

# An Approach to Increase the Lifetime of a Linear Array of Wireless Sensor Nodes

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**Abstract** The nodes in a wireless sensor network are generally energy constrained. The lifetime of such a network is limited by the energy dissipated by individual nodes during signal processing and communication with other nodes. The issues of modeling a sensor network and assessment of its lifetime have received considerable attention in recent years. This paper provides an analytical framework for placing a number of nodes in a linear array such that each node dissipates the same energy per data gathering cycle. This approach ensures that all nodes run out of battery energy almost simultaneously. It is shown that the network lifetime almost doubles with the proposed scheme as compared to other reported schemes. However, in practice, the nodes are not expected to be placed as per this theoretical requirement. The issue of random placement of nodes has also been investigated to obtain the statistics of energy consumption of a node. The analytical results for random node placement are validated through simulation studies.

**Keywords** Data gathering sensor network · Multi-hop · Inter-node distance · Network lifetime · Random node placement

## 1 Introduction

A wireless sensor network consists of energy-constrained nodes that are deployed for monitoring multiple phenomena of interest. A sensor node consists of a sensing unit, a processing unit, a radio transceiver and a power management unit [1]. Sensor nodes produce some measurable responses to changes in physical or chemical conditions and transmit these responses to a common sink in the form of packets of data over a wireless channel. There are three broad classes of sensor networks, viz. (i) data gathering (clock-driven), (ii) event-driven and (iii) demand-driven [2]. All these categories have been widely considered for studying several issues such as assessment of network lifetime, energy-aware routing procedures, data aggregation and traffic modeling.

The sensor nodes are sometimes deployed in adverse conditions with limited energy that may not be replenished. An accepted definition of lifetime of a sensor network is the time span from the instant when the network is deployed to the instant when the network is considered to be non-functional. A network is considered to be non-functional (a) even when a single sensor node dies, or (b) a percentage of the nodes die or (c) when a loss of coverage occurs due to mobility of the nodes or due to failure of a node [2–4].

In a multi-hop linear wireless sensor network, the nodes closer to the sink may have higher load of relaying packets as compared to the distant nodes. Hence the nodes closer to the sink are likely to get over-burdened and run out of their battery energy sooner. This type of linear sensor network

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has applications in highway traffic monitoring, border line surveillance, oil and natural gas pipeline monitoring etc. to mention a few. Bhardwaj et al. [5] have considered a linear multi-hop network and proposed an upper bound on the lifetime of the network for an optimum number of intermediate nodes. However, this analysis is not applicable to situations where each node in the network senses and transmits its own packet, in addition to the packets received from other nodes.

Shelby et al. [6] have considered a linear many-to-one multi-hop sensor network. The issue of optimal spacing between consecutive nodes in a data gathering network over a given distance has been addressed while minimizing the overall energy consumption during a data gathering cycle. It has been shown that the node farthest from the sink consumes maximum energy as compared to the nodes nearer to the sink. However, in the case of a weighted spacing arrangement, nodes nearer to the sink consume more energy than the distant nodes. So the problem of non-uniform energy consumption by the nodes still exists.

Several authors have focused on the issue of minimization of total energy consumed by the network in a data gathering cycle while allowing dissimilar energy consumption by the nodes. This approach results in faster burn out of nodes far away from the sink. A different approach has been reported in [7], where nodes are intelligently placed in a linear array ensuring that all nodes run out of energy at the same time.

Haenggi [8] has found the inter-node distance for a linear sensor network ensuring equal energy consumption. This analysis has been done considering only the energy required to transmit a packet. Energy requirement for receiving a packet and idle state energy are not included in the analysis.

It may be noted that, none of the reported work has addressed the issue of random node placement and its effect on energy dissipation profile of the array. These issues are important in the practical scenarios. Another common feature of the related reported work is that the total energy consumed by the overall network is minimized. This implies that the nodes dissipate different amount of energy over a data gathering cycle. In our work, we enforce equal energy dissipation by each and every node except the sink in our network. This approach helps to eliminate the non-uniform energy consumption pattern in

the network. We give an explicit analysis of node placement strategy ensuring same energy dissipation by all nodes in a data gathering cycle. An expression for network lifetime is also developed. We have compared our results with [6] and [8]. With our scheme, the lifetime is seen to increase significantly. In the process, we have introduced a new performance measure of the whole network, viz. energy utilization ratio ( $\eta$ ), which indicates the percentage of energy of the network that is consumed when the network dies. It is shown that energy utilization as per our scheme is much higher than other comparable schemes. We also address the issue of random placement of nodes to obtain the statistics of energy consumption of a node. The analytical results are also validated through simulation studies.

The rest of the paper is organized as follows. Section 2 gives the system description and problem formulation. Section 3 illustrates the proposed scheme. Random node placement is analyzed in Sect. 4. In Sect. 5, we present extensive analytical and simulation results. Finally, Sect. 6 concludes the paper.

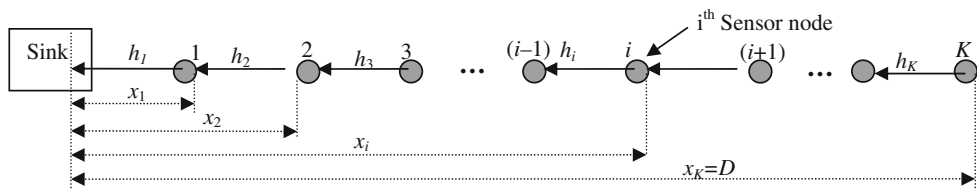
### 2 System Description

A linear array of  $K$  wireless sensor nodes is considered with the sink at one end (Fig. 1). We assume that all the  $K$  nodes have same initial energy of  $E_0$  units. The distance between  $i$ th node and  $(i-1)$ th node is indicated as  $h_i$  units for  $2 \leq i \leq K$ . The distance between the sink and the 1st node is denoted as  $h_1$ . The farthest  $K$ th node is at a distance of  $D$  units from the sink. For this model

$$\sum_{i=1}^K h_i = D \tag{1}$$

Let  $x_i$  be the distance between the sink and the  $i$ th node where  $x_i = \sum_{j=1}^i h_j$  for  $1 \leq i \leq K$ . We consider a data-gathering network where each node generates one packet of equal length ( $B$  bits) over a data gathering cycle of  $T_d$  sec. We have assumed that a sensor node has adjustable sensing range and is equal to half of the corresponding maximum inter-node distance. This assumption requires maximum inter-node distance. This assumption requires maximum sensing range for the node farthest from the sink. We also assume that sensed data can fit into one data gathering cycle.

**Fig. 1** Linear array of wireless sensor nodes



A node sends a packet to the sink by using the nearest neighbour towards the sink as a repeater. Nodes closer to the sink are expected to forward all the packets towards the sink. One can incorporate data aggregation at the node level by incorporating extra processing software to all the nodes. In our model, no data aggregation is assumed for simplicity. We assume that each node can deal with maximum  $P$  packets/second. This implies that  $P \cdot T_d \geq K$ .

### 2.1 Energy Dissipation Model of a Node

The energy dissipation model (Fig. 2) for radio communication is assumed similar to [9] and [10], following which the energy consumed by the  $i$ th node for transmitting a packet to the  $(i-1)$ th node over a distance  $h_i$  is

$$E_{t,i} = e_t + e_d h_i^n \quad (2)$$

In  $E_{t,i}$ , the suffix  $t$  denotes the transmission and  $i$  denotes the node number and it varies between 1 and  $K$ . The term  $e_t$  is the amount of energy spent per packet in the transmitter electronics circuitry and  $e_d h_i^n$  is the amount of energy necessary for transmitting a packet satisfactorily to the  $(i-1)$ th node over a distance  $h_i$ . The constant ' $e_d$ ' is dependent on the transmit amplifier efficiency, antenna gains and other system parameters. The path loss exponent is  $n$  (usually  $2.0 \leq n \leq 4.0$ ) [11]. On the receiving end, the amount of energy spent to capture an incoming packet of  $B$  bits is  $e_r$  units. The radio is assumed to consume energy even during idle state, i.e., when the radio neither transmits nor receives. The idle state energy consumption is equal to  $e_{id} T_{id} P$ , where  $T_{id}$  is the idle time and  $e_{id} = c \cdot e_r$  is the idle state energy spent per packet duration, where  $0 < c \leq 1.0$  [9].

Let the maximum radio range of a sensor node be  $R_r$  units. Perfect power control is employed i.e., the radio of the node is capable of adjusting its transmitting power to reach a node at a distance less than  $R_r$  from it. We assume an idealistic channel where the channel can be represented by radio disc model. The uncertainty in received signal strength due to fading channel has not been considered in our analysis. As a result, there is no message failure due to insufficient signal to noise ratio within the radio range from a transmitting node. Further, we assume that there is no

collision between the transmitted packets. We have also assumed that  $h_i \leq R_r$ , for  $1 \leq i \leq K$ .

In the next section, we determine the distance  $h_i$  between neighboring sensor nodes ( $1 \leq i \leq K$ ) such that each node spends same energy over a data gathering cycle. This constraint ensures that all the nodes get exhausted of their stored battery energy almost simultaneously. The lifetime that can be achieved by such an array of sensor nodes is of interest.

### 3 Regular Placement of Sensor Nodes for Equal Energy Dissipation

According to the system model, the number of packets  $A_r(i)$  received by the  $i$ th node per data gathering cycle is

$$A_r(i) = K - i, \text{ for } 1 \leq i \leq K \quad (3)$$

The number of packets  $A_t(i)$  transmitted by the  $i$ th node including its own packet per data gathering cycle is

$$A_t(i) = (K - i) + 1 = A_r(i) + 1, \text{ for } 1 \leq i \leq K \quad (4)$$

The idle time,  $T_{id}(i)$  i.e., the fraction of a data gathering cycle ( $T_d$ ) over which the radio of the  $i$ th sensor node neither transmits nor receives any packets may be expressed as

$$T_{id}(i) = \left( T_d - \frac{2(K - i) + 1}{P} \right), \text{ for } 1 \leq i \leq K \quad (5)$$

where,  $P$  is the packet dealing rate of a node and the term  $[2(K-i) + 1]/P$  is the busy time for  $i$ th node over  $T_d$ . The scheduling time has not been considered in our analysis.

Following the energy consumption model, the total amount of energy  $E(i)$  spent by the  $i$ th node per data gathering cycle is,

$$E(i) = E_1 - iE_2 + e_d(K - i + 1)h_i^n, \text{ for } 1 \leq i \leq K \quad (6)$$

where,  $E_1$  and  $E_2$  are constants and defined as

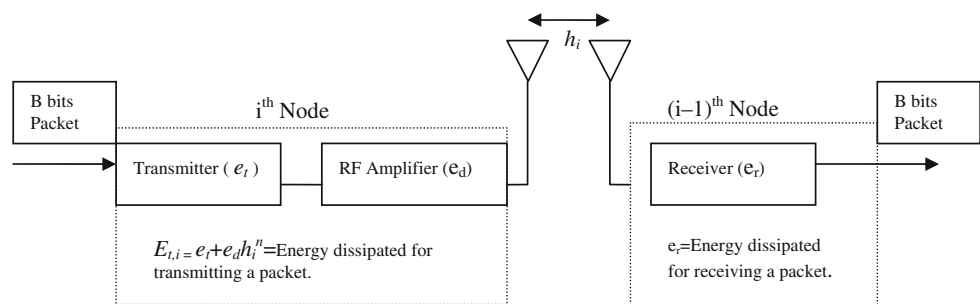
$$E_1 = e_t(K + 1) + e_r K + e_{id}(PT_d - 2K - 1) \quad (7)$$

and

$$E_2 = (e_t + e_r - 2e_{id}) \quad (8)$$

Now, imposing the condition that all the nodes dissipate same energy  $E$  joule per data gathering cycle

**Fig. 2** Radio energy consumption model



i.e.,  $E(i) = E$ , for  $1 \leq i \leq K$ , the inter-node distance  $h_i$  can be expressed as

$$h_i = \left[ \frac{1}{(K-i+1)e_d} [E + (K-i)(2e_{id} - e_r) - (PT_d - 1)e_{id} - e_r(K-i+1)] \right]^{\frac{1}{n}}, \quad 1 \leq i \leq K \tag{9}$$

The distance  $h_i$  can be obtained from (9) normalized to  $h_K$ , the inter-node distance between  $K$ th node and  $(K-1)$ th node. A solution for  $h_K$  can be obtained from (10) graphically:

$$C_1 h_K^n - D h_K^{n-1} - C_2 = 0 \tag{10}$$

where,  $C_1 = \sum_{i=1}^K i^{-1/n}$  and  $C_2 = \frac{(e_r + e_r - 2e_{id})}{n e_d} \sum_{i=1}^K (i-1) i^{-1/n}$

It is interesting to note that each node dissipates a minimum energy  $E_{min}$  joule based on the values of  $D, K, e_r, e_r, e_d$  and  $e_{id}$  as obtained from (9)

$$E_{min} = K e_r + (K-1) e_r + (PT_d - 2K + 1) e_{id} \tag{11}$$

For known values of radio parameters, the feasible solution of  $h_i$  can be found when  $E > E_{min}$ , as has been implicitly assumed in (9).

In the next section, we present an analysis about the effects of random node placement on the overall energy dissipation of the array.

#### 4 Analysis for Random Node Placement

In the previous section we have derived the exact position of  $K$  nodes over a distance ( $D$ ) to ensure equal energy dissipation by each node. However, in practice, the nodes

distance  $D$  into  $(K-1)$  unequal bin width (Fig. 3). The width of each bin is decided by  $x_i$  and  $h_i$ . We assume that each bin contains one node. Let the position of the  $K$ th node be fixed at distance  $D$  from the sink.

We assume that the position of the  $i$ th node is a random variable ( $Y_i$ ) and it varies between  $(x_i - \frac{h_i}{2})$  and  $(x_i + \frac{h_{i+1}}{2})$ , for  $i = 1$  to  $(K-1)$ . Assuming  $Y_i$  follows a uniform probability density function (pdf),  $f_{Y_i}(y_i)$ ,

$$f_{Y_i}(y_i) = \begin{cases} \frac{2}{h_i + h_{i+1}}, & \left(x_i - \frac{h_i}{2}\right) < y_i \leq \left(x_i + \frac{h_{i+1}}{2}\right) \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

where,  $1 \leq i \leq K-1$

Now, the inter-node distance  $Z_i$  may be expressed as

$$Z_i = \begin{cases} Y_i, & i = 1 \\ Y_i - Y_{i-1}, & 2 \leq i \leq K-1 \\ D - Y_{i-1}, & i = K \end{cases} \tag{13}$$

The energy consumption ( $\varepsilon_i$ ) of the  $i$ th node may be expressed from (6) as:

$$\varepsilon_i = E_3(i) + a(i) Z_i^n, \quad \text{for } 1 \leq i \leq K \tag{14}$$

where,

$$E_3(i) = E_1 - i E_2 \text{ and } a(i) = e_d (K - i + 1), \quad \text{for } 1 \leq i \leq K \tag{15}$$

The probability density function (pdf) of  $\varepsilon_i$  can be obtained as

For  $i = 1$ :

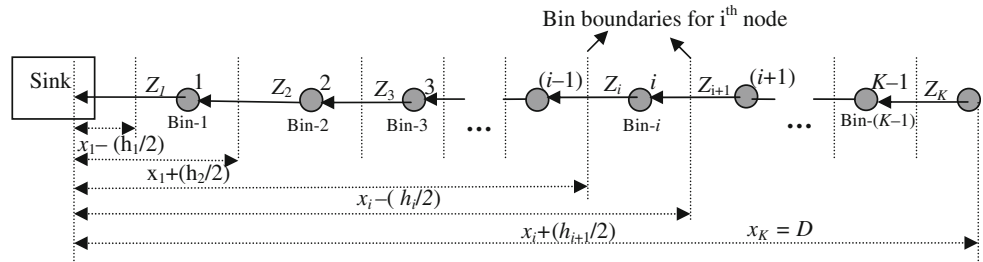
$$f_{\varepsilon_i}(\varepsilon_i) = \begin{cases} \frac{2}{n t_i a(i)} \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{1}{n}-1\right)}, & E_3(i) + a(i) \left(\frac{h_i}{2}\right)^n < \varepsilon_i \leq E_3(i) + a(i) \left(h_i + \frac{h_{i+1}}{2}\right)^n \\ 0, & \text{otherwise} \end{cases}$$

are not expected to be placed so ideally. In this section we consider a situation wherein the position of a node is random within a defined distance bin. We divide the total link

For  $2 \leq i \leq K-1$ :

$$f_{\varepsilon_i}(\varepsilon_i) = \begin{cases} \frac{4}{n s_i t_i a(i)} \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{2}{n}-1\right)}, & E_3(i) < \varepsilon_i \leq E_3(i) + a(i) \left(\frac{s_i}{2}\right)^n \\ \frac{2}{n t_i a(i)} \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{1}{n}-1\right)}, & E_3(i) + a(i) \left(\frac{s_i}{2}\right)^n < \varepsilon_i \leq E_3(i) + a(i) \left(\frac{t_i}{2}\right)^n \\ -\frac{4}{n s_i t_i a(i)} \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{2}{n}-1\right)} + \frac{2}{n a(i)} \left( \frac{1}{s_i} + \frac{1}{t_i} \right) \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{1}{n}-1\right)}, & E_3(i) + a(i) \left(\frac{t_i}{2}\right)^n < \varepsilon_i \leq E_3(i) + a(i) \left(\frac{s_i + t_i}{2}\right)^n \\ 0, & \text{otherwise} \end{cases}$$

**Fig. 3** Scheme for random node placement with variable bin width



For  $i = K$ :

$$f_{\bar{\varepsilon}_i}(e_i) = \begin{cases} \frac{2}{ns_i a(i)} \left( \frac{e_i - E_3(i)}{a(i)} \right)^{\left(\frac{1}{n}-1\right)}, E_3(i) + a(i) \left( \frac{h_i}{2} \right)^n < e_i \leq E_3(i) + a(i) \left( h_i + \frac{h_{i-1}}{2} \right)^n \\ 0, \text{ otherwise} \end{cases} \tag{16}$$

where,  $s_i = h_i + h_{i-1}$ , for  $2 \leq i \leq K$ ; and  $t_i = h_i + h_{i+1}$ , for  $1 \leq i \leq K-1$ .

The mean of  $\varepsilon_i$  may be expressed as:

$$\bar{\varepsilon}_i = \begin{cases} E_3(i) + \frac{2a(i)}{t_i(n+1)} \left\{ \left( \frac{t_i+h_i}{2} \right)^{n+1} - \left( \frac{h_i}{2} \right)^{n+1} \right\}, i=1 \\ E_3(i) + \frac{a(i)2^{-n} \{ (s_i+t_i)^{n+2} - s_i^{n+2} - t_i^{n+2} \}}{(n+1)(n+2)s_i t_i}, 2 \leq i \leq K-1 \\ E_3(i) + \frac{2a(i)}{s_i(n+1)} \left\{ \left( \frac{s_i+h_i}{2} \right)^{n+1} - \left( \frac{h_i}{2} \right)^{n+1} \right\}, i=K \end{cases} \tag{17}$$

The variance of  $\varepsilon_i$ ,  $\sigma_{\varepsilon_i}^2 = \overline{\varepsilon_i^2} - (\bar{\varepsilon}_i)^2$  is

$$\sigma_{\varepsilon_i}^2 = \begin{cases} \frac{2a(i)^2}{t_i} \left[ \frac{1}{(2n+1)} \left\{ \left( \frac{h_i+t_i}{2} \right)^{2n+1} - \left( \frac{h_i}{2} \right)^{2n+1} \right\} - \frac{2}{t_i(n+1)^2} \left\{ \left( \frac{h_i+t_i}{2} \right)^{n+1} - \left( \frac{h_i}{2} \right)^{n+1} \right\}^2 \right], i=1 \\ \frac{a(i)^2}{2^{2n}s_i t_i} \left[ \frac{(s_i+t_i)^{2(n+1)} - s_i^{2(n+1)} - t_i^{2(n+1)}}{2(n+1)(2n+1)} - \frac{1}{s_i t_i} \left\{ \frac{(s_i+t_i)^{n+2} - s_i^{n+2} - t_i^{n+2}}{(n+1)(n+2)} \right\}^2 \right], 2 \leq i \leq K-1 \\ \frac{2a(i)^2}{s_i} \left[ \frac{1}{(2n+1)} \left\{ \left( \frac{h_i+s_i}{2} \right)^{2n+1} - \left( \frac{h_i}{2} \right)^{2n+1} \right\} - \frac{2}{s_i(n+1)^2} \left\{ \left( \frac{h_i+s_i}{2} \right)^{n+1} - \left( \frac{h_i}{2} \right)^{n+1} \right\}^2 \right], i=K \end{cases} \tag{18}$$

Mean and variance of  $\varepsilon_i$  depend on the radio parameters,  $K$ ,  $n$  and  $h_i$ . In the next section, we present numerical and simulation results.

### 5 Results and Case Studies

#### 5.1 Regular Node Placement

In this sub-section we present some numerical examples to study the issue of node placement and its effects on

network lifetime. Specifically, the following issues are considered:

- (i) Node placement strategies to obtain desired energy consumption pattern (equal energy dissipation).
- (ii) Energy consumption pattern of each node to give an insight of how placement of a node affects its energy consumption profile in a data gathering cycle.
- (iii) Comparison of network lifetime and energy utilization ratio ( $\eta$ ) achieved by different node placement schemes.

Let  $E_{th}$  be the threshold of residual energy below which a sensor node becomes non-functional. Also, let  $E_{max}$

denote the maximum energy consumed by a node over a data gathering cycle in a sensor network. Since the duration of each data gathering cycle is  $T_d$  units, the network lifetime ( $T_{life}$ ) is

$$T_{life} = T_d \left( \frac{E_0 - E_{th}}{E_{max}} \right) \tag{19}$$

Energy utilization ratio ( $\eta$ ) is defined as

$$\eta = \frac{E_{used}}{E_T} \times 100\% \tag{20}$$

**Table 1** System parameters considered for performance analysis

Parameter	Value
Link distance, $D$	3,000 m
Packet length, $B$	512 bits
Packet dealing rate of radio, $P$	1 Packet/s
Duration of data gathering cycle, $T_d$	500 s
Path loss exponent, $n$	2.0
$e_t$	25.6 $\mu$ J/packet
$e_r$	25.6 $\mu$ J/packet
$e_d$ (for $n = 2$ )	51.2 nJ/packet/m <sup>2</sup>
$e_{id}$ (assuming $c = 0.9$ )	23.04 $\mu$ J/packet
Initial battery energy, $E_0$ (assuming 0.5 A-h and 3 V battery)	5.4 KJ
Threshold energy, $E_{th}$	15 mJ
Radio Range, $R_r$	300 m

where,  $E_{used}$  is the total energy utilized by the network during its lifetime and  $E_T$  is the total deployed energy. For  $K$  nodes  $E_T$  equals to  $KE_0$ .

For all the studies we consider a typical set of parameters as shown in Table 1. Following three schemes have been used for performance comparison:

*Scheme a*—All nodes have equal inter-node spacing i.e.  $h_i = D/K$  [6].

*Scheme b*—Nodes are placed for minimizing the overall energy dissipation in a data gathering cycle [6].

*Scheme c*—This is our proposed scheme. Here, the nodes are placed so that each node dissipates equal amount of energy in a data gathering cycle.

First we consider free space communication (i.e. path loss exponent,  $n = 2.0$ ) for ease of explanation. Typical values of other relevant parameters in Table 1 have been chosen following [9] closely. The radio parameters are given on a per packet basis.

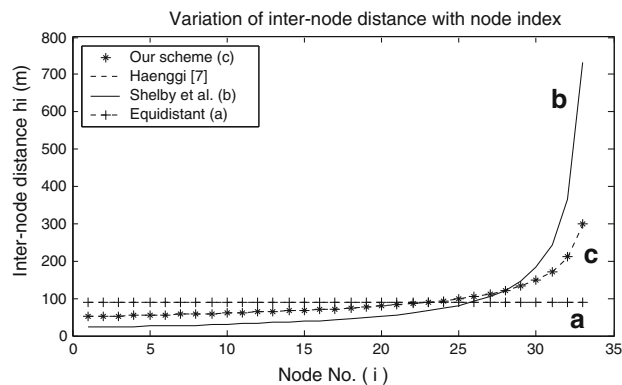
Using the constraint  $h_K \leq R_r$  we get form (10)

$$\sum_{i=1}^K (2R_r^2 - (i - 1)e')/\sqrt{i} \geq 2R_rD \tag{21}$$

where, the constant  $e' = (e_t + e_r - 2e_{id})/e_d$ .

The minimum value of  $K$  satisfying the inequality (21) is 33 for the chosen parameter values.

Figure 4 shows comparison of inter-node distance among schemes-*a*, *b* and *c*. Curves-*a*, *b* and *c* describe the inter-node distance for schemes-*a*, *b* and *c*, respectively. Nodes are equally spaced for scheme-*a* and it is reflected from curve-*a*. For curves-*b* and *c* the inter-node distance increases as node index ( $i$ ) increases. For node index  $i < 28$ , the inter-node distance for scheme-*b* is smaller than that of the scheme-*c*. For node index  $i \geq 28$ , the inter-node



**Fig. 4** Inter-node distance for different node placement ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) for scheme-*a*; (b) for scheme-*b*; (c) for scheme-*c*

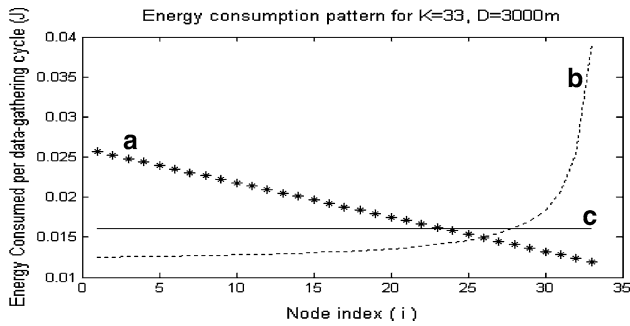
distance for scheme-*b* is larger than that of our scheme-*c*. Inter-node distance for the farthest node from the sink has highest value for both the schemes-*b* and *c*. In light of connectivity of the array, the farthest node has major role. The inter-node distances for the farthest node from the sink are 725 and 300 m for schemes-*b* and *c*, respectively. Thus higher radio range or more powerful radio is required for scheme-*b* to cover a given distance. On the other hand scheme-*c* requires moderate radio range to cover the same distance. Usually a sensor node has low or moderate radio range. Thus in the context of connectivity of the array, our scheme-*c* outperforms scheme-*b*.

We have also compared our result with [8]. In [8], only the energy consumed in the transmitter amplifier has been considered for balancing energy consumption. In this paper we have included other possible energy consumption components like transmitter electronics energy, receiving energy and idle state energy while maintaining the energy balancing conditions. There is a slight difference in inter-node distance between our scheme and [8]. However, this is not clear from Fig. 4 since the ratio of the difference to the maximum value of inter-node distance is very small.

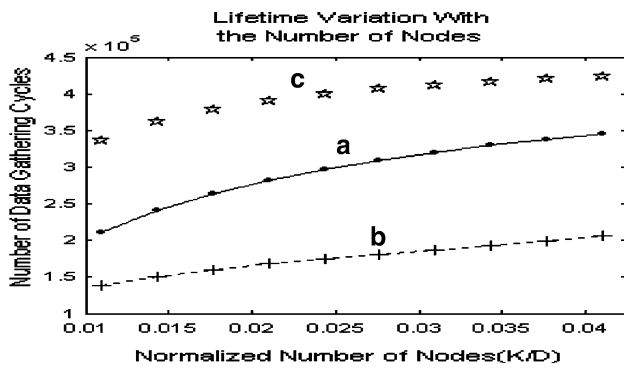
Figure 5 shows the variation of energy consumption pattern of each node per data gathering cycle. Curves-*a*, *b* and *c* show the patterns for the schemes-*a*, *b* and *c* respectively. Curves-*a* and *b* reflect non-uniform energy dissipation profile while curve-*c* shows the uniform one. The node nearest to the sink limits the lifetime of the network for scheme-*a* while the farthest node from the sink limits it for scheme-*b*. Curve-*c* shows uniform energy dissipation pattern where each node dissipates equal energy per data gathering cycle. This ensures that all nodes will run out their battery energy almost simultaneously.

Figure 6 compares the number of data gathering cycles with normalized number of nodes ( $K/D$ ) for different node placement strategies. It shows that lifetime increases with the increase of number of nodes,  $K$  (for a fixed link distance





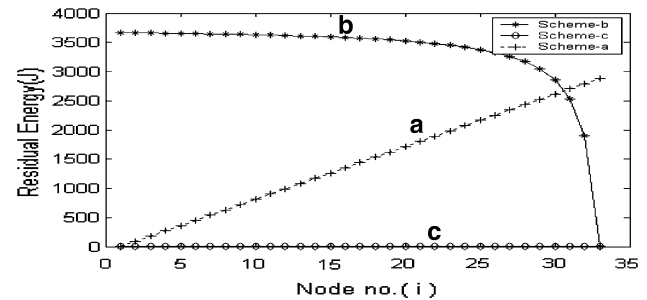
**Fig. 5** Energy consumption pattern for different node placement ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) for scheme-a; (b) for scheme-b; (c) for scheme-c



**Fig. 6** Lifetime variation with number of nodes deployed ( $D = 3,000$  m,  $n = 2.0$ ): (a) for scheme-a; (b) for scheme-b; (c) for scheme-c

$D$ ). However, for higher value of  $K$  the increment in network lifetime is marginally low because the hop distances are becoming smaller and smaller as  $K$  increases and receiving energy for packets are dominating. Curve-c shows the highest number of data gathering cycles that can be achieved when the nodes are placed according to scheme-c. Scheme-b results in lower number of data gathering cycles (i.e. smaller lifetime) for the same set of system parameters. As an example, for  $K/D = 0.03$ , scheme-c offers about 2.5 and 1.3 times lifetime improvement while comparing with schemes-b and a respectively. Scheme a, though simple, performs moderately.

Figure 7 shows the variation of residual energy of the nodes for different schemes. It indicates that for scheme-c there is hardly any residual energy left to the nodes when the network dies. We have derived the value of the performance measure energy utilization ratio ( $\eta$ ) for all the schemes. Numerical results show that 73.29% and 38.43% of the deployed energy is utilized for schemes-a and b respectively. On the other hand our proposed scheme has the highest value of  $\eta$  and it is about 100%. Thus by ensuring equal energy consumption one can gain two things: effectively no residual energy when network dies and the consequence of that is gaining a maximum network



**Fig. 7** Residual energy of the network ( $D = 3,000$  m,  $K = 33$ ,  $n = 2.0$ ): (a) for scheme-a; (b) for scheme-b; (c) for scheme-c

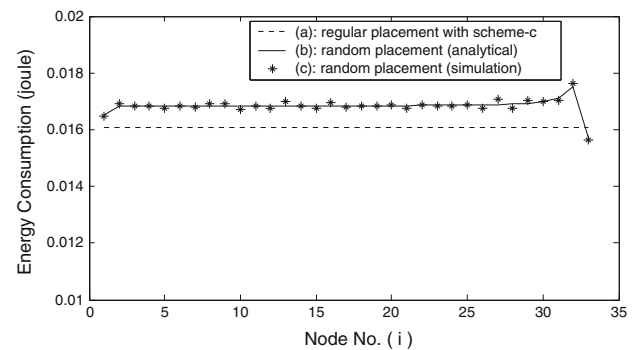
lifetime. In the next sub-section we present the results with random node placement.

## 5.2 Random node placement

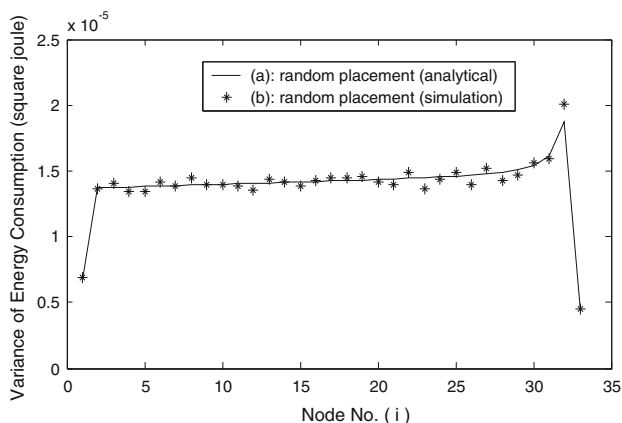
In this sub-section we present numerical and simulation results for random node placement. We show results for two types of random node placement. In case-1, we present results by considering the variable bin width while in case-2 we present that for fixed bin width.

*Case-1 (Variable bin width):* In this case we present the effect of random node placement with variable bin width on overall energy dissipation of the array. In the previous sub-section, we have seen that scheme-c only provides uniform energy dissipation over the array. Thus, we present the results only for scheme-c. The analytical results are validated through the simulation studies using MATLAB. For simulation, we have generated  $(K-1)$  independent uniformly distributed random numbers. The generated random numbers represent the position of nodes on the array. In our simulation we have assumed fixed position of the  $K$ th node at a distance  $D$  from the sink. The result is obtained by running the simulation for 2,000 times.

Figure 8 compares the energy consumption profile between regular and random placement of nodes. The



**Fig. 8** Comparison of energy consumption profile of nodes ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) regular placement (scheme-c); (b) analytical result (scheme-c) for random placement with variable bin width; (c) simulation result (scheme-c) for random placement with variable bin width



**Fig. 9** Variance of energy consumed by nodes ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) analytical result for random placement with variable bin width; (b) simulation result for random placement with variable bin width

simulation results agree with the analytical results. By introducing randomness in node placement the energy consumption profile is not significantly changed from its regularly placed counterpart. It is interesting to note that by introducing randomness in node placement most of the nodes spend about 1 mJ more energy per data gathering cycle in comparison to regular placement. The  $(K-1)$ th node dissipates maximum energy per data gathering cycle for random placement. However, the difference of energy consumption from other nodes is not so significant. The discrepancy in energy consumption by the 1st node and  $K$ th node from the rest of the nodes is due to inter-node distance. According to the present random node placement strategy, the distance between 1st node and sink i.e.  $Z_1$  and distance between  $K$ th node and  $(K-1)$ th node i.e.  $Z_K$  are lesser than the other values of  $Z_i$  for  $2 \leq i \leq K-1$ . This study shows that equal energy dissipation condition is achieved for the case where nodes are placed randomly.

Figure 9 shows the variance of energy consumed by nodes for random placement with variable bin width. The analytical result for variance of  $\epsilon_i$  is also validated through the simulation studies. The profile of the energy variance is similar to mean energy dissipation profile. In case-2, we present results for another strategy of random node placement.

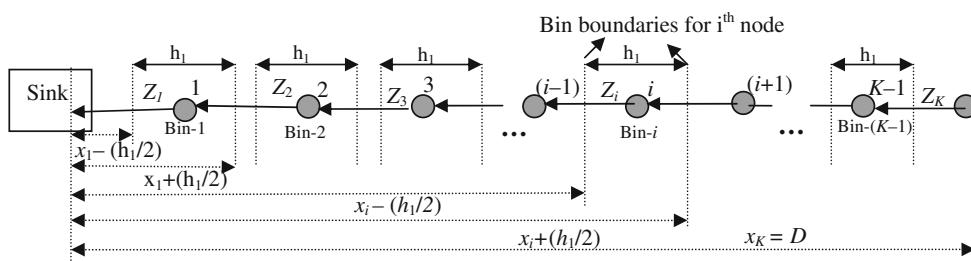
**Case-2 (Fixed bin width):** Another interesting random node placement is shown in Fig. 10. Here, all the bin widths are equal to the smallest inter-node distance i.e.  $h_1$ . The bin boundaries for the  $i$ th node are  $[x_i - (h_1/2)]$  and  $[x_i + (h_1/2)]$  for  $1 \leq i \leq K-1$ . In this case we assume that each bin contains one node and its position is uniformly distributed within the bin boundaries where the mean position is the exact position of the node for regular placement with scheme-c. The performance of this type of random node placement is compared with the previous type of node placement strategy.

Figures 11 and 12 show the variation of mean and variance of energy with node number. The mean of energy consumption for random placement with fixed bin width is decreasing with node number and it converges to the regular placement for the nodes far away from the sink. This convergence takes place due to the introduction of fixed bin width i.e. node placement error is fixed for all the  $K$  nodes. The placement error affects the statistics of the nodes nearer to the sink while it does not affect that of the nodes far away from the sink. It is clear from Fig. 12 that the variance is decreasing with node number for fixed bin width while it is almost same for variable bin width. The random node placement scheme with fixed bin width outperforms the variable bin width scheme.

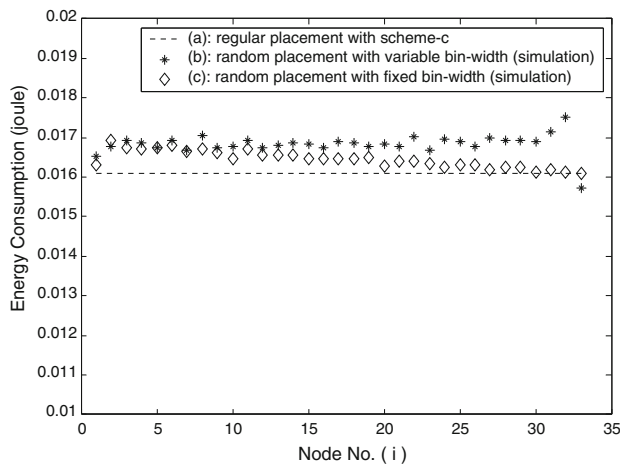
### 6 Conclusion

In this paper we have considered a linear array of  $K$  wireless sensor nodes over a distance ( $D$ ). An exact placement of nodes has been obtained in order to ensure equal energy dissipation by each node in a data gathering cycle. We have compared network lifetime that provides our proposed scheme with two other schemes. It is found that maximum network lifetime is achieved when each node dissipates same energy per data gathering cycle. The residual energy in the network is also minimized in the process. We have also addressed the issue of random node placement and its effect on energy consumption profile. It is evident from our study that equal energy consumption condition can also be achieved for restricted random node placement. The result of random analysis can be used for

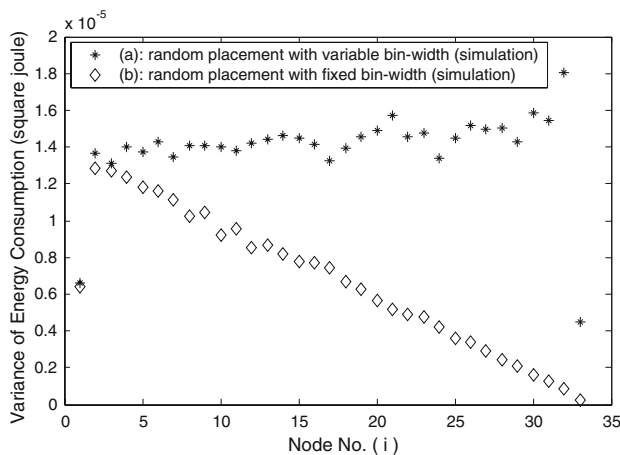
**Fig. 10** Random node placement with fixed bin width (Case-2)







**Fig. 11** Variation of mean energy with node number ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) regular placement with scheme-c; (b) random placement with variable bin width; (c) random placement with fixed bin width



**Fig. 12** Variation of energy variance with node number ( $K = 33$ ,  $D = 3,000$  m,  $n = 2.0$ ): (a) random placement with variable bin width; (b) random placement with fixed bin width

mobile wireless sensor network with restricted movement of nodes.

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