} = (L_{RA}^2 - 1) \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \tag{14}$$

4.3 Alltop Combined with ZC Sequence(*aZC*)

Alltop sequences as defined in [23] are cubic phase sequences. One form of such sequences are defined in [5, 25] as follows:

$$\begin{aligned} a_{\lambda,w}[n] &= \exp \left\{ -j2\pi \frac{(n+w)^3 + \lambda n}{L_{RA}} \right\}, \quad 0 \leq n \leq L_{RA} - 1 \\ &0 \leq \lambda \leq L_{RA-1} \\ &0 \leq w \leq L_{RA-1} \end{aligned} \tag{15}$$

Some important properties of Alltop sequences are given below:

- *a_{λ,w}[n]* is a cyclic shifted version of the sequence *a_λ[n]*.
- Two sequences *a_{λ,w}[n]* and *a_{λ',w}[n]* are orthogonal to each other for *λ* ≠ *λ'*.
- Whereas, *a_{λ,w}[n]* have very low cross correlation with *a_{λ,w}[n]* and is equal to $\sqrt{L_{RA}}$.

Applying the sequence in (15) as a cover to ZC sequence introduces ambiguity in resulting sequences. Pitaval et al. in [5] discusses this in details and proposes a new cover sequence to be used as:

$$z_{l,\lambda,w,\mu,v} = (a_{\lambda,w}[n])^l \cdot * x_\mu^v[n], \quad 0 \leq l \leq L_{RA} - 1 \tag{16}$$

Preamble capacity of *aZC* sequence is shown to be [5]

$$PrCapacity^{aZC} = (L_{RA}^2 - 1) \left[\frac{L_{RA}}{N_{CS}} \right] \tag{17}$$

5 Proposed mALL Preamble Sequence

In Sects. 4.2 and 4.3, we saw that both mZC and aZC sequences have the same PrCapacity given by (14) and (17). It is observed that for aZC sequences, there can be multiple sequences which are ambiguous for an arbitrary choice of l, λ, w, μ, ν , reason being λ, w and μ parameters modify the phases of sequences simultaneously [5]. So we need to be careful in the selection of such sequences. In worst case scenario that can be obtained by substituting $\left[\frac{L_{RA}}{N_{CS}} \right] = 1$ in (14) and (17), we have PrCapacity in order of 10^5 . However the requirement for mMTC is in order of 10^6 per km^2 [1].

For Alltop sequences defined in (15) and (16), we find that for a fixed value of l and λ , the w parameter makes a cyclic shifted version of the sequence. Thus taking correlation of two Alltop sequences $(a_{\lambda,w}[n])^l$ and $(a_{\lambda,w'}[n])^l$ will give us a high correlation peak, which limits the possible number of unique sequences that can be generated based on l and λ only. To make the correlation peak of Alltop sequences independent of w , we make use of m-sequences as cover sequences.

We propose a new combination sequence called mALL sequence by using m-sequence as a cover sequence for Alltop sequence. Therefore, from equations (15) and (9), we define mALL sequence to be,

$$z_{l,\lambda,w,t} = (a_{\lambda,w}[n])^l \cdot * x'_m[n] \tag{18}$$

$$0 \leq l, \lambda, w, t \leq L_{RA} - 1$$

We have generated the preamble sequences defined by (18) in a simulation setup using MATLAB and have explored the auto-correlation and cross-correlation properties of this sequence. We run the simulation extensively for all possible combinations of the parameters, l, λ, w, t . Fig. 1 shows the correlation results for different combinations of l, λ, w, t . Our findings are presented below:

- The auto-correlation peak has a value equal to the length of a sequence $L_{RA} = 139$ with all other samples in LCZ.
- The average maximum cross-correlation value between two different sequences is 27.37 which is less than $3\sqrt{L_{RA}}$.
- For a fixed t and varying l, λ, w we find that all sequences generated are non-ambiguous where their cross-correlation values are always in LCZ.

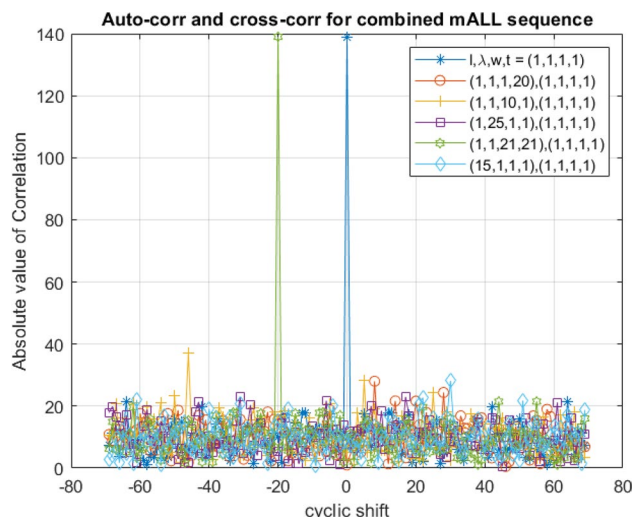


Fig. 1 Periodic Correlation of mALL sequences. We represent the auto-correlation result in blue line, where the sequence $z_{l,\lambda,w,t}$ is generated from Eq. (18) with $\{l, \lambda, w, t\} = \{1, 1, 1, 1\}$. Other results represented in the legend are cross-correlation results between $z_{1,1,1,1}$ and $z_{l,\lambda,w,t}$ for arbitrary choice of $\{l, \lambda, w, t\}$ values

- For fixed values of l, λ and w , different values of t lead to ambiguity in the sequences resulting in high correlation peak.

It is observed in Fig. 1, that the peak for auto-correlation of $z_{1,1,1,1}$ occurs at zero cyclic shift value and has a value equal to length of sequence ($L_{RA} = 139$). It is observed that there exist another sequence $z_{1,1,21,21}$ which also shows peak of value of 139. Such sequences are ambiguous and can result in mis-detections. To avoid such cases, we simply keep t constant and vary only l, λ, w to obtain a larger set. For other cases of cross-correlation, it is observed that the values remain sufficiently lower, such that detection of sequences using the method described in Sect. 6 holds good. Therefore, each sequence obtained by varying l, λ, w and keeping t constant, we will have a set of unambiguous sequences. The number of sequences that can be generated from the range of values taken by l, λ, w in equation (18) we have L_{RA}^3 unique sequences.

To further increase the capacity we include cyclic shift of each sequence in the set, depending on zeroCorrelationZoneConfig. For each combination of l, λ, w , with a fixed t , we include the cyclic shift parameter ν such that

$$z_{l,\lambda,w,t}^\nu = z_{l,\lambda,w,t}[(n + C_\nu) \bmod L_{RA}] \tag{19}$$

where C_ν is the cyclic shift defined in TS 38.211 ([7], Sec. 6.3.3.1). Let $\{z_{l,\lambda,w,t}^\nu[n]\}$ be the set of all non-ambiguous sequences for a given ν obeying (19). Let us define a set of such sequences as:

$$s_v^{mALL} = \bigcup_{l,\lambda,w} \{z_{l,\lambda,w,t}^v[n]\} \tag{20}$$

From (20), we get L_{RA}^3 sequences for each value of v and all possible combination of l, λ, w , with a fixed t . Union of all such sets in (20) over v will form a complete set of sequences as given below.

$$S^{mALL} = \bigcup_{v=0}^{\lfloor \frac{L_{RA}}{N_{CS}} \rfloor - 1} s_v^{mALL} \tag{21}$$

Taking all combinations into account we can calculate PRACH capacity for the proposed sequence:

$$\begin{aligned} PrCapacity^{mALL} &= L_{RA} \times L_{RA} \times L_{RA} \times \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \\ &= L_{RA}^3 \left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor \end{aligned} \tag{22}$$

We see from (22), that the proposed $mALL$ sequence has a much larger $PrCapacity$ than the previously identified sequences namely ZC, mZC and aZC .

6 Preamble Detection Algorithm

For each cell there are 64^1 preambles available at UE for transmission. This set of 64 sequences are generated according to standard specifications [7]. The gNB has the information about which root sequences are present in all 64 preamble from which UE transmits one of the preamble randomly. To calculate the periodic correlation we make use of FFT (fast fourier transform) for faster calculations. As we know the convolution of two sequences in time domain is equivalent to the operation of element wise multiplication of the FFT of sequences. Let the received sequence be $s[n]$ and the root sequences present at gNB be $k_\mu[n]$.

$$\mathcal{F}(s[n]) = S[k], \quad \mathcal{F}(k_\mu[n]) = K_\mu[k] \tag{23}$$

where \mathcal{F} represents the Discreet Fourier Transform operator.

$$\begin{aligned} R_{s,k_\mu}[\tau] &= \sum_{n=0}^{L-1} s[n]k_\mu^*[(n + \tau) \bmod L] \\ &= \mathcal{F}^{-1}(K_\mu^*[k].S[k]) \end{aligned} \tag{24}$$

Algorithm 1 Preamble Detection Algorithm

Require:Received Signal $s_i[n]$ for i^{th} antenna

Require:Root sequences $k_\mu[n]$

Require:Threshold $\rightarrow \eta^{N_{Ant}}, N_{CS}$

Ensure: $L_{RA} = 139$

```

1: Start:
2:  $S_i[k] \leftarrow \mathcal{F}(s_i[n])$                                 ▶ FFT of received signal
3:  $K_\mu[k] \leftarrow \mathcal{F}(k_\mu[n])$                         ▶ FFT of Root Sequences
4:  $P_\mu \leftarrow 0$                                        ▶ Initialise PDP matrix
5: for  $i^{th}$  antenna do
6:   for each row ( $\mu$ ) of  $K_\mu[k]$  do
7:      $R_{s_i,k_\mu} \leftarrow \mathcal{F}^{-1}(K_\mu^*[k].S_i[k])$     ▶ periodic correlation
8:      $P_\mu^i \leftarrow |R_{s_i,k_\mu}|^2$                        ▶ Calculate PDP
9:   end for
10:   $P_\mu \leftarrow P_\mu + P_\mu^i$                           ▶ Accumulation of PDP
11: end for
12:  $\bar{P}_\mu \leftarrow mean\{P_\mu\}$ 
13: for each row  $\mu$  do
14:   for  $i = 0; i < L_{RA}; i = i + N_{CS}$  do
15:      $window \leftarrow P_\mu[i : i + N_{CS} - 1]$           ▶ Window of length  $N_{CS}$ 
16:     if  $MAX\{window\} \geq \eta^{N_{Ant}} \bar{P}_\mu$  then           ▶ Compare Threshold
17:        $DETECT \leftarrow 1$                                ▶ Decision for Detection
18:     else
19:        $DETECT \leftarrow 0$                                ▶ Not Detected
20:     end if
21:   end for
22: end for

```

¹ This is valid for $L_{RA} = 139$. For $L_{RA} = 839$ there are restricted sets which limit the number of available preamble for transmission

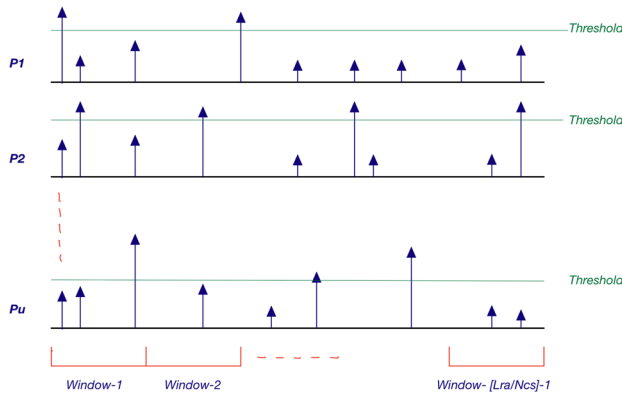


Fig. 2 Illustration of detection algorithm

The detection algorithm adopted by us based on [5, 19] is given in Algorithm 1:

- First we calculate FFT of received signal $s_i[n]$ for i^{th} antenna and Root sequences $k_\mu[n]$
- Next we calculate the periodic correlation between $s_i[n]$ and $k_\mu[n]$ based on Eq. (24). This results into a matrix R_{s_i, k_μ} of dimension $\mu \times L_{RA}$ where $1 \leq \mu \leq \left\lfloor \frac{64}{\left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor} \right\rfloor$.
- Taking absolute squared value of R_{s_i, k_μ} , results into PDP matrix P_μ^i . Here for each antenna we accumulate the P_μ^i matrix. The accumulated matrix is of dimension $\mu \times L_{RA}$.
- Fig. 2 shows an example of a accumulated PDP where P_1, P_2, \dots, P_μ correspond to the rows of P_μ matrix of length L_{RA} . The amplitudes denote the absolute squared values of the periodic correlation R_{s_i, k_μ} . Each row is divided into $\left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor$ number of windows of length N_{CS} . We declare the detection of preamble if the peak value in any window corresponding to any row is greater than the threshold (shown by green lines).
- Then we calculate mean of P_μ obtained after accumulation as \bar{P}_μ .
- Next step is to divide each row of P_μ into $\left\lfloor \frac{L_{RA}}{N_{CS}} \right\rfloor$ different windows of length N_{CS} as shown in Fig. 2.
- We then decide a threshold for each row as $\eta^{N_{Ant}} \bar{P}_\mu$, shown as a green line in Fig. 2.
- If the peak value in any given window across the rows of P_μ is greater than $\eta^{N_{Ant}} \bar{P}_\mu$, we say that a preamble is detected. Based on the row and window number we decide which preamble out of 64 preambles was transmitted.

Table 1 Generation of ZC, mZC, aZC and mALL sequences where $L_{RA} = 139$ and $N_{CS} = 23$

Sequences	Variation of parameters { $v = 1$ to 5, $N_{CS} = 23$ }	Equations
ZC	$\mu = \{1 \text{ to } 11\}$	(1)
mZC	$l = 1, \mu = \{1 \text{ to } 11\}$	(11)
aZC	$l = 1, \lambda = 1, \mu = \{1 \text{ to } 11\}, w = 1$	(16)
mALL	$l = 1, \lambda = \{1 \text{ to } 11\}, w = 1, t = 1$	(19)

6.1 Diversity Combining

Equal gain combining [26], is a linear diversity combining technique which is very simple to implement. It requires the received noisy signals from different antennas to be simply added before any processing. This technique can give us a significant SNR gain [27]. In [28] the use of equal gain combining for mmWave and MIMO application have also been discussed.

Let N_{Ant} denote the number of receiver antennas at gNB. After equal gain combining at the receiver, the PDPs is given by:

$$P_\mu^{N_{Ant}} = \sum_{i=1}^{N_{Ant}} P_\mu^i \quad (25)$$

6.2 Threshold Measurement

Each of the PDP obtained from (25) is normalised by its average value $\bar{P}_\mu^{N_{Ant}}$.

$$\bar{P}_\mu^{N_{Ant}} = \frac{1}{L_{RA}} \sum_{k=0}^{L_{RA}-1} P_\mu^{N_{Ant}}[k] \quad (26)$$

Therefore from (25) and (26) we have normalised PDP

$$P_\mu^{Norm} = \frac{P_\mu^{N_{Ant}}}{\bar{P}_\mu^{N_{Ant}}} \quad (27)$$

Compare the value obtained in (27) with a decided value of η . If the value of P_μ^{Norm} is greater than η then preamble is detected else it is not.

The value of η is decided such that the condition for $P(\text{false})$ defined in 2.1.5 is satisfied.

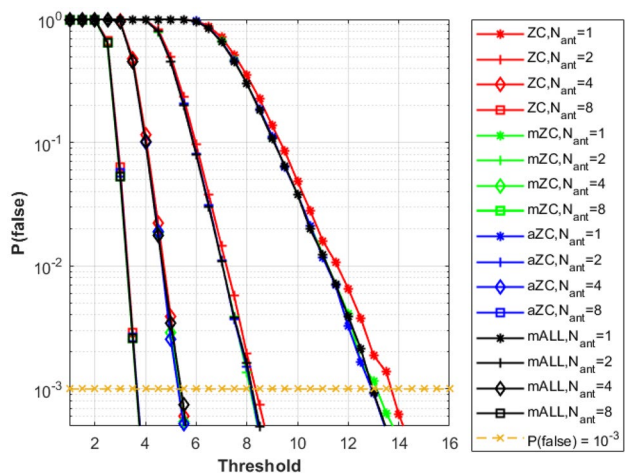


Fig. 3 Estimating threshold for $P(\text{false}) \leq 0.1\%$

Table 2 Threshold (η) satisfying $P(\text{false})$ condition

Sequences	$\eta(N_{Ant}=1)$	$\eta(N_{Ant}=2)$	$\eta(N_{Ant}=4)$	$\eta(N_{Ant}=8)$
ZC	13.7	8.4	5.4	3.8
mZC	13.1	8.2	5.3	3.8
aZC	13.0	8	5.3	3.8
mALL	13.0	8.4	5.4	3.8

7 Simulations and Results

7.1 Simulation Methodology

We perform the simulations of random access procedure in 5G MATLAB 5G toolbox to evaluate the performance of our proposed preamble sequence. We generate different sequences using Eq. (11), (16) and (19). We generate a set of 64 preamble sequences set according to in TS 38.211 ([7], Sec. 6.3.3.1). We use the value for the parameter *zeroCorrelationZoneConfig* as 11 which translates to N_{CS} value of 23. The preamble set of 64 sequences is generated by varying the associated parameters for the sequences as given in TABLE 1².

We use the AWGN channel for comparison of detection performance. For detection performance we use the method described in Sect. 6. We run the simulation for 10^5 times for each of the sequences, for every antenna to get $P(\text{detect})$.

² Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

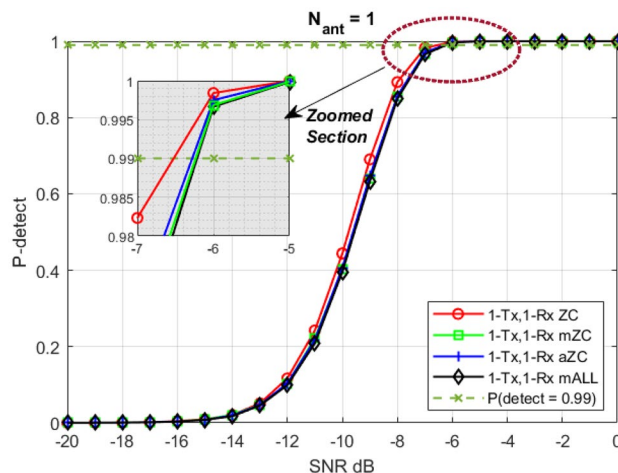


Fig. 4 $P(\text{detection})$ for ZC, mZC, aZC, mALL for 1-TX, 1-RX configuration

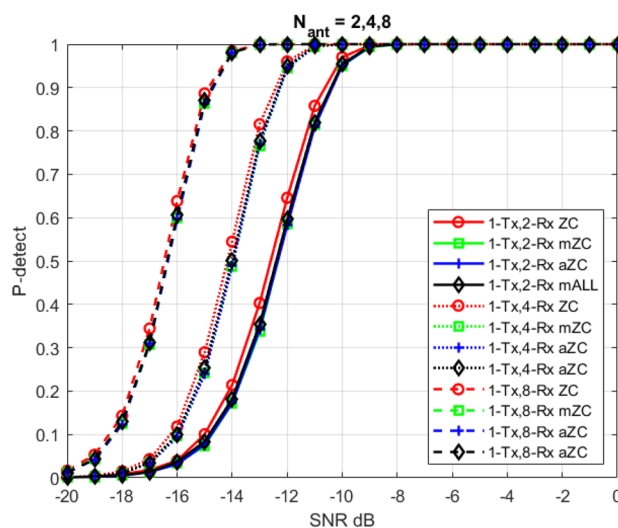


Fig. 5 Comparison of $P(\text{detection})$ for ZC, mZC, aZC, mALL sequences varying SNR and N_{Ant}

7.2 Threshold for False Alarm Probability

Based on Sect. 6.1.1, we determine the threshold to be set which satisfies the conditions for $P(\text{false})$ given in Sect. 2.1.5. For simulation we follow the steps using Algorithm. 1:

- The received signal $s[n]$ in Algorithm. 1 is a complex AWGN noise.
- We correlate the received noise signal $s[n]$ with all root sequences $k_\mu[n]$.
- We decide the threshold value $\eta^{N_{Ant}}$ based on the number of antennas at the receiver.
- Based on $\eta^{N_{Ant}}$, we make a decision for detection.

- We then have the probability of detection when input is AWGN noise for all sequences $\{ZC, mZC, aZC, mALL\}$ for each antenna configuration $\{N_{Ant}\}$.

In Fig. 3, we observe that as the Threshold increases the $P(false)$ decreases. It is found that to satisfy the condition of $P(false) \leq 0.1\%$, we have Threshold defined in Table 2. We observe that there is a variation of η for lower N_{Ant} values but as N_{Ant} increase, the η is same for all four sequences.

7.3 Detection Probability

We use the a single transmit and single receive antenna configuration for the analysis of detection probability. The parameter *zeroCorrelationZoneConfig* is set to 11 that translates to $N_{CS} = 23$. The $P(detection)$ is calculated using the detection method described in Sect. 6 and values of η taken from TABLE 2. In Fig. 4, we observe that ZC , mZC , aZC and $mALL$ sequences achieve $P(detect)$ of 99% at -6.5 dB, -6.2 dB, -6.3 dB and -6.2 dB respectively. It is observed that the ZC sequence detection performance is better than rest of the sequences, and mZC and $mALL$ sequences perform equally. Difference between the best performing ZC sequence and proposed sequence $mALL$ is only 0.3 dB, which is very low.

7.3.1 Diversity Combining

In Fig. 5, the detection performance has been observed in AWGN channel for a single user case, varying N_{Ant} as 2, 4 and 8. We wanted to study the properties of the combination sequences $\{mZC, aZC$ and $mALL\}$ and compare their properties and detection performance with the ZC sequence. In order to do that the preamble sequences are not repeated unlike the formats prescribed by 3GPP in [7]. Threshold η is taken from TABLE 2. We observe that there is no deviation in performance of detection with respect to varying SNR for all the four sequences considered namely ZC , mZC , aZC and $mALL$. But varying N_{Ant} has effect on the performance. All the four sequences perform in a similar manner for a given threshold calculated based on the number of antennas. However, as the number of antennas increase we find improvement in detection performance. This behaviour can be attributed to the diversity combining of antennas that results in more accurate detection.

In Fig. 5, for lower SNR in the range of -20 to -16 dB, the performance of the system for the cases of $N_{ant} = 8$ and $N_{ant} = 4$ is 3dB and 1dB better than that of the $N_{ant} = 2$ case, respectively. When achieving the $P(detect)$ greater than 99%, $N_{ant} = 8$ and $N_{ant} = 4$ systems perform better than $N_{ant} = 2$ system, by 4.6dB and 2.1dB, respectively. This is a significant improvement that we obtain using the

diversity combining. Note, that the performance for legacy ZC sequence is always marginally better than that of other three sequences and the proposed $mALL$ sequence also performs marginally better than mZC and aZC sequences. It is also observed that as N_{Ant} increases, the performance gap of $mZC, aZC, mALL$ with ZC sequence reduces.

Observations from Fig. 5, show that the detection performance of the proposed $mALL$ sequence is at par with ZC sequence and slightly better than aZC and mZC sequences. The advantage of proposed $mALL$ sequence is that, it provides a higher preamble capacity without the degradation of detection performance.

7.4 PAPR and CUBIC METRIC (CM)

PAPR and CM are the metrics which indicate the efficiency of Power Amplifier (PA) used to transmit the signals. Generally, higher the PAPR or CM, lower is the efficiency of the PA. PAPR gives us the measure of power backoff required. For a transmitted signal $x(t)$ PAPR is defined in [29] as:

$$PAPR[x(t)]dB = 10 \log \frac{\max[x^2(t)]}{\text{mean}[x^2(t)]} \quad (28)$$

However, recent studies suggest that the CM is a better performance metric than the PAPR metric [30–32] as it considers the effect of 3rd order harmonics introduced due to the non-linearity in PA. In general, the amplifier voltage can be written as

$$v_0(t) = g_1 v_i(t) + g_3 v_i^3(t) \quad (29)$$

where g_1 and g_3 are linear and non-linear gains respectively. These gains are inherent to PAs and do not change with the type of signal. The cubic term in (29) introduces distortions in the transmitted signal, resulting in erroneous detection at receiver. Moreover, it also introduces third harmonics which interfere with adjacent channels. In order to suppress these distortion we need to suppress the non-linear gain (g_3). The CM for a transmitted signal $x(t)$ is given by [29]:

$$CM[x(t)] = \frac{20 \log \text{rms}[x_{norm}^3(t)] - 1.52}{1.56} \quad (30)$$

where $x_{norm}(t) = \frac{\text{mod } x(t)}{\text{rms}[x(t)]}$

Through simulations we have obtained the CDF of CM and PAPR for all the sequences discussed.

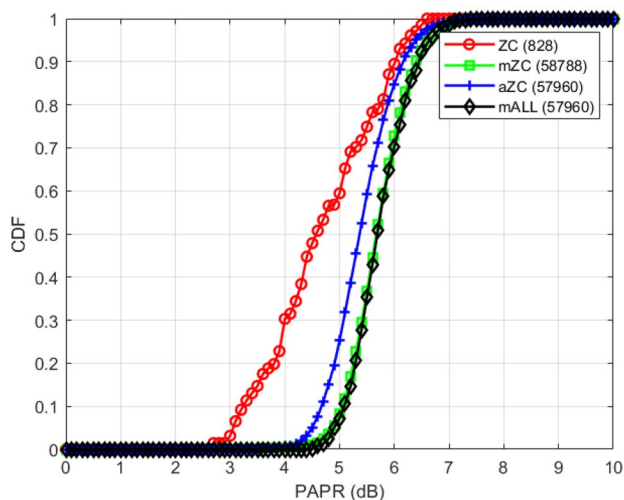


Fig. 6 CDF of PAPR for ZC, mZC, aZC, mALL sequences

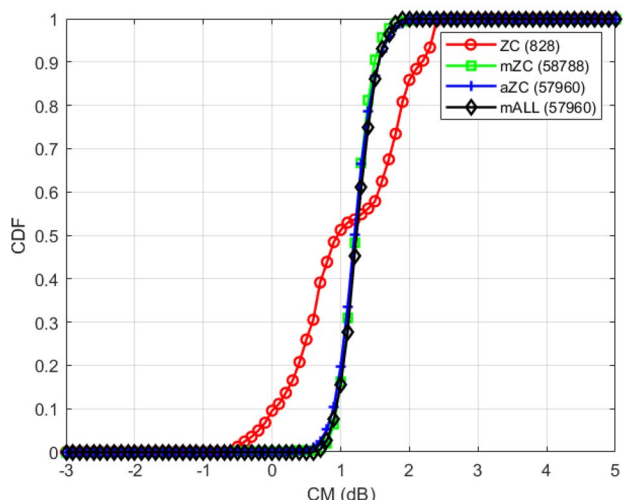


Fig. 7 CDF of cubic metric (CM) for ZC, mZC, aZC, mALL sequences

- For ZC sequence we have set *zeroCorrelationZoneConfig* = 11, which translates to $N_{CS} = 23$ and $L_{RA} = 139$. From (3), we obtain the preamble capacity of 828 sequences.
- For mZC, defined by (14) we have considered m-seq with cyclic shifts of 2 samples. This results in $N_m = 70$ different M-sequences, we obtain the preamble capacity of 58788 sequences.
- For aZC, from (17), similar to mZC we generate 57960 different sequence.
- For mALL, in (18), we generate limited number of sequences 57960 by keeping l constant and varying λ from 0 to 137, w from 0 to 69 and $N_{CS} = 23$. For differ-

ent values of l we have same results for PAPR and CM measurements.

Note that in above process, we have not considered all possible combinations of the parameters defined in Eq. (1), (11), (16) and (19) due to resource limitations for computations. Considering the length of sequences $L_{RA} = 139$, the maximum number of different sequences that can be generated from ZC, mZC, aZC and mALL sequences using the Preamble capacities defined in Eq. (3), (14), (17) and (22) are summarised in TABLE- 3. At $N_{CS} = 2$, as defined in [7] for all cases, we can obtain maximum number of different sequences.

We have simulated PAPR and CM measurements using following steps:

- Considering time domain sequences defined in (1), (10), (16) and (18), calculate its FFT of length 139 to convert them into frequency domain.
- Map these to 4096 sub-carriers to be generated as OFDM symbols and take IFFT to form time domain signal.
- Add the cyclic-prefix (CP) of length 288 which will give us the final time domain signal to be transmitted.
- Based on the transmitted time domain signal calculate PAPR and CM using (28) and (30).

In Fig. 6, we see that the ZC sequence performs better than mZC, aZC and mALL sequences. At 99% usage of sequences we have PAPR of 6.6dB, 6.8dB, 7.1dB and 7.05 dB for ZC, aZC, mZC, mALL sequences respectively. This is a very marginal change and can be accommodated when handling other data signals. In Fig. 7, we observe that for CM values between -1dB to 1dB, ZC sequences perform better than mZC, ZC and mALL. But considering at 50% usage of sequences, all sequences perform equally well at 1.2dB. We see a difference in performance at 99% usage of sequences, where mZC and mALL sequence achieve a CM value of 1.8dB, aZC sequence achieves a CM value of 1.9dB and ZC sequence achieves a CM value of 2.4dB.

From Figs. 6 and 7, it is observed that the proposed mALL sequence has an increased capacity for preambles with a negligible loss in PAPR performance. Also, the CM performance of mZC, aZC and mALL is better than ZC by 0.6dB, 0.5dB and 0.6dB respectively.

7.5 Effect of Carrier Frequency Offset [CFO]

CFO is one of the non-idealities faced by OFDM system. We know that OFDM system is sensitive to timing and frequency synchronisations. CFO occurs when the frequency

of local oscillator is not in sync with the carrier frequency. This can occur due to mismatch in transmitter and receiver oscillators and the Doppler effect due to movement of UE. For a received time domain signal $r[n]$, the CFO results in the phase shift of the signal given by [11]:

$$r_{f_0}[n] = r[n] * \exp\left\{\frac{j2\pi f_0 n}{L_{RA}}\right\} \quad (31)$$

where $0 \leq n \leq L_{RA}-1$

where f_0 is the normalised frequency offset due to local oscillator error and Doppler shift. The presence of CFO destroys the orthogonality between sub-carriers and introduces Inter-Carrier Interference (ICI) [33]. From (31), the expression used for calculating the periodic auto-correlation in Eq.(2) becomes:

$$R_{x,x}^f[\tau] = \sum_{n=0}^{L-1} x[n]x^*[(n+\tau) \bmod L_{RA}]e^{\left\{\frac{j2\pi f_0 n}{L_{RA}}\right\}} \quad (32)$$

The effect of frequency offset is more pronounced for ZC sequences [5, 11]. We make use of Eq. (32) to see the effect of CFO on the auto-correlation properties of ZC, mZC, aZC and mALL sequences. Fig. 8, shows the auto-correlation result of ZC sequence. We observe that varying the CFO results into high correlation peaks at non-zero values of cyclic shift (τ). This means that in presence of CFO a transmitted ZC sequence with no cyclic shift can be detected as a cyclic shifted version of itself, which is not desirable as it may lead to false-alarm events. Also it can lead to time estimation errors. The mZC and aZC sequences described in [5] are not sensitive to CFO. This can be seen in Fig. 9 and 10, where we observe the peak auto-correlation values at $\tau = 0$ and CFO = 0. Except this, we see low correlation values at all other values ($\tau \neq 0$ and CFO $\neq 0$). This means that we will not detect ambiguous sequences in presence of CFO. For the proposed mALL sequence in Sect. 5, we simulate the auto-correlations in presence of CFO and the results are shown in Fig. 11. We observe similar results compared to mZC and aZC sequences, where the auto-correlation peak is independent of CFO effect and we face no ambiguity when detecting the transmitted sequence.

8 Conclusion

We have compared the detection performance of our proposed mALL sequence with legacy ZC sequence and other proposed sequences in literature namely, mZC and aZC. We find that the detection performance of ZC sequence has no advantage over the proposed mALL sequence. Moreover the

Table 3 Maximum number of sequences that can be generated where $L_{RA} = 139$ and $N_{CS} = 2$

Sequences	Maximum number of sequences
ZC	9522
mZC	$\approx 1.333 \times 10^6$
aZC	$\approx 1.333 \times 10^6$
mALL	$\approx 1.853 \times 10^8$

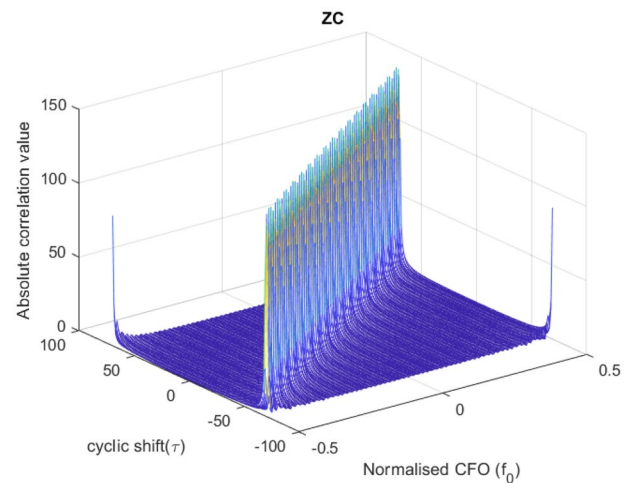


Fig. 8 Periodic correlation of ZC sequence- x_2^0 from Eq. (1) with CFO effect

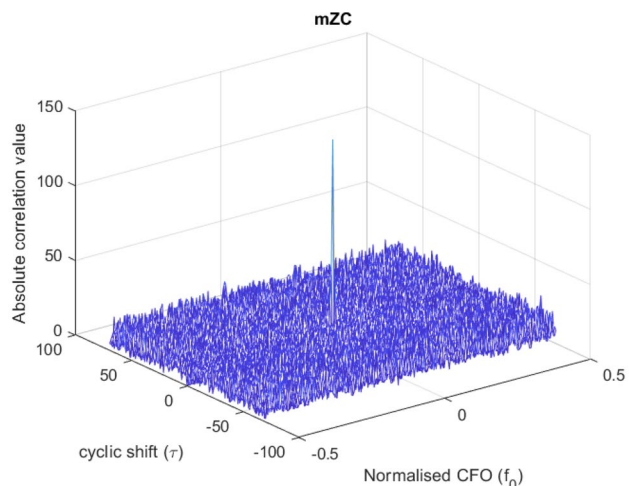


Fig. 9 Periodic correlation of mZC sequence- $z_{1,2,0}$ from Eq. (11) with CFO effect

detection performance of the mALL sequence is at par with other candidate sequences considered and complies with the 3GPP standards. The detection method of preambles

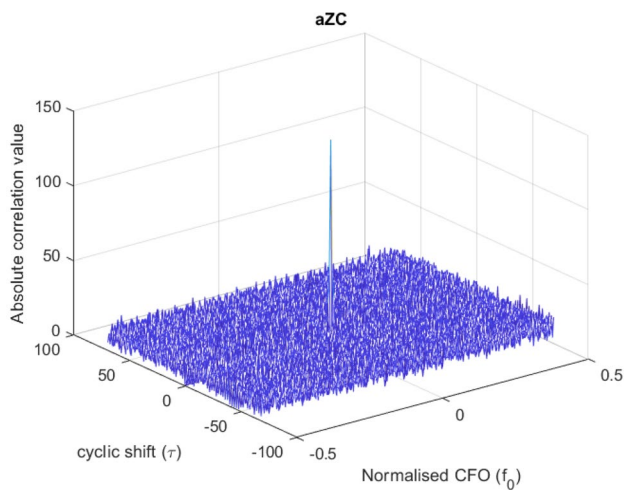


Fig. 10 Periodic correlation of aZC sequence- $z_{1,1,2,1,0}$ from Eq. (16) with CFO effect

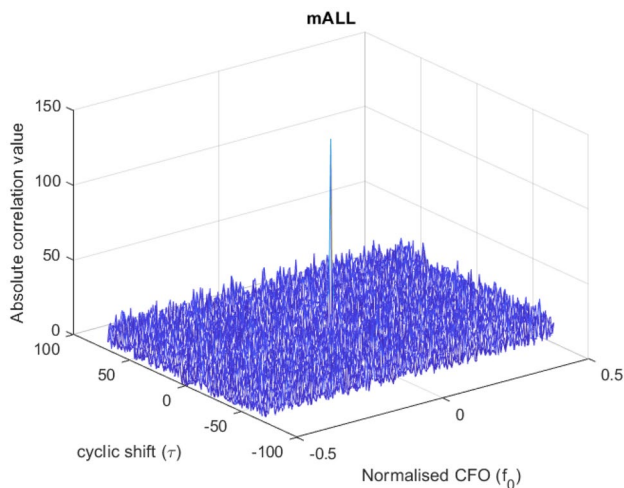


Fig. 11 Periodic correlation of mALL sequence- $z_{1,2,1,0}$ from Eq. (18) with CFO effect

is same for all sequences. This result is expected as the sequences considered in this study have a property of ZCZ or LCZ. The PAPR and CM performance of *mALL* sequence is compared with that of other sequences such as ZC, *mZC* and *aZC*, where the CM performance of *mALL* at 99% usage is very similar to *mZC* and *aZC* with, ZC sequence performing the worst. Thus the proposed *mALL* sequence provides better metric than the legacy ZC sequence. PAPR performance at 99% usage of sequence for proposed sequence is comparable to other sequences considered. We also observe that in presence of CFO, the ZC sequence detection is ambiguous and may lead to false

detection. In contrast the proposed *mALL* sequence detection is non-ambiguous and provides a single peak which may result in reduced false detection. This can be attributed to the fact that the *m-sequence*, used as cover sequence, being robust against the CFO [11]. Above results suggest that the proposed *mALL* sequence provides much higher preamble capacity than the legacy ZC sequences, *mZC*, and *aZC* without any deterioration in performance metrics, which is ideal for the high density connection requirements in mMTC. The preamble capacity of proposed *mALL* sequence is two orders of magnitude larger than *mZC*, *aZC* and four orders of magnitude larger than ZC sequence (Table 3). Thus *mALL* sequence is a big step forward in providing high preamble capacity, considering no degradation in detection performance, improved CM performance and unambiguity in presence of CFO. This increased preamble capacity is expected to improve the SINR (Signal to Interference and Noise Ratio) and reduce the probability of collision. Detailed exploration of the above features in the multi user scenario can be taken up as a future extension of our work.

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