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Quantum Features of the Nonlinear Dissipative Cavity Interacting with a Two-Level Atom under the Influence of Stark Shift

S Mirzaei[1] · A Chenaghlou[1] · Y Alishamsi[1]

Abstract

Generating the entanglement and controlling the non-classical properties are an important topic in quantum optics. In this paper, we consider a system consisting of two-photon Jaynes-Cummings model with an intensity dependent coupling and Kerr term in the presence of the Stark shift and in dispersive approximation. It is assumed that the cavity field is initially provided in the coherent state and the atom is in the superposition state, $|\psi\rangle_{af} = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$. We obtain the exact energy spectrum of the deformed model and examine the dependence of Stark shifts, deformation parameter and atom-field coupling constant on the level crossing phenomenon. Dissipation due to the coupling of the system with its surrounding medium is an unavoidable phenomenon in quantum systems. Thus for deformed JCM under nonlinear quantum dissipation, we solve the deformed Master equation in dispersive approximation. The non-classical properties of the time-evolved atom-field states are exhibited through evaluating the linear entropy measure and Mandel parameter. We examine the significance of damping and deformation parameters on the dynamical properties of atom-field system. It will be shown that the cavity damping and deformation parameter control the non-classical features of the field throughout the interaction time.

Keywords Dissipation · Entanglement · Linear entropy · Mandel parameter · Nonlinear Jaynes-Cummings model · Stark shift

1 Introduction

In quantum optics, the Jaynes-Cummings model (JCM) is a candidate for the simplest and most effective theoretical models which studies the interaction of a single mode quantized

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electromagnetic field with a two-level atom [1]. This model as an exactly solvable model explains various interesting quantum effects such as Rabi oscillations [2–5], collapse and revival of the atomic population inversion [6–9], sub-Poissonian statistics and squeezing of the radiation field [8–12]. The significance of the JCM in quantum optics leads to the generalization of this model to the q-deformed [13–15], f-deformed [16] and parity λ-deformed JCMs [17, 18].

An outstanding aspect of quantum mechanics is the entanglement which was introduced by Einstein et al. [19]. Quantum entanglement plays an important role in quantum computation [20] and quantum information processing [21]. Quantum entanglement is widely applied in quantum cryptography [22] and teleportation [23, 24]. In order to prepare the entangled state, many schemes are proposed such as the JCM [1]. Many authors have studied the entanglement in this model, such as Refs. [25–29]. In all of the papers, the influence of the environment has been ignored. However due to the coupling of the system with its surrounding environment, dissipation is an inevitable phenomenon.

The dissipative dynamics of the entanglement in the single and double JCMs in the dispersive approximation has been studied in Refs. [30, 31], respectively. Zhou et al studied the dissipative dynamics of two-photon JCM with Stark shift [32] and the non-classical properties of this model under Kerr like medium have been examined in Ref. [33]. The dissipative characteristic of the system in the two-photon JCM with degenerate atomic levels have been analyzed in Ref. [34]. Naderi et al. [35] studied the dynamics of the atom-field system described by the f-deformed JCM with nonlinear quantum dissipation and in the large detuning approximation.

In what follows, we study the two-photon JCM in the appearance of the Stark shift and in dispersive approximation. Assuming that at first, the field is prepared in the coherent state and the atom is arranged in a superposition of its ground $|g\rangle$ and excited $|e\rangle$ states, we calculate the deformed Master equation in dispersive approximation. The dynamics of the non-classical features of atom-field states is evaluated by using the linear entropy measure and Mandel parameter. We analyze the effect of the deformation parameter and cavity damping on the non-classical features of the field during the interaction time.

The organization of this paper has the following steps: The nonlinear JCM in the appearance of the Stark shift is reviewed in Section 2. In Section 3, we solve the deformed master equation in the dispersive approximation, and obtain the matrix elements of the atom-field density operator. The dynamics of the linear entropy and the Mandel parameter are studied in Section 4. Finally, Section 5 involves the results.

2 Model Description

JCM studies the interaction between a single two-level atom with ground state $|g\rangle$ and excited state $|e\rangle$ coupled to a single-mode quantized electromagnetic field [1]. In the rotating wave and dipole approximations, the JCM Hamiltonian is

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-), \tag{1}$$

where ω and ω_0 are the field and atomic transition frequencies, respectively and $\hat{a}(\hat{a}^\dagger)$ is the annihilation (creation) operators for the field quanta. The operators $\hat{\sigma}_\pm$ and $\hat{\sigma}_z$ are atomic operators for the two-level atom which satisfy the commutation relations $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$

and $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$. Also, g is the atom-field coupling constant. Let us introduce the operators [36, 37]

$$\hat{K}_+ := \hat{a}^\dagger \sqrt{1 + \chi \hat{a}^\dagger \hat{a}}, \quad \hat{K}_- := \sqrt{1 + \chi \hat{a}^\dagger \hat{a}} \, \hat{a}, \tag{2}$$

where the operators $\hat{K}_+$ and $\hat{K}_-$ form a closed algebra $[\hat{K}_-, \hat{K}_+] = 2\hat{K}_0$ and $[\hat{K}_0, \hat{K}_\pm] = \pm \chi \hat{K}_\pm$ with $\hat{K}_0 = \chi \hat{a}^\dagger \hat{a} + \frac{1}{2}$. The number operator is defined as $\hat{n} = \hat{a}^\dagger \hat{a}$, which satisfies the following commutation relations

$$[\hat{n}, \hat{K}_+] = \hat{K}_+, \quad [\hat{n}, \hat{K}_-] = -\hat{K}_-. \tag{3}$$

Clearly for $\chi = 0$, the commutation relations form the Heisenberg-Weyl algebra for $\hat{a}$ and $\hat{a}^\dagger$. Thus, a more general Hamiltonian consisting of the nonlinear JCM in terms of operators $\hat{K}_\pm$ is obtained as follows:

$$\hat{H} = \hbar \omega \hat{K}_+ \hat{K}_- + \frac{\hbar \omega_0}{2} \hat{\sigma}_z + \hbar g (\hat{K}_+ \hat{\sigma}_- + \hat{K}_- \hat{\sigma}_+). \tag{4}$$

In this work, we consider the Hamiltonian of the nonlinear two-photon JCM in the appearance of the Stark shift under the rotating-wave and dipole approximations which is defined as ($\hbar = 1$)

$$\hat{H} = \omega \hat{K}_+ \hat{K}_- + \frac{\omega_0}{2} \hat{\sigma}_z + g(\hat{K}_+^2 \hat{\sigma}_- + \hat{K}_-^2 \hat{\sigma}_+) + \hat{K}_+ \hat{K}_- (\beta_1 |g\rangle \langle g| + \beta_2 |e\rangle \langle e|), \tag{5}$$

where $\beta_i = \frac{g_i^2}{\delta} (i = 1, 2)$ are the Stark shift parameters which are connected to the virtual transition to the intermediate level, and may occur for two-photon transition [38, 39]. $\delta = \omega_0 - 2\omega$ is the detuning parameter. g_1 and g_2 illustrate the coupling strengths of the intermediate state to the states $|e\rangle$ and $|g\rangle$, respectively. In fact, we want to examine the dynamics of a two-level atom interacting with the field in a dissipative cavity as well as in the appearance of Stark shift. Besides, we analyze the effect of the dissipation on the entanglement dynamics and Mandel parameter. The matrix form of the Hamiltonian in (5) with respect to the bases $|e, n\rangle$ and $|g, n + 2\rangle$ takes the following form

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}, \tag{6}$$

where

$$h_{11} = \frac{\omega_0}{2} + (\omega + \beta_2)(n + \chi n(n-1)),$$

$$h_{12} = h_{21} = g\sqrt{(n+1)(n+2)(1+\chi n)(1+\chi(n+1))}, \tag{7}$$

$$h_{22} = \frac{\omega_0}{2} - \delta + \omega(n + \chi(n+1)(n+2)) + \beta_1(n+2+\chi(n+1)(n+2)).$$

The eigenvalues of this Hamiltonian are given by

$$E_n^\pm = \omega n + \frac{\omega_0}{2} - \frac{\delta}{2} + \frac{\beta_1(n+2)(1+\chi(n+1))}{2} + \frac{\beta_2(n+1)(1+\chi n)}{2} + \omega\chi(1+n(n+1))$$

$$-\beta_2 \chi n - \frac{\beta_2}{2} \pm \frac{1}{2}\{4g^2(n+1)(n+2)(1+\chi n)(1+\chi(n+1))$$

$$+[\delta + \beta_2(n+1)(1+\chi n) - \beta_1(n+2)(1+\chi(n+1)) - 2\chi(\beta_2 n + \omega(1+2n)) - \beta_2]^2\}^{\frac{1}{2}}. \tag{8}$$

The energy eigenvalues $E_n^\pm$ versus detuning parameter δ are plotted in Fig. 1, for $n = 1$, $\omega = 1$, $\chi = g = 0.5$, and three different values of β_1 and β_2.
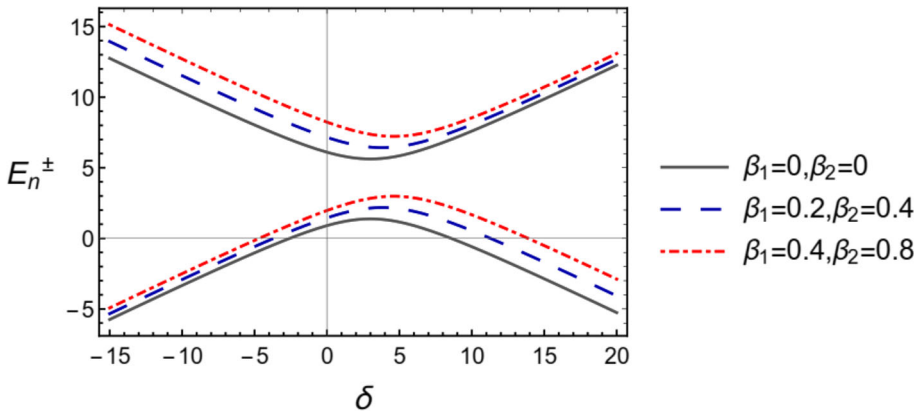
Fig. 1 Energy eigenvalues $E_n^{\pm}$ versus detuning parameter δ for $n = 1$, $\omega = 1$ and $\chi = g = 0.5$, when $\beta_1 = \beta_2 = 0$ (solid curve), $\beta_1 = 0.2$, $\beta_2 = 0.4$ (dashed curve), and $\beta_1 = 0.4$, $\beta_2 = 0.8$(dotdashed curve). The lower portion of the figure is for E_n^- and the upper portion is for E_n^+

Figure 1 denotes that in the appearance of Stark shifts, the magnitude of the energies increases, and the extremum of $E_n^{\pm}$ shifts to the positive detuning parameter $\delta > 0$. On the other hand, we plot the diagrams of the energy eigenvalues $E_n^{\pm}$ versus δ for $\beta_1 = 0.02$, $\beta_2 = 0.04$, $n = 1$, and $\omega = 1$ for given coupling constant g and deformation parameter χ. Figure 2 indicates that when the coupling between the atom and the field becomes zero, i.e. when $g = 0$, the energy eigenvalues will be degenerate for exact resonance, i.e. $\delta = 0$. The energy levels E_n^+ and E_n^- are split for non-zero coupling between the atom and the field, that is, the degeneracy is broken. Furthermore, the comparison of Fig. 2(a) and (b) indicates that the energy eigenvalues move to the larger values as the deformation parameter χ becomes larger. For nonlinear JCM ($\chi \neq 0$), the level crossing moves to positive detuning parameter. Furthermore, the gap between the energies E_n^+ and E_n^- increases when the deformation parameter and coupling constant increase.

In the dispersive limits of a two-photon process, i.e.

$$\frac{\beta_1 (n + 2)(1 + \chi (n + 1))}{|\delta - 2\chi (\beta_2 n + \omega(1 + 2n)) - \beta_2|} \ll 1, \tag{9a}$$

$$\frac{\beta_2 (n + 1)(1 + \chi n)}{|\delta - 2\chi (\beta_2 n + \omega(1 + 2n)) - \beta_2|} \ll 1. \tag{9b}$$
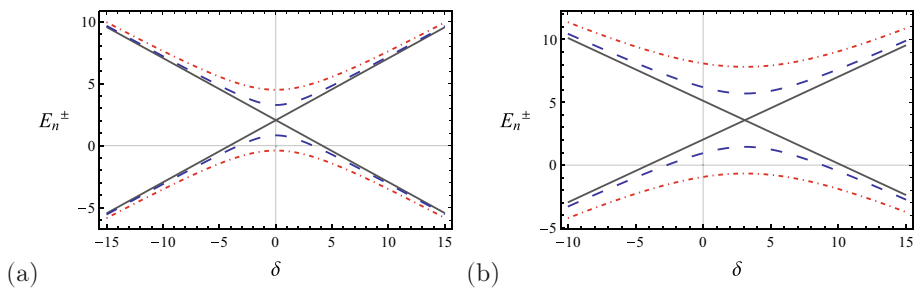

(a) (b)

Fig. 2 Energy eigenvalues $E_n^{\pm}$ versus detuning parameter δ for $\beta_1 = 0.02$, $\beta_2 = 0.04$, $n = 1$, and $\omega = 1$, when $g = 0$ (full line), $g = 0.5$ (dashed curve) and $g = 1$(dotdashed curve): (a) $\chi = 0$, and (b) $\chi = 0.5$. The lower portion of the figure is for E_n^- and the upper portion is for E_n^+

the eigenvalues are rewritten as

$$E_n^+ = \frac{\omega_0}{2} + \omega n + \beta_2 n(1 + \chi(n-1)) + \omega\chi n(n-1) + \Omega(n+1)(n+2)(1+\chi n)(1+\chi(n+1)),$$

$$E_n^- = -\frac{\omega_0}{2} + \omega(n+2) + \beta_1(n+2)(1+\chi(n+1)) - \Omega(n+1)(n+2)(1+\chi n)(1+\chi(n+1))$$

$$+ \omega\chi(n+1)(n+2),$$

(10)

in which $\Omega = \frac{\beta_1\beta_2}{|\delta - 2\chi(\beta_2 n + \omega(1+2n)) - \beta_2|}$. If the condition in (9) is fulfilled for all values of n, the effective Hamiltonian is rewritten as

$$\hat{H}_{eff} = \frac{\omega_0}{2}\hat{\sigma}_z + \omega\hat{K}_+\hat{K}_- + \hat{K}_+\hat{K}_-(\beta_1 |g\rangle\langle g| + \beta_2 |e\rangle\langle e|)$$

$$+ \Omega[(\hat{a}^\dagger\hat{a}+1)(\hat{a}^\dagger\hat{a}+2)(1+\chi\hat{a}^\dagger\hat{a})(1+\chi(\hat{a}^\dagger\hat{a}+1))|e\rangle\langle e|$$

(11)

$$- (\hat{a}^\dagger\hat{a}-1)\hat{a}^\dagger\hat{a}\left(1+\chi(\hat{a}^\dagger\hat{a}-2)\right)\left(1+\chi(\hat{a}^\dagger\hat{a}-1)\right)|g\rangle\langle g|].$$

3 Solution of Deformed Master Equation for Density Matrix

We now study the nonlinear dissipative cavity interacting with a two-level atom along with Stark shift and coupled to a zero temperature reservoir. In fact, Isar et al. [40] have obtained a master equation for the deformed harmonic oscillator under a dissipative environment. Regarding the damped deformed oscillator in the environment as a thermal bath at equilibrium temperature T, the master equation in the interaction picture under the Born-Markov approximation is given by [41]

$$\frac{d\hat{\rho}_{D.O}(t)}{dt} = \frac{\lambda}{2}\left(\left[\left[\coth\frac{\hbar\omega\Omega(\hat{n})}{2k_BT}\hat{K}_-, \hat{\rho}_{D.O}(t)\right], \hat{K}_+\right] - \left[\hat{K}_+, \{\hat{K}_-, \hat{\rho}_{D.O}(t)\}\right] + H.C.\right),$$

(12)

where, $\hat{\rho}_{D.O}(t)$ is the reduced density operator for the deformed oscillator with frequency ω, λ is the damping parameter, k_B is the Boltzman constant and by definition

$$\Omega(\hat{n}) = \frac{1}{2}((\hat{n}+2)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}-1)),$$

(13)

in which $f(\hat{n}) = \sqrt{1+\chi\hat{n}}$. Thus, the evolution of the total atom-field system in a dispersive deformed JCM at zero temperature in the interaction picture becomes ($\hbar = 1$) [35, 42]

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_{eff}, \hat{\rho}(t)] - \hat{\mathcal{L}}\hat{\rho}(t),$$

(14)

where $\hat{\rho}$ is the atom-field density operator and $\hat{\mathcal{L}}$ is the nonlinear superoperator which can be defined as follows

$$\hat{\mathcal{L}}\cdot = \lambda(\hat{K}_+\hat{K}_-\cdot + \cdot\hat{K}_+\hat{K}_- - 2\hat{K}_-\cdot\hat{K}_+).$$

(15)

The Liouvillian superoperator $\hat{\mathcal{L}}$ is a linear combination of bosonic superoperators [43, 44] which constitute a finite Lie algebra under the commutation relation. The application of creation and annihilation operators of the harmonic oscillator, i.e. $\hat{a}$ and $\hat{a}^\dagger$ on an operator $\hat{O}$ is represented by the bosonic superoperators:

$$(\hat{a}\cdot)\hat{O} = \hat{a}\hat{O}, \quad (\cdot\hat{a})\hat{O} = \hat{O}\hat{a},$$

$$(\hat{a}^\dagger\cdot)\hat{O} = \hat{a}^\dagger\hat{O}, \quad (\cdot\hat{a}^\dagger)\hat{O} = \hat{O}\hat{a}^\dagger.$$

(16)

Considering the fundamental commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$, the bosonic superoperators satisfy the following commutation relations

$$[(\hat{a}\cdot), (\hat{a}^\dagger\cdot)] = 1, \quad [(\cdot\hat{a}), (\cdot\hat{a}^\dagger)] = -1. \tag{17}$$

These superoperators construct the following bilinear products

$$\mathcal{M} = (\hat{a}^\dagger \hat{a}\cdot) = (\hat{a}^\dagger\cdot)(\hat{a}\cdot),$$

$$\mathcal{P} = (\cdot\hat{a}^\dagger a) = (\cdot\hat{a}^\dagger)(\cdot\hat{a}), \tag{18}$$

$$\mathcal{F} = (\hat{a} \cdot \hat{a}^\dagger) = (\hat{a}\cdot)(\cdot\hat{a}^\dagger) = (\cdot\hat{a}^\dagger)(\hat{a}\cdot).$$

The density operator $\hat{\rho}(t)$ belongs to the Hilbert space $\mathcal{H}_a \otimes \mathcal{H}_f$ of the atom and field. Therefore, the reduced field operator is defined as

$$\hat{\rho}_f(t) = Tr_a[\hat{\rho}(t)] = \hat{\rho}_{gg}(t) + \hat{\rho}_{ee}(t), \tag{19}$$

where $\hat{\rho}_{gg}(t) = \langle g| \hat{\rho}(t) |g\rangle$ and $\hat{\rho}_{ee}(t) = \langle e| \hat{\rho}(t) |e\rangle$.

By substituting (11) and (15) in the master equation (14), the Liouvillians associated with the matrix elements $\hat{\rho}_{ee}(t)$ and $\hat{\rho}_{gg}(t)$ are given by

$$\hat{\mathcal{L}}_{ee} = i\beta_2 \left[(\mathcal{P}^2 - \mathcal{M}^2)\chi + (\mathcal{M} - \mathcal{P})\chi + (\mathcal{P} - \mathcal{M})\right]$$

$$+i\Omega[(\mathcal{P} + 1)(\mathcal{P} + 2)(1 + \chi\mathcal{P})(1 + \chi(\mathcal{P} + 1))$$

$$-(\mathcal{M} + 1)(\mathcal{M} + 2)(1 + \chi\mathcal{M})(1 + \chi(\mathcal{M} + 1))] \tag{20}$$

$$+\lambda(2\mathcal{F}A(\mathcal{M}, \mathcal{P}) - \mathcal{M}(1 + \chi(\mathcal{M} - 1)) - \mathcal{P}(1 + \chi(\mathcal{P} - 1))),$$

and

$$\hat{\mathcal{L}}_{gg} = i\beta_1 \left[(\mathcal{P}^2 - \mathcal{M}^2)\chi + (\mathcal{M} - \mathcal{P})\chi + (\mathcal{P} - \mathcal{M})\right]$$

$$-i\Omega[\mathcal{P}(\mathcal{P} - 1)(1 + \chi(\mathcal{P} - 2)(1 + \chi(\mathcal{P} - 1))$$

$$-\mathcal{M}(\mathcal{M} - 1)(1 + \chi(\mathcal{M} - 2)(1 + \chi(\mathcal{M} - 1))] \tag{21}$$

$$+\lambda(2\mathcal{F}A(\mathcal{M}, \mathcal{P}) - \mathcal{M}(1 + \chi(\mathcal{M} - 1)) - \mathcal{P}(1 + \chi(\mathcal{P} - 1))),$$

with $A(\mathcal{M}, \mathcal{P}) = \sqrt{1 + \chi(\mathcal{M} - 1)}\sqrt{1 + \chi\mathcal{P}}$.

By using the dynamical symmetry technique proposed in Ref. [45], one may solve the Master equation as

$$\hat{\rho}_{ee}(t) = e^{\hat{\mathcal{L}}_{ee}t} \hat{\rho}_{ee}(0)$$

$$= \exp\left[i\beta_2 t[(\mathcal{P}^2 - \mathcal{M}^2)\chi + (\mathcal{M} - \mathcal{P})\chi + (\mathcal{P} - \mathcal{M})]\right.$$

$$+i\Omega t[(\mathcal{P} + 1)(\mathcal{P} + 2)(1 + \chi\mathcal{P})(1 + \chi(\mathcal{P} + 1))$$

$$-(\mathcal{M} + 1)(\mathcal{M} + 2)(1 + \chi\mathcal{M})(1 + \chi(\mathcal{M} + 1))] \tag{22}$$

$$-\lambda t[\mathcal{M}(1 + \chi(\mathcal{M} - 1)) + \mathcal{P}(1 + \chi(\mathcal{P} - 1))]] \times \exp[2\lambda\mathcal{F}A(\mathcal{M}, \mathcal{P})t]\hat{\rho}_{ee}(0),$$

and

$$\hat{\rho}_{gg}(t) = e^{\hat{\mathcal{L}}_{gg}t} \hat{\rho}_{gg}(0)$$

$$= \exp\left[i\beta_1 t[(\mathcal{P}^2 - \mathcal{M}^2)\chi + (\mathcal{M} - \mathcal{P})\chi + (\mathcal{P} - \mathcal{M})]\right.$$

$$-i\Omega t[\mathcal{P}(\mathcal{P} - 1)(1 + \chi(\mathcal{P} - 2)(1 + \chi(\mathcal{P} - 1)) \tag{23}$$

$$-\mathcal{M}(\mathcal{M} - 1)(1 + \chi(\mathcal{M} - 2)(1 + \chi(\mathcal{M} - 1)))]$$

$$\left.-\lambda t[\mathcal{M}(1 + \chi(\mathcal{M} - 1)) + \mathcal{P}(1 + \chi(\mathcal{P} - 1)))]\right] \times \exp[2\lambda\mathcal{F}A(\mathcal{M}, \mathcal{P})t]\hat{\rho}_{ee}(0).$$

It is supposed that at first the atom is arranged in a superposition of the excited and ground states, and the cavity field is in the coherent state:

$$\left|\psi_{af}(0)\right\rangle = \frac{1}{\sqrt{2}}\left(|e\rangle + |g\rangle\right) \otimes |\alpha\rangle, \tag{24}$$

where $|\alpha\rangle = \exp[\frac{-|\alpha|^2}{2}] \sum_m \frac{\alpha^m}{\sqrt{m!}} |m\rangle$, and for clarity, we will suppose that α is real parameter. Therefore, when $t = 0$ then $\hat{\rho}_{ee}(0) = \hat{\rho}_{gg}(0) = \frac{1}{2} |\alpha\rangle\langle\alpha|$. Substituting (24) in (22) and (23) and after some calculation, the following formulae for the atom-field density operator $\hat{\rho}_{ee}(t)$ and $\hat{\rho}_{gg}(t)$ in the matrix representation will be obtained as

$$\hat{\rho}_{ee}(t) = \frac{1}{2}\exp[-|\alpha|^2]\sum_{n,m}\frac{\alpha^m\alpha^{*n}}{\sqrt{m!n!}} \times \exp[\Gamma_{nm}(t) + i\Theta_{enm}(t)]|m\rangle\langle n|, \tag{25}$$

and

$$\hat{\rho}_{gg}(t) = \frac{1}{2}\exp[-|\alpha|^2]\sum_{n,m}\frac{\alpha^m\alpha^{*n}}{\sqrt{m!n!}} \times \exp[\Gamma_{nm}(t) + i\Theta_{gnm}(t)]|m\rangle\langle n|, \tag{26}$$

in which

$$\Theta_{enm}(t) = \beta_2 t[(n^2 - m^2)\chi + (m - n)\chi + (n - m)]$$

$$+\Omega t[(n + 1)(n + 2)(1 + \chi n)(1 + \chi(n + 1))$$

$$-(m + 1)(m + 2)(1 + \chi m)(1 + \chi(m + 1))] \tag{27}$$

$$+\frac{\lambda A(m,n)|\alpha|^2}{\lambda^2 A^2(m,n) + \Omega^2(m-n)^2}\left(\Omega(m - n) - e^{-2\lambda A(m,n)t}\right.$$

$$\times[\Omega(m - n)\cos 2\Omega t(m - n) + \lambda A(m, n)\sin 2\Omega t(m - n)]),$$

and

$$\Theta_{gnm}(t) = \beta_1 t[(n^2 - m^2)\chi + (m - n)\chi + (n - m)]$$

$$-\Omega t[n(n - 1)(1 + \chi(n - 2))(1 + \chi(n - 1))$$

$$-m(m - 1)(1 + \chi(m - 2))(1 + \chi(m - 1))] \tag{28}$$

$$+\frac{\lambda A(m,n)|\alpha|^2}{\lambda^2 A^2(m,n) + \Omega^2(m-n)^2}\left(\Omega(m - n) - e^{-2\lambda A(m,n)t}\right.$$

$$\times[\Omega(m - n)\cos 2\Omega t(m - n) + \lambda A(m, n)\sin 2\Omega t(m - n)]),$$

and

$$\Gamma_{nm}(t) = -\lambda t(m(1 + \chi(m-1)) + n(1 + \chi(n-1)))$$

$$+\frac{\lambda A(m,n)|\alpha|^2}{\lambda^2 A^2(m,n) + \Omega^2(m-n)^2} \times (\lambda A(m,n) - e^{-2\lambda A(m,n)t}$$

(29)

$$\times[\lambda A(m,n)\cos 2\Omega t(m-n) - \Omega(m-n)\sin 2\Omega t(m-n)]),$$

clearly, the lack of cavity damping effect, i.e. $\lambda = 0$ leads to $\Gamma = 0$.

4 Dynamical Characteristics of the Model

Here, we investigate the effect of dissipation and nonlinear parameter on the time evolution of linear entropy and Mandel parameter of the atom-field system with Stark shift in dispersive approximation.

4.1 Linear Entropy

The coherence features of the atom-field system are examined by means of the linear entropy [46]. For the reduced field density matrix, the linear entropy is introduced as:

$$S_f(t) = 1 - Tr[\hat{\rho}_f^2(t)],$$

(30)

where $Tr[\hat{\rho}_f^2(t)]$ measures the purity of the state and varies from 1 for a pure state to $\frac{1}{N}$ for a completely mixed state with dimension N. The dynamics of the field linear entropy shows the time evolution of the degree of entanglement. Substituting (25) and (26) in (19) and using (30), the linear entropy will be obtained. We plotted the linear entropy $S_f(t)$ versus scaled time Ωt for fixed values of $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\alpha = 1$, $\chi = 0.5$ and different values of damping parameter λ in Fig. 3.

From Fig. 3, we see that at time $t = 0$ the field linear entropy is zero , i.e $S_f = 0$. This confirms that at initial time, the field is in the pure coherent state and the atom-field
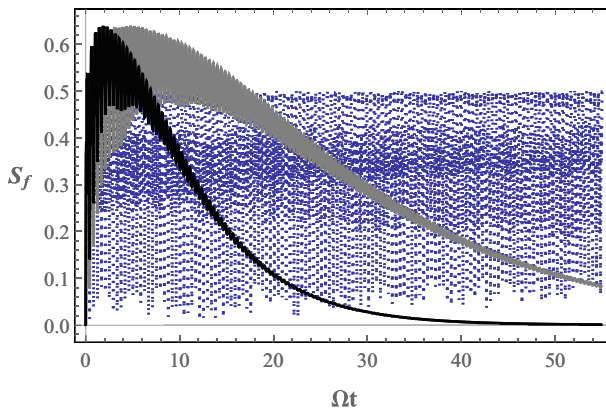

Fig. 3 Linear entropy versus scaled time Ωt for $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\alpha = 1$, $\chi = 0.5$ and different values of dissipation parameter: $\lambda = 10^{-5}\Omega$ (dotted curve), $\lambda = 0.04\Omega$ (gray curve) and $\lambda = 0.1\Omega$ (black curve)
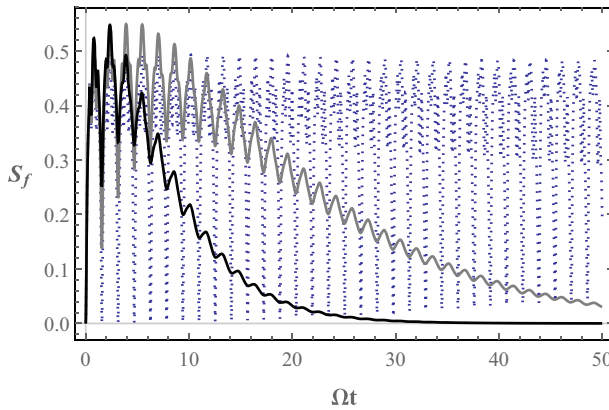
Fig. 4 Linear entropy versus scaled time Ωt for $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\alpha = 1$, $\chi = 0$ and different values of dissipation parameter: $\lambda = 10^{-5}\Omega$ (dotted curve), $\lambda = 0.04\Omega$ (gray curve) and $\lambda = 0.1\Omega$ (black curve)

system is separable. In the absence of dissipation, the linear entropy acts periodically between a maximum value 0.5 and a minimum value 0, which corresponds to the entangled and separable states, respectively. After adding decay terms, the field linear entropy has damped oscillation. By increasing the dissipation parameter λ, the amplitude of the local maxima and minima reduces with time and the linear entropy decreases quickly, i.e. the system reaches quickly to a pure state.

In Fig. 4, the linear entropy is plotted for $\chi = 0$, and the other parameters are the same with Fig. 3. Figure 4 indicates that for small values of the Kerr-like parameter, the linear entropy still illustrates local maximum and local minimum. A comparison of Figs. 3 and 4 shows that the nonlinear interaction of the Kerr medium with the field gives rise to growing the maximum and minimum values of linear entropy.

Moreover, we plot the linear entropy for $\alpha = 1$ and $\alpha = \sqrt{2}$. From Fig. 5, it is seen that as the amplitude of coherent field increases then the amplitude of the local maxima and minima decreases and the maximum value of the atom-field entanglement increases.
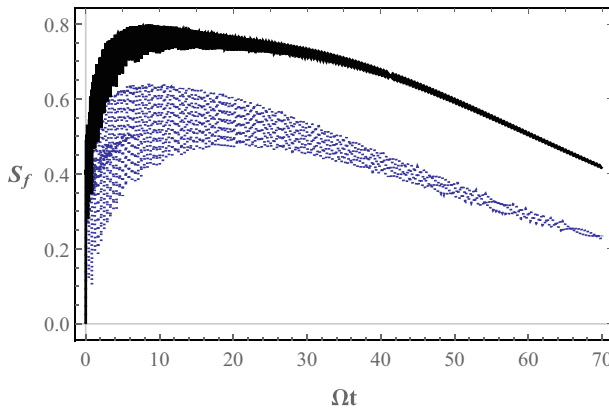

Fig. 5 Linear entropy versus scaled time Ωt for $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\chi = 0.5$ and different values α: $\alpha = 1$ (dotted curve), and $\alpha = \sqrt{2}$ (black curve)

4.2 Mandel Parameter

One way to examine the photons distribution in the light field is the deformed Mandel parameter [47–49]. It is defined as

$$Q(t) = \frac{\left\langle \hat{N}^2 \right\rangle}{\left\langle \hat{N} \right\rangle} - \left\langle \hat{N} \right\rangle - 1, \tag{31}$$

with $\hat{N} = \hat{K}_+ \hat{K}_-$. $Q(t) > 0$ confirms the super-Poissonian statistics and $Q(t) = 0$ corresponds to the coherent states. $Q(t) < 0$ indicates the sub-Poissonian statistics, where the sub-Poissonian photon statistics is one of the most significant non-classical characteristics of a quantum system. Now, regarding the dynamics of the photon counting statistics, we demonstrate the diagram of the deformed Mandel parameter $Q(t)$ versus the scaled time Ωt for $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\alpha = 1$, and for $\lambda = 0.5\Omega$, and $\chi = 0.2$ in Fig. 6(a) and (b), respectively.

Figure 6(a) for usual (undeformed) dissipative JCM in dispersive approximation, i.e. $\chi = 0$ shows that at the initial time the Mandel parameter is zero ($Q = 0$) which corresponds to the Poissonian photon statistics. It is seen that the super-Poissonian property of the cavity field increases with growing χ. As time goes on, the behavior of the field approaches the sub-Poissonian statistics. Moreover, by increasing the deformation parameter χ, the statistics more rapidly tends to the sub-Poissonian statistics which corresponds to the non-classical state. Considering the cavity damping effect, it is seen from Fig. 6(b) that the Mandel parameter decreases and tends to negative values. In other words, the field statistics shifts from the super-Poissonian to sub-Poissonian photon distribution. The increasing of the cavity damping accelerates the non-classicality of the field. Therefore, Fig. 6(a) and (b) show that the deformation and damping parameters control the non-classical characteristic of the field throughout the interaction time.

5 Conclusion

In this paper, we examined the two-photon JCM in the appearance of the Stark shift and under dispersive approximation. Firstly, we obtained the energy eigenvalues of the Hamiltonian, i.e. $E_n^{\pm}$. We found that the magnitude of the energies increases in the presence of Stark shifts. For usual (nondeformed) JCM, i.e. $\chi = 0$, with no coupling between the atom and the field,
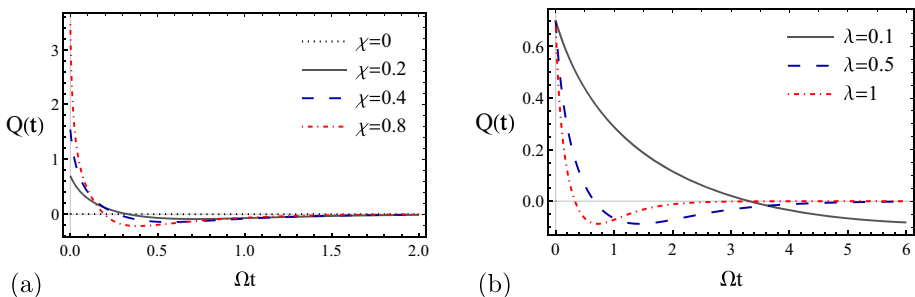

Fig. 6 Deformed Mandel parameter $Q(t)$ as a function of the scaled time Ωt for $\beta_1 = 0.02\Omega$, $\beta_2 = 0.04\Omega$, $\alpha = 1$: (a) $\lambda = 0.5\Omega$, and (b) $\chi = 0.2$

the level crossing was observed in exact resonance ($\delta = 0$). By growing the deformation parameter from zero to $\chi > 0$, the level crossing between the excited and ground states shifts to positive detuning. The degeneracy between the energy levels E_n^+ and E_n^- was disappeared for non-zero coupling. The gap between the two energy levels increases by increasing the deformation parameter and coupling constant. Then, we solved the master equation for the deformed JCM in the appearance of damping effect under the dispersive approximation. Assuming at first, the atom is provided in the superposition state, $|\psi\rangle_{atom} = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and the cavity field is arranged in the coherent state, we obtained the matrix elements associated with the atom-field density operator in dispersive approximation. Using the linear entropy, the time evolution of the system entanglement has been explored. It has been shown that, when $\lambda = 0$, the linear entropy treats periodically between a maximum value 0.5 and a minimum value 0, which corresponds to the entangled and separable state, respectively. However when $\lambda \neq 0$, the linear entropy has damped oscillation. By increasing the dissipation parameter, the domain of the entanglement reduces when the time goes on and the system reaches rapidly to a pure state. Moreover, the effect of the Kerr medium interaction with the field gives rise to growing the maximum and minimum values of the linear entropy. The plots of the linear entropy versus scaled time Ωt for the distinct values of α indicate that if the intensity of the coherent field becomes larger, the atom-field entanglement will be enhanced. The influences of the deformation parameter χ and damping parameter on the field statistics dynamics were examined by applying Mandel parameter. The Mandel parameter is applied as a statistical measure of the non-classical features of the field. It is found that at initial time, for usual dissipative JCM in dispersive approximation, i.e. $\chi = 0$, the Mandel parameter is zero which corresponds to the Poissonian photon statistics. As time evolves, by increasing the deformation parameter, the statistics more rapidly tends to the sub-Poissonian statistics. Furthermore under cavity damping effect, the field statistics shifts from the super-Poissonian to sub-Poissonian photon distribution and the growing of the cavity damping accelerates the non-classical features of the field. Hence, we found out that the cavity damping and deformation parameter control the non-classical characteristics of the field throughout the interaction time.

Declarations

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