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Rapid short communication

Mathematical model for the viscoelastic properties of dura mater

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Introduction

It is becoming increasingly important to produce accurate mathematical models of human tissue for use in finite element analysis and the development of replacement materials. In particular, accurate models of the cord and surrounding tissues are required to increase the level of understanding of spinal cord injury (SCI) and to develop improved methods of protection. The dura mater provides essential protection to the cord, and its behavior both under insult and during subsequent relaxation is of primary importance when modeling SCIs.

Several studies have investigated the mechanical behavior of cranial dura mater, including its properties under tensile load[11] and biaxial tension[2] and the viscoelastic characteristics.[4,7] Tencer et al.[10] and Runza et al.[9] studied dura samples from the spine to determine the tensile properties, but no one appears to have investigated the viscoelastic behavior of spinal dura mater. This study reports a method and the preliminary results of an investigation into the time-dependent characteristics of this tissue under circumferential and longitudinal loading.

Methods

Experimental study

Samples of bovine spinal cord were obtained from an abattoir; the dura mater was removed and frozen within 2h of slaughter. Prior to testing, the dura was thawed and sliced open along one side. Samples of dimensions 6–8 × 30–40mm were cut in either the circumferential or the longitudinal direction. The method of gripping the specimen was similar to that developed by Bilston and Thibault[1] for spinal cord samples. The ends were dried and bonded to strips of high-density polyethylene using cyanoacrylate adhesive; the central section was kept moist by placing the sample on a swab soaked in saline. To measure the strain, a matrix of varnish dots was applied along the length of the specimen. The sample was then transferred to a bench-top tensile testing machine (United Calibration, Stanton, CA, USA), and the plastic strips at either end were gripped in the rig jaws. Care was taken not to stretch the sample while inserting it into the rig. The specimen was held in a continual flow of saline solution heated to 37°C to maintain the moisture content and to keep the specimen at body temperature during the test.

The specimens were preconditioned at the same strain rate and to the same maximum strain as for the subsequent test. To determine the optimum number of preconditioning cycles, the percentage difference in the maximum force generated on two consecutive cycles was calculated. After 10 cycles, it was found that the change had dropped to less than 1% in the circumferential direction and 3% in the longitudinal direction. Further preconditioning did not significantly alter these values and added to the risk of specimen dehydration or tearing at the grip interface.

After the 10 preconditioning cycles, each specimen was extended at a constant strain rate (range 0.03–$0.13\,s^{-1}$) to a specific strain (range 2%–46%) and held for up to 30min. During the loading period, the matrix of dots on the specimen was filmed with a video camera (Panasonic WV BC200; Matsushita, Osaka, Japan). Image analysis software (Image Pro Plus 3.0; Media Cybernetics, Silver Spring, MD, USA) was used to detect the center of each dot on every frame. The

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relative displacement between dots was then used to calculate the axial strain.

Mathematical model

The mathematical model was based on the quasilinear theory developed by Fung[3] in which it is assumed that the relaxation function can be separated into a reduced relaxation function $G(t)$ and an elastic response $\sigma^e(\varepsilon)$, which is a function of the strain alone. By using the general theory of linear viscoelasticity, the stress as a response to a strain history $\varepsilon(t)$ is given by

$$\sigma(t) = \int_{-\infty}^{t} G(t-\tau)\frac{\partial\sigma^e[\varepsilon(\tau)]}{\partial\varepsilon}\cdot\frac{\partial\varepsilon}{\partial\tau}\cdot d\tau \tag{1}$$

Several forms of elastic response are documented in the literature for soft tissues.[6] In this case, because of the predominance of collagen fibers in the dura,[9] the response used by Haut and Little[5] was assumed

$$\sigma^e = C\varepsilon^2 \tag{2}$$

where C is a constant with the units of stress.

The relaxation function was taken to be of the form

$$G(t) = A\ln t + B \tag{3}$$

where A and B are dimensionless constants derived by approximating the reduced relaxation function.[3] The strain in this case is given by

$$\varepsilon(t) = \beta[t - (t - t_1)u(t - t_1)] \tag{4}$$

where $u(t)$ is the unit step function, and β is the strain rate.

By replacing Eqs. (2) to (4) into Eq. (1), the stress as a function of time becomes

$$\sigma(t) = k\beta^2\mu\left[\frac{t^2}{2}\ln(t) - \frac{3}{4}t^2\right] + \frac{k\beta^2t^2}{2} \text{ for } t \leqslant t_1 \tag{5a}$$

$$\sigma(t) = k\beta^2\mu\left[\frac{(t_1^2 - t^2)}{2}\ln(t - t_1) - \frac{1}{4}(t_1^2 + 2t_1t) + \frac{1}{2}t^2\ln(t)\right] + \frac{k\beta^2t_1^2}{2} \text{ for } t > t_1 \tag{5b}$$

where k equals $2BC$, and μ equals A/B.

Results

Nine tests were carried out: six in the longitudinal direction and three in the circumferential direction. The data from each test were fitted to the model using a generalized reduced gradient non-linear optimization algorithm (Microsoft Excel Solver; Microsoft, Redmond, WA, USA) to obtain the values for k and μ that minimized the sum of the errors squared. The results of the tests are summarized in Table 1, and a typical plot is shown in Fig. 1. In all cases, the model specified by Eq. 5 fitted the data extremely well, as indicated by the high values of R^2 shown in the table. Both k and μ were greater in the circumferential direction, with the elastic constant k being more than two orders of magnitude lower in the longitudinal direction. The relaxation time in the circumferential tests was also longer than in the longitudinal direction (average times

Table 1. Summary of results

Direction	k (MPa)	μ	R^2
Longitudinal	1.2 (0.2–3.4)	−0.04 (−0.02 to −0.05)	0.81 (0.77–0.98)
Circumferential	138 (92–380)	−0.09 (−0.07 to −0.09)	0.95 (0.94–0.97)

The median values of the two coefficients and the coefficient of determination R^2 are shown (ranges are in parentheses)

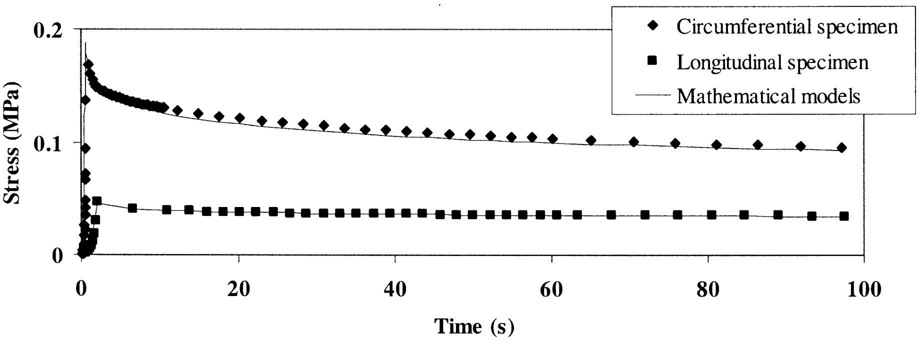

Fig. 1. Typical stress–time plots for the first 100 s of specimens tested in the longitudinal and circumferential directions

for stress to change by less than 0.01% per second were 380 and 35 s, respectively).

Discussion

The results of the experimental tests demonstrate the highly viscoelastic nature of the dura mater in both longitudinal and circumferential directions. The use of the elastic stress-strain equation developed for collagen appeared to fit the data well, although the constant k calculated for the dura was substantially lower than that documented for pure collagen fibers.[5] This indicates the composite form of the dura, with the collagen fibers suspended in a lower modulus matrix.

Previous microscopy has shown the collagen fibers to be organized in a corrugated fashion.[11] However, at the epidural surface, the fibers tend to be arranged in a more isotropic manner.[8] In the longitudinal direction, the maximum gradient observed was of the order of 0.1 MPa — several orders of magnitude lower than that reported by Tencer et al.[10] and Runza et al.,[9] who tested to higher strains. This suggests that most of the collagen fibers aligned longitudinally have yet to become taut at the maximum strains used in this study. The greater stiffness observed circumferentially indicates that the fibers aligned in this direction are less corrugated and become taut at lower strains.

The equation fitted the data extremely well during both loading and relaxation periods, and the lower values of R^2 in the circumferential direction were due in part to the material being less stiff and there being greater scatter of the load readings. This equation will be of use when modeling the SCI process in events such as burst fractures, where both the impact on the dura by the bone fragments and the subsequent relaxation of the system are of interest.

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