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RESEARCH

Bidirectional Quantum Teleportation of GHZ and EPR States Through Entanglement Swapping Utilizing a Pre-established GHZ Channel

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Abstract

In this paper, we present a novel bidirectional quantum teleportation protocol facilitating the simultaneous teleportation of pairs of EPR and GHZ states through entanglement swapping within a six-qubit GHZ channel. This protocol leverages pre-existing GHZ states within the channel, alongside the EPR and GHZ states intended for teleportation, to generate a new entangled state. Subsequently, this state undergoes measurement by Alice and Bob, with the measurement outcomes determining the teleported states. Finally, the successful teleportation of the EPR and GHZ states to each other is ensured through the utilization of CNOT operators, single-qubit and two-qubit measurements, and the application of the identity operator. The proposed protocol presents several advantages over existing protocols, including its bidirectional nature and higher efficiency.

Keywords Bidirection quantum teleportation · GHZ state · EPR state · Entanglement swapping

1 Introduction

Quantum entanglement serves as a cornerstone in quantum information theory, catalyzing significant progress in various protocols such as quantum teleportation (QT) [1, 2], quantum cryptography (QC) [2], quantum secret sharing (QSS) [3], and quantum key distribution (QKD) [4, 5]. Bennett and colleagues pioneered quantum teleportation in 1993, teleporting a single-qubit state using classical bits in a Bell-state channel [1]. Since then, researchers have expanded on this concept, introducing diverse quantum teleportation protocols. For instance, in 2002, Bao et al. proposed a protocol teleporting a single-qubit state through a W-state channel [6]. Advancements included protocols for transferring single qubits in both W and EPR channels [7–9], as well as protocols for teleporting states composed of two to six qubits through Bell and W channels [9–14].

In 2010, Liu et al. proposed a protocol enabling controlled teleportation of an arbitrary two-particle state using a five-qubit cluster state [15]. In 2017, Sadiqzadeh and colleagues introduced a remote transfer protocol for an arbitrary two-qubit state in an eight-qubit channel,

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relying solely on single-qubit measurements [16, 17]. Subsequently, multi-stage teleportation schemes with specific objectives emerged. This paper focuses on bidirectional quantum teleportation (BQT), facilitating the teleportation of unknown two- and three-qubit states, including EPR and GHZ states, between Alice and Bob using a six-qubit GHZ channel. This entanglement swapping-based transmission preserves quantum information, ensuring higher security and efficiency.

The paper is structured as follows:

- Section 2: Provides a comprehensive description of the protocol, including its theoretical foundations, entanglement swapping procedure, system preparation, relevant operator application, measurement scheme based on specific bases, and thorough data analysis.
- Section 3: Presents the efficiency calculations of the protocol and compares them to other reported schemes using established metrics.
- Section 4: Explains the methodology and implementation of quantum circuit simulation.

2 Protocol Description

As mentioned, this protocol constitutes a bidirectional quantum teleportation (BQT) scheme allowing Alice and Bob to simultaneously teleport EPR and GHZ states to each other in an indeterminate manner, represented as follows:

$$|GHZ\rangle a_1a_2a_3 = \alpha_0|000\rangle + \alpha_1|111\rangle, \tag{1}$$
$$|EPR\rangle a_4a_5 = \alpha_2|00\rangle + \alpha_3|11\rangle.$$

$$|GHZ\rangle b_1b_2b_3 = \beta_0|000\rangle + \beta_1|111\rangle, \tag{2}$$

$$|EPR\rangle b_4b_5 = \beta_2|00\rangle + \beta_3|11\rangle.$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$ are arbitrary coefficients with

$$|\alpha_0|^2 + |\alpha_1|^2 = 1, \quad |\alpha_2|^2 + |\alpha_3|^2 = 1, \quad |\beta_0|^2 + |\beta_1|^2 = 1, \quad |\beta_2|^2 + |\beta_3|^2 = 1.$$

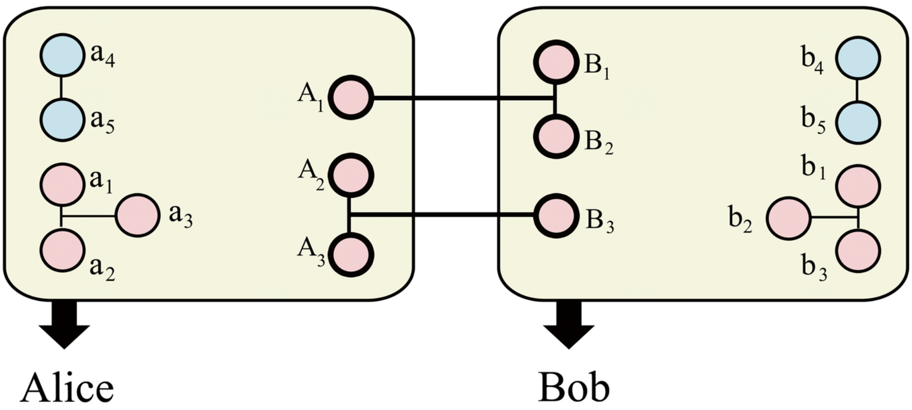

Fig. 1 In the overall system structure, qubits $a_1a_2a_3a_4a_5$ belong to Alice, qubits $b_1b_2b_3b_4b_5$ belong to Bob, and qubits $A_1A_2A_3B_1B_2B_3$ are shared as a channel by Alice and Bob

The preparation of this protocol involves several steps, which will be explained in the following sections.

2.1 System Preparation

To initiate the teleportation process by Alice and Bob, a quantum channel is established, comprising a six-qubit state formed from two pairs of GHZ, as follows:

$$|Q_C\rangle_{A_1B_1B_2A_2A_3B_3} = 1/\sqrt{2}(|000\rangle + |111\rangle) \otimes 1/\sqrt{2}(|000\rangle + |111\rangle). \tag{3}$$

Equation (3) can be expressed by changing the qubits positions as follows:

$$|Q_C\rangle_{A_1A_2A_3B_1B_2B_3} = 1/2(|000\rangle|000\rangle + |011\rangle|001\rangle + |100\rangle|110\rangle + |111\rangle|111\rangle).$$

As depicted in Fig. 1, within this channel, qubits A_1, A_2, andA_3 are allocated to Alice, while three qubits B_1, B_2, and B_3 are assigned to Bob. It's noteworthy that the selected GHZ states within this channel signify a particular state characterized by maximum three-qubit entanglement (Fig. 1).

2.1.1 The Analysis of Entanglement Transfer Within the Channel

Before proceeding with the teleportation steps, the primary emphasis is on entanglement swapping within the channel [18]. The central focus is on the impact of EPR and GHZ states on entanglement transfer. To achieve this, the CNOT operator is applied in two steps: initially on qubits a_1, a_2 and a_4, which control qubits A_1, A_2, and A_3 respectively, and then on qubits B_1, B_2, and B_4, serving as control qubits for B_1, B_2, and B_3 [19].

$$|EPR\rangle_{a_4a_5b_4b_5} = |EPR\rangle_{a_4a_5} \bigotimes |EPR\rangle_{b_4b_5} = \alpha_2\beta_2|0000\rangle + \alpha_2\beta_3|0011\rangle + \alpha_3\beta_2|1100\rangle + \alpha_3\beta_3|1111\rangle. \tag{4}$$

$$|GHZ\rangle_{a_1a_2a_3b_1b_2b_3} = |GHZ\rangle_{a_1a_2a_3} \bigotimes |GHZ\rangle_{b_1b_2b_3} = \alpha_0\beta_0|000000\rangle + \alpha_0\beta_1|000111\rangle + \alpha_1\beta_0|111000\rangle + \alpha_1\beta_1|111111\rangle. \tag{5}$$

GHZ-type quantum entanglement is characterized by a quantum superposition system. The GHZ state encompasses three qubits and is represented as follows [20–23]:

$$|\psi(i, j, k)\rangle = 1/\sqrt{2}(|i\rangle|j\rangle|k\rangle + |\bar{i}\rangle|\bar{j}\rangle|\bar{k}\rangle)_{Q_1Q_2Q_3} \tag{6}$$

In which i, j, k ∈ {0,1}, $\bar{i}$ = i ⊕1 = (i + 1) mod 2, Q1, Q2 and Q3 are GHZ involved particles. In (6), i, j and k are set the possible value in {0,1}, respectively. The possible GHZ states are achieved as that:

$$|\psi_0\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle), \tag{7}$$

$$|\psi_1\rangle = 1/\sqrt{2}(|000\rangle - |111\rangle),$$

$$|\psi_2\rangle = 1/\sqrt{2}(|100\rangle + |011\rangle),$$
$$|\psi_3\rangle = 1/\sqrt{2}(|100\rangle - |011\rangle),$$
$$|\psi_6\rangle = 1/\sqrt{2}(|110\rangle + |011\rangle),$$
$$|\psi_7\rangle = 1/\sqrt{2}(|110\rangle - |011\rangle).$$

We can elucidate (7) as follows:

$$\begin{aligned}
|000\rangle &= 1/\sqrt{2}(|\psi_0\rangle + |\psi_1\rangle), \\
|111\rangle &= 1/\sqrt{2}(|\psi_0\rangle - |\psi_1\rangle), \\
|100\rangle &= 1/\sqrt{2}(|\psi_2\rangle + |\psi_3\rangle), \\
|011\rangle &= 1/\sqrt{2}(|\psi_2\rangle - |\psi_3\rangle), \\
|110\rangle &= 1/\sqrt{2}(|\psi_6\rangle + |\psi_7\rangle), \\
|001\rangle &= 1/\sqrt{2}(|\psi_6\rangle - |\psi_7\rangle).
\end{aligned} \tag{8}$$

The changes in the channel state after the influence of the EPR state are as follows [20]:

$$|Q\rangle_c \bigotimes |EPR\rangle_{a_4a_5b_4b_5} = |Q'\rangle_c = |\varphi\rangle_1 + |\varphi\rangle_2 + |\varphi\rangle_3 + |\varphi\rangle_4 \tag{9}$$

The values of $|\varphi\rangle_1, |\varphi\rangle_2, |\varphi\rangle_3, |\varphi\rangle_4$ can be calculated as follows:

$$\begin{aligned}
|\varphi\rangle_1 &= 1/2(|\psi_0\rangle_{A_1A_2A_3}|\psi_0\rangle_{B_1B_2B_3} + |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle\alpha_2\beta_2|0000\rangle), \\
|\varphi\rangle_2 &= 1/2(|\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle)\alpha_2\beta_3|0011\rangle, \\
|\varphi\rangle_3 &= 1/2(|\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle + |\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle)\alpha_3\beta_2|1100\rangle, \\
|\varphi\rangle_4 &= 1/2(|\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle)\alpha_3\beta_3|1111\rangle].
\end{aligned} \tag{10}$$

It can be observed that the EPR state within the channel results in the transfer of entangled states. In the second step, the GHZ state is applied to the channel, and the changes in the basis and state transfer are analyzed.

$$\begin{aligned}
&|Q'\rangle_c \bigotimes |GHZ\rangle_{a_1a_2a_3b_1b_2b_3} = \\
&|\varphi\rangle_{11} + |\varphi\rangle_{12} + |\varphi\rangle_{13} + |\varphi\rangle_{14} + |\varphi\rangle_{21} + |\varphi\rangle_{22} + |\varphi\rangle_{23} + |\varphi\rangle_{24} + \\
&|\varphi\rangle_{31} + |\varphi\rangle_{32} + |\varphi\rangle_{33} + |\varphi\rangle_{34} + |\varphi\rangle_{41} + |\varphi\rangle_{42} + |\varphi\rangle_{43} + |\varphi\rangle_{44}
\end{aligned} \tag{11}$$

After performing the necessary calculations, we obtain the values of the sentences:

$$\begin{aligned}
|\varphi\rangle_{11} &= 1/2(|\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle)\varepsilon_0|0000\rangle|000000\rangle, \\
|\varphi\rangle_{12} &= 1/2(|\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle)\varepsilon_2|0000\rangle|000111\rangle, \\
|\varphi\rangle_{13} &= 1/2(|\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle + |\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle)\varepsilon_8|0000\rangle|111000\rangle, \\
|\varphi\rangle_{14} &= 1/2(|\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle)\varepsilon_{10}|0000\rangle|111111\rangle, \\
|\varphi\rangle_{21} &= 1/2(|\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle)\varepsilon_1|0011\rangle|000000\rangle, \\
|\varphi\rangle_{22} &= 1/2(|\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle)\varepsilon_3|0011\rangle|000111\rangle, \\
|\varphi\rangle_{23} &= 1/2(|\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle)\varepsilon_9|0011\rangle|111000\rangle,
\end{aligned}$$

$|\varphi\rangle_{24} = 1/2(|\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle + |\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle)\varepsilon_{11}|0011\rangle|111111\rangle,$
$|\varphi\rangle_{31} = 1/2(|\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle + |\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle)\varepsilon_4|1100\rangle|000000\rangle,$
$|\varphi\rangle_{32} = 1/2(|\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle)\varepsilon_6|1100\rangle|000111\rangle,$
$|\varphi\rangle_{33} = 1/2(|\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_2\rangle - |\psi_3\rangle|\psi_7\rangle)\varepsilon_{12}|1100\rangle|111000\rangle,$
$|\varphi\rangle_{34} = 1/2(|\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle)\varepsilon_{14}|1100\rangle|000000\rangle,$
$|\varphi\rangle_{41} = 1/2(|\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle)\varepsilon_5|1111\rangle|000000\rangle,$
$|\varphi\rangle_{42} = 1/2(|\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_0\rangle + |\psi_6\rangle|\psi_1\rangle + |\psi_7\rangle|\psi_7\rangle)\varepsilon_7|1111\rangle|000111\rangle,$
$|\varphi\rangle_{43} = 1/2(|\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle)\varepsilon_{13}|1111\rangle|111000\rangle,$
$|\varphi\rangle_{44} = 1/2(|\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle)\varepsilon_{15}|1111\rangle|111111\rangle].$

The entanglement of the channel transitions from one basis to another, progressing from one point to an intermediate point while preserving quantum information, even during longer distance transfers. Ultimately, Alice and Bob can independently reconstruct the final states of the teleported pairs $|EPR\rangle$ and $|GHZ\rangle$ accurately by measuring in their respective bases and utilizing classical bits, as described below:

2.1.2 Introduction of System Components

Following channel preparation, Alice and Bob transmit a five-qubit state, comprised of EPR and GHZ states, to the opposing party (Table 1):

$$|Q\rangle_T = |Q\rangle_t \otimes |Q\rangle_c. \tag{12}$$

$|Q_t\rangle = a_1a_2a_3a_4a_5\, b_1b_2b_3b_4b_5 = |GHZ\rangle_{a_1a_2a_3} \otimes |EPR\rangle_{a_4a_5} \otimes |GHZ\rangle_{b_1b_2b_3} \otimes |EPR\rangle_{b_4b_5}$
$|Q_t\rangle = (\epsilon_0|00000\ 00000\rangle + \epsilon_1|00000\ 00011\rangle + \epsilon_2|00000\ 11100\rangle + \epsilon_3|00000\ 11111\rangle + (\epsilon_4|00011\ 00\ 000\rangle + \epsilon_5|00011\ 00011\rangle + \epsilon_6|00011\ 11100\rangle + \epsilon_7|00011\ 11111\rangle + \epsilon_8|11100\ 00000\rangle + \epsilon_9|11100\ 00011\rangle + \epsilon_{10}|11100\ 11100\rangle + \epsilon_{11}|11100\ 11111\rangle + \epsilon_{12}|11111\ 00000\rangle + \epsilon_{13}|11111\ 00011\rangle + \epsilon_{14}|11111\ 00011\rangle + \epsilon_{15}|11111\ 11111\rangle).$

$$|Q\rangle_T = |Q\rangle_t \otimes (|000\ 000\rangle + |011\ 001\rangle + |100\ 110\rangle + |111\ 111\rangle)$$

$$|Q\rangle_{T_1} = |Q\rangle_t \otimes (|000\ 000\rangle,$$

$$|Q\rangle_{T_2} = |Q\rangle_t \otimes (|011\ 001\rangle,$$

$$|Q\rangle_{T_3} = |Q\rangle_t \otimes (|100\ 110\rangle,$$

$$|Q\rangle_{T_4} = |Q\rangle_t \otimes (|111\ 111\rangle.$$

Table 1 Complex coefficients satisfying the normalization condition

$\epsilon_0 = \alpha_0\alpha_2\beta_0\beta_2$	$\epsilon_1 = \alpha_0\alpha_2\beta_0\beta_3$	$\epsilon_2 = \alpha_0\alpha_2\beta_1\beta_2$	$\epsilon_3 = \alpha_0\alpha_2\beta_1\beta_3$
$\epsilon_4 = \alpha_0\alpha_3\beta_0\beta_2$	$\epsilon_5 = \alpha_0\alpha_3\beta_0\beta_3$	$\epsilon_6 = \alpha_0\alpha_3\beta_1\beta_2$	$\epsilon_7 = \alpha_0\alpha_3\beta_1\beta_3$
$\epsilon_8 = \alpha_1\alpha_2\beta_0\beta_2$	$\epsilon_9 = \alpha_1\alpha_2\beta_0\beta_3$	$\epsilon_{10} = \alpha_1\alpha_2\beta_1\beta_2$	$\epsilon_{11} = \alpha_1\alpha_2\beta_1\beta_3$
$\epsilon_{12} = \alpha_1\alpha_3\beta_0\beta_2$	$\epsilon_{13} = \alpha_1\alpha_3\beta_0\beta_3$	$\epsilon_{14} = \alpha_1\alpha_3\beta_1\beta_2$	$\epsilon_{15} = \alpha_1\alpha_3\beta_1\beta_3$

Table 2 The (X-basis) measurement results of users and the corresponding collapsed states

$\|\phi_{T11}\rangle = 1/\sqrt{2}($
$\varepsilon_0\|00\rangle\|00\rangle\|00000\,00000\rangle + \varepsilon_0\|00\rangle\|00\rangle\|00000\,00011\rangle + \varepsilon_0\|00\rangle\|11\rangle\|00000\,11100\rangle + \varepsilon_0\|00\rangle\|11\rangle\|00000\,11111\rangle +$
$\varepsilon_0\|00\rangle\|00\rangle\|00011\,00000\rangle + \varepsilon_0\|00\rangle\|00\rangle\|00011\,00011\rangle + \varepsilon_0\|00\rangle\|11\rangle\|00011\,11100\rangle + \varepsilon_0\|00\rangle\|11\rangle\|00011\,11111\rangle +$
$\varepsilon_0\|11\rangle\|00\rangle\|11100\,00000\rangle + \varepsilon_0\|11\rangle\|00\rangle\|11100\,00011\rangle + \varepsilon_0\|11\rangle\|11\rangle\|11100\,11100\rangle + \varepsilon_0\|11\rangle\|11\rangle\|11100\,11111\rangle +$
$\varepsilon_0\|11\rangle\|00\rangle\|11111\,00000\rangle + \varepsilon_0\|11\rangle\|00\rangle\|11111\,00011\rangle + \varepsilon_0\|11\rangle\|11\rangle\|11111\,11100\rangle + \varepsilon_0\|11\rangle\|11\rangle\|11111\,11111\rangle)$.

2.2 Applying Operators

In this section, the $CNOT$ operation is executed by Alice and Bob in such a manner that $a_1, a_2, a_4, b_1, b_2, b_3$ and $A_1, A_2, A_3, B_1, B_2, B_3$ represent the control qubits and target qubits, respectively. Upon applying CNOT, the overall state of the system undergoes changes. A detailed explanation is provided in the appendix of the article. Here, we present an examination of only one aspect of the four-part process (Fig. 2):

$$|\phi\rangle_{T1} = 1/2(\varepsilon_0|000\rangle|000\rangle|00000\,00000\rangle + \varepsilon_1|000\rangle|001\rangle|00000\,00011\rangle +$$
$$\varepsilon_2|000\rangle|110\rangle|00000\,11100\rangle + \varepsilon_3|000\rangle|111\rangle|00000\,11111\rangle +$$
$$\varepsilon_4|001\rangle|000\rangle|00011\,00000\rangle + \varepsilon_5|001\rangle|001\rangle|00011\,00011\rangle +$$
$$\varepsilon_6|001\rangle|110\rangle|00011\,11100\rangle + \varepsilon_7|001\rangle|111\rangle|00011\,11111\rangle +$$
$$\varepsilon_8|110\rangle|000\rangle|11100\,00000\rangle + \varepsilon_9|110\rangle|001\rangle|11100\,00011\rangle +$$
$$\varepsilon_{10}|110\rangle|110\rangle|11100\,11100\rangle + \varepsilon_{11}|110\rangle|111\rangle|11100\,11111\rangle +$$
$$\varepsilon_{12}|111\rangle|000\rangle|11111\,00000\rangle + \varepsilon_{13}|111\rangle|001\rangle|11111\,00011\rangle +$$
$$\varepsilon_{14}|111\rangle|110\rangle|11111\,11100\rangle + \varepsilon_{15}|111\rangle|111\rangle|11111\,11111\rangle))$$

2.3 Measurement

2.3.1 Measurement in the X-Basis

In this step, Alice and Bob conduct measurements on the single qubits a_4 and b_4 in the X-basis. They exchange the measurement outcomes with each other using two classical bits. Following the measurement, the system's state collapses to $(a_1a_2b_1b_2A_2A_3a_3a_5A_1b_3B_1B_2b_5B_3)$ [24, 25].

The measurement of one state is presented as follows (Table 2) (refer to the appendix for the complete description of measurements):

2.3.2 Von Neumann Basis Measurement

Following the X-basis measurement, Alice and Bob measure qubits (a_1, a_2) and (b_1, b_2) in the Von Neumann basis, resulting in the system collapsing to the state $(A_1A_2a_3a_5A_1b_3B_1B_2b_5B_3)$. They then exchange four additional classical bits to communicate their respective measurement outcomes. By combining the information from these two measurement steps, conveyed through the transmission of six classical bits, Alice and Bob can reconstruct the initial state by applying appropriate identity operations [26] (Table 3).

Table 3 X-basis and Von Neumann basis measurement results

Alice	Bob	Collapsed state	Alice's operator	Bob's operator
$\vert+\rangle\vert0\rangle$	$\vert+\rangle\vert0\rangle$	$\epsilon_0\vert00000\ 00000\rangle+\epsilon_1\vert00000\ 00011\rangle+\epsilon_2\vert00000\ 11100\rangle+\epsilon_3\vert00000\ 11111\rangle+$ $\epsilon_4\vert00011\ 00000\rangle+\epsilon_5\vert00011\ 00011\rangle+\epsilon_6\vert00011\ 11100\rangle+\epsilon_7\vert00011\ 11111\rangle+$ $\epsilon_8\vert11100\ 00000\rangle+\epsilon_9\vert11100\ 00011\rangle+\epsilon_{1_0}\vert11100\ 11100\rangle+\epsilon_{1_1}\vert11100\ 11111\rangle+$ $\epsilon_{1_2}\vert11111\ 00000\rangle+\epsilon_{1_3}\vert11111\ 00011\rangle+\epsilon_{1_4}\vert11111\ 11100\rangle+\epsilon_{1_5}\vert11111\ 11111\rangle$	IIIII	IIIII

3 Comparison and Efficiency

In the preceding sections, bidirectional quantum teleportation of ten qubits in a six-qubits channel, facilitated by six classical bits for various states has been achieved.

The protocol's efficiency can be computed using the formula [27]:

$$\eta = c/(p+q), \tag{13}$$

where represents efficiency, c is the number of sent qubits, p is the number of channel qubits, and q is the number of classical bits [28]. Here, $\eta = 83.3$ indicates that the proposed protocol has higher efficiency compared to other protocols (as shown in Table 4). Furthermore, its effectiveness is further emphasized due to the advantages associated with entanglement swapping in the channel [14, 29].

Advantages of Entangled State Transfer [20]:

- Greater efficiency.
- Higher security.
- Preservation of quantum information is more accessible.
- Expanded applicability in quantum networks.
- the initial teleportation qubits are not destroyed.

4 Circuit Simulation

The entire sequence of operations performed in this protocol, from its initiation to the final measurements, has been meticulously replicated in the circuit simulation (Fig. 2):

Table 4 Comparison of different protocols with our protocols in efficiency

Design	Bidirectional transfer of qubits	Quantum channel	η
BCQT [30]	1↔1	5 qubit	28
BCQT [13]	5↔4	6 qubits cluster	25
BCQT [12]	1↔1	7 qubits	22
BCQT [31]	1↔1	6 qubits	25
BCQT [31]	1↔2	7 qubits	21
BCQT [32]	2↔1	6 qubits cluster	25
BCQT [33]	3↔2	9 qubits cluster	7.27
ABCQT [14]	2↔3	8 qubits cluster	38
BCQT [34]	2↔3	6 qubits	45
BCQT [35]	3↔3	6 qubits	50
BCQT [35]	N↔N	2N qubits	50
BCQT [36]	N↔N	4 qubits cluster	66
BCQT[presented work]	**5↔5**	**6 qubits**	**83.3**

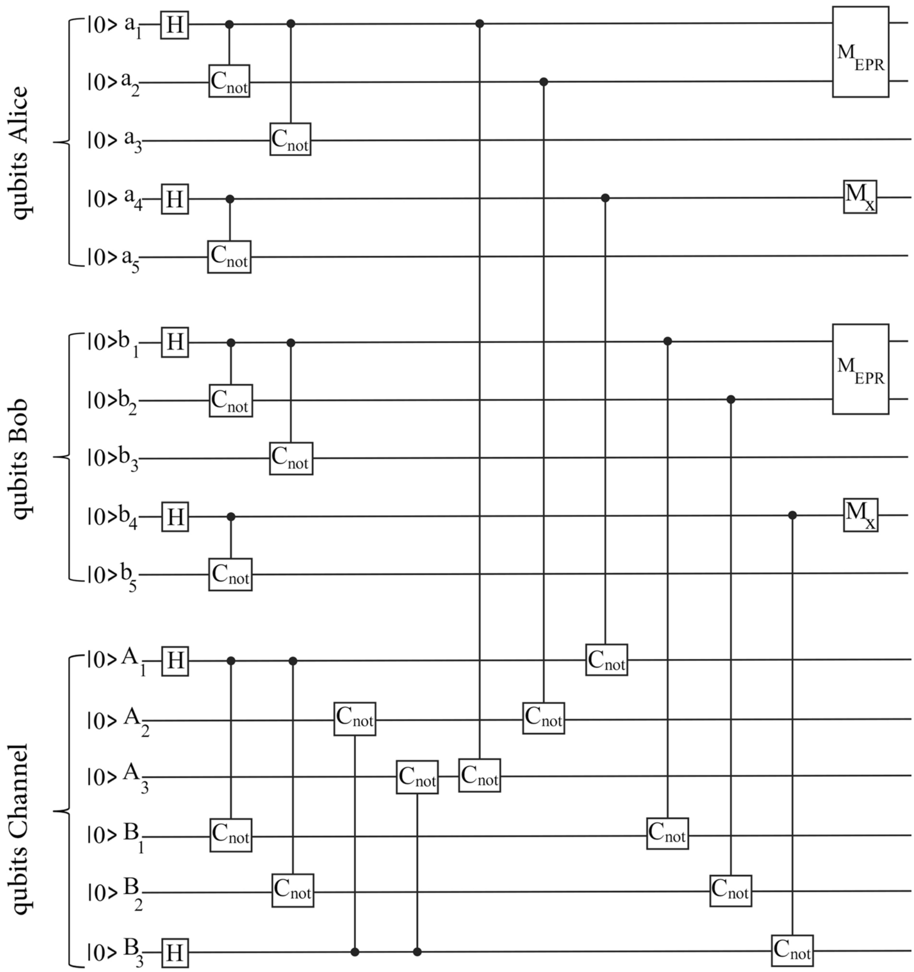

Fig. 2 The protocol architecture proposed by Alice and Bob

5 Conclusion

This paper introduces a novel bidirectional quantum teleportation scheme that utilizes entanglement swapping to achieve the simultaneous teleportation of two pairs of GHZ and EPR states between Alice and Bob using a GHZ-state channel. The proposed protocol exploits entanglement swapping to generate a new entangled state, subsequently measured by Alice and Bob to determine the teleported states. This process facilitates the efficient transfer of a total of ten qubits between the two parties. Moreover, the protocol accommodates the transfer of both GHZ and EPR states, thereby broadening its applicability. Furthermore, the proposed scheme lays the groundwork for extending to multi-step teleportation for N qubits, thereby expanding its capabilities.

Appendix A Attachment:

Table 5 X-basis and Von Neumann basis complete measurement results

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle + \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle + \epsilon_9\|11100\ 00011\rangle + \epsilon_{10}\|11100\ 11100\rangle + \epsilon_{11}\|11100\ 11111\rangle + \epsilon_{12}\|11111\ 00000\rangle + \epsilon_{13}\|11111\ 00011\rangle + \epsilon_{14}\|11111\ 11100\rangle + \epsilon_{15}\|11111\ 11111\rangle$	IIIII	IIIII
$\|+\rangle\|0\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle + \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle - \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle - \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle + \epsilon_9\|11100\ 00011\rangle - \epsilon_{10}\|11100\ 11100\rangle - \epsilon_{11}\|11100\ 11111\rangle + \epsilon_{12}\|11111\ 00000\rangle + \epsilon_{13}\|11111\ 00011\rangle - \epsilon_{14}\|11111\ 11100\rangle - \epsilon_{15}\|11111\ 11111\rangle$	IIIII	ZIIII
$\|+\rangle\|1\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle + \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle - \epsilon_{10}\|11100\ 11100\rangle - \epsilon_{11}\|11100\ 11111\rangle - \epsilon_{12}\|11111\ 00000\rangle - \epsilon_{13}\|11111\ 00011\rangle - \epsilon_{14}\|11111\ 11100\rangle - \epsilon_{15}\|11111\ 11111\rangle$	ZIIII	IIIII
$\|+\rangle\|1\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle + \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle - \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle - \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle + \epsilon_{10}\|11100\ 11100\rangle + \epsilon_{11}\|11100\ 11111\rangle - \epsilon_{12}\|11111\ 00000\rangle - \epsilon_{13}\|11111\ 00011\rangle + \epsilon_{14}\|11111\ 11100\rangle + \epsilon_{15}\|11111\ 11111\rangle$	ZIIII	ZIIII

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle - \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle - \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle - \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle + \epsilon_{1_0}\|11100\ 11100\rangle - \epsilon_{1_1}\|11100\ 11111\rangle + \epsilon_{1_2}\|11111\ 00000\rangle - \epsilon_{1_3}\|11111\ 00011\rangle + \epsilon_{1_4}\|11111\ 11100\rangle - \epsilon_{1_5}\|11111\ 11111\rangle$	IIIII	IIIIZ
$\|+\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle - \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle - \epsilon_{1_0}\|11100\ 11100\rangle + \epsilon_{1_1}\|11100\ 11111\rangle + \epsilon_{1_2}\|11111\ 00000\rangle - \epsilon_{1_3}\|11111\ 00011\rangle - \epsilon_{1_4}\|11111\ 11100\rangle + \epsilon_{1_5}\|11111\ 11111\rangle$	IIIII	ZIIIZ
$\|+\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle + \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle - \epsilon_{1_0}\|11100\ 11100\rangle - \epsilon_{1_1}\|11100\ 11111\rangle - \epsilon_{1_2}\|11111\ 00000\rangle - \epsilon_{1_3}\|11111\ 00011\rangle - \epsilon_{1_4}\|11111\ 11100\rangle - \epsilon_{1_5}\|11111\ 11111\rangle$	ZIIII	IIIIZ
$\|+\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle + \epsilon_4\|00011\ 00000\rangle - \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle + \epsilon_9\|11100\ 00011\rangle + \epsilon_{1_0}\|11100\ 11100\rangle - \epsilon_{1_1}\|11100\ 11111\rangle - \epsilon_{1_2}\|11111\ 00000\rangle + \epsilon_{1_3}\|11111\ 00011\rangle + \epsilon_{1_4}\|11111\ 11100\rangle - \epsilon_{1_5}\|11111\ 11111\rangle$	ZIIII	ZIIIZ

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00000\ 00000\rangle + \epsilon_1\vert 00000\ 00011\rangle + \epsilon_2\vert 00000\ 11100\rangle + \epsilon_3\vert 00000\ 11111\rangle - \epsilon_4\vert 00011\ 00000\rangle - \epsilon_5\vert 00011\ 00011\rangle - \epsilon_6\vert 00011\ 11100\rangle - \epsilon_7\vert 00011\ 11111\rangle + \epsilon_8\vert 11100\ 00000\rangle + \epsilon_9\vert 11100\ 00011\rangle + \epsilon_{1_0}\vert 11100\ 11100\rangle + \epsilon_{1_1}\vert 11100\ 11111\rangle - \epsilon_{1_2}\vert 11111\ 00000\rangle - \epsilon_{1_3}\vert 11111\ 00011\rangle - \epsilon_{1_4}\vert 11111\ 11100\rangle - \epsilon_{1_5}\vert 11111\ 11111\rangle$	ZIIII	IIIII
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00000\ 00000\rangle + \epsilon_1\vert 00000\ 00011\rangle - \epsilon_2\vert 00000\ 11100\rangle - \epsilon_3\vert 00000\ 11111\rangle - \epsilon_4\vert 00011\ 00000\rangle - \epsilon_5\vert 00011\ 00011\rangle + \epsilon_6\vert 00011\ 11100\rangle + \epsilon_7\vert 00011\ 11111\rangle + \epsilon_8\vert 11100\ 00000\rangle + \epsilon_9\vert 11100\ 00011\rangle - \epsilon_{1_0}\vert 11100\ 11100\rangle - \epsilon_{1_1}\vert 11100\ 11111\rangle - \epsilon_{1_2}\vert 11111\ 00000\rangle - \epsilon_{1_3}\vert 11111\ 00011\rangle + \epsilon_{1_4}\vert 11111\ 11100\rangle + \epsilon_{1_5}\vert 11111\ 11111\rangle$	ZIIII	ZIIII
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00000\ 00000\rangle + \epsilon_1\vert 00000\ 00011\rangle + \epsilon_2\vert 00000\ 11100\rangle + \epsilon_3\vert 00000\ 11111\rangle - \epsilon_4\vert 00011\ 00000\rangle - \epsilon_5\vert 00011\ 00011\rangle - \epsilon_6\vert 00011\ 11100\rangle - \epsilon_7\vert 00011\ 11111\rangle - \epsilon_8\vert 11100\ 00000\rangle - \epsilon_9\vert 11100\ 00011\rangle - \epsilon_{1_0}\vert 11100\ 11100\rangle - \epsilon_{1_1}\vert 11100\ 11111\rangle + \epsilon_{1_2}\vert 11111\ 00000\rangle + \epsilon_{1_3}\vert 11111\ 00011\rangle + \epsilon_{1_4}\vert 11111\ 11100\rangle + \epsilon_{1_5}\vert 11111\ 11111\rangle$	ZZIII	IIIII
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00000\ 00000\rangle + \epsilon_1\vert 00000\ 00011\rangle - \epsilon_2\vert 00000\ 11100\rangle - \epsilon_3\vert 00000\ 11111\rangle - \epsilon_4\vert 00011\ 00000\rangle - \epsilon_5\vert 00011\ 00011\rangle + \epsilon_6\vert 00011\ 11100\rangle + \epsilon_7\vert 00011\ 11111\rangle - \epsilon_8\vert 11100\ 00000\rangle - \epsilon_9\vert 11100\ 00011\rangle + \epsilon_{1_0}\vert 11100\ 11100\rangle + \epsilon_{1_1}\vert 11100\ 11111\rangle + \epsilon_{1_2}\vert 11111\ 00000\rangle + \epsilon_{1_3}\vert 11111\ 00011\rangle - \epsilon_{1_4}\vert 11111\ 11100\rangle - \epsilon_{1_5}\vert 11111\ 11111\rangle$	ZZIII	ZIIII

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|-\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle - \epsilon_3\|00000\ 11111\rangle - \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle + \epsilon_{1_0}\|11100\ 11100\rangle - \epsilon_{1_1}\|11100\ 11111\rangle + \epsilon_{1_2}\|11111\ 00000\rangle + \epsilon_{1_3}\|11111\ 00011\rangle - \epsilon_{1_4}\|11111\ 11100\rangle + \epsilon_{1_5}\|11111\ 11111\rangle$	ZIIII	IIIIZ
$\|-\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle - \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle - \epsilon_7\|00011\ 11111\rangle + \epsilon_8\|11100\ 00000\rangle - \epsilon_9\|11100\ 00011\rangle - \epsilon_{1_0}\|11100\ 11100\rangle + \epsilon_{1_1}\|11100\ 11111\rangle + \epsilon_{1_2}\|11111\ 00000\rangle + \epsilon_{1_3}\|11111\ 00011\rangle + \epsilon_{1_4}\|11111\ 11100\rangle - \epsilon_{1_5}\|11111\ 11111\rangle$	ZIIII	ZIIIZ
$\|-\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle - \epsilon_2\|00000\ 11100\rangle - \epsilon_3\|00000\ 11111\rangle - \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle - \epsilon_6\|00011\ 11100\rangle + \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle + \epsilon_9\|11100\ 00011\rangle - \epsilon_{1_0}\|11100\ 11100\rangle + \epsilon_{1_1}\|11100\ 11111\rangle - \epsilon_{1_2}\|11111\ 00000\rangle - \epsilon_{1_3}\|11111\ 00011\rangle + \epsilon_{1_4}\|11111\ 11100\rangle - \epsilon_{1_5}\|11111\ 11111\rangle$	ZZIII	IIIIZ
$\|-\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00000\ 00000\rangle - \epsilon_1\|00000\ 00011\rangle + \epsilon_2\|00000\ 11100\rangle + \epsilon_3\|00000\ 11111\rangle - \epsilon_4\|00011\ 00000\rangle + \epsilon_5\|00011\ 00011\rangle + \epsilon_6\|00011\ 11100\rangle - \epsilon_7\|00011\ 11111\rangle - \epsilon_8\|11100\ 00000\rangle + \epsilon_9\|11100\ 00011\rangle + \epsilon_{1_0}\|11100\ 11100\rangle - \epsilon_{1_1}\|11100\ 11111\rangle - \epsilon_{1_2}\|11111\ 00000\rangle - \epsilon_{1_3}\|11111\ 00011\rangle - \epsilon_{1_4}\|11111\ 11100\rangle + \epsilon_{1_5}\|11111\ 11111\rangle$	ZZIII	ZIIIZ

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\lvert+\rangle\lvert 0\rangle$	$\lvert+\rangle\lvert 0\rangle$	$\epsilon_0\lvert 11000\ 00001\rangle + \epsilon_1\lvert 11000\ 00010\rangle + \epsilon_2\lvert 11000\ 11101\rangle + \epsilon_3\lvert 11000\ 11110\rangle + \epsilon_4\lvert 11011\ 00001\rangle + \epsilon_5\lvert 11011\ 00010\rangle + \epsilon_6\lvert 11011\ 11101\rangle + \epsilon_7\lvert 11011\ 11110\rangle + \epsilon_8\lvert 00100\ 00001\rangle + \epsilon_9\lvert 00100\ 00010\rangle + \epsilon_{10}\lvert 00100\ 11101\rangle + \epsilon_{11}\lvert 00100\ 11110\rangle + \epsilon_{12}\lvert 00111\ 00001\rangle + \epsilon_{13}\lvert 00111\ 00010\rangle + \epsilon_{14}\lvert 00111\ 11101\rangle + \epsilon_{15}\lvert 00111\ 11110\rangle$	XXIII	IIIIX
$\lvert+\rangle\lvert 0\rangle$	$\lvert+\rangle\lvert 1\rangle$	$\epsilon_0\lvert 11000\ 00001\rangle + \epsilon_1\lvert 11000\ 00010\rangle - \epsilon_2\lvert 11000\ 11101\rangle - \epsilon_3\lvert 11000\ 11110\rangle + \epsilon_4\lvert 11011\ 00001\rangle + \epsilon_5\lvert 11011\ 00010\rangle - \epsilon_6\lvert 11011\ 11101\rangle - \epsilon_7\lvert 11011\ 11110\rangle + \epsilon_8\lvert 00100\ 00001\rangle + \epsilon_9\lvert 00100\ 00010\rangle - \epsilon_{10}\lvert 00100\ 11101\rangle - \epsilon_{11}\lvert 00100\ 11110\rangle + \epsilon_{12}\lvert 00111\ 00001\rangle + \epsilon_{13}\lvert 00111\ 00010\rangle - \epsilon_{14}\lvert 00111\ 11101\rangle - \epsilon_{15}\lvert 00111\ 11110\rangle$	XXIII	ZIIIX
$\lvert+\rangle\lvert 1\rangle$	$\lvert+\rangle\lvert 0\rangle$	$\epsilon_0\lvert 11000\ 00001\rangle + \epsilon_1\lvert 11000\ 00010\rangle + \epsilon_2\lvert 11000\ 11101\rangle + \epsilon_3\lvert 11000\ 11110\rangle + \epsilon_4\lvert 11011\ 00001\rangle + \epsilon_5\lvert 11011\ 00010\rangle + \epsilon_6\lvert 11011\ 11101\rangle + \epsilon_7\lvert 11011\ 11110\rangle - \epsilon_8\lvert 00100\ 00001\rangle - \epsilon_9\lvert 00100\ 00010\rangle - \epsilon_{10}\lvert 00100\ 11101\rangle - \epsilon_{11}\lvert 00100\ 11110\rangle - \epsilon_{12}\lvert 00111\ 00001\rangle - \epsilon_{13}\lvert 00111\ 00010\rangle - \epsilon_{14}\lvert 00111\ 11101\rangle - \epsilon_{15}\lvert 00111\ 11110\rangle$	XXZII	IIIIX
$\lvert+\rangle\lvert 1\rangle$	$\lvert+\rangle\lvert 1\rangle$	$\epsilon_0\lvert 11000\ 00001\rangle + \epsilon_1\lvert 11000\ 00010\rangle - \epsilon_2\lvert 11000\ 11101\rangle - \epsilon_3\lvert 11000\ 11110\rangle + \epsilon_4\lvert 11011\ 00001\rangle + \epsilon_5\lvert 11011\ 00010\rangle - \epsilon_6\lvert 11011\ 11101\rangle - \epsilon_7\lvert 11011\ 11110\rangle - \epsilon_8\lvert 00100\ 00001\rangle - \epsilon_9\lvert 00100\ 00010\rangle + \epsilon_{10}\lvert 00100\ 11101\rangle + \epsilon_{11}\lvert 00100\ 11110\rangle - \epsilon_{12}\lvert 00111\ 00001\rangle - \epsilon_{13}\lvert 00111\ 00010\rangle + \epsilon_{14}\lvert 00111\ 11101\rangle + \epsilon_{15}\lvert 00111\ 11110\rangle$	XXZII	ZIIIX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle - \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle - \epsilon_3\|11000\ 11110\rangle + \epsilon_4\|11011\ 00001\rangle - \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle - \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle - \epsilon_9\|00100\ 00010\rangle + \epsilon_{10}\|00100\ 11101\rangle - \epsilon_{11}\|00100\ 11110\rangle + \epsilon_{12}\|00111\ 00001\rangle - \epsilon_{13}\|00111\ 00010\rangle + \epsilon_{14}\|00111\ 11101\rangle - \epsilon_{15}\|00111\ 11110\rangle$	XXIII	IIIZX
$\|+\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle + \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle - \epsilon_{10}\|00100\ 11101\rangle + \epsilon_{11}\|00100\ 11110\rangle + \epsilon_{12}\|00111\ 00001\rangle + \epsilon_{13}\|00111\ 00010\rangle - \epsilon_{14}\|00111\ 11101\rangle + \epsilon_{15}\|00111\ 11110\rangle$	XXIII	IIZZX
$\|+\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle + \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle - \epsilon_{10}\|00100\ 11101\rangle + \epsilon_{11}\|00100\ 11110\rangle - \epsilon_{12}\|00111\ 00001\rangle + \epsilon_{13}\|00111\ 00010\rangle - \epsilon_{14}\|00111\ 11101\rangle + \epsilon_{15}\|00111\ 11110\rangle$	XXZII	IIIZX
$\|+\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle + \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle + \epsilon_{10}\|00100\ 11101\rangle + \epsilon_{11}\|00100\ 11110\rangle - \epsilon_{12}\|00111\ 00001\rangle + \epsilon_{13}\|00111\ 00010\rangle + \epsilon_{14}\|00111\ 11101\rangle + \epsilon_{15}\|00111\ 11110\rangle$	XXZII	IIZZX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|-\rangle\|0\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle - \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle - \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle + \epsilon_{1_0}\|00100\ 11101\rangle + \epsilon_{1_1}\|00100\ 11110\rangle - \epsilon_{1_2}\|00111\ 00001\rangle - \epsilon_{1_3}\|00111\ 00010\rangle - \epsilon_{1_4}\|00111\ 11101\rangle - \epsilon_{1_5}\|00111\ 11110\rangle$	XXIIZ	IIIIX
$\|-\rangle\|0\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle - \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle - \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle - \epsilon_{1_0}\|00100\ 11101\rangle - \epsilon_{1_1}\|00100\ 11110\rangle - \epsilon_{1_2}\|00111\ 00001\rangle - \epsilon_{1_3}\|00111\ 00010\rangle + \epsilon_{1_4}\|00111\ 11101\rangle + \epsilon_{1_5}\|00111\ 11110\rangle$	XXIIZ	ZIIIX
$\|-\rangle\|1\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle - \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle - \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle - \epsilon_9\|00100\ 00010\rangle - \epsilon_{1_0}\|00100\ 11101\rangle - \epsilon_{1_1}\|00100\ 11110\rangle + \epsilon_{1_2}\|00111\ 00001\rangle + \epsilon_{1_3}\|00111\ 00010\rangle + \epsilon_{1_4}\|00111\ 11101\rangle + \epsilon_{1_5}\|00111\ 11110\rangle$	XXZIZ	IIIIX
$\|-\rangle\|1\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle + \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle - \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle - \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle - \epsilon_9\|00100\ 00010\rangle + \epsilon_{1_0}\|00100\ 11101\rangle + \epsilon_{1_1}\|00100\ 11110\rangle + \epsilon_{1_2}\|00111\ 00001\rangle + \epsilon_{1_3}\|00111\ 00010\rangle - \epsilon_{1_4}\|00111\ 11101\rangle - \epsilon_{1_5}\|00111\ 11110\rangle$	XXZIZ	ZIIIX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|-\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle - \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle - \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle - \epsilon_9\|00100\ 00010\rangle + \epsilon_{1_0}\|00100\ 11101\rangle - \epsilon_{1_1}\|00100\ 11110\rangle - \epsilon_{1_2}\|00111\ 00001\rangle + \epsilon_{1_3}\|00111\ 00010\rangle - \epsilon_{1_4}\|00111\ 11101\rangle + \epsilon_{1_5}\|00111\ 11110\rangle$	XXIIZ	IIIZX
$\|-\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle - \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle - \epsilon_7\|11011\ 11110\rangle + \epsilon_8\|00100\ 00001\rangle - \epsilon_9\|00100\ 00010\rangle - \epsilon_{1_0}\|00100\ 11101\rangle + \epsilon_{1_1}\|00100\ 11110\rangle - \epsilon_{1_2}\|00111\ 00001\rangle + \epsilon_{1_3}\|00111\ 00010\rangle + \epsilon_{1_4}\|00111\ 11101\rangle - \epsilon_{1_5}\|00111\ 11110\rangle$	XXIIZ	ZIIZX
$\|-\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11000\ 00001\rangle - \epsilon_1\|11000\ 00010\rangle + \epsilon_2\|11000\ 11101\rangle - \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle - \epsilon_6\|11011\ 11101\rangle + \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle - \epsilon_{1_0}\|00100\ 11101\rangle + \epsilon_{1_1}\|00100\ 11110\rangle + \epsilon_{1_2}\|00111\ 00001\rangle - \epsilon_{1_3}\|00111\ 00010\rangle + \epsilon_{1_4}\|00111\ 11101\rangle - \epsilon_{1_5}\|00111\ 11110\rangle$	XXZIZ	IIIZX
$\|-\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11000\ 00001\rangle - \epsilon_1\|11000\ 00010\rangle - \epsilon_2\|11000\ 11101\rangle + \epsilon_3\|11000\ 11110\rangle - \epsilon_4\|11011\ 00001\rangle + \epsilon_5\|11011\ 00010\rangle + \epsilon_6\|11011\ 11101\rangle - \epsilon_7\|11011\ 11110\rangle - \epsilon_8\|00100\ 00001\rangle + \epsilon_9\|00100\ 00010\rangle + \epsilon_{1_0}\|00100\ 11101\rangle - \epsilon_{1_1}\|00100\ 11110\rangle + \epsilon_{1_2}\|00111\ 00001\rangle - \epsilon_{1_3}\|00111\ 00010\rangle - \epsilon_{1_4}\|00111\ 11101\rangle + \epsilon_{1_5}\|00111\ 11110\rangle$	XXZIZ	ZIIZX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\vert +\rangle\vert 0\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle + \epsilon_2\vert 00001\ 10000\rangle + \epsilon_3\vert 00001\ 10011\rangle + \epsilon_4\vert 00010\ 01100\rangle + \epsilon_5\vert 00010\ 01111\rangle + \epsilon_6\vert 00010\ 10000\rangle + \epsilon_7\vert 00010\ 10011\rangle + \epsilon_8\vert 11101\ 01100\rangle + \epsilon_9\vert 11101\ 01111\rangle + \epsilon_{1_0}\vert 11101\ 10000\rangle + \epsilon_{1_1}\vert 11101\ 10011\rangle + \epsilon_{1_2}\vert 11110\ 01100\rangle + \epsilon_{1_3}\vert 11110\ 01111\rangle + \epsilon_{1_4}\vert 11110\ 10000\rangle + \epsilon_{1_5}\vert 11110\ 10011\rangle$	IIIIX	IXXII
$\vert +\rangle\vert 0\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle - \epsilon_2\vert 00001\ 10000\rangle - \epsilon_3\vert 00001\ 10011\rangle + \epsilon_4\vert 00010\ 01100\rangle + \epsilon_5\vert 00010\ 01111\rangle - \epsilon_6\vert 00010\ 10000\rangle - \epsilon_7\vert 00010\ 10011\rangle + \epsilon_8\vert 11101\ 01100\rangle + \epsilon_9\vert 11101\ 01111\rangle - \epsilon_{1_0}\vert 11101\ 10000\rangle - \epsilon_{1_1}\vert 11101\ 10011\rangle + \epsilon_{1_2}\vert 11110\ 01100\rangle + \epsilon_{1_3}\vert 11110\ 01111\rangle - \epsilon_{1_4}\vert 11110\ 10000\rangle - \epsilon_{1_5}\vert 11110\ 10011\rangle$	IIIIX	ZXXII
$\vert +\rangle\vert 1\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle + \epsilon_2\vert 00001\ 10000\rangle + \epsilon_3\vert 00001\ 10011\rangle + \epsilon_4\vert 00010\ 01100\rangle + \epsilon_5\vert 00010\ 01111\rangle + \epsilon_6\vert 00010\ 10000\rangle + \epsilon_7\vert 00010\ 10011\rangle - \epsilon_8\vert 11101\ 01100\rangle - \epsilon_9\vert 11101\ 01111\rangle - \epsilon_{1_0}\vert 11101\ 10000\rangle - \epsilon_{1_1}\vert 11101\ 10011\rangle + \epsilon_{1_2}\vert 11110\ 01100\rangle + \epsilon_{1_3}\vert 11110\ 01111\rangle - \epsilon_{1_4}\vert 11110\ 10000\rangle - \epsilon_{1_5}\vert 11110\ 10011\rangle$	ZIIIX	IXXII
$\vert +\rangle\vert 1\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle - \epsilon_2\vert 00001\ 10000\rangle - \epsilon_3\vert 00001\ 10011\rangle + \epsilon_4\vert 00010\ 01100\rangle + \epsilon_5\vert 00010\ 01111\rangle - \epsilon_6\vert 00010\ 10000\rangle - \epsilon_7\vert 00010\ 10011\rangle - \epsilon_8\vert 11101\ 01100\rangle - \epsilon_9\vert 11101\ 01111\rangle + \epsilon_{1_0}\vert 11101\ 10000\rangle + \epsilon_{1_1}\vert 11101\ 10011\rangle - \epsilon_{1_2}\vert 11110\ 01100\rangle - \epsilon_{1_3}\vert 11110\ 01111\rangle + \epsilon_{1_4}\vert 11110\ 10000\rangle + \epsilon_{1_5}\vert 11110\ 10011\rangle$	ZXXII	ZXXII

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle + \epsilon_2\|00001\ 10000\rangle - \epsilon_3\|00001\ 10011\rangle + \epsilon_4\|00010\ 01100\rangle - \epsilon_5\|00010\ 01111\rangle + \epsilon_6\|00010\ 10000\rangle - \epsilon_7\|00010\ 10011\rangle + \epsilon_8\|11101\ 01100\rangle - \epsilon_9\|11101\ 01111\rangle + \epsilon_{10}\|11101\ 10000\rangle - \epsilon_{11}\|11101\ 10011\rangle + \epsilon_{12}\|11110\ 01100\rangle - \epsilon_{13}\|11110\ 01111\rangle + \epsilon_{14}\|11110\ 10000\rangle - \epsilon_{15}\|11110\ 10011\rangle$	IIIIX	ZXXII
$\|+\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle - \epsilon_2\|00001\ 10000\rangle + \epsilon_3\|00001\ 10011\rangle + \epsilon_4\|00010\ 01100\rangle - \epsilon_5\|00010\ 01111\rangle - \epsilon_6\|00010\ 10000\rangle + \epsilon_7\|00010\ 10011\rangle + \epsilon_8\|11101\ 01100\rangle - \epsilon_9\|11101\ 01111\rangle - \epsilon_{10}\|11101\ 10000\rangle + \epsilon_{11}\|11101\ 10011\rangle + \epsilon_{12}\|11110\ 01100\rangle - \epsilon_{13}\|11110\ 01111\rangle - \epsilon_{14}\|11110\ 10000\rangle + \epsilon_{15}\|11110\ 10011\rangle$	IIIIX	$\mathrm{ZX\underline{ZX}II}$
$\|+\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle + \epsilon_2\|00001\ 10000\rangle - \epsilon_3\|00001\ 10011\rangle + \epsilon_4\|00010\ 01100\rangle - \epsilon_5\|00010\ 01111\rangle + \epsilon_6\|00010\ 10000\rangle - \epsilon_7\|00010\ 10011\rangle - \epsilon_8\|11101\ 01100\rangle + \epsilon_9\|11101\ 01111\rangle - \epsilon_{10}\|11101\ 10000\rangle + \epsilon_{11}\|11101\ 10011\rangle - \epsilon_{12}\|11110\ 01100\rangle + \epsilon_{13}\|11110\ 01111\rangle - \epsilon_{14}\|11110\ 10000\rangle + \epsilon_{15}\|11110\ 10011\rangle$	ZIIIX	ZXXII
$\|+\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle - \epsilon_2\|00001\ 10000\rangle + \epsilon_3\|00001\ 10011\rangle + \epsilon_4\|00010\ 01100\rangle - \epsilon_5\|00010\ 01111\rangle - \epsilon_6\|00010\ 10000\rangle + \epsilon_7\|00010\ 10011\rangle - \epsilon_8\|11101\ 01100\rangle + \epsilon_9\|11101\ 01111\rangle + \epsilon_{10}\|11101\ 10000\rangle - \epsilon_{11}\|11101\ 10011\rangle - \epsilon_{12}\|11110\ 01100\rangle + \epsilon_{13}\|11110\ 01111\rangle + \epsilon_{14}\|11110\ 10000\rangle - \epsilon_{15}\|11110\ 10011\rangle$	ZIIIX	$\mathrm{ZX\underline{ZX}II}$

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle + \epsilon_2\vert 00001\ 10000\rangle + \epsilon_3\vert 00001\ 10011\rangle - \epsilon_4\vert 00010\ 01100\rangle - \epsilon_5\vert 00010\ 01111\rangle - \epsilon_6\vert 00010\ 10000\rangle - \epsilon_7\vert 00010\ 10011\rangle + \epsilon_8\vert 11101\ 01100\rangle + \epsilon_9\vert 11101\ 01111\rangle + \epsilon_{10}\vert 11101\ 10000\rangle + \epsilon_{11}\vert 11101\ 10011\rangle - \epsilon_{12}\vert 11110\ 01100\rangle - \epsilon_{13}\vert 11110\ 01111\rangle - \epsilon_{14}\vert 11110\ 10000\rangle - \epsilon_{15}\vert 11110\ 10011\rangle$	IIIZX	IXXII
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle - \epsilon_2\vert 00001\ 10000\rangle - \epsilon_3\vert 00001\ 10011\rangle - \epsilon_4\vert 00010\ 01100\rangle - \epsilon_5\vert 00010\ 01111\rangle + \epsilon_6\vert 00010\ 10000\rangle + \epsilon_7\vert 00010\ 10011\rangle + \epsilon_8\vert 11101\ 01100\rangle + \epsilon_9\vert 11101\ 01111\rangle - \epsilon_{10}\vert 11101\ 10000\rangle - \epsilon_{11}\vert 11101\ 10011\rangle - \epsilon_{12}\vert 11110\ 01100\rangle - \epsilon_{13}\vert 11110\ 01111\rangle + \epsilon_{14}\vert 11110\ 10000\rangle + \epsilon_{15}\vert 11110\ 10011\rangle$	IIIZX	ZXXII
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle + \epsilon_2\vert 00001\ 10000\rangle + \epsilon_3\vert 00001\ 10011\rangle - \epsilon_4\vert 00010\ 01100\rangle - \epsilon_5\vert 00010\ 01111\rangle - \epsilon_6\vert 00010\ 10000\rangle - \epsilon_7\vert 00010\ 10011\rangle - \epsilon_8\vert 11101\ 01100\rangle - \epsilon_9\vert 11101\ 01111\rangle - \epsilon_{10}\vert 11101\ 10000\rangle - \epsilon_{11}\vert 11101\ 10011\rangle + \epsilon_{12}\vert 11110\ 01100\rangle + \epsilon_{13}\vert 11110\ 01111\rangle + \epsilon_{14}\vert 11110\ 10000\rangle + \epsilon_{15}\vert 11110\ 10011\rangle$	IIZZX	IXXII
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 00001\ 01100\rangle + \epsilon_1\vert 00001\ 01111\rangle - \epsilon_2\vert 00001\ 10000\rangle - \epsilon_3\vert 00001\ 10011\rangle - \epsilon_4\vert 00010\ 01100\rangle - \epsilon_5\vert 00010\ 01111\rangle + \epsilon_6\vert 00010\ 10000\rangle + \epsilon_7\vert 00010\ 10011\rangle + \epsilon_8\vert 11101\ 01100\rangle + \epsilon_9\vert 11101\ 01111\rangle + \epsilon_{10}\vert 11101\ 10000\rangle + \epsilon_{11}\vert 11101\ 10011\rangle - \epsilon_{12}\vert 11110\ 01100\rangle - \epsilon_{13}\vert 11110\ 01111\rangle - \epsilon_{14}\vert 11110\ 10000\rangle - \epsilon_{15}\vert 11110\ 10011\rangle$	IIZZX	ZXXII

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|-\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle + \epsilon_2\|00001\ 10000\rangle - \epsilon_3\|00001\ 10011\rangle - \epsilon_4\|00010\ 01100\rangle + \epsilon_5\|00010\ 01111\rangle - \epsilon_6\|00010\ 10000\rangle + \epsilon_7\|00010\ 10011\rangle + \epsilon_8\|11101\ 01100\rangle - \epsilon_9\|11101\ 01111\rangle + \epsilon_{10}\|11101\ 10000\rangle - \epsilon_{11}\|11101\ 10011\rangle - \epsilon_{12}\|11110\ 01100\rangle + \epsilon_{13}\|11110\ 01111\rangle - \epsilon_{14}\|11110\ 10000\rangle + \epsilon_{15}\|11110\ 10011\rangle$	IIIZX	ZXXII
$\|-\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle - \epsilon_2\|00001\ 10000\rangle + \epsilon_3\|00001\ 10011\rangle - \epsilon_4\|00010\ 01100\rangle + \epsilon_5\|00010\ 01111\rangle + \epsilon_6\|00010\ 10000\rangle - \epsilon_7\|00010\ 10011\rangle + \epsilon_8\|11101\ 01100\rangle - \epsilon_9\|11101\ 01111\rangle - \epsilon_{10}\|11101\ 10000\rangle + \epsilon_{11}\|11101\ 10011\rangle - \epsilon_{12}\|11110\ 01100\rangle + \epsilon_{13}\|11110\ 01111\rangle + \epsilon_{14}\|11110\ 10000\rangle - \epsilon_{15}\|11110\ 10011\rangle$	IIIZX	ZXXII
$\|-\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle + \epsilon_2\|00001\ 10000\rangle - \epsilon_3\|00001\ 10011\rangle - \epsilon_4\|00010\ 01100\rangle + \epsilon_5\|00010\ 01111\rangle - \epsilon_6\|00010\ 10000\rangle + \epsilon_7\|00010\ 10011\rangle - \epsilon_8\|11101\ 01100\rangle + \epsilon_9\|11101\ 01111\rangle - \epsilon_{10}\|11101\ 10000\rangle + \epsilon_{11}\|11101\ 10011\rangle + \epsilon_{12}\|11110\ 01100\rangle - \epsilon_{13}\|11110\ 01111\rangle + \epsilon_{14}\|11110\ 10000\rangle - \epsilon_{15}\|11110\ 10011\rangle$	IIZZX	ZXXII
$\|-\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|00001\ 01100\rangle - \epsilon_1\|00001\ 01111\rangle - \epsilon_2\|00001\ 10000\rangle + \epsilon_3\|00001\ 10011\rangle - \epsilon_4\|00010\ 01100\rangle + \epsilon_5\|00010\ 01111\rangle + \epsilon_6\|00010\ 10000\rangle - \epsilon_7\|00010\ 10011\rangle - \epsilon_8\|11101\ 01100\rangle + \epsilon_9\|11101\ 01111\rangle + \epsilon_{10}\|11101\ 10000\rangle - \epsilon_{11}\|11101\ 10011\rangle + \epsilon_{12}\|11110\ 01100\rangle - \epsilon_{13}\|11110\ 01111\rangle - \epsilon_{14}\|11110\ 10000\rangle + \epsilon_{15}\|11110\ 10011\rangle$	IIZZX	ZX<u>ZX</u>II

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|11001\ 01101\rangle + \epsilon_1\|11001\ 01110\rangle + \epsilon_2\|11001\ 10001\rangle + \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle + \epsilon_5\|11010\ 01110\rangle + \epsilon_6\|11010\ 10001\rangle + \epsilon_7\|11010\ 10010\rangle + \epsilon_8\|00101\ 01101\rangle + \epsilon_9\|00101\ 01110\rangle + \epsilon_{1_0}\|00101\ 10001\rangle + \epsilon_{1_1}\|00101\ 10010\rangle + \epsilon_{1_2}\|00110\ 01101\rangle + \epsilon_{1_3}\|00110\ 01110\rangle + \epsilon_{1_4}\|00110\ 10001\rangle + \epsilon_{1_5}\|00110\ 10010\rangle$	XXIIX	IXXIX
$\|+\rangle\|0\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|11001\ 01101\rangle + \epsilon_1\|11001\ 01110\rangle - \epsilon_2\|11001\ 10001\rangle - \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle + \epsilon_5\|11010\ 01110\rangle - \epsilon_6\|11010\ 10001\rangle - \epsilon_7\|11010\ 10010\rangle + \epsilon_8\|00101\ 01101\rangle + \epsilon_9\|00101\ 01110\rangle - \epsilon_{1_0}\|00101\ 10001\rangle - \epsilon_{1_1}\|00101\ 10010\rangle + \epsilon_{1_2}\|00110\ 01101\rangle + \epsilon_{1_3}\|00110\ 01110\rangle - \epsilon_{1_4}\|00110\ 10001\rangle - \epsilon_{1_5}\|00110\ 10010\rangle$	XXIIX	ZXXIX
$\|+\rangle\|1\rangle$	$\|+\rangle\|0\rangle$	$\epsilon_0\|11001\ 01101\rangle + \epsilon_1\|11001\ 01110\rangle + \epsilon_2\|11001\ 10001\rangle + \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle + \epsilon_5\|11010\ 01110\rangle + \epsilon_6\|11010\ 10001\rangle + \epsilon_7\|11010\ 10010\rangle - \epsilon_8\|00101\ 01101\rangle - \epsilon_9\|00101\ 01110\rangle - \epsilon_{1_0}\|00101\ 10001\rangle - \epsilon_{1_1}\|00101\ 10010\rangle - \epsilon_{1_2}\|00110\ 01101\rangle - \epsilon_{1_3}\|00110\ 01110\rangle - \epsilon_{1_4}\|00110\ 10001\rangle - \epsilon_{1_5}\|00110\ 10010\rangle$	XXZIX	IXXIX
$\|+\rangle\|1\rangle$	$\|+\rangle\|1\rangle$	$\epsilon_0\|11001\ 01101\rangle + \epsilon_1\|11001\ 01110\rangle - \epsilon_2\|11001\ 10001\rangle - \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle + \epsilon_5\|11010\ 01110\rangle - \epsilon_6\|11010\ 10001\rangle - \epsilon_7\|11010\ 10010\rangle - \epsilon_8\|00101\ 01101\rangle - \epsilon_9\|00101\ 01110\rangle + \epsilon_{1_0}\|00101\ 10001\rangle + \epsilon_{1_1}\|00101\ 10010\rangle - \epsilon_{1_2}\|00110\ 01101\rangle - \epsilon_{1_3}\|00110\ 01110\rangle + \epsilon_{1_4}\|00110\ 10001\rangle + \epsilon_{1_5}\|00110\ 10010\rangle$	XXZIX	ZXXIX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\|+\rangle\|0\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11001\ 01101\rangle - \epsilon_1\|11001\ 01110\rangle + \epsilon_2\|11001\ 10001\rangle - \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle - \epsilon_5\|11010\ 01110\rangle + \epsilon_6\|11010\ 10001\rangle - \epsilon_7\|11010\ 10010\rangle + \epsilon_8\|00101\ 01101\rangle - \epsilon_9\|00101\ 01110\rangle + \epsilon_{10}\|00101\ 10001\rangle - \epsilon_{11}\|00101\ 10010\rangle + \epsilon_{12}\|00110\ 01101\rangle - \epsilon_{13}\|00110\ 01110\rangle + \epsilon_{14}\|00110\ 10001\rangle - \epsilon_{15}\|00110\ 10010\rangle$	XXIIX	IXXZX
$\|+\rangle\|0\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11001\ 01101\rangle - \epsilon_1\|11001\ 01110\rangle - \epsilon_2\|11001\ 10001\rangle + \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle - \epsilon_5\|11010\ 01110\rangle - \epsilon_6\|11010\ 10001\rangle + \epsilon_7\|11010\ 10010\rangle + \epsilon_8\|00101\ 01101\rangle - \epsilon_9\|00101\ 01110\rangle - \epsilon_{10}\|00101\ 10001\rangle + \epsilon_{11}\|00101\ 10010\rangle + \epsilon_{12}\|00110\ 01101\rangle - \epsilon_{13}\|00110\ 01110\rangle - \epsilon_{14}\|00110\ 10001\rangle + \epsilon_{15}\|00110\ 10010\rangle$	XXIIX	ZXXZX
$\|+\rangle\|1\rangle$	$\|-\rangle\|0\rangle$	$\epsilon_0\|11001\ 01101\rangle - \epsilon_1\|11001\ 01110\rangle + \epsilon_2\|11001\ 10001\rangle - \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle - \epsilon_5\|11010\ 01110\rangle + \epsilon_6\|11010\ 10001\rangle - \epsilon_7\|11010\ 10010\rangle - \epsilon_8\|00101\ 01101\rangle + \epsilon_9\|00101\ 01110\rangle - \epsilon_{10}\|00101\ 10001\rangle + \epsilon_{11}\|00101\ 10010\rangle - \epsilon_{12}\|00110\ 01101\rangle + \epsilon_{13}\|00110\ 01110\rangle - \epsilon_{14}\|00110\ 10001\rangle + \epsilon_{15}\|00110\ 10010\rangle$	XXZIX	IXXZX
$\|+\rangle\|1\rangle$	$\|-\rangle\|1\rangle$	$\epsilon_0\|11001\ 01101\rangle - \epsilon_1\|11001\ 01110\rangle + \epsilon_2\|11001\ 10001\rangle - \epsilon_3\|11001\ 10010\rangle + \epsilon_4\|11010\ 01101\rangle - \epsilon_5\|11010\ 01110\rangle + \epsilon_6\|11010\ 10001\rangle - \epsilon_7\|11010\ 10010\rangle + \epsilon_8\|00101\ 01101\rangle - \epsilon_9\|00101\ 01110\rangle + \epsilon_{10}\|00101\ 10001\rangle - \epsilon_{11}\|00101\ 10010\rangle + \epsilon_{12}\|00110\ 01101\rangle - \epsilon_{13}\|00110\ 01110\rangle + \epsilon_{14}\|00110\ 10001\rangle - \epsilon_{15}\|00110\ 10010\rangle$	XXZIX	ZXXZX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 11001\ 01101\rangle + \epsilon_1\vert 11001\ 01110\rangle + \epsilon_2\vert 11001\ 10001\rangle + \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle - \epsilon_5\vert 11010\ 01110\rangle - \epsilon_6\vert 11010\ 10001\rangle - \epsilon_7\vert 11010\ 10010\rangle + \epsilon_8\vert 00101\ 01101\rangle + \epsilon_9\vert 00101\ 01110\rangle + \epsilon_{10}\vert 00101\ 10001\rangle + \epsilon_{11}\vert 00101\ 10010\rangle - \epsilon_{12}\vert 00110\ 01101\rangle - \epsilon_{13}\vert 00110\ 01110\rangle - \epsilon_{14}\vert 00110\ 10001\rangle - \epsilon_{15}\vert 00110\ 10010\rangle$	XXIZX	IXXIX
$\vert -\rangle\vert 0\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 11001\ 01101\rangle + \epsilon_1\vert 11001\ 01110\rangle - \epsilon_2\vert 11001\ 10001\rangle - \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle - \epsilon_5\vert 11010\ 01110\rangle + \epsilon_6\vert 11010\ 10001\rangle + \epsilon_7\vert 11010\ 10010\rangle + \epsilon_8\vert 00101\ 01101\rangle + \epsilon_9\vert 00101\ 01110\rangle - \epsilon_{10}\vert 00101\ 10001\rangle - \epsilon_{11}\vert 00101\ 10010\rangle - \epsilon_{12}\vert 00110\ 01101\rangle - \epsilon_{13}\vert 00110\ 01110\rangle + \epsilon_{14}\vert 00110\ 10001\rangle + \epsilon_{15}\vert 00110\ 10010\rangle$	XXIZX	ZXXIX
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 0\rangle$	$\epsilon_0\vert 11001\ 01101\rangle + \epsilon_1\vert 11001\ 01110\rangle + \epsilon_2\vert 11001\ 10001\rangle + \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle - \epsilon_5\vert 11010\ 01110\rangle - \epsilon_6\vert 11010\ 10001\rangle - \epsilon_7\vert 11010\ 10010\rangle - \epsilon_8\vert 00101\ 01101\rangle - \epsilon_9\vert 00101\ 01110\rangle - \epsilon_{10}\vert 00101\ 10001\rangle - \epsilon_{11}\vert 00101\ 10010\rangle + \epsilon_{12}\vert 00110\ 01101\rangle + \epsilon_{13}\vert 00110\ 01110\rangle + \epsilon_{14}\vert 00110\ 10001\rangle + \epsilon_{15}\vert 00110\ 10010\rangle$	XXZZX	IXXIX
$\vert -\rangle\vert 1\rangle$	$\vert +\rangle\vert 1\rangle$	$\epsilon_0\vert 11001\ 01101\rangle + \epsilon_1\vert 11001\ 01110\rangle - \epsilon_2\vert 11001\ 10001\rangle - \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle - \epsilon_5\vert 11010\ 01110\rangle + \epsilon_6\vert 11010\ 10001\rangle + \epsilon_7\vert 11010\ 10010\rangle - \epsilon_8\vert 00101\ 01101\rangle - \epsilon_9\vert 00101\ 01110\rangle + \epsilon_{10}\vert 00101\ 10001\rangle + \epsilon_{11}\vert 00101\ 10010\rangle + \epsilon_{12}\vert 00110\ 01101\rangle + \epsilon_{13}\vert 00110\ 01110\rangle - \epsilon_{14}\vert 00110\ 10001\rangle - \epsilon_{15}\vert 00110\ 10010\rangle$	XXZZX	ZXXIX

Table 5 continued

Alice	Bob	Collapsed state	Alice's operat	Bob's operat
$\vert -\rangle\vert 0\rangle$	$\vert -\rangle\vert 0\rangle$	$\epsilon_0\vert 11001\ 01101\rangle - \epsilon_1\vert 11001\ 01110\rangle + \epsilon_2\vert 11001\ 10001\rangle - \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle + \epsilon_5\vert 11010\ 01110\rangle - \epsilon_6\vert 11010\ 10001\rangle + \epsilon_7\vert 11010\ 10010\rangle + \epsilon_8\vert 00101\ 01101\rangle - \epsilon_9\vert 00101\ 01110\rangle + \epsilon_{10}\vert 00101\ 10001\rangle - \epsilon_{11}\vert 00101\ 10010\rangle - \epsilon_{12}\vert 00110\ 01101\rangle + \epsilon_{13}\vert 00110\ 01110\rangle - \epsilon_{14}\vert 00110\ 10001\rangle + \epsilon_{15}\vert 00110\ 10010\rangle$	XXIZX	IXXZX
$\vert -\rangle\vert 0\rangle$	$\vert -\rangle\vert 1\rangle$	$\epsilon_0\vert 11001\ 01101\rangle - \epsilon_1\vert 11001\ 01110\rangle - \epsilon_2\vert 11001\ 10001\rangle + \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle + \epsilon_5\vert 11010\ 01110\rangle + \epsilon_6\vert 11010\ 10001\rangle - \epsilon_7\vert 11010\ 10010\rangle + \epsilon_8\vert 00101\ 01101\rangle - \epsilon_9\vert 00101\ 01110\rangle - \epsilon_{10}\vert 00101\ 10001\rangle + \epsilon_{11}\vert 00101\ 10010\rangle - \epsilon_{12}\vert 00110\ 01101\rangle + \epsilon_{13}\vert 00110\ 01110\rangle + \epsilon_{14}\vert 00110\ 10001\rangle - \epsilon_{15}\vert 00110\ 10010\rangle$	XXIZX	ZXXZX
$\vert -\rangle\vert 1\rangle$	$\vert -\rangle\vert 0\rangle$	$\epsilon_0\vert 11001\ 01101\rangle - \epsilon_1\vert 11001\ 01110\rangle + \epsilon_2\vert 11001\ 10001\rangle - \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle + \epsilon_5\vert 11010\ 01110\rangle - \epsilon_6\vert 11010\ 10001\rangle + \epsilon_7\vert 11010\ 10010\rangle - \epsilon_8\vert 00101\ 01101\rangle + \epsilon_9\vert 00101\ 01110\rangle - \epsilon_{10}\vert 00101\ 10001\rangle + \epsilon_{11}\vert 00101\ 10010\rangle + \epsilon_{12}\vert 00110\ 01101\rangle - \epsilon_{13}\vert 00110\ 01110\rangle + \epsilon_{14}\vert 00110\ 10001\rangle - \epsilon_{15}\vert 00110\ 10010\rangle$	XXZZX	IXXZX
$\vert -\rangle\vert 1\rangle$	$\vert -\rangle\vert 1\rangle$	$\epsilon_0\vert 11001\ 01101\rangle - \epsilon_1\vert 11001\ 01110\rangle - \epsilon_2\vert 11001\ 10001\rangle + \epsilon_3\vert 11001\ 10010\rangle - \epsilon_4\vert 11010\ 01101\rangle + \epsilon_5\vert 11010\ 01110\rangle + \epsilon_6\vert 11010\ 10001\rangle - \epsilon_7\vert 11010\ 10010\rangle - \epsilon_8\vert 00101\ 01101\rangle + \epsilon_9\vert 00101\ 01110\rangle + \epsilon_{10}\vert 00101\ 10001\rangle - \epsilon_{11}\vert 00101\ 10010\rangle + \epsilon_{12}\vert 00110\ 01101\rangle - \epsilon_{13}\vert 00110\ 01110\rangle - \epsilon_{14}\vert 00110\ 10001\rangle + \epsilon_{15}\vert 00110\ 10010\rangle$	XXZZX	ZXXZX

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Declarations

Conflicts of interest The authors declare no competing interests.

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