RESEARCH



Bidirectional Quantum Teleportation of GHZ and EPR States Through Entanglement Swapping Utilizing a Pre-established GHZ Channel

Behzad Alipour¹ · Ahmad Akhound¹

Received: 25 February 2024 / Accepted: 24 April 2024 / Published online: 29 May 2024 © The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2024

Abstract

In this paper, we present a novel bidirectional quantum teleportation protocol facilitating the simultaneous teleportation of pairs of EPR and GHZ states through entanglement swapping within a six-qubit GHZ channel. This protocol leverages pre-existing GHZ states within the channel, alongside the EPR and GHZ states intended for teleportation, to generate a new entangled state. Subsequently, this state undergoes measurement by Alice and Bob, with the measurement outcomes determining the teleported states. Finally, the successful teleportation of the EPR and GHZ states to each other is ensured through the utilization of CNOT operators, single-qubit and two-qubit measurements, and the application of the identity operator. The proposed protocol presents several advantages over existing protocols, including its bidirectional nature and higher efficiency.

Keywords Bidirection quantum teleportation · GHZ state · EPR state · Entanglement swapping

1 Introduction

Quantum entanglement serves as a cornerstone in quantum information theory, catalyzing significant progress in various protocols such as quantum teleportation (QT) [1, 2], quantum cryptography (QC) [2], quantum secret sharing (QSS) [3], and quantum key distribution (QKD) [4, 5]. Bennett and colleagues pioneered quantum teleportation in 1993, teleporting a single-qubit state using classical bits in a Bell-state channel [1]. Since then, researchers have expanded on this concept, introducing diverse quantum teleportation protocols. For instance, in 2002, Bao et al. proposed a protocol teleporting a single-qubit state through a W-state channel [6]. Advancements included protocols for transferring single qubits in both W and EPR channels [7–9], as well as protocols for teleporting states composed of two to six qubits through Bell and W channels [9–14].

In 2010, Liu et al. proposed a protocol enabling controlled teleportation of an arbitrary two-particle state using a five-qubit cluster state [15]. In 2017, Sadiqzadeh and colleagues introduced a remote transfer protocol for an arbitrary two-qubit state in an eight-qubit channel,

Ahmad Akhound aakhound@pnu.ac.ir

¹ Department of Physics, Payame Noor University, P.O.BOX 19395-3697, Tehran, Iran

relying solely on single-qubit measurements [16, 17]. Subsequently, multi-stage teleportation schemes with specific objectives emerged. This paper focuses on bidirectional quantum teleportation (BQT), facilitating the teleportation of unknown two- and three-qubit states, including EPR and GHZ states, between Alice and Bob using a six-qubit GHZ channel. This entanglement swapping-based transmission preserves quantum information, ensuring higher security and efficiency.

The paper is structured as follows:

- Section 2: Provides a comprehensive description of the protocol, including its theoretical foundations, entanglement swapping procedure, system preparation, relevant operator application, measurement scheme based on specific bases, and thorough data analysis.

- Section 3: Presents the efficiency calculations of the protocol and compares them to other reported schemes using established metrics.

- Section 4: Explains the methodology and implementation of quantum circuit simulation.

2 Protocol Description

As mentioned, this protocol constitutes a bidirectional quantum teleportation (BQT) scheme allowing Alice and Bob to simultaneously teleport EPR and GHZ states to each other in an indeterminate manner, represented as follows:

$$|GHZ\rangle a_1 a_2 a_3 = \alpha_0 |000\rangle + \alpha_1 |111\rangle,$$
(1)
$$|EPR\rangle a_4 a_5 = \alpha_2 |00\rangle + \alpha_3 |11\rangle.$$

$$|GHZ\rangle b_1 b_2 b_3 = \beta_0 |000\rangle + \beta_1 |111\rangle, \tag{2}$$

$$|EPR\rangle b_4b_5 = \beta_2|00\rangle + \beta_3|11\rangle$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$ are arbitrary coefficients with

 $|\alpha_0|^2 + |\alpha_1|^2 = 1, \quad |\alpha_2|^2 + |\alpha_3|^2 = 1, \quad |\beta_0|^2 + |\beta_1|^2 = 1, \quad |\beta_2|^2 + |\beta_3|^2 = 1.$



Fig. 1 In the overall system structure, qubits $a_1a_2a_3a_4a_5$ belong to Alice, qubits $b_1b_2b_3b_4b_5$ belong to Bob, and qubits $A_1A_2A_3B_1B_2B_3$ are shared as a channel by Alice and Bob

The preparation of this protocol involves several steps, which will be explained in the following sections.

2.1 System Preparation

To initiate the teleportation process by Alice and Bob, a quantum channel is established, comprising a six-qubit state formed from two pairs of GHZ, as follows:

$$|Q_C\rangle_{A_1B_1B_2A_2A_3B_3} = 1/\sqrt{2}(|000\rangle + |111\rangle) \otimes 1/\sqrt{2}(|000\rangle + |111\rangle).$$
(3)

Equation (3) can be expressed by changing the qubits positions as follows:

$$|Q_C\rangle_{A_1A_2A_3B_1B_2B_3} = 1/2(|000\rangle|000\rangle + |011\rangle|001\rangle + |100\rangle|110\rangle + |111\rangle|111\rangle).$$

As depicted in Fig. 1, within this channel, qubits A_1 , A_2 , and A_3 are allocated to Alice, while three qubits B_1 , B_2 , and B_3 are assigned to Bob. It's noteworthy that the selected GHZ states within this channel signify a particular state characterized by maximum three-qubit entanglement (Fig. 1).

2.1.1 The Analysis of Entanglement Transfer Within the Channel

Before proceeding with the teleportation steps, the primary emphasis is on entanglement swapping within the channel [18]. The central focus is on the impact of EPR and GHZ states on entanglement transfer. To achieve this, the CNOT operator is applied in two steps: initially on qubits a_1 , a_2 and a_4 , which control qubits A_1 , A_2 , and A_3 respectively, and then on qubits B_1 , B_2 , and B_4 , serving as control qubits for B_1 , B_2 , and B_3 [19].

$$|EPR\rangle_{a_4a_5b_4b_5} = |EPR\rangle_{a_4a_5} \bigotimes |EPR\rangle_{b_4b_5} = \alpha_2\beta_2|0000\rangle + \alpha_2\beta_3|0011\rangle + \alpha_3\beta_2|1100\rangle + \alpha_3\beta_3|1111\rangle.$$
(4)

$$|GHZ\rangle_{a_{1}a_{2}a_{3}b_{1}b_{2}b_{3}} = |GHZ\rangle_{a_{1}a_{2}a_{3}} \bigotimes |GHZ\rangle_{b_{1}b_{2}b_{3}} = \alpha_{0}\beta_{0}|000000\rangle + \alpha_{0}\beta_{1}|000111\rangle + \alpha_{1}\beta_{0}|111000\rangle + \alpha_{1}\beta_{1}|11111\rangle.$$
(5)

GHZ-type quantum entanglement is characterized by a quantum superposition system. The GHZ state encompasses three qubits and is represented as follows [20–23]:

$$|\psi(i,j,k)\rangle = 1/\sqrt{2}(|i\rangle|j\rangle|k\rangle + |\bar{i}\rangle|\bar{j}\rangle|\bar{k}\rangle)_{Q_1Q_2Q_3} \tag{6}$$

In which i, j, k $\in \{0,1\}$, $\overline{i} = i \oplus 1 = (i + 1) \mod 2$, Q1, Q2 and Q3 are GHZ involved particles. In (6), i, j and k are set the possible value in $\{0,1\}$, respectively. The possible GHZ states are achieved as that:

$$|\psi_0\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle),$$
 (7)
 $|\psi_1\rangle = 1/\sqrt{2}(|000\rangle - |111\rangle),$

$$\begin{split} |\psi_2\rangle &= 1/\sqrt{2}(|100\rangle + |011\rangle), \\ |\psi_3\rangle &= 1/\sqrt{2}(|100\rangle - |011\rangle), \\ |\psi_6\rangle &= 1/\sqrt{2}(|110\rangle + |011\rangle), \\ |\psi_7\rangle &= 1/\sqrt{2}(|110\rangle - |011\rangle). \end{split}$$

We can elucidate (7) as follows:

$$|000\rangle = 1/\sqrt{2}(|\psi_{0}\rangle + |\psi_{1}\rangle),$$
(8)

$$|111\rangle = 1/\sqrt{2}(|\psi_{0}\rangle - |\psi_{1}\rangle),$$
(9)

$$|100\rangle = 1/\sqrt{2}(|\psi_{2}\rangle + |\psi_{3}\rangle),$$
(9)

$$|011\rangle = 1/\sqrt{2}(|\psi_{2}\rangle - |\psi_{3}\rangle),$$
(10)

$$|110\rangle = 1/\sqrt{2}(|\psi_{6}\rangle + |\psi_{7}\rangle),$$
(9)

$$|001\rangle = 1/\sqrt{2}(|\psi_{6}\rangle - |\psi_{7}\rangle).$$
(9)

The changes in the channel state after the influence of the EPR state are as follows [20]:

$$|Q\rangle_c \bigotimes |EPR\rangle_{a_4a_5b_4b_5} = |Q'\rangle_c = |\varphi\rangle_1 + |\varphi\rangle_2 + |\varphi\rangle_3 + |\varphi\rangle_4 \tag{9}$$

The values of $|\varphi\rangle_1$, $|\varphi\rangle_2$, $|\varphi\rangle_3$, $|\varphi\rangle_4$ can be calculated as follows:

$$\begin{split} |\varphi\rangle_{1} &= 1/2(|\psi_{0}\rangle_{A_{1}A_{2}A_{3}}|\psi_{0}\rangle_{B_{1}B_{2}B_{3}} + |\psi_{1}\rangle|\psi_{1}\rangle + |\psi_{2}\rangle|\psi_{6}\rangle + |\psi_{3}\rangle|\psi_{7}\rangle\alpha_{2}\beta_{2}|0000\rangle), \quad (10) \\ |\varphi\rangle_{2} &= 1/2(|\psi_{0}\rangle|\psi_{6}\rangle - |\psi_{1}\rangle|\psi_{7}\rangle + |\psi_{2}\rangle|\psi_{0}\rangle - |\psi_{3}\rangle|\psi_{1}\rangle)\alpha_{2}\beta_{3}|0011\rangle, \\ |\varphi\rangle_{3} &= 1/2(|\psi_{2}\rangle|\psi_{0}\rangle + |\psi_{3}\rangle|\psi_{1}\rangle + |\psi_{0}\rangle|\psi_{6}\rangle + |\psi_{1}\rangle|\psi_{7}\rangle)\alpha_{3}\beta_{2}|1100\rangle, \\ |\varphi\rangle_{4} &= 1/2(|\psi_{2}\rangle|\psi_{6}\rangle - |\psi_{3}\rangle|\psi_{7}\rangle + |\psi_{0}\rangle|\psi_{0}\rangle - |\psi_{1}\rangle|\psi_{1}\rangle)\alpha_{3}\beta_{3}|111\rangle]. \end{split}$$

It can be observed that the EPR state within the channel results in the transfer of entangled states. In the second step, the GHZ state is applied to the channel, and the changes in the basis and state transfer are analyzed.

$$|Q'\rangle_{c} \bigotimes |GHZ\rangle_{a_{1}a_{2}a_{3}b_{1}b_{2}b_{3}} =$$

$$|\varphi\rangle_{11} + |\varphi\rangle_{12} + |\varphi\rangle_{13} + |\varphi\rangle_{14} + |\varphi\rangle_{21} + |\varphi\rangle_{22} + |\varphi\rangle_{23} + |\varphi\rangle_{24} +$$

$$|\varphi\rangle_{31} + |\varphi\rangle_{32} + |\varphi\rangle_{33} + |\varphi\rangle_{34} + |\varphi\rangle_{41} + |\varphi\rangle_{42} + |\varphi\rangle_{43} + |\varphi\rangle_{44}$$
(11)

After performing the necessary calculations, we obtain the values of the sentences:

$$\begin{split} |\varphi\rangle_{11} &= 1/2(|\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle\rangle\varepsilon_0|000\rangle|00000\rangle, \\ |\varphi\rangle_{12} &= 1/2(|\psi_0\rangle|\psi_6\rangle + |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle + |\psi_3\rangle|\psi_1\rangle)\varepsilon_2|000\rangle|000111\rangle, \\ |\varphi\rangle_{13} &= 1/2(|\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle + |\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle)\varepsilon_8|000\rangle|111000\rangle, \\ |\varphi\rangle_{14} &= 1/2(|\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle)\varepsilon_1|0000\rangle|111111\rangle, \\ |\varphi\rangle_{21} &= 1/2(|\psi_0\rangle|\psi_6\rangle - |\psi_1\rangle|\psi_7\rangle + |\psi_2\rangle|\psi_0\rangle - |\psi_3\rangle|\psi_1\rangle)\varepsilon_1|0011\rangle|00000\rangle, \\ |\varphi\rangle_{22} &= 1/2(|\psi_0\rangle|\psi_0\rangle - |\psi_1\rangle|\psi_1\rangle + |\psi_2\rangle|\psi_6\rangle - |\psi_3\rangle|\psi_7\rangle)\varepsilon_3|0011\rangle|000111\rangle, \\ |\varphi\rangle_{23} &= 1/2(|\psi_2\rangle|\psi_6\rangle + |\psi_3\rangle|\psi_7\rangle + |\psi_0\rangle|\psi_0\rangle + |\psi_1\rangle|\psi_1\rangle)\varepsilon_9|0011\rangle|111000\rangle, \end{split}$$

$ \varphi\rangle_{24}$	=	1/2	$(\psi_2\rangle$	$ \psi_0 angle$	+	$ \psi_3\rangle$	$ \psi_1\rangle$	+	$ \psi_0 angle$	$ \psi_6\rangle$	+	$ \psi_1\rangle$	$ \psi_7\rangle$	$\rangle)\varepsilon_{11}$	00	11)	11	11	11),
$ \varphi\rangle_{31}$	=	1/2	$(\psi_2\rangle$	$ \psi_0 angle$	$^+$	$ \psi_3\rangle$	$ \psi_1\rangle$	+	$ \psi_0 angle$	$ \psi_6\rangle$	+	$ \psi_1\rangle$	$ \psi_7\rangle$	$\rangle)\varepsilon_4$	110	$ \langle 00\rangle$	000)00	$0\rangle$,
$ \varphi\rangle_{32}$	=	1/2	$(\psi_2\rangle$	$ \psi_6\rangle$	+	$ \psi_3\rangle$	$ \psi_7\rangle$	+	$ \psi_0 angle$	$ \psi_0 angle$	+	$ \psi_1\rangle$	$ \psi_1\rangle$	$\rangle)\varepsilon_{6}$	110	$ \langle 00\rangle$	000)11	1>,
$ \varphi\rangle_{33}$	=	1/2	$(\psi_0\rangle$	$ \psi_0 angle$	—	$ \psi_1\rangle$	$ \psi_1\rangle$	+	$ \psi_2\rangle$	$ \psi_2\rangle$	—	$ \psi_3\rangle$	$ \psi_7\rangle$	$\rangle)\varepsilon_{12}$	11	00)	11	100	$ 00\rangle$,
$ \varphi\rangle_{34}$	=	1/2	$(\psi_0\rangle$	$ \psi_6\rangle$	—	$ \psi_1\rangle$	$ \psi_7\rangle$	+	$ \psi_2\rangle$	$ \psi_0 angle$	-	$ \psi_3\rangle$	$ \psi_1\rangle$	$\rangle)\varepsilon_{14}$	11	00)	00	000	$ 00\rangle$,
$ \varphi\rangle_{41}$	=	1/2	$(\psi_2\rangle$	$ \psi_6\rangle$	—	$ \psi_3\rangle$	$ \psi_7\rangle$	+	$ \psi_0 angle$	$ \psi_0 angle$	—	$ \psi_1\rangle$	$ \psi_1\rangle$	$\rangle)\varepsilon_{5}$	111	$ 1\rangle$	000)00	$0\rangle$,
$ \varphi\rangle_{42}$	=	1/2	$(\psi_2\rangle$	$ \psi_0 angle$	—	$ \psi_3\rangle$	$ \psi_0 angle$	+	$ \psi_6\rangle$	$ \psi_1\rangle$	+	$ \psi_7\rangle$	$ \psi_7\rangle$	$\rangle)\varepsilon_7$	111	$ 1\rangle$	000)11	1>,
$ \varphi\rangle_{43}$	=	1/2	$(\psi_0\rangle$	$ \psi_6\rangle$	+	$ \psi_1\rangle$	$ \psi_7\rangle$	$^+$	$ \psi_2\rangle$	$ \psi_0 angle$	+	$ \psi_3\rangle$	$ \psi_1\rangle$	$\rangle)\varepsilon_{13}$	11	11)	11	100	00),
$ \varphi\rangle_{44}$	=	1/2	$\langle \psi_0 \rangle$	$ \psi_0\rangle$	+	$ \psi_1\rangle$	$ \psi_1\rangle$	+	$ \psi_2\rangle$	$ \psi_6\rangle$	+	$ \psi_3\rangle$	$ \psi_7\rangle$	$\rangle)\varepsilon_{15}$	11	11)	11	11	$ 11\rangle$

The entanglement of the channel transitions from one basis to another, progressing from one point to an intermediate point while preserving quantum information, even during longer distance transfers. Ultimately, Alice and Bob can independently reconstruct the final states of the teleported pairs $|EPR\rangle$ and $|GHZ\rangle$ accurately by measuring in their respective bases and utilizing classical bits, as described below:

2.1.2 Introduction of System Components

Following channel preparation, Alice and Bob transmit a five-qubit state, comprised of EPR and GHZ states, to the opposing party (Table 1):

$$|Q\rangle_T = |Q\rangle_t \otimes |Q\rangle_c. \tag{12}$$

$$\begin{split} |Q_t\rangle &= a_1 a_2 a_3 a_4 a_5 \, b_1 b_2 b_3 b_4 b_5 = |GHZ\rangle_{a_1 a_2 a_3} \otimes |EPR\rangle_{a_4 a_5} \otimes |GHZ\rangle_{b_1 b_2 b_3} \otimes |EPR\rangle_{b_4 b_5} \\ |Q_t\rangle &= (\epsilon_0 |00000 \ 00000\rangle + \epsilon_1 |00000 \ 00011\rangle + \epsilon_2 |00000 \ 11100\rangle + \epsilon_3 |00000 \ 11111\rangle + (\epsilon_4 |00011 \ 00000\rangle + \epsilon_5 |00011 \ 00011\rangle + \epsilon_6 |00011 \ 11100\rangle + \epsilon_7 |00011 \ 11111\rangle + \epsilon_8 |11100 \ 00000\rangle + \epsilon_9 |11100 \ 00011\rangle + \epsilon_{10} |11100 \ 11100\rangle + \epsilon_{11} |11100 \ 11111\rangle + \epsilon_{12} |11111 \ 00000\rangle + \epsilon_{13} |11111 \ 00011\rangle + \epsilon_{15} |11111 \ 1111\rangle. \end{split}$$

$$\begin{split} |Q\rangle_T &= |Q\rangle_t \otimes (|000\ 000\rangle + |011\ 001\rangle + |100\ 110\rangle + |111\ 111\rangle) \\ |Q\rangle_{T_1} &= |Q\rangle_t \otimes (|000\ 000\rangle, \\ |Q\rangle_{T_2} &= |Q\rangle_t \otimes (|011\ 001\rangle, \\ |Q\rangle_{T_3} &= |Q\rangle_t \otimes (|100\ 110\rangle, \\ |Q\rangle_{T_4} &= |Q\rangle_t \otimes (|111\ 111\rangle. \end{split}$$

Table 1	Complex	coefficients	satisfying	the	normalization	condition
---------	---------	--------------	------------	-----	---------------	-----------

$\epsilon_0 = \alpha_0 \alpha_2 \beta_0 \beta_2$	$\epsilon_1 = \alpha_0 \alpha_2 \beta_0 \beta_3$	$\epsilon_2 = \alpha_0 \alpha_2 \beta_1 \beta_2$	$\epsilon_3 = \alpha_0 \alpha_2 \beta_1 \beta_3$
$\epsilon_4 = \alpha_0 \alpha_3 \beta_0 \beta_2$	$\epsilon_5 = \alpha_0 \alpha_3 \beta_0 \beta_3$	$\epsilon_6 = \alpha_0 \alpha_3 \beta_1 \beta_2$	$\epsilon_7 = \alpha_0 \alpha_3 \beta_1 \beta_3$
$\epsilon_8 = \alpha_1 \alpha_2 \beta_0 \beta_2$	$\epsilon_9 = \alpha_1 \alpha_2 \beta_0 \beta_3$	$\epsilon_{10} = \alpha_1 \alpha_2 \beta_1 \beta_2$	$\epsilon_{11} = \alpha_1 \alpha_2 \beta_1 \beta_3$
$\epsilon_{12} = \alpha_1 \alpha_3 \beta_0 \beta_2$	$\epsilon_{13} = \alpha_1 \alpha_3 \beta_0 \beta_3$	$\epsilon_{14} = \alpha_1 \alpha_3 \beta_1 \beta_2$	$\epsilon_{15} = \alpha_1 \alpha_3 \beta_1 \beta_3$

Table 2 The (X-basis) measurement results of users and the corresponding collapsed states

2.2 Applying Operators

In this section, the *CNOT* operation is executed by Alice and Bob in such a manner that $a_1, a_2, a_4, b_1, b_2, b_3$ and $A_1, A_2, A_3, B_1, B_2, B_3$ represent the control qubits and target qubits, respectively. Upon applying CNOT, the overall state of the system undergoes changes. A detailed explanation is provided in the appendix of the article. Here, we present an examination of only one aspect of the four-part process (Fig. 2):

$$\begin{split} |\phi\rangle_{T1} &= 1/2(\varepsilon_0|000\rangle|000\rangle|00000000\rangle + \varepsilon_1|000\rangle|001\rangle|0000000011\rangle + \\ \varepsilon_2|000\rangle|110\rangle|0000011100\rangle + \varepsilon_3|000\rangle|111\rangle|0000011111\rangle + \\ \varepsilon_4|001\rangle|000\rangle|0001100000\rangle + \varepsilon_5|001\rangle|001\rangle|0001100011\rangle + \\ \varepsilon_6|001\rangle|110\rangle|0001111100\rangle + \varepsilon_7|001\rangle|111\rangle|000111111\rangle + \\ \varepsilon_8|110\rangle|000\rangle|111000000\rangle + \\ \varepsilon_9|110\rangle|001\rangle|1110\rangle|1110\rangle + \\ \varepsilon_{10}|110\rangle|110\rangle|1110\rangle + \\ \varepsilon_{11}|110\rangle|111\rangle|1110\rangle + \\ \varepsilon_{12}|111\rangle|000\rangle|11110\rangle + \\ \varepsilon_{13}|111\rangle|001\rangle|111110011\rangle + \\ \varepsilon_{14}|111\rangle|110\rangle|11111110\rangle + \\ \varepsilon_{15}|111\rangle|111\rangle|11111111\rangle)) \end{split}$$

2.3 Measurement

2.3.1 Measurement in the X-Basis

In this step, Alice and Bob conduct measurements on the single qubits a_4 and b_4 in the X-basis. They exchange the measurement outcomes with each other using two classical bits. Following the measurement, the system's state collapses to $(a_1a_2b_1b_2A_2A_3a_3a_5A_1b_3B_1B_2b_5B_3)$ [24, 25].

The measurement of one state is presented as follows (Table 2) (refer to the appendix for the complete description of measurements):

2.3.2 Von Neumann Basis Measurement

Following the X-basis measurement, Alice and Bob measure qubits (a_1, a_2) and (b_1, b_2) in the Von Neumann basis, resulting in the system collapsing to the state $(A_1A_2a_3a_5A_1b_3B_1B_2b_5B_3)$. They then exchange four additional classical bits to communicate their respective measurement outcomes. By combining the information from these two measurement steps, conveyed through the transmission of six classical bits, Alice and Bob can reconstruct the initial state by applying appropriate identity operations [26] (Table 3).

Table 3 X-basis	and Von Neumann b	sis measurement results		
Alice	Bob	Collapsed state	Alice's operator	Bob's operator
(0)(+)	(01(+ 1	$\begin{split} \epsilon_0 & 00000 \ 00000\rangle + \epsilon_1 & 00000 \ 00011\rangle + \epsilon_2 & 00000 \ 11100\rangle + \epsilon_3 & 00000 \ 11111\rangle + \\ \epsilon_4 & 00011 \ 00000\rangle + \epsilon_5 & 00011 \ 00011\rangle + \epsilon_6 & 00011 \ 11100\rangle + \epsilon_7 & 00011 \ 11111\rangle + \\ \epsilon_8 & 11100 \ 00000\rangle + \epsilon_9 & 11100 \ 00011\rangle + \epsilon_{1_0} & 11100 \ 11100\rangle + \epsilon_{1_1} & 11100 \ 11111\rangle + \\ \epsilon_{1_2} & 11111 \ 00000\rangle + \epsilon_{1_3} & 11111 \ 00011\rangle + \epsilon_{1_4} & 11111 \ 11100\rangle + \epsilon_{1_5} & 11111 \ 11111\rangle \end{split}$	I	I

3 Comparison and Efficiency

In the preceding sections, bidirectional quantum teleportation of ten qubits in a six-qubits channel, facilitated by six classical bits for various states has been achieved.

The protocol's efficiency can be computed using the formula [27]:

$$\eta = c/(p+q),\tag{13}$$

where represents efficiency, *c* is the number of sent qubits, *p* is the number of channel qubits, and *q* is the number of classical bits [28]. Here, $\eta = 83.3$ indicates that the proposed protocol has higher efficiency compared to other protocols (as shown in Table 4). Furthermore, its effectiveness is further emphasized due to the advantages associated with entanglement swapping in the channel [14, 29].

Advantages of Entangled State Transfer [20]:

- Greater efficiency.
- Higher security.
- Preservation of quantum information is more accessible.
- Expanded applicability in quantum networks.
- the initial teleportation qubits are not destroyed.

4 Circuit Simulation

The entire sequence of operations performed in this protocol, from its initiation to the final measurements, has been meticulously replicated in the circuit simulation (Fig. 2):

Design	Bidirectional transfer of qubits	Quantum channel	η
BCQT [30]	1⇔1	5 qubit	28
BCQT [13]	5⇔4	6 qubits cluster	25
BCQT [12]	$1 \leftrightarrow 1$	7 qubits	22
BCQT [31]	$1 \leftrightarrow 1$	6 qubits	25
BCQT [31]	$1 \leftrightarrow 2$	7 qubits	21
BCQT [32]	$2 \leftrightarrow 1$	6 qubits cluster	25
BCQT [33]	3⇔2	9 qubits cluster	7.27
ABCQT [14]	2⇔3	8 qubits cluster	38
BCQT [34]	2⇔3	6 qubits	45
BCQT [35]	3⇔3	6 qubits	50
BCQT [35]	$N \leftrightarrow N$	2N qubits	50
BCQT [36]	N↔N	4 qubits cluster	66
BCQT[presented work]	5⇔5	6 qubits	83.3

Table 4 Comparison of different protocols with our protocols in efficiency



Fig. 2 The protocol architecture proposed by Alice and Bob

5 Conclusion

This paper introduces a novel bidirectional quantum teleportation scheme that utilizes entanglement swapping to achieve the simultaneous teleportation of two pairs of GHZ and EPR states between Alice and Bob using a GHZ-state channel. The proposed protocol exploits entanglement swapping to generate a new entangled state, subsequently measured by Alice and Bob to determine the teleported states. This process facilitates the efficient transfer of a total of ten qubits between the two parties. Moreover, the protocol accommodates the transfer of both GHZ and EPR states, thereby broadening its applicability. Furthermore, the proposed scheme lays the groundwork for extending to multi-step teleportation for N qubits, thereby expanding its capabilities.

Appendix A Attachment:

Table 5 X-basis and	Von Neumann basis comp	olete measurement results		
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
(1+)10	() +)IO	$\begin{split} \varepsilon_0 & \left 00000 \ 00000 \right\rangle + \varepsilon_1 & \left 00000 \ 00011 \right\rangle + \varepsilon_2 & \left 00000 \ 11110 \right\rangle + \\ \varepsilon_3 & \left 00000 \ 11111 \right\rangle + \varepsilon_4 & \left 00011 \ 00000 \right\rangle + \varepsilon_5 & \left 00011 \ 00001 \right\rangle + \\ \varepsilon_6 & \left 00011 \ 11100 \right\rangle + \varepsilon_7 & \left 00011 \ 11111 \right\rangle + \varepsilon_8 & \left 11100 \ 00000 \right\rangle + \\ \varepsilon_9 & \left 11100 \ 00001 \right\rangle + \varepsilon_{1_3} & \left 11110 \ 01110 \right\rangle + \varepsilon_{1_4} & \left 11110 \ 11111 \right\rangle + \\ \varepsilon_{1_2} & \left 11111 \ 00000 \right\rangle + \varepsilon_{1_3} & \left 11111 \ 00011 \right\rangle + \varepsilon_{1_4} & \left 11111 \ 11100 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11110 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11110 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11110 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 11111 \right\rangle + \\ \varepsilon_{1_6} & \left 11111 \ 111111 \right\rangle + \\ \varepsilon_{1_6} & \left 111111 \ 11111$	IIII	I
(ol(+ 1	(11) + 1	$\begin{split} \varepsilon_{0} & 00000 & 00000 \end{pmatrix} + \varepsilon_{1} & 00000 & 00011 \end{pmatrix} - \varepsilon_{2} & 00000 & 11100 \end{pmatrix} - \\ \varepsilon_{3} & 00000 & 11111 \end{pmatrix} + \varepsilon_{4} & 00001 & 00000 \end{pmatrix} + \varepsilon_{5} & 00011 & 000011 \end{pmatrix} - \\ \varepsilon_{6} & 00011 & 11100 \end{pmatrix} - \varepsilon_{7} & 00001 & 11111 \end{pmatrix} + \varepsilon_{8} & 111100 & 00000 \end{pmatrix} + \\ \varepsilon_{9} & 11110 & 00001 \end{pmatrix} - \varepsilon_{1_{3}} & 111110 & 11110 \end{pmatrix} - \\ \varepsilon_{1_{2}} & 11111 & 00000 \end{pmatrix} + \varepsilon_{1_{3}} & 11111 & 00011 \end{pmatrix} - \varepsilon_{1_{4}} & 111111 & 11100 \end{pmatrix} - \\ \varepsilon_{1_{5}} & 11111 & 11110 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11110 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11110 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11100 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11100 \end{pmatrix} - \\ \varepsilon_{1_{5}} & 11111 & 11110 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11111 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 111111 & 11111 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 11111 & 11111 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 111111 & 11111 \end{pmatrix} + \\ \varepsilon_{1_{5}} & 111111 & 111111 \end{pmatrix} + \\ \varepsilon_{1_{5}} & $	Ш	ZIII
+) 1}	(0)(+1)	$\begin{split} \varepsilon_0 & \left[00000 \ 00000 \right] + \varepsilon_1 & \left[00000 \ 00011 \right] + \varepsilon_2 & \left[00000 \ 11110 \right] + \varepsilon_4 & \left[00001 \ 00000 \right] + \varepsilon_5 & \left[00001 \ 00011 \right] + \varepsilon_6 & \left[00011 \ 111100 \right] + \varepsilon_6 & \left[00011 \ 111100 \right] + \varepsilon_6 & \left[111100 \ 00000 \right] - \varepsilon_{11} & \left[111100 \ 00000 \right] - \varepsilon_{12} & \left[1111100 \ 11111 \right] - \varepsilon_{12} & \left[111111 \ 00000 \right] - \varepsilon_{12} & \left[111111 \ 00000 \right] - \varepsilon_{12} & \left[111111 \ 11110 \right] - \varepsilon_{12} & \left[111111 \ 11110 \right] - \varepsilon_{13} & \left[111111 \ 00011 \right] - \varepsilon_{14} & \left[111111 \ 11110 \right] - \varepsilon_{15} & \left[111111 \ 11111 \right] - \varepsilon_{15} & \left[111111 \ 111111 \right] - \varepsilon_{15} & \left[1111111 \ 111111 \right] - \varepsilon_{15} & \left[1111111 \ 111111 \right] - \varepsilon_{15} & \left[11111$	ZIII	
(11)	(11) + 1	$ \begin{array}{l} \varepsilon_{0} \\ \varepsilon_{0} \\ (00000 \ 00000) \\ \varepsilon_{3} \\ (00001 \ 11110) \\ \varepsilon_{6} \\ (00011 \ 11100) \\ \varepsilon_{7} \\ \varepsilon_{10} \\ (11100 \ 00011) \\ \varepsilon_{12} \\ (11110 \ 00000) \\ \varepsilon_{13} \\ (11111 \ 00000) \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ \varepsilon_{13} \\ (11111 \ 00011) \\ \varepsilon_{13} \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ \varepsilon_{13} \\ (11111 \ 00001) \\ \varepsilon_{13} \\ $	ZIII	ZIII

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
(0)(+1	1-)10)	$\begin{split} & \epsilon_0 00000 \ 00000 \rangle - \epsilon_1 00000 \ 00011 \rangle + \epsilon_2 00000 \ 1110 \rangle - \\ & \epsilon_3 00000 \ 11111 \rangle + \epsilon_4 00011 \ 00000 \rangle - \epsilon_5 00011 \ 00011 \rangle + \\ & \epsilon_6 00011 \ 11100 \rangle - \epsilon_7 00011 \ 11111 \rangle + \epsilon_8 11100 \ 00000 \rangle - \\ & \epsilon_9 11100 \ 00011 \rangle + \epsilon_{10} 11100 \ 11100 \rangle - \epsilon_{11} 11100 \ 11111 \rangle + \\ & \epsilon_{12} 11111 \ 00000 \rangle - \epsilon_{13} 11111 \ 00011 \rangle + \epsilon_{14} 11111 \ 11100 \rangle - \\ & \epsilon_{15} 11111 \ 11111 \rangle \end{split}$	Ш	ZIII
(0)(+1	1	$\begin{split} \epsilon_0 & \left[00000 \ 00000 \right] - \epsilon_1 & \left[00000 \ 00011 \right] - \epsilon_2 & \left[00000 \ 11100 \right] + \\ \epsilon_3 & \left[00000 \ 11111 \right] + \epsilon_4 & \left[00011 \ 00000 \right] - \epsilon_5 & \left[00011 \ 00011 \right] - \\ \epsilon_6 & \left[00011 \ 111100 \right] + \epsilon_7 & \left[00011 \ 11111 \right] + \epsilon_8 & \left[11100 \ 111110 \right] + \\ \epsilon_{12} & \left[11111 \ 00000 \right] - \epsilon_{13} & \left[11110 \ 01110 \right] + \\ \epsilon_{12} & \left[11111 \ 00000 \right] - \epsilon_{13} & \left[11111 \ 00011 \right] - \epsilon_{14} & \left[11111 \ 11110 \right] + \\ \epsilon_{15} & \left[11111 \ 11111 \right] \end{split}$		ZIIIZ
+)[1]	I –)I0)	$\begin{split} \epsilon_0 & \left[00000 \ 00000 \right] + \epsilon_1 & \left[00000 \ 00011 \right] + \epsilon_2 & \left[00000 \ 11100 \right] + \\ \epsilon_3 & \left[00000 \ 11111 \right] + \epsilon_4 & \left[00011 \ 00000 \right] + \epsilon_5 & \left[00011 \ 00011 \right] + \\ \epsilon_6 & \left[00011 \ 111100 \right] + \epsilon_7 & \left[00011 \ 11111 \right] - \epsilon_8 & \left[111100 \ 00000 \right] - \\ \epsilon_9 & \left[111100 \ 00011 \right] - \epsilon_{10} & \left[111100 \ 11110 \right] - \\ \epsilon_{12} & \left[11111 \ 00000 \right] - \epsilon_{13} & \left[11111 \ 00011 \right] - \epsilon_{14} & \left[11111 \ 11100 \right] - \\ \epsilon_{15} & \left[11111 \ 11111 \right] \end{split}$	IIIIZ	ZIIII
(11)	(-)II	$ \begin{split} \varepsilon_0 & \left[00000 \ 00000 \right] - \varepsilon_1 & \left[00000 \ 00011 \right] - \varepsilon_2 & \left[00000 \ 11101 \right] + \varepsilon_4 & \left[00011 \ 00000 \right] - \varepsilon_5 & \left[00011 \ 00011 \right] - \varepsilon_6 & \left[00011 \ 11100 \right] + \varepsilon_7 & \left[00011 \ 11111 \right] - \varepsilon_8 & \left[111100 \ 00000 \right] + \varepsilon_9 & \left[111100 \ 00011 \right] + \varepsilon_{10} & \left[111100 \ 11110 \right] - \varepsilon_{12} & \left[111110 \ 011111 \right] - \varepsilon_{12} & \left[111110 \ 011111 \right] - \varepsilon_{12} & \left[111111 \ 00000 \right] + \varepsilon_{13} & \left[111110 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 11100 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 111100 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 111100 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 111100 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[111111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 0000 \right] + \varepsilon_{12} & \left[11111 \ 00011 \right] + \varepsilon_{14} & \left[111111 \ 0000 \right] - \varepsilon_{12} & \left[11111 \ 00000 \right] + $	ZIII	ZIIIZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
1 —)IO)	1+ /10)	$\begin{array}{l} \epsilon_0 00000 \ 00000 \rangle \ + \ \epsilon_1 00000 \ 00011 \rangle \ + \ \epsilon_2 00000 \ 11100 \rangle \ + \\ \epsilon_3 00000 \ 11111 \rangle \ - \ \epsilon_4 00011 \ 00000 \rangle \ - \ \epsilon_5 00011 \ 00011 \rangle \ - \\ \epsilon_6 00011 \ 11100 \rangle \ - \ \epsilon_7 00011 \ 11111 \rangle \ + \ \epsilon_8 11100 \ 00000 \rangle \ + \\ \epsilon_9 11100 \ 00001 \rangle \ + \ \epsilon_{1_3} 11110 \ 011100 \rangle \ + \ \epsilon_{1_4} 11111 \ 11100 \rangle \ - \\ \epsilon_{1_2} 11111 \ 00000 \rangle \ - \ \epsilon_{1_3} 11111 \ 00011 \rangle \ - \ \epsilon_{1_4} 11111 \ 11100 \rangle \ - \\ \epsilon_{1_6} 11111 \ 11110 \rangle \ - \\ \epsilon_{1_6} 11111 \ 11100 \rangle \ - \\ \epsilon_{1_6} 11111 \ 1110 \rangle \ - \\ \epsilon_{1_6} 11111 \ 1110 \rangle \ - \\ \epsilon_{1_6} 11111 \ 11110 \rangle \ - \\ \epsilon_{1_6} 11111 \ - \\ \epsilon_{1_6} 111111 \ - \\ \epsilon_{1_6} 11111 \ - \\ \epsilon_{1_6} 11111 \ - \\$	IIIIZ	Ш
I –)IO)	1+)11>	$\begin{array}{l} \epsilon_0 \Big] 00000 \ 00000 \Big\rangle + \epsilon_1 \Big] 00000 \ 00011 \Big\rangle - \epsilon_2 \Big] 00000 \ 11100 \Big\rangle - \\ \epsilon_3 \Big] 00000 \ 11111 \Big\rangle + \epsilon_4 \Big] 00001 \ 00000 \Big\rangle - \\ \epsilon_5 \Big] 00001 \ 111100 \Big\rangle + \epsilon_7 \Big] 00011 \ 11111 \Big\rangle + \epsilon_8 \Big] 11100 \ 00000 \Big\rangle + \\ \epsilon_9 \Big] 11100 \ 00011 \Big\rangle - \epsilon_{10} \Big] 11110 \ 11100 \ - \epsilon_{11} \Big] 111100 \ 11111 \Big\rangle - \\ \epsilon_{12} \Big] 11111 \ 00000 \Big\rangle - \epsilon_{13} \Big] 11111 \ 00011 \Big\rangle + \epsilon_{14} \Big] 11111 \ 11100 \Big\rangle + \\ \epsilon_{15} \Big] 11111 \ 11111 \Big\rangle \end{array}$	ZIII	ZIII
(11)	(1+)10	$\begin{split} \epsilon_0 & \left[00000 \ 00000 \right] + \epsilon_1 & \left[00000 \ 00011 \right] + \epsilon_2 & \left[00000 \ 11100 \right] + \\ \epsilon_3 & \left[00000 \ 11111 \right] - \epsilon_4 & \left[00011 \ 00000 \right] - \epsilon_5 & \left[00011 \ 00011 \right] - \\ \epsilon_6 & \left[00011 \ 11100 \right] - \epsilon_7 & \left[00011 \ 11111 \right] - \epsilon_8 & \left[11110 \ 00000 \right] - \\ \epsilon_9 & \left[11100 \ 00011 \right] - \epsilon_{1_3} & \left[11110 \ 01110 \right] + \\ \epsilon_{1_2} & \left[11111 \ 00000 \right] + \epsilon_{1_3} & \left[11111 \ 00011 \right] + \\ \epsilon_{1_4} & \left[11111 \ 11110 \right] + \\ \epsilon_{1_5} & \left[11111 \ 11111 \right] \end{split}$	IIIZZ	Ш
(II) - I	1+)11	$ \begin{array}{l} \varepsilon_{0} \left[00000 \ 00000 \right] + \varepsilon_{1} \left[00000 \ 00011 \right] & -\varepsilon_{2} \left[00000 \ 11100 \right] & -\varepsilon_{3} \left[00001 \ 11110 \right] \\ \varepsilon_{3} \left[00001 \ 11110 \right] + \varepsilon_{7} \left[00011 \ 00000 \right] & -\varepsilon_{8} \left[10011 \ 00011 \right] \\ \varepsilon_{6} \left[00011 \ 11100 \right] + \varepsilon_{10} \left[11100 \ 11100 \right] + \varepsilon_{11} \left[11100 \ 00000 \right] \\ \varepsilon_{9} \left[11110 \ 00001 \right] + \varepsilon_{13} \left[11111 \ 00011 \right] & -\varepsilon_{14} \left[111111 \ 11100 \right] \\ \varepsilon_{12} \left[11111 \ 100000 \right] + \varepsilon_{13} \left[11111 \ 00011 \right] & -\varepsilon_{14} \left[111111 \ 11100 \right] \\ \end{array} \right] $	IIIZZ	ZIII

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
-)IO)	1-)10)	$\begin{split} & \epsilon_0 \big 00000 \ 00000 \big\rangle - \epsilon_1 \big 00000 \ 00011 \big\rangle - \epsilon_2 \big 00000 \ 11100 \big\rangle - \\ & \epsilon_3 \big 00000 \ 11111 \big\rangle - \epsilon_4 \big 00011 \ 00000 \big\rangle + \epsilon_5 \big 00011 \ 00011 \big\rangle - \\ & \epsilon_6 \big 00011 \ 11100 \big\rangle + \epsilon_7 \big 00011 \ 11111 \big\rangle + \epsilon_8 \big 11100 \ 00000 \big\rangle - \\ & \epsilon_9 \big 11100 \ 00011 \big\rangle + \epsilon_{13} \big 11110 \ 01110 \big\rangle - \epsilon_{14} \big 11110 \ 11111 \big\rangle + \\ & \epsilon_{12} \big 11111 \ 00000 \big\rangle + \epsilon_{13} \big 11111 \ 00011 \big\rangle - \epsilon_{14} \big 11111 \ 11100 \big\rangle + \\ & \epsilon_{16} \big 11111 \ 11111 \big\rangle \end{split}$	ZIIII	ZIIII
-) 0)	- 1	$\begin{array}{l} \epsilon_0 \Big[00000 \ 00000 \Big] - \epsilon_1 \Big] 00000 \ 00011 \Big\} + \epsilon_2 \Big] 00000 \ 11100 \Big\} + \\ \epsilon_3 \Big] 00000 \ 11111 \Big\} - \epsilon_4 \Big] 00011 \ 00000 \Big\} + \epsilon_5 \Big] 00011 \ 00011 \Big\} + \\ \epsilon_6 \Big] 000011 \ 11100 \Big\} - \epsilon_7 \Big] 00011 \ 11111 \Big\} + \epsilon_8 \Big] 11100 \ 00000 \Big\} - \\ \epsilon_9 \Big] 11100 \ 00011 \Big\} - \epsilon_{1_0} \Big] 11110 \ 011100 \Big\} + \epsilon_{1_4} \Big] 11110 \ 11111 \Big\} + \\ \epsilon_{1_2} \Big] 11111 \ 00000 \Big\} + \epsilon_{1_3} \Big] 11111 \ 00011 \Big\} + \epsilon_{1_4} \Big] 11111 \ 11100 \Big\} - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \Big\} - \\ \epsilon_{1_5} \Big] 11111 \ 11110 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11110 \Big\} - \\ \epsilon_{1_5} \Big] 11111 \ 11110 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11110 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 111111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 11111 \Big] - \\ \epsilon_{1_5} \Big] 11111 \ 11111 \ 111111 \ 111111 \Big] - \\ \epsilon_{1_5} \Big] 111111 \ 111111 \ 11111 \ 111111 \ $	ZIII	ZIIIZ
(II) - I	I –)10)	$\begin{array}{l} \epsilon_0 \left[00000 \ 00000 \right] & - \epsilon_1 \left[00000 \ 00011 \right] & - \epsilon_2 \left[00000 \ 11100 \right] & - \\ \epsilon_3 \left[00000 \ 11111 \right] & - \epsilon_4 \left[00011 \ 00000 \right] & + \\ \epsilon_5 \left[00011 \ 11100 \right] & + \\ \epsilon_7 \left[00011 \ 11100 \right] & + \\ \epsilon_7 \left[11110 \ 00001 \right] & - \\ \epsilon_1_2 \left[11111 \ 00001 \right] & - \\ \epsilon_{13} \left[11111 \ 00011 \right] & + \\ \epsilon_{14} \left[11111 \ 1110 \right] & - \\ \epsilon_{15} \left[11111 \ 11110 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11110 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11110 \right] & - \\ \epsilon_{15} \left[11111 \ 11110 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right] & - \\ \epsilon_{15} \left[11111 \ 11110 \right] & - \\ \epsilon_{15} \left[11111 \ 11111 \right$	IIIZZ	ZIIII
-) 1)	- 1	$\begin{array}{l} \epsilon_0 \left[00000 \ 00000 \right] & - \epsilon_1 \left[00000 \ 00011 \right] & + \epsilon_2 \left[00000 \ 11100 \right] & + \\ \epsilon_3 \left[00000 \ 11111 \right] & - \epsilon_4 \left[00011 \ 00000 \right] & + \epsilon_5 \left[00011 \ 00011 \right] & + \\ \epsilon_6 \left[00011 \ 11100 \right] & - \epsilon_7 \left[00011 \ 11111 \right] & - \\ \epsilon_8 \left[111100 \ 00001 \right] & + \\ \epsilon_1 \left[111100 \ 00001 \right] & - \\ \epsilon_1 \left[11111 \ 00000 \right] & - \\ \epsilon_1 \left[11111 \ 00011 \right] & - \\ \epsilon_1 \left[11111 \ 11110 \right] & - \\ \epsilon_1 \left[11111 \ 11110 \right] & - \\ \epsilon_1 \left[11111 \ 11110 \right] & - \\ \epsilon_1 \left[11111 \ 00011 \right] & - \\ \epsilon_1 \left[11111 \ 11110 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11110 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[111111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 111111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 111111 \right] & - \\ \epsilon_1 \left[11111 \ 111111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1 \left[11111 \ 11111 \right] & - \\ \epsilon_1$	IIIZZ	ZIIIZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
{o}(+ +)	()(+)	$ \begin{split} \epsilon_0 11000 \ 00001 \rangle &+ \epsilon_1 11000 \ 00010 \rangle &+ \epsilon_2 11000 \ 11101 \rangle &+ \\ \epsilon_3 11000 \ 11110 \rangle &+ \epsilon_4 11011 \ 00001 \rangle &+ \epsilon_5 11011 \ 00010 \rangle &+ \\ \epsilon_6 11011 \ 11101 \rangle &+ \epsilon_7 11011 \ 11110 \rangle &+ \epsilon_8 00100 \ 00001 \rangle &+ \\ \epsilon_9 00100 \ 00010 \rangle &+ \epsilon_{10} 00101 \ 11101 \rangle &+ \\ \epsilon_{12} 00111 \ 00001 \rangle &+ \epsilon_{13} 00111 \ 00010 \rangle &+ \\ \epsilon_{14} 00111 \ 11101 \rangle &+ \\ \end{split}$	IIIXX	XIIII
(0)	+ 1	$ \begin{array}{l} \epsilon_{0} \\ \epsilon_{1} \\ 11000 \ 00001 \\ + \epsilon_{1} \\ 11000 \ 11110 \\ + \epsilon_{4} \\ 11011 \ 11101 \\ + \epsilon_{3} \\ 11011 \ 11101 \\ + \epsilon_{3} \\ 111110 \\ + \epsilon_{3} \\ 111110 \\ + \epsilon_{3} \\ 10110 \ 00010 \\ + \epsilon_{1_{3}} \\ 100101 \ 00010 \\ + \epsilon_{1_{3}} \\ 100111 \ 10010 \\ - \epsilon_{1_{4}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ - \epsilon_{1_{4}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11110 \\ + \epsilon_{1_{3}} \\ 100111 \ 11110 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_{1_{3}} \\ 100111 \ 11100 \\ + \epsilon_{1_{3}} \\ 100111 \ 11101 \\ + \epsilon_$	IIIXX	ZIIIX
1+)11)	(01(+1	$\begin{split} \epsilon_0 & 11000 \ 00001 \rangle + \epsilon_1 & 11000 \ 00010 \rangle + \epsilon_2 & 11000 \ 11101 \rangle + \\ \epsilon_3 & 11000 \ 11110 \rangle + \epsilon_4 & 11011 \ 00001 \rangle + \epsilon_5 & 1011 \ 00010 \rangle + \\ \epsilon_6 & 11011 \ 11101 \rangle + \epsilon_7 & 11011 \ 11110 \rangle - \epsilon_8 & 00100 \ 00001 \rangle - \\ \epsilon_9 & 00100 \ 00010 \rangle - \epsilon_{10} & 00101 \ 11110 \rangle - \\ \epsilon_{12} & 00111 \ 00001 \rangle - \epsilon_{13} & 00111 \ 00010 \rangle - \\ \epsilon_{14} & 00111 \ 11101 \rangle - \\ \epsilon_{16} & 00111 \ 11110 \rangle - \\ \epsilon_{16} & 00111 \ 11101 \rangle - \\ \epsilon_{16} & 00111 \ 11100 \rangle - \\ \epsilon_{16} & 00111 \ 11101 \rangle - \\ \epsilon_{16} & 00111 \ 11101 \rangle - \\ \epsilon_{16} & 00111 \ 11100 \rangle - \\ \epsilon_{16} & 00110 \ 11100 \rangle - \\ \epsilon_{16} & 00100 \ 11100 \rangle - \\ \epsilon_{16} & 00100 \ 1100 \rangle - \\ \epsilon_{16} & 00100 \ 11$	IIZXX	XIIII
(11)	+ 1	$\begin{array}{l} \epsilon_{0} \\ 11000 \ 00001 \\ + \epsilon_{1} \\ 11000 \ 11110 \\ + \epsilon_{4} \\ 11011 \ 00011 \\ + \epsilon_{5} \\ 11011 \ 00010 \\ + \epsilon_{1} \\ 11011 \\ - \epsilon_{7} \\ 11011 \ 11110 \\ + \epsilon_{1} \\ 100100 \ 00010 \\ + \epsilon_{10} \\ 100101 \ 11101 \\ + \epsilon_{11} \\ 100100 \ 11110 \\ + \epsilon_{12} \\ 100111 \ 00001 \\ + \epsilon_{13} \\ 100111 \ 00010 \\ + \epsilon_{14} \\ 100111 \ 11101 \\ + \epsilon_{15} \\ 100111 \ 1110 \\ + \epsilon_{15} \\ 100111 \ 1110 \\ + \epsilon_{15} \\ 100111 \ 00010 \\ + \epsilon_{15} \\ 100111 \ 00010 \\ + \epsilon_{14} \\ 100111 \ 1110 \\ + \epsilon_{15} \\ 100111 \ 1110 \\ + \epsilon_{15} \\ 100111 \ 00010 \\ + \epsilon_{15} \\ 100111 \ 00010 \\ + \epsilon_{15} \\ 100111 \ 00010 \\ + \epsilon_{15} \\ 100111 \ 1110 \\ + \epsilon_{15} \\ 10010 \\ + \epsilon_{15} \\ +$	IIZXX	ZIIIZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
{o (+	(0)	$\begin{array}{llllllllllllllllllllllllllllllllllll$	IIIXX	XZIII
(0)(+1	(11)	$ \begin{array}{l} \epsilon_{0} \\ \epsilon_{0} \\ 11000 \ 00001 \\ + \epsilon_{1} \\ 11000 \ 11110 \\ + \epsilon_{4} \\ 11011 \ 11101 \\ + \epsilon_{4} \\ 11011 \ 11101 \\ + \epsilon_{7} \\ 11011 \ 11110 \\ + \epsilon_{7} \\ 11011 \ 11110 \\ + \epsilon_{1} \\ 100100 \ 00001 \\ - \epsilon_{10} \\ 100101 \ 11110 \\ + \epsilon_{1} \\ 100111 \ 10010 \\ + \epsilon_{1} \\ 100111 \ 11101 \\ + \epsilon_{1} \\ 10$	IIIXX	XZZII
(+)	⟨ol(− 1	$ \begin{array}{llllllllllllllllllllllllllllllllllll$	IIZXX	XZIII
+) 1)		$ \begin{split} \epsilon_0 & 11000 \ 00001 \rangle + \epsilon_1 & 11000 \ 00010 \rangle - \epsilon_2 & 11000 \ 11101 \rangle + \\ \epsilon_3 & 11000 \ 111101 \rangle + \epsilon_4 & 11011 \ 00001 \rangle + \epsilon_5 & 11011 \ 00010 \rangle - \\ \epsilon_6 & 1011 \ 11101 \rangle + \epsilon_7 & 11011 \ 11110 \rangle - \epsilon_8 & 00100 \ 00001 \rangle + \\ \epsilon_9 & 00100 \ 00001 \rangle + \epsilon_{10} & 00101 \ 11101 \rangle + \\ \epsilon_{12} & 00111 \ 00001 \rangle + \epsilon_{13} & 00111 \ 00010 \rangle + \\ \epsilon_{13} & 00111 \ 100001 \rangle + \\ \epsilon_{13} & 00111 \ 11101 \rangle + \\ \end{split} $	IIZXX	XZZII

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
1-)10	1+)10)	$\begin{array}{l} \epsilon_0 11000 \ 00001 \rangle + \epsilon_1 11000 \ 00010 \rangle + \epsilon_2 11000 \ 11101 \rangle + \\ \epsilon_3 11000 \ 11110 \rangle - \epsilon_4 11011 \ 00001 \rangle - \epsilon_5 11011 \ 00010 \rangle - \\ \epsilon_6 11011 \ 11101 \rangle - \epsilon_7 11011 \ 11110 \rangle + \epsilon_8 00100 \ 00001 \rangle + \\ \epsilon_9 00100 \ 00010 \rangle + \epsilon_{10} 00101 \ 11101 \rangle + \epsilon_{11} 00100 \ 11110 \rangle - \\ \epsilon_{12} 00111 \ 00001 \rangle - \epsilon_{13} 00111 \ 00010 \rangle - \epsilon_{14} 00111 \ 11101 \rangle - \\ \epsilon_{14} 00111 \ 11110 \rangle \end{array}$	XXIIZ	XIIII
1-)10	(11(+ 1	$ \begin{split} \epsilon_0 & 11000 \ 00001 \rangle + \epsilon_1 & 11000 \ 00010 \rangle - \epsilon_2 & 11000 \ 11101 \rangle - \\ \epsilon_3 & 11000 \ 11110 \rangle - \epsilon_4 & 11011 \ 00001 \rangle - \epsilon_5 & 11011 \ 00010 \rangle + \\ \epsilon_6 & 1011 \ 11101 \rangle + \epsilon_7 & 11011 \ 11110 \rangle + \epsilon_8 & 00100 \ 00001 \rangle + \\ \epsilon_9 & 00100 \ 00010 \rangle - \epsilon_{10} & 00100 \ 11110 \rangle - \\ \epsilon_{12} & 00111 \ 00001 \rangle - \epsilon_{13} & 00111 \ 00010 \rangle + \epsilon_{14} & 001111 \ 11101 \rangle + \\ \epsilon_{13} & 00111 \ 11110 \rangle \end{split} $	XXIIZ	ZIIIX
-) 1)	(ol(+ 1	$\begin{split} \epsilon_0 & 11000 \ 00001 \rangle + \epsilon_1 & 11000 \ 00010 \rangle + \epsilon_2 & 11000 \ 11101 \rangle + \\ \epsilon_3 & 11000 \ 11110 \rangle - \epsilon_4 & 11011 \ 00001 \rangle - \\ \epsilon_5 & 10111 \ 11101 \rangle - \epsilon_7 & 11011 \ 11110 \rangle - \epsilon_8 & 00100 \ 00001 \rangle - \\ \epsilon_9 & 00100 \ 00010 \rangle - \epsilon_{10} & 00100 \ 11110 \rangle - \\ \epsilon_{12} & 00111 \ 00001 \rangle + \epsilon_{13} & 00111 \ 00010 \rangle + \epsilon_{14} & 00111 \ 11101 \rangle + \\ \epsilon_{13} & 00111 \ 10001 \rangle + \\ \epsilon_{14} & 00111 \ 11100 \rangle + \\ \epsilon_{14} & 00111 \ 11101 \rangle + \\ \epsilon_{14} & 00111 \ 11100 \rangle + \\ \epsilon_{14} & 00111 \ 11100 \rangle + \\ \epsilon_{15} & 00111 \ 11100 \rangle + \\ \epsilon_{15} & 00111 \ 11100 \rangle + \\ \epsilon_{16} & 00110 \ 1100 \rangle + \\ \epsilon_{16} & 00100 \ 1100 \rangle + \\ \epsilon_{16} & 00100 \$	ZIZXX	XIIII
-) 1)	(11)	$ \begin{split} \varepsilon_0 & \left[11000 \ 00001 \right] + \varepsilon_1 & \left[11000 \ 00010 \right] - \varepsilon_2 & \left[11000 \ 11101 \right] - \varepsilon_3 \\ \varepsilon_1 & \left[11000 \ 111101 \right] - \varepsilon_4 & \left[11011 \ 00001 \right] - \varepsilon_5 & \left[11011 \ 00010 \right] + \varepsilon_6 \\ \varepsilon_6 & \left[11011 \ 11101 \right] + \varepsilon_7 & \left[11011 \ 11110 \right] - \varepsilon_8 & \left[00100 \ 00001 \right] - \varepsilon_6 \\ \varepsilon_9 & \left[00100 \ 00010 \right] + \varepsilon_{13} & \left[00110 \ 11101 \right] + \varepsilon_{14} & \left[00100 \ 11110 \right] + \varepsilon_{15} \\ \varepsilon_{12} & \left[00111 \ 00001 \right] + \varepsilon_{13} & \left[00111 \ 00010 \right] - \varepsilon_{14} & \left[00111 \ 11101 \right] - \varepsilon_{15} \\ \varepsilon_{15} & \left[00111 \ 11100 \right] \end{split} $	ZIZXX	ZIIIZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
(0)(1-)10)	$ \begin{split} \epsilon_0 & 11000 \ 00001 \rangle & - \epsilon_1 & 11000 \ 00010 \rangle & + \epsilon_2 & 11000 \ 11101 \rangle & - \\ \epsilon_3 & 11000 \ 11110 \rangle & - \epsilon_4 & 11011 \ 00001 \rangle & + \epsilon_5 & 11011 \ 00010 \rangle & - \\ \epsilon_6 & 11011 \ 11110 \rangle & + \epsilon_7 & 11011 \ 11110 \rangle & + \epsilon_8 & 00100 \ 00001 \rangle & - \\ \epsilon_9 & 00100 \ 00010 \rangle & + \epsilon_{13} & 00111 \ 00010 \rangle & - \\ \epsilon_{12} & 00111 \ 00001 \rangle & + \epsilon_{13} & 001111 \ 00010 \rangle & - \\ \epsilon_{14} & 001111 \ 11101 \rangle & + \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & + \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & + \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 00111 \ 11110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 \ 1110 \rangle & - \\ \epsilon_{16} & 001111 $	ZIIXX	XZIII
-)Io/	-)11	$ \begin{array}{l} \epsilon_0 \\ \epsilon_1 \\ 11000 \ 00001 \\ \epsilon_3 \\ 11000 \ 11110 \\ \epsilon_4 \\ 11011 \ 11101 \\ \epsilon_5 \\ \epsilon_1 \\ 10101 \ 11101 \\ \epsilon_7 \\ \epsilon_1 \\ 10100 \ 00010 \\ \epsilon_1 \\ \epsilon_1 \\ 10111 \ 11101 \\ \epsilon_1 \\ 10110 \ 00011 \\ \epsilon_1 \\ 10111 \ 00010 \\ \epsilon_1 \\ 100111 \ 00010 \\ \epsilon_1 \\ 100111 \ 00010 \\ \epsilon_1 \\ 100111 \ 10010 \\ \epsilon_1 \\ 100111 \ 11101 \\ \end{array} \right) \\ - \begin{array}{l} \epsilon_2 \\ \epsilon_1 \\ 100111 \ 00010 \\ \epsilon_1 \\ 100111 \ 00010 \\ \epsilon_1 \\ 100111 \ 11101 \\ \end{array} \right) \\ - \begin{array}{l} \epsilon_1 \\ \epsilon_1 \\ 100111 \ 11101 \\ \epsilon_1 \\ 100101 \ 11110 \\ \epsilon_1 \\ 100101 \ 11101 \\ \epsilon_1 \\ 100101 \ 11101 \\ \epsilon_1 \\ 100101 \ 11110 \\ \epsilon_1 \\ 100101 \ 11110 \\ \epsilon_1 \\ \epsilon_1 \\ 100111 \ 11101 \\ \epsilon_1 \\ \epsilon_1 \\ \epsilon_1 \\ \epsilon_1 \\ 100111 \ 11101 \\ \epsilon_1 \\ \epsilon$	ZIIXX	XZIIZ
-) 1)	1- 10	$\begin{array}{l} \epsilon_0 \\ \epsilon_1 \\ 11000 \ 00001 \\ \end{array} - \epsilon_1 \\ \epsilon_1 \\ 11000 \ 1110 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 11011 \ 1110 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ \epsilon_1 \\ 11011 \ 1110 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 11010 \ 00010 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 100100 \ 00001 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 100101 \ 00001 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 100111 \ 1110 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 100111 \ 1110 \\ \end{array} + \epsilon_1 \\ \epsilon_1 \\ 100111 \ 1110 \\ \end{array}$	ZIZX	XZIII
-) 1)) 1 >	$\begin{split} \epsilon_0 & \left[11000 \ 00001 \right] - \epsilon_1 & \left[11000 \ 00010 \right] + \epsilon_2 & \left[11000 \ 11101 \right] + \\ \epsilon_3 & \left[11000 \ 111101 \right] - \epsilon_4 & \left[11011 \ 00001 \right] + \epsilon_5 & \left[11011 \ 00010 \right] + \\ \epsilon_6 & \left[11011 \ 11101 \right] - \epsilon_7 & \left[11011 \ 11110 \right] - \epsilon_8 & \left[00100 \ 00001 \right] + \\ \epsilon_9 & \left[00100 \ 00001 \right] + \epsilon_{10} & \left[00100 \ 11110 \right] - \\ \epsilon_{12} & \left[00111 \ 00001 \right] - \epsilon_{13} & \left[001111 \ 00010 \right] - \\ \epsilon_{12} & \left[001111 \ 00001 \right] - \epsilon_{13} & \left[001111 \ 00010 \right] - \\ \epsilon_{15} & \left[001111 \ 11101 \right] \end{split}$	ZIZX	XZIIZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
(0)(+1	1+)10)	$\begin{array}{l} \epsilon_0 \Big 00001 \ 01100 \Big\rangle + \epsilon_1 \Big 00001 \ 01111 \Big\rangle + \epsilon_2 \Big 00001 \ 0000 \Big\rangle + \\ \epsilon_3 \Big 00001 \ 10011 \Big\rangle + \epsilon_4 \Big 00010 \ 01100 \Big\rangle + \epsilon_5 \Big 00010 \ 01111 \Big\rangle + \\ \epsilon_6 \Big 00010 \ 10000 \Big\rangle + \epsilon_7 \Big 00010 \ 10011 \Big\rangle + \epsilon_8 \Big 11101 \ 01100 \Big\rangle + \\ \epsilon_9 \Big 11101 \ 01111 \Big\rangle + \epsilon_{10} \Big 11101 \ 10000 \Big\rangle + \epsilon_{11} \Big 11101 \ 10011 \Big\rangle + \\ \epsilon_{12} \Big 11110 \ 01100 \Big\rangle + \epsilon_{13} \Big 11110 \ 01111 \Big\rangle + \epsilon_{14} \Big 11110 \ 10000 \Big\rangle + \\ \epsilon_{14} \Big 11110 \ 10000 \Big\rangle + \\ \epsilon_{16} \Big 11110 \ 10001 \Big\rangle + \\ \epsilon_{16} \Big 11110 \ 10000 \Big\rangle + \\ \epsilon_{16} \Big 1110 \ 10000 \Big\rangle + \\ \epsilon_{16} \Big 1100 \ 10000 \Big\rangle + \\ \epsilon$	XIIII	IIXXI
(01(+1	(+)	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XIII	IIXXZ
(+)	(ol(+ 1	$\begin{array}{l} \epsilon_0 \left 00001 \ 01100 \right\rangle + \epsilon_1 \left 00001 \ 01111 \right\rangle + \epsilon_2 \left 00001 \ 00000 \right\rangle + \\ \epsilon_3 \left 00001 \ 10011 \right\rangle + \epsilon_4 \left 00010 \ 01100 \right\rangle + \epsilon_5 \left 00010 \ 01111 \right\rangle + \\ \epsilon_6 \left 00010 \ 10000 \right\rangle + \epsilon_7 \left 00010 \ 10011 \right\rangle - \epsilon_8 \left 11110 \ 01100 \right\rangle - \\ \epsilon_9 \left 11110 \ 01111 \right\rangle - \epsilon_{10} \left 11101 \ 10000 \right\rangle - \\ \epsilon_{12} \left 11110 \ 01100 \right\rangle + \epsilon_{13} \left 11110 \ 01111 \right\rangle - \epsilon_{14} \left 11110 \ 10000 \right\rangle - \\ \epsilon_{14} \left 11110 \ 10000 \right\rangle - \\ \epsilon_{14} \left 11110 \ 10000 \right\rangle - \\ \epsilon_{16} \left 11110 \ 10001 \right\rangle + \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle $	ZIIIX	IIXXI
(+)	{11}	$\begin{array}{l} \epsilon_0 \left[00001 \ 01100 \right) \ + \ \epsilon_1 \left[00001 \ 01111 \right) \ - \ \epsilon_2 \left[00001 \ 00000 \right] \ - \ \epsilon_3 \left[00001 \ 00011 \right) \ + \ \epsilon_4 \left[00010 \ 01100 \right) \ + \ \epsilon_5 \left[00010 \ 01111 \right) \ - \ \epsilon_6 \left[00010 \ 10000 \right] \ - \ \epsilon_7 \left[00010 \ 10010 \right] \ - \ \epsilon_8 \left[11101 \ 01100 \right] \ - \ \epsilon_8 \left[11101 \ 01100 \right] \ - \ \epsilon_8 \left[11101 \ 01100 \right] \ - \ \epsilon_8 \left[11101 \ 100110 \right] \ - \ \epsilon_8 \left[11101 \ 100110 \right] \ - \ \epsilon_8 \left[111101 \ 100110 \right] \ - \ \epsilon_8 \left[111101 \ 100110 \right] \ - \ \epsilon_8 \left[111101 \ 100110 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[11110 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10011 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[111101 \ 10001 \right] \ - \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11110 \ 10000 \right] \ + \ \epsilon_8 \left[11100 \ 10000 \right] \ + \ \epsilon_8 \left[11000 \ 1000 \ - \ 1000 \ - \ 10000 \ - \ 1000 \ - \ 10000 \ - \ 10000 \ - \ 10000 \ - \ 10000 \ - \ 10000 \ - \ 10000 \ - \ 100000 \ - \ 10000 \$	IIXXZ	IIXXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
{o}(+ +)	1-)10)	$\begin{array}{l} \epsilon_{0} \left 00001 \ 01100 \right\rangle & - \epsilon_{1} \left 00001 \ 01111 \right\rangle & + \epsilon_{2} \left 00001 \ 00000 \right\rangle & - \\ \epsilon_{3} \left 00001 \ 00011 \right\rangle & + \epsilon_{4} \left 00010 \ 01100 \right\rangle & - \epsilon_{5} \left 00010 \ 01111 \right\rangle & + \\ \epsilon_{6} \left 00010 \ 10000 \right\rangle & - \epsilon_{7} \left 00010 \ 10011 \right\rangle & + \epsilon_{8} \left 11101 \ 01100 \right\rangle & - \\ \epsilon_{9} \left 11101 \ 01111 \right\rangle & + \epsilon_{10} \left 11101 \ 10000 \right\rangle & - \epsilon_{11} \left 11101 \ 10011 \right\rangle & + \\ \epsilon_{12} \left 11110 \ 01100 \right\rangle & - \epsilon_{13} \left 11110 \ 01111 \right\rangle & + \epsilon_{14} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{15} \left 11110 \ 10011 \right\rangle & - \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_$	ХШ	IIXXZ
1+)10	(11(- 1	$\begin{array}{l} \epsilon_{0} \left[00001 \ 01100 \right) - \epsilon_{1} \left[00001 \ 01111 \right) - \epsilon_{2} \left[00001 \ 0000 \right] + \\ \epsilon_{3} \left[00001 \ 10011 \right) + \epsilon_{4} \left[00010 \ 01100 \right) - \epsilon_{5} \left[00010 \ 01111 \right) - \\ \epsilon_{6} \left[00010 \ 10000 \right] + \epsilon_{7} \left[00010 \ 10011 \right] + \epsilon_{8} \left[11101 \ 01100 \right] - \\ \epsilon_{9} \left[11101 \ 01111 \right] - \epsilon_{10} \left[11101 \ 10000 \right] + \epsilon_{1_{1}} \left[11101 \ 10011 \right] + \\ \epsilon_{1_{2}} \left[11110 \ 01100 \right] - \epsilon_{1_{3}} \left[11110 \ 01111 \right] - \epsilon_{1_{4}} \left[11110 \ 10000 \right] + \\ \epsilon_{1_{4}} \left[11110 \ 10000 \right] + \\ \epsilon_{1_{5}} \left[1110 \ 10000 \right] + \\ \epsilon_{1_{5}} \left[1100 \ 10000 \right] + \\ \epsilon_{1_{5}} \left[1100 \ 10000 \right] + \\ \epsilon_{1_{$	ЛПХ	IIXZXZ
[++](1)	lo)(– 1	$\begin{array}{l} \epsilon_{0} \left[00001 \ 01100 \right) - \epsilon_{1} \left[00001 \ 01111 \right) + \epsilon_{2} \left[00001 \ 0000 \right) - \\ \epsilon_{3} \left[00001 \ 10011 \right) + \epsilon_{4} \left[00010 \ 01100 \right) - \epsilon_{5} \left[00010 \ 01111 \right) + \\ \epsilon_{6} \left[00010 \ 10000 \right) - \epsilon_{7} \left[00010 \ 10011 \right] - \\ \epsilon_{9} \left[11101 \ 01111 \right] - \epsilon_{10} \left[11101 \ 10000 \right) + \\ \epsilon_{1} \left[11110 \ 01100 \right] + \\ \epsilon_{1} \left[11110 \ 01100 \right] + \\ \epsilon_{1} \left[11110 \ 10000 \right] + \\ \epsilon_{1} \left[1110 \ 10000 \right]$	ZIIIX	IIXXZ
[+]]1}	- 1	$\begin{array}{l} \epsilon_{0} \left[00001 \ 01100 \right) & \epsilon_{1} \left[00001 \ 01111 \right) & \epsilon_{2} \left[00001 \ 00000 \right) \\ \epsilon_{3} \left[00001 \ 10011 \right) & + \epsilon_{4} \left[00010 \ 01100 \right) & - \epsilon_{5} \left[00010 \ 01111 \right) \\ \epsilon_{6} \left[00010 \ 10000 \right) & + \epsilon_{7} \left[00010 \ 10011 \right) & - \epsilon_{8} \left[11101 \ 01100 \right) \\ \epsilon_{9} \left[11101 \ 01111 \right) & + \epsilon_{10} \left[11101 \ 10000 \right) & - \epsilon_{1_{1}} \left[111101 \ 10011 \right) \\ \epsilon_{1_{2}} \left[11110 \ 01110 \right) & + \epsilon_{1_{3}} \left[11110 \ 01111 \right] \\ + \epsilon_{1_{4}} \left[11110 \ 10000 \right) & - \epsilon_{1_{5}} \left[11110 \ 10000 \right) \\ \end{array}$	ZIIIX	IIXZXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
1-)10	(0)(+ 1	$\begin{array}{l} \epsilon_0 \left 00001 \ 01100 \right\rangle \ + \ \epsilon_1 \left 00001 \ 01111 \right\rangle \ + \ \epsilon_2 \left 00001 \ 0000 \right\rangle \ + \\ \epsilon_3 \left 00001 \ 10011 \right\rangle \ - \ \epsilon_4 \left 00010 \ 01100 \right\rangle \ - \ \epsilon_5 \left 00010 \ 01111 \right\rangle \ - \\ \epsilon_6 \left 00010 \ 10000 \right\rangle \ - \ \epsilon_7 \left 00010 \ 10011 \right\rangle \ + \ \epsilon_8 \left 11101 \ 01100 \right\rangle \ + \\ \epsilon_9 \left 11101 \ 01111 \right\rangle \ + \ \epsilon_{10} \left 111101 \ 10000 \right\rangle \ + \ \epsilon_{11} \left 11101 \ 10011 \right\rangle \ - \\ \epsilon_{12} \left 11110 \ 01100 \right\rangle \ - \ \epsilon_{13} \left 11110 \ 01111 \right\rangle \ - \ \epsilon_{14} \left 11110 \ 10000 \right\rangle \ - \\ \epsilon_{15} \left 11110 \ 10011 \right\rangle \ - \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1110 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1100 \ 10000 \right\rangle \ - \\ \epsilon_{16} \left 1100 \ 10000 \ - \\ \epsilon_{16} \left 1100 \ - \\ \epsilon_{16} \left $	ХZШ	IIXXI
1-)10)	+ 1	$\begin{array}{l} \epsilon_0 \Big[00001 \ 01100 \Big) \ + \ \epsilon_1 \Big] 00001 \ 01111 \Big) \ - \ \epsilon_2 \Big] 00001 \ 00000 \Big) \ - \\ \epsilon_3 \Big] 00001 \ 10011 \Big) \ - \ \epsilon_4 \Big] 00010 \ 01100 \Big) \ - \ \epsilon_5 \Big] 00010 \ 01111 \Big) \ + \\ \epsilon_6 \Big] 00010 \ 10000 \Big) \ + \ \epsilon_7 \Big] 00010 \ 10011 \Big) \ + \ \epsilon_8 \Big] 11101 \ 01110 \Big) \ + \\ \epsilon_9 \Big] 11101 \ 01111 \Big) \ - \ \epsilon_{1_3} \Big] 111101 \ 10000 \Big) \ - \ \epsilon_{1_4} \Big] 111101 \ 10011 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 01110 \Big) \ - \ \epsilon_{1_3} \Big] 111110 \ 01111 \Big) \ + \ \epsilon_{1_4} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_5} \Big] 11110 \ 10000 \Big) \ - \ \epsilon_{1_3} \Big] 11110 \ 01111 \Big) \ + \ \epsilon_{1_4} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_5} \Big] 11110 \ 10001 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 10001 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 10001 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_5} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_6} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_6} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ - \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 11110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big) \ + \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big] \ + \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big] \ + \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big] \ + \\ \epsilon_{1_7} \Big] 1110 \ - \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big] \ + \\ \epsilon_{1_7} \Big] 1110 \ 10000 \Big] \ + \\ \epsilon_{1_7} \Big] 1110 \ - \\ \epsilon_{1_7} \Big] 1110 \ - \\ \epsilon_{1_7} \Big] 1100 \ - $	XZIII	IIXXZ
-) 1)	(0)(+ 1	$\begin{array}{l} \epsilon_0 \left[00001 \ 01100 \right] + \epsilon_1 \left 00001 \ 01111 \right] + \epsilon_2 \left 00001 \ 00000 \right] + \\ \epsilon_3 \left 00001 \ 10011 \right] - \epsilon_4 \left 00010 \ 01100 \right] - \\ \epsilon_5 \left 00010 \ 10000 \right] - \epsilon_7 \left 00010 \ 10011 \right] - \\ \epsilon_9 \left 11101 \ 01111 \right] - \epsilon_{10} \left 11101 \ 10000 \right] - \\ \epsilon_{12} \left 11110 \ 01110 \right] + \epsilon_{13} \left 11110 \ 10000 \right] + \\ \epsilon_{14} \left 11110 \ 10000 \right] + \\ \epsilon_{15} \left 11110 \ 10011 \right] \end{array}$	XZZII	IIXXI
1)	(+)(1)	$ \begin{array}{l} \epsilon_0 \left[00001 \ 01100 \right] + \epsilon_1 \left[00001 \ 01111 \right] & - \epsilon_2 \left[00001 \ 00000 \right] & - \\ \epsilon_3 \left[00001 \ 10011 \right] & - \epsilon_4 \left[00010 \ 01100 \right] & - \\ \epsilon_6 \left[00010 \ 10000 \right] + \epsilon_7 \left[00010 \ 10011 \right] & + \\ \epsilon_8 \left[11101 \ 01111 \right] & + \epsilon_{10} \left[11101 \ 10000 \right] & + \\ \epsilon_{12} \left[111101 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 01110 \right] & - \\ \epsilon_{13} \left[11110 \ 01010 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 10000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 01000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 01000 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{13} \left[11110 \ 01111 \right] & - \\ \epsilon_{14} \left[1110 \ 0100 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1110 \ 0111 \right] & - \\ \epsilon_{15} \left[1111 \ 0110 \ 011 \right]$	XZZII	IIXXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
{o (1-)10>	$\begin{array}{llllllllllllllllllllllllllllllllllll$	ШZX	IIXXZ
-)10)	-	$\begin{array}{l} \epsilon_{0} \left 00001 \ 01100 \right\rangle - \epsilon_{1} \left 00001 \ 01111 \right\rangle - \epsilon_{2} \left 00001 \ 0000 \right\rangle + \\ \epsilon_{3} \left 00001 \ 10011 \right\rangle - \epsilon_{4} \left 00010 \ 01100 \right\rangle + \epsilon_{5} \left 00010 \ 01111 \right\rangle + \\ \epsilon_{6} \left 00010 \ 10000 \right\rangle - \epsilon_{7} \left 00010 \ 10011 \right\rangle + \epsilon_{8} \left 11101 \ 01100 \right\rangle - \\ \epsilon_{9} \left 11101 \ 01111 \right\rangle - \epsilon_{10} \left 11101 \ 10000 \right\rangle + \epsilon_{1} \left 11101 \ 10011 \right\rangle - \\ \epsilon_{12} \left 11110 \ 01100 \right\rangle + \epsilon_{13} \left 11110 \ 01111 \right\rangle + \epsilon_{14} \left 11110 \ 10000 \right\rangle - \\ \epsilon_{13} \left 11110 \ 10011 \right\rangle \end{array}$	ХZШ	IIXXZ
-) 1}	1-/10	$\begin{array}{l} \epsilon_0 \left 00001 \ 01100 \right\rangle & \epsilon_1 \left 00001 \ 01111 \right\rangle & \epsilon_2 \left 00001 \ 00000 \right\rangle & - \\ \epsilon_3 \left 00001 \ 10011 \right\rangle & - \epsilon_4 \left 00010 \ 01100 \right\rangle & + \epsilon_5 \left 00010 \ 01111 \right\rangle & - \\ \epsilon_6 \left 00010 \ 10000 \right\rangle & + \epsilon_7 \left 00010 \ 10011 \right\rangle & - \epsilon_8 \left 11101 \ 01100 \right\rangle & + \\ \epsilon_9 \left 11101 \ 01111 \right\rangle & - \epsilon_{10} \left 11101 \ 10000 \right\rangle & + \epsilon_{14} \left 11110 \ 10011 \right\rangle & + \\ \epsilon_{12} \left 11110 \ 01100 \right\rangle & - \epsilon_{13} \left 11110 \ 01111 \right\rangle & + \epsilon_{14} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{14} \left 11110 \ 10010 \right\rangle & - \\ \epsilon_{14} \left 11110 \ 10010 \right\rangle & - \\ \epsilon_{16} \left 11110 \ 10011 \right\rangle & - \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10001 \right\rangle & - \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10001 \right\rangle & - \\ \epsilon_{16} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 11110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1110 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left 1100 \ 10000 \right\rangle & - \\ \epsilon_{17} \left$	XZZII	IIXXZ
-) 1)	-	$\begin{array}{l} \epsilon_0 00001 \ 01100\rangle & -\epsilon_1 00001 \ 01111\rangle & -\epsilon_2 00001 \ 10000\rangle \\ \epsilon_3 00001 \ 10011\rangle & -\epsilon_4 00010 \ 01100\rangle & +\epsilon_5 00010 \ 01111\rangle \\ +\epsilon_6 00010 \ 10000\rangle & -\epsilon_7 00010 \ 10011\rangle & -\epsilon_8 11101 \ 01100\rangle \\ +\epsilon_{12} 11101 \ 01111\rangle & +\epsilon_{10} 11101 \ 10000\rangle & -\epsilon_{11} 11101 \ 10011\rangle \\ +\epsilon_{12} 11110 \ 01100\rangle & -\epsilon_{13} 11110 \ 01111\rangle & -\epsilon_{14} 11110 \ 10000\rangle \\ +\epsilon_{13} 11110 \ 10011\rangle \end{array}$	XZZII	IIXZXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
{o}{+	()(+)	$ \begin{split} \epsilon_0 11001 \ 01101 \rangle + \epsilon_1 11001 \ 01110 \rangle + \epsilon_2 11001 \ 10001 \rangle + \\ \epsilon_3 11001 \ 10010 \rangle + \epsilon_4 11010 \ 01101 \rangle + \epsilon_5 11010 \ 01110 \rangle + \\ \epsilon_6 11010 \ 10001 \rangle + \epsilon_7 11010 \ 10010 \rangle + \epsilon_8 00101 \ 01101 \rangle + \\ \epsilon_9 00101 \ 01110 \rangle + \epsilon_{13} 00111 \ 00001 \rangle + \epsilon_{14} 00111 \ 10010 \rangle + \\ \epsilon_{12} 00110 \ 01101 \rangle + \epsilon_{13} 00110 \ 01110 \rangle + \epsilon_{14} 00110 \ 10001 \rangle + \\ \epsilon_{16} 00110 \ 10010 \rangle \end{split} $	XIIX	IXXIX
(0)(+1	+ 1	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XIIX	ZXXIX
1+)11)	(01(+1	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XIZXX	XIXXI
(11)	(+) 1)	$\begin{array}{l} \epsilon_{0} \\ \epsilon_{1} \\ 11001 \ 01101 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 11001 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 11010 \ 10001 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 10101 \ 10001 \end{pmatrix} - \epsilon_{1} \\ \epsilon_{1} \\ 10101 \ 01010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01110 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01010 \end{pmatrix} - \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10010 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10001 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 10000 \end{pmatrix} + \epsilon_{1} \\ \epsilon_{1} \\ 10000 \ 1000 \\ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ 1000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 10000 \\ + \epsilon_{1} \\ 10000 \ 1000 \\ + \epsilon_{1}$	XIZXX	XIXXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
(0)	-)10)	$\begin{array}{l} \epsilon_0 11001 \ 01101 \rangle & -\epsilon_1 11001 \ 01110 \rangle + \epsilon_2 11001 \ 10001 \rangle + \\ \epsilon_3 11001 \ 10010 \rangle + \epsilon_4 11010 \ 01101 \rangle - \\ \epsilon_5 11010 \ 10001 \rangle & -\epsilon_7 11010 \ 10010 \rangle + \epsilon_8 00101 \ 01110 \rangle + \\ \epsilon_9 00101 \ 01110 \rangle + \epsilon_{10} 00101 \ 10001 \rangle - \\ \epsilon_{12} 00110 \ 01110 \rangle - \\ \epsilon_{13} 00110 \ 01110 \rangle + \\ \epsilon_{14} 00110 \ 10001 \rangle - \\ \epsilon_{15} 00110 \ 10010 \rangle + \\ \epsilon_{16} 00110 \ 10010 \rangle - \\ \epsilon_{16} 00110 \ 10010 \rangle + \\ \epsilon_{16} 00110 \ 10010 \rangle - \\ \epsilon_{16} 00110 \ 100010 \rangle - \\ \epsilon_{16} 00110 \ 10000 \rangle - \\ \epsilon_{$	XIIXX	IXXXX
(o)(+ 1	(T)($\begin{split} \epsilon_0 & \left[11001 \ 01101 \right] - \epsilon_1 & \left[11001 \ 01110 \right] - \epsilon_2 & \left[11001 \ 10001 \right] + \\ \epsilon_3 & \left[11001 \ 10010 \right] + \epsilon_4 & \left[11010 \ 01101 \right] - \epsilon_5 & \left[11010 \ 01110 \right] - \\ \epsilon_6 & \left[11010 \ 10001 \right] + \epsilon_7 & \left[11010 \ 10010 \right] + \epsilon_8 & \left[00101 \ 01101 \right] - \\ \epsilon_9 & \left[00101 \ 01110 \right] - \epsilon_{13} & \left[00101 \ 10001 \right] + \epsilon_{14} & \left[00101 \ 10010 \right] + \\ \epsilon_{12} & \left[00110 \ 01101 \right] - \epsilon_{13} & \left[00110 \ 01110 \right] - \epsilon_{14} & \left[00110 \ 10001 \right] + \\ \epsilon_{15} & \left[00110 \ 10010 \right] - \\ \epsilon_{15} & \left[00110 \ 10010 \right] - \\ \epsilon_{15} & \left[00110 \ 10010 \right] - \\ \epsilon_{15} & \left[00110 \ 10010 \right] - \\ \epsilon_{15} & \left[00110 \ 10010 \right] - \\ \epsilon_{15} & \left[00110 \ 10010 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00110 \ 10000 \right] + \\ \epsilon_{15} & \left[00100 \ 10000 \right] + \\ \epsilon_{15} & \left[00000 \ 1000 \right] + \\ \epsilon_{15} & \left[00$	XIIIX	XZXXZ
(+)	-) 0)	$\begin{split} \epsilon_0 & \left[11001 \ 01101 \right] - \epsilon_1 & \left[11001 \ 01110 \right] + \epsilon_2 & \left[11001 \ 10001 \right] + \epsilon_3 \\ \epsilon_3 & \left[11001 \ 10010 \right] + \epsilon_4 & \left[11010 \ 01101 \right] - \epsilon_5 & \left[11010 \ 01110 \right] + \epsilon_6 & \left[11010 \ 10001 \right] + \epsilon_7 & \left[100101 \ 01010 \right] + \epsilon_9 & \left[00101 \ 01110 \right] - \epsilon_{10} & \left[00101 \ 10001 \right] + \epsilon_{11} & \left[00101 \ 10010 \right] - \epsilon_{12} & \left[00110 \ 01110 \right] + \epsilon_{13} & \left[00110 \ 01110 \right] - \epsilon_{14} & \left[00110 \ 10001 \right] + \epsilon_{15} & \left[00110 \ 10001 \right] + \epsilon_{16} & \left[000110 \ 10001 \right] + \epsilon_{16} & \left[00110 \ 10001 \right] + \epsilon_{16} & \left[001$	XIZXX	XXXI
[+]1]	(11)	$\begin{array}{l} \epsilon_0 \\ \epsilon_0 \\ 11001 \ 01101 \\ \end{array} - \epsilon_1 \\ 11001 \ 10000 \\ + \epsilon_4 \\ 11010 \ 10001 \\ \end{array} + \epsilon_4 \\ 11010 \ 10001 \\ + \epsilon_1 \\ 100101 \ 01100 \\ + \epsilon_1 \\ 000101 \ 01110 \\ + \epsilon_1 \\ 000101 \ 01110 \\ \end{array} + \epsilon_1 \\ 000101 \ 01101 \\ + \epsilon_1 \\ 000101 \ 01010 \\ + \epsilon_1 \\ 000101 \ 0001 \\ + \epsilon_1 \\ 000101 \ 0001 \\ \end{array} + \epsilon_1 \\ 000101 \ 0000 \\ + \epsilon_1 \\ 00000 \\ + \epsilon_1 \\ 0000 \\ + \epsilon_1 \\ 00000 \\ + \epsilon_1 \\ 00000 \\ + \epsilon_1 \\ + \epsilon_1 \\ 00000 \\ + \epsilon_1 \\ 0000 \\ + \epsilon_1 \\ 00000 \\ + \epsilon_1 \\ 0000 \\ + \epsilon_1 \\ 00000 \\ +$	XIZXX	XZXXZ

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
1-)10	1+)10)	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XXIXX	IXXIX
1-)10)	(11) + 1	$ \begin{array}{l} \epsilon_{0} \\ \epsilon_{1} \\ 11001 \ 01101 \\ \epsilon_{3} \\ 11001 \ 10010 \\ \epsilon_{4} \\ 11001 \ 10001 \\ \epsilon_{4} \\ \epsilon_{1} \\ 11010 \ 10001 \\ \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01100 \\ \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01101 \\ \epsilon_{1} \\ 100101 \ 01101 \\ \epsilon_{1} \\ 100110 \ 01101 \\ \epsilon_{1} \\ 100110 \ 10010 \\ \epsilon_{1} \\ 100110 \ 10001 \\ \epsilon_{1} \\ 100110 \ 10000 \\ \epsilon_{1} \\ 10000 \\ \epsilon_{1} \\ 100100 \ 10000 \\ \epsilon_{1} \\ 10000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 10000 \\ 1$	XXIXX	ZXXIX
1)	(ol(+ 1	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XZZXX	XIXXI
(11)	(+) 1)	$ \begin{array}{l} \epsilon_0 11001 \ 01101 \rangle + \epsilon_1 11001 \ 01110 \rangle - \epsilon_2 11001 \ 10001 \rangle - \\ \epsilon_3 11001 \ 10010 \rangle - \epsilon_4 11010 \ 01101 \rangle - \\ \epsilon_5 11010 \ 10001 \rangle + \epsilon_7 11010 \ 10010 \rangle - \\ \epsilon_8 00101 \ 01110 \rangle + \epsilon_{10} 00101 \ 10001 \rangle + \\ \epsilon_{12} 00110 \ 01110 \rangle + \epsilon_{13} 00110 \ 01110 \rangle - \\ \epsilon_{12} 00110 \ 01101 \rangle + \\ \epsilon_{13} 00110 \ 01101 \rangle + \\ \epsilon_{13} 00110 \ 01110 \rangle - \\ \epsilon_{14} 00110 \ 10001 \rangle - \\ \end{array} $	XZZXX	ZXXIX

Table 5 continued				
Alice	Bob	Collapsed state	Alice's operat	Bob's operat
-)IO	1-/10	$ \begin{array}{llllllllllllllllllllllllllllllllllll$	XXIZX	IXXXX
1-)10)		$\begin{array}{llllllllllllllllllllllllllllllllllll$	XZIXX	XZXXZ
-) 1)	1)10>	$\begin{array}{llllllllllllllllllllllllllllllllllll$	XZZXX	XXXI
(11)	(11)	$ \begin{array}{l} \epsilon_{0} \\ \epsilon_{0} \\ 11001 \ 01101 \\ \epsilon_{3} \\ 11001 \ 10010 \\ \epsilon_{3} \\ \epsilon_{3} \\ 11010 \ 10010 \\ \epsilon_{4} \\ \epsilon_{1} \\ 10010 \ 01110 \\ \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01110 \\ \epsilon_{1} \\ \epsilon_{1} \\ 100101 \ 01110 \\ \epsilon_{1} \\ \epsilon_{1} \\ 100110 \ 01101 \\ \epsilon_{1} \\ 100110 \ 01101 \\ \epsilon_{1} \\ 100110 \ 01010 \\ \epsilon_{1} \\ 100110 \ 10001 \\ \epsilon_{1} \\ 10001 \\ \epsilon_{1} \\ 10001 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 10000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 10000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 10000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 1000000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 100000 \\ \epsilon_{1} \\ 10$	XZZXX	XZXXZ

Acknowledgements Lastly, I extend my utmost gratitude to the esteemed reviewers of the International Journal of Theoretical Physics for their invaluable insights, which have significantly enhanced the quality of the manuscript.

Author Contributions All authors reviewed the manuscript

Data Availability No datasets were generated or analysed during the current study.

Declarations

Conflicts of interest The authors declare no competing interests.

References

- Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels. Phys. Rev. Letters 70(13), 1895 (1993)
- Ekert, A.K.: Quantum cryptography based on bell's theorem. Phys. Rev. Lett. 67, 661–663 (1991). https:// doi.org/10.1103/PhysRevLett.67.661
- Hillery, M., Bužek, V., Berthiaume, A.: Quantum secret sharing. Phys. Rev. A 59, 1829–1834 (1999). https://doi.org/10.1103/PhysRevA.59.1829
- Cao, Y., Zhao, Y., Wang, Q., Zhang, J., Ng, S.X., Hanzo, L.: The evolution of quantum key distribution networks: On the road to the qinternet. IEEE Commun. Surv. Tutor. 24(2), 839–894 (2022)
- Scarani, V., Bechmann-Pasquinucci, H., Cerf, N.J., Dušek, M., Lütkenhaus, N., Peev, M.: The security of practical quantum key distribution. Rev. Modern Phys. 81(3), 1301 (2009)
- Shi, B.-S., Tomita, A.: Teleportation of an unknown state by w state. Phys. Lett. A 296(4), 161–164 (2002). https://www.sciencedirect.com/science/article/pii/S0375960102002578
- Agrawal, P., Pati, A.: Perfect teleportation and superdense coding with w states. Phys. Rev. A 74(6), 062320 (2006)
- Kim, Y.-H., Kulik, S.P., Shih, Y.: Quantum teleportation of a polarization state with a complete bell state measurement. Phys. Rev. Lett. 86(7), 1370 (2001)
- Yang, K., Huang, L., Yang, W., Song, F.: Quantum teleportation via ghz-like state. Int. J. Theor. Phys. 48, 516–521 (2009)
- Hassanpour, S., Houshmand, M.:Bidirectional quantum teleportation via entanglement swapping. In 2015 23rd Iranian conference on electrical engineering. IEEE, pp. 501–503 (2015)
- Zha, X.-W., Zou, Z.-C., Qi, J.-X., Song, H.-Y.: Bidirectional quantum controlled teleportation via fivequbit cluster state. Int. J. Theor. Phys. 52, 1740–1744 (2013)
- Duan, Y.-J., Zha, X.-W., Sun, X.-M., Xia, J.-F.: Bidirectional quantum controlled teleportation via a maximally seven-qubit entangled state. Int. J. Theor. Phys. 53, 2697–2707 (2014)
- Yan, A.: Bidirectional controlled teleportation via six-qubit cluster state. Int. J. Theor. Phys. 52, 3870–3873 (2013)
- Kazemikhah, P., Tabalvandani, M.B., Mafi, Y., Aghababa, H.: Asymmetric bidirectional controlled quantum teleportation using eight qubit cluster state. Int. J. Theor. Phys. 61(2), 17 (2022)
- Liu, J.-C., Li, Y.-H., Nie, Y.-Y.: Controlled teleportation of an arbitrary two-particle pure or mixed state by using a five-qubit cluster state. Int. J. Theor. Phys. 49, 1976–1984 (2010)
- Sadeghi Zadeh, M.S., Houshmand, M., Aghababa, H.: Bidirectional teleportation of a two-qubit state by using eight-qubit entangled state as a quantum channel. Int. J. Theor. Phys. 56, 2101–2112 (2017)
- Zangi, S., Li, J.-L., Qiao, C.-F.: Quantum state concentration and classification of multipartite entanglement. Phys. Rev. A 97(1), 012301 (2018)
- Zangi, S.M., Shukla, C., Ur Rahman, A., Zheng, B.: Entanglement swapping and swapped entanglement. Entropy 25(3), 415 (2023)
- Verma, V.: Bidirectional quantum teleportation by using two ghz-states as the quantum channel. IEEE Commun. Lett. 25(3), 936–939 (2020)
- Du, Z., Li, X., Liu, X.: Bidirectional quantum teleportation with ghz states and epr pairs via entanglement swapping. Int. J. Theor. Phys. 59, 622–631 (2020)

- Sadeghi-Zadeh, M.S., Houshmand, M., Aghababa, H., Kochakzadeh, M.H., Zarmehi, F.: Bidirectional quantum teleportation of an arbitrary number of qubits over noisy channel. Quantum Inf. Process. 18, 1–19 (2019)
- Wang, M., Li, H.-S.: Bidirectional quantum teleportation using a five-qubit cluster state as a quantum channel. Quantum Inf. Process. 21(2), 44 (2022)
- Zangi, S., Qiao, C.-F.: Robustness of 2× n× m entangled states against qubit loss. Phys. Lett. A 400, 127322 (2021)
- Zadeh, M.S.S., Houshmand, M., Aghababa, H.: Bidirectional quantum teleportation of a class of n-qubit states by using (2 n+ 2)-qubit entangled states as quantum channel. Int. J. Theor. Phys. 57, 175–183 (2018)
- Ahmadkhaniha, A., Mafi, Y., Kazemikhah, P., Aghababa, H., Barati, M., Kolahdouz, M.: Enhancing quantum teleportation: an enable-based protocol exploiting distributed quantum gates. Opt. Quantum Electron. 55(12), 1079 (2023)
- Zhou, R.-G., Xu, R., Lan, H.: Bidirectional quantum teleportation by using six-qubit cluster state. Ieee Access 7, 44 269-44 275 (2019)
- Chen, J., Li, D., Liu, M., Yang, Y.: Bidirectional quantum teleportation by using a four-qubit ghz state and two bell states. IEEE Access 8, 28 925-28 933 (2020)
- Podoshvedov, S.A.: Efficient quantum teleportation of unknown qubit based on dv-cv interaction mechanism. Entropy 21(2), 150 (2019)
- Mahjoory, A., Kazemikhah, P., Aghababa, H., Kolahdouz, M.: Asymmetric tridirectional quantum teleportation using seven-qubit cluster states. Physica Scripta 98(8), 085218 (2023)
- Li, Y.-H., Nie, L.-P.: Bidirectional controlled teleportation by using a five-qubit composite ghz-bell state. Int. J. Theor. Phys. 52, 1630–1634 (2013)
- Chen, Y.: Bidirectional quantum controlled teleportation by using a genuine six-qubit entangled state. Int. J. Theor. Phys. 54, 269–272 (2015)
- Li, Y.-H., Nie, L.-P., Li, X.-L., Sang, M.-H.: Asymmetric bidirectional controlled teleportation by using six-qubit cluster state. Int. J. Theor. Phys. 55, 3008–3016 (2016)
- Choudhury, B.S., Samanta, S.: Asymmetric bidirectional 3 2 qubit teleportation protocol between alice and bob via 9-qubit cluster state. Int. J. Theor. Phys. 56, 3285–3296 (2017)
- Zhou, R.-G., Li, X., Qian, C., Ian, H.: Quantum bidirectional teleportation 2 2 or 2 3 qubit teleportation protocol via 6-qubit entangled state. Int. J. Theor. Phys. 59, 166–172 (2020)
- Malik, J.A., Lone, M.Q., Malla, R.A.: Symmetric bidirectional quantum teleportation using a six-qubit cluster state as a quantum channel. Pramana 97(1), 50 (2023)
- 36. Kazemikhah, P., Aghababa, H.: Bidirectional quantum teleportation of an arbitrary number of qubits by using four qubit cluster state. Int. J. Theor. Phys. **60**, 378–386 (2021)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.