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RESEARCH

Influence of the Stark Shift and Field Nonclassicality on the Dynamics of Non-classical Correlations of N two-level Atomic System

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Abstract
We investigate the dynamical evolution of the global quantum discord (GQD) and the von Neumann entropy (VNE) of a moving two-, three-, and four two-level atomic system (TLS) ($N = 2, 3, 4$). These systems interact with a single-mode Fock and coherent fields in the cavity under the influence of the Stark effect. According to the evolution of the GQD and VNE, quantum correlations for both the Fock and coherent fields decrease with an increase in the Stark parameter. Quantum correlations deplete more rapidly in the presence of coherent field as compared to the Fock field with increasing value of the Stark parameter. The maximum amount of quantum entanglement that the quantum system can achieve with an increase in the Stark shift parameter is seen to decrease more quickly in the presence of the Fock field than it does in the coherent field. Moreover, the large N systems are more prone to the increasing values of the Stark shift parameter. The GQD increases with the number of atoms N for both the Fock and the coherent field while the VNE increases only with the Fock field. Additionally, it has been found that for the larger N atomic systems, atomic motion has no effect on the period of entanglement oscillations as the number of atoms increases. Periodic behavior for the GQD and VNE is seen for both the initial mixed and pure states in the presence of the Stark shift for the Fock and coherent fields, respectively.

Keywords Quantum entanglement (QE) · Global quantum discord (GQD) · Stark effect · Von neumann entropy (VNE)

1 Introduction

Quantum entanglement is the most fascinating phenomenon linked to a composite quantum system. If the combined state of two particles cannot be described as the sum of the states of their individual subsystems, they are said to be entangled. The first to demonstrate some nontrivial effects of quantum entanglement on the ontology of quantum theory were Einstein,

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Podolsky, and Rosen [1]. By creating and manipulating these entangled states, one can better understand quantum phenomena. For tests of quantum nonlocality, for instance, complex entangled states are employed, such as the Greenberger, Horne, and Zeilinger [2] triplets of the particles [3]. Beyond these fundamental components, entanglement has developed rapidly in recent years to become a crucial resource in quantum information processing [4].

The interaction between a TLS with a one-mode cavity field in quantum optics is described by the Jaynes-Cummings model, one of the precisely solvable models (JCM) [5]. This model has been discussed from a different point of view, such as multi-level atoms [6], multi-mode fields [7] and multi-atoms interaction [8]. The effect of detuning and time-dependent coupling and Kerr-like medium on the behavior of quantum properties in the general setting of a TLS with a single-mode cavity field has been discussed [9]. It has been studied the interaction of a TLS with a quantized cavity field and how the atom field coupling is affected by light intensity [10]. Both theoretical and experimental physics have extensively studied this model for the interaction of various three-level atom types with a quantized field. A three-level atom in motion that interacts with a single-mode field in an optical cavity and an intensity-dependent coupling regime is studied in Ref. [11].

The Stark effect, a widely researched phenomenon in the field of quantum optics, is another notable feature of the interaction of light with matter. Strong and ultra-strong light-matter interaction patterns can now be observed experimentally thanks to recent developments in the field of cavity quantum electrodynamics [12]. Numerous methods for plotting the Stark effect in quantum optical applications have emerged in recent years. To name a few, schemes have been put forth for using both dc and ac Stark shifts to implement quantum logic gates and algorithms [13–15], as well as for enhancing photon sources for interferometers [16]. Investigating whether atom-atom entanglement can also be strengthened by the Stark shift is intriguing as an outcome.

According to widely held belief, quantum entanglement is a special quantum correlation that allows us to go beyond the limitations of classical computational models [17–19]. Nevertheless, it has recently been demonstrated that there are other quantum correlations that can be used to achieve a quantum speedup in addition to entanglement; examples include the deterministic quantum computation with one qubit, which uses only separable states [20–22]. Quantum discord (QD) is the source that is thought to improve the computational task [23, 24]. The QD takes into account a set of local measurements on one subsystem, an asymmetric extension of the QD, known as global quantum discord (GQD) [26], has been proposed. This extension has analytical expressions for some classes of quantum states [27].

Over the past 20 years, there has been a lot of focus on the features of the Jaynes-Cummings model (JCM) for moving atoms. The experimental advancements in cavity QED have sparked theoretical efforts. In addition to the experimental drive, the atomic motion effect should be included in the JCM because its dynamics become more fascinating. Handling the JCM when there is atomic motion, several authors have used numerical calculations and analytic approximations [28–32]. For cold atoms, the solution in the presence of atomic motion is significant from both a theoretical and practical standpoint. Recently the effect of Stark and Kerr-like medium on the quantum entanglement dynamics of moving two three-level atomic systems is studied [33] and a moving three-level atom in the presence of intensity-dependent atom and field coupling is studied in Ref. ([34]). Quantum Fisher information of moving TLS studied in Ref. [35] takes into account the thermal field, intrinsic decoherence, Stark effect, and Kerr-like medium. Jamal et. al. studied the quantum entanglement dynamics of moving N-level atomic systems in the presence of the Stark effect and non-linear Kerr medium [36]. In Ref. [37], it is investigated how moving two TLS affects their quantum entanglement and quantum Fisher information.

Investigating the quantum entanglement and quantum correlation dynamics of multi-moving TLS in the presence of the Stark effect interacting with the Fock and coherent field is the main goal of the paper. We investigate the Stark effect's effects on the dynamics of the GQD and VNE for the two, three, and four TLS. The Stark effect is seen to predominate the time evolution of the GQD and VNE for moving multi-TLS. Both the dynamics of GQD and the dynamics of the VNE are significantly impacted by the Stark effect.

The paper is structured as follows: in Section 2, we present the model Hamiltonian as well as the dynamics of the interactions between the Fock and coherent fields of moving two, three, and four TLS influenced by the Stark effect. The numerical results and discussions are presented in Section 3. Section 4 is where we wrap up our findings.

2 Hamiltonian Model

Since the Tavis-Cummings (TC) model has been extensively studied and is regarded as the best example of tripartite quantum systems, we take it into consideration. The reference [38] provides the TC Hamiltonian describing two identical TLS A and B coupled with the single-mode field C. Under the rotating wave approximation, the total Hamiltonian of the system made up of N TLS $\hat{H}_T$ can be written as

$$\hat{H}_T = \frac{\omega_0}{2}\sum_{i=1}^{N}\hat{\sigma}_i^z + \omega\hat{a}^\dagger\hat{a} + g\sum_{i=1}^{N}\left(\hat{a}\hat{\sigma}_i^+ + \hat{a}^\dagger\hat{\sigma}_i^-\right), \quad (1)$$

The cavity field's annihilation and creation operators are $\hat{a}$ and $\hat{a}^\dagger$. The atomic inversion operator is $\hat{\sigma}^z$ and g is the coupling constant of the atoms and field. The atomic transition and field frequencies are, respectively, ω_0 and ω, and $\hat{\sigma}^+$ and $\hat{\sigma}^-$ are the atomic raising and lowering operators of the atom.

The influence of the Stark shift on a TLS is also an important topic of study [39]. The Stark shift caused by an electric field is proportional to the photon numbers $\hat{a}^\dagger\hat{a}$ inside the cavity. Furthermore, the Stark shift is also proportional to the polarizabilities of the two resonant states of a TLS. Therefore, we incorporate such shifts as intensity-dependent corrections $\hbar\beta_j\hat{a}^\dagger\hat{a}$ to the energy of the system, with β_j, is the Stark parameter corresponding to the state $|j\rangle$ of a TLS. The Stark Hamiltonian is, therefore, given as

$$\hat{H}_{\text{Stark}} = \hbar\sum_{j=0,1}\beta_j\hat{a}^\dagger\hat{a}\hat{R}_{jj} \quad (2)$$

where the atomic operator $\hat{R}_{jj}$ is defined as

$$\sum_{j=0,1}\hat{R}_{jj} = |j\rangle\langle j| \quad (3)$$

Assuming that the atoms resonantly interact with the single-mode field, and the ground state $|g\rangle$ is coupled with the electric field, the interaction Hamiltonian of the system is, $(\hbar = 1)$

$$\hat{H}_I = \Omega(t)\sum_{i=1}^{N}\left(\hat{a}\hat{\sigma}_i^+ + \hat{a}^\dagger\hat{\sigma}_i^-\right) + \sum_{i=1}^{N}\beta\hat{a}^\dagger\hat{a}|g_i\rangle\langle g_i| \quad (4)$$

We set $\Omega(t) = g\,\sin(p\pi vt/L)$ which describes the cavity field shape function. In this case (4) holds with,

$$\Omega_1(t) = \int_0^t \Omega(\tau)d\tau = \frac{1}{\eta}(1-\cos(\eta\pi vt/L) \text{ for } \eta \neq 0, \tag{5}$$

$$= gt \qquad \text{for } \eta = 0. \tag{6}$$

The atomic motion parameter is η, and v is the speed of the moving atoms. The shape function with atomic motion, i.e. $\eta \neq 0$ whereas $\Omega(t) = g$ in the absence of atomic motion, i.e., $\eta = 0$ [40]. Along the z-axis, the atomic motion is considered. η is related to the permitted wavelengths of the atoms in the cavity. The term η denotes the number of half wavelengths of the mode in the cavity. The cavity length in the z-direction is denoted by the symbol L. The atom moves at a velocity of $v = gL/\pi$.

The (5), (6) gives a description of the probability distribution of the atoms inside the cavity. When the atoms are moving within the cavity, it has variable Ω as opposed to the static situation where it is equal to the gt. As the direct result of the field and a mixed state of N atoms, the entire system is taken into consideration. The initial optimal state of the atomic system interacting with the field is,

$$\hat{\rho}\,(0) = [(1-p)\,|\psi\rangle\langle\psi| + \; p|g_1g_2.....g_N\rangle\langle g_1g_2.....g_N|] \otimes \hat{\rho}_E, \tag{7}$$

Here $|\psi\rangle = \cos(\theta)\,|g_1g_2.....g_N\rangle + \sin(\theta)\,|e_1e_2.....e_N\rangle$ where $0\leq\theta\leq\pi$. With $0 \leq p \leq 1$, and $\hat{\rho}_E$ denoting the coherent state of the field written as

$$\hat{\rho}_E = \sum_n p_n|n\rangle\langle n| \tag{8}$$

where

$$p_n = 1 \tag{9}$$

for the Fock field,

$$p_n = \frac{|\alpha|^{2n}\,e^{-|\alpha|^{2n}}}{n!}$$

for the Coherent field and

$$\langle n\rangle = |\alpha|^2 \tag{10}$$

where n refers to the photons that are present inside the cavity.
For the permitted basis states, it is possible to write $\{|\psi_i\rangle\}$

$$\{|\psi_i\rangle\} = |g_1g_2.....g_{N,}n+N\rangle, |e_1g_2.....g_N, n+N-1\rangle, ..., |e_1e_2.....e_N, n\rangle. \tag{11}$$

where $|g_i\rangle$ and $|e_i\rangle$ are the ground and excited state of the ith atom.

According to the Markovian approximation, the system's time evolution is given by [41],

$$\dot{\hat{\rho}}(t) = -i[\hat{H}, \hat{\rho}(t)] - \frac{\gamma}{2}[\hat{H}, [\hat{H}, \hat{\rho}(t)]] \tag{12}$$

where ρ is the density matrix and γ is the intrinsic decoherence parameter. The formal solution of (12) is given by,

$$\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}\hat{M}^k(t)\hat{\rho}(0)\hat{M}^{k\dagger}(t), \tag{13}$$

with

$$\hat{M}^k(t) = \hat{H}^k\exp(-i\hat{H}t)\exp(-\gamma t\hat{H}^2/2), \tag{14}$$

where $\hat{\rho}(0)$ is the initial state of the system. For $\gamma = 0$, (13) and (7) in the allowable basis $\{|\psi_i\langle\}$ are used to obtain the final state of the system at time t

$$\hat{\rho}_{AF}(t) = \sum_{i,j}^{N} \exp\left(-i\left(E_i - E_j\right)t\right) \times \langle\psi_i|\hat{\rho}(0)|\psi_j\rangle|\psi_i\rangle\langle\psi_j|, \tag{15}$$

where E_i, E_j and $|\psi_i\rangle$, $|\psi_j\rangle$ are the density matrix's $\rho(0)$ eigenvalues and eigenvectors. After taking a trace over the field, the final state of the atomic system is, $\hat{\rho}_T(t) = Tr_F\left[\hat{\rho}_{AF}(t)\right]$.

To calculate the quantum correlations in our multipartite system we can use the form of the GQD [42] given below

$$\text{GQD}(\rho_T) = \min_{\{\Pi^k\}} \left\{ \sum_{j=1}^{N} \sum_{l=0}^{1} \tilde{\rho}_j^{ll} \log_2 \tilde{\rho}_j^{ll} - \sum_{k=0}^{2^N-1} \tilde{\rho}_T^{kk} \log_2 \tilde{\rho}_T^{kk} \right\} + \sum_{j=1}^{N} S(\rho_j) - S(\rho_T), \tag{16}$$

where

$$\tilde{\rho}_T^{kk} = \langle k|\hat{R}^t \rho_T \hat{R}|k\rangle \text{ and } \tilde{\rho}_j^{ll} = \langle l|\hat{\rho}_j|l\rangle \text{ and } \Pi^k = \hat{R}|k\rangle\langle k|\hat{R}^t, \tag{17}$$

where $|k\rangle$ are the eigenstates of $\otimes_{j=1}^{N} \hat{\sigma}_j^z$ and $\hat{R}$ is the local qubit rotational operator acting on the jth qubit and expressed as

$$\hat{R} = \otimes_{j=1}^{N} \hat{R}_j(\theta_j, \phi_j), \tag{18}$$

with

$$\hat{R}_j(\theta_j, \phi_j) = \cos\theta_j \hat{1} + i \sin\theta_j \cos\phi_j \hat{\sigma}_y + i \sin\theta_j \sin\phi_j \hat{\sigma}_x \tag{19}$$

In (16)

$$S(\hat{\rho}_j) = -Tr[\hat{\rho}_j \log_2 \hat{\rho}_j] \text{ and } S(\hat{\rho}_T) = -Tr[\hat{\rho}_T \log_2 \hat{\rho}_T] \tag{20}$$

are the VNE of the subsystem j and the total system respectively as the VNE can be defined as [43]

$$S(\rho) = -\text{Tr}(\rho \log_2 \rho) = -\sum_i r_i \ln r_i, \tag{21}$$

where the eigenvalues of the time-evolved atomic density matrix ρ_T are represented by r_i.

3 Results and Discussions

Figure 1 shows the effects of two different fields on the quantifiers for a two TLS for $\eta = 0$ and different β values. A comparison is presented between the Fock and coherent field with the same number and an average number of photons. We have considered the initial mixed atomic state with $p = 0.5$ and $\theta = 3\pi/4$. In the case of the Fock field, the Stark shift parameter has shown prominent and significant effects on the quantifiers. By increasing the Stark shift parameter β, both quantifiers decrease with time evolution. Furthermore, the rapid oscillations of the quantifiers increase, and the time period of oscillations decrease with the increasing value of β. In the case of the coherent field, collapses and revivals are observed. The number of revivals increases by increasing the value of β, and on the other hand, the amplitude of revival and magnitude of the GQD and VNE decrease with β. When we compare both fields, it is observed that fields have different effects on GQD and VNE. For the Fock field, we observe that the maximum values of the magnitude of the GQD have decreased by an order of magnitude for $\beta = 3$ as compared to $\beta = 0.3$ with time evolution. The same is the case of the coherent field, the decrease of GQD is of the same order of magnitude.

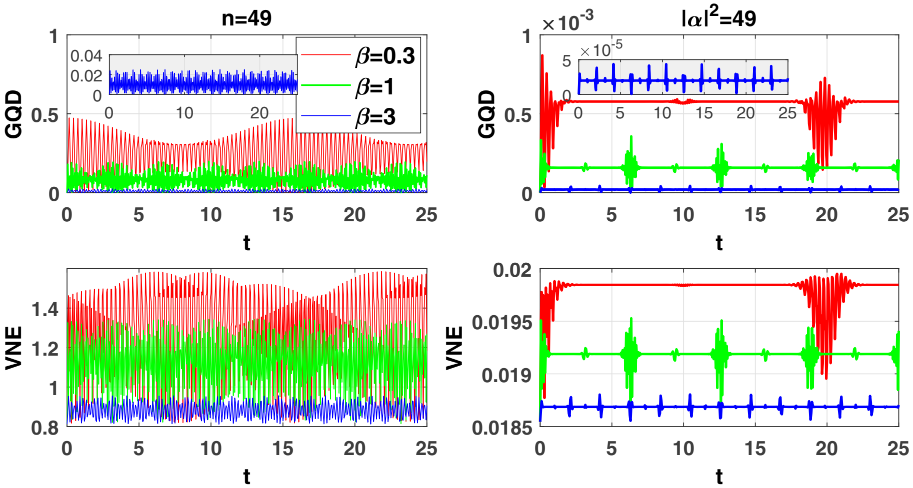

Fig. 1 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a two TLS (N=2) are plotted with varying β values. The parameters used throughout the data are $\eta = 0$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

There is a sharp decrease in the GQD maximum value, however, the VNE dynamics remain unchanged with the change of β. It is found that for the Fock field, the maximum value of VNE decreases by a factor of 2 for $\beta = 3$ as compared to $\beta = 0.3$, and for the coherent field, there is a slight decrease of VNE. This change in the magnitude of the quantifiers in the presence of the Stark shift suggests that quantum correlations decrease with the increase of β for both Fock and coherent fields. The system loses quantum correlations more rapidly in the presence of the coherent field as compared to the Fock field with an increasing value

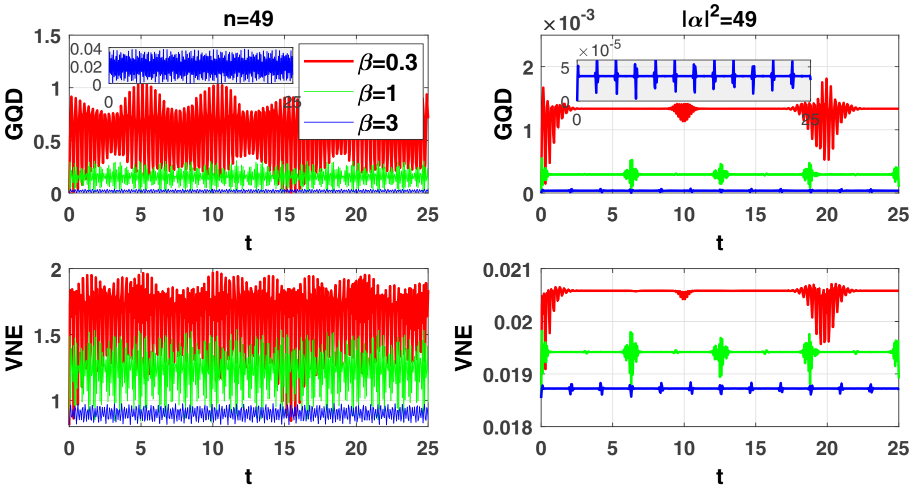

Fig. 2 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a three TLS (N=3) are plotted with varying β values. The parameters used throughout the data are $\eta = 0$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

of β. On the other hand, the quantum entanglement(QE) remains almost the same value in the coherent field when β is increased and drops by a factor of 2 for the case of the Fock field. Hence, it is seen that in the presence of the Stark shift, the GQD is decreased more for the coherent field as compared to the Fock field, and the increase of the Stark shift parameter β suppresses the GQD more in the presence of the coherent field as compared to the Fock field, and we observed that the Fock field sustains the quantum correlations. The maximum of QE is decreased more rapidly for the Fock field with an increase of β as compared to the coherent field.

In Figs. 2 and 3, the effect of β on the quantifiers is presented for Fock and the coherent field in the case of three (N=3) and four (N=4) TLS, respectively. We have taken the initial mixed atomic state with $p = 0.5$ and $\theta = 3\pi/4$. The quantifiers' values are increased as the system gets more TLS. Furthermore, in the case of the Fock field, we have seen rapid oscillations for all the values of β. The period of these rapid oscillations is also decreased for the large N atomic system and the higher values of β. In the case of a coherent field, the GQD and VNE increase for large N systems. However, for the coherent field, for N=3 and N=4 the revivals remain the same as for N=2. When we increase the value of β from 0.3 to 3 under the influence of the Fock field for $N = 3$, we observed an order of magnitude decrease in the value of GQD and the same is true for the coherent field. On the other hand, for the Fock field, the magnitude of VNE is decreased by half and for the coherent field, there is a slight decrease. When $N = 4$ and the field is Fock or coherent, the decrease in the maximum values of GQD at $\beta = 0.3$ to $\beta = 3$ is n order of magnitude. The dynamics of VNE for $N = 4$ shows that the maximum values of VNE decrease by half for the case of Fock and no change for coherent field with the increase in β from 0.3 to 3. For our system composed of N TLS, for large N, the quantifiers are increased in the presence of the Stark shifted medium. Nonclassical fields have different trends on the quantifiers on their increase. The increase of GQD and VNE in the Fock field is not linear, and the higher N systems have a nonlinear increase of GQD and VNE for various values of β. Furthermore, as β increases, the values of GQD and VNE decrease. For the coherent field, with the increase in N, the

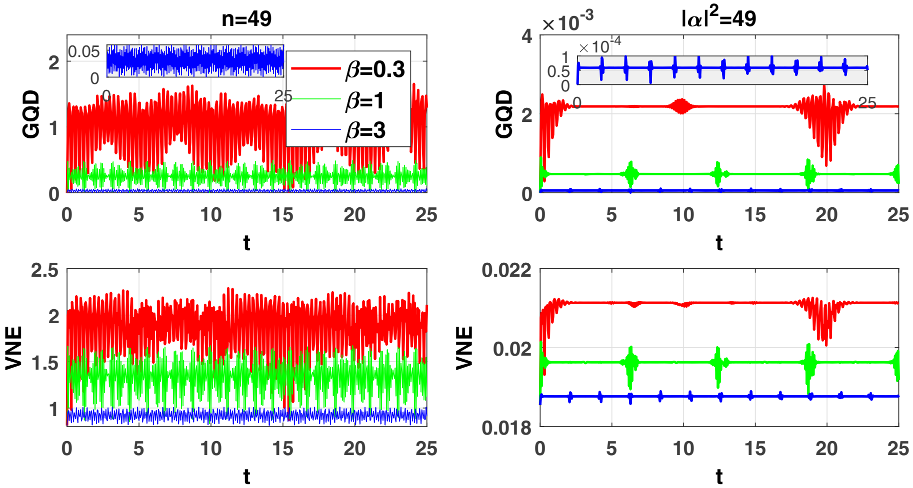

Fig. 3 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a four TLS (N=4) are plotted with varying β values. The parameters used throughout the data are $\eta = 0$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

GQD increases non-linearly for the different values of the Stark shift parameter β. For the coherent field, the VNE increases linearly with the increase in N for $\beta = 0.3$ and remains the same with the increase in N for $\beta = 3$. Varying β affects the correlations of the N TLS. The higher N systems are more prone to the increasing values of β and the magnitude of the GQD is more suppressed as compared to the case of $N = 2$. In comparison, the system has increased GQD for the Fock field as compared to the coherent field and the GQD is slightly more suppressed with the increase of β for the coherent field as compared to the Fock field. The dynamics of the VNE show that QE decreases with the increase of β and QE and non-classical correlations are affected in a different way for the Fock and the coherent field. The GQD increases with the N for both the Fock and the coherent field while the VNE increases only for the Fock field. In the case of the coherent field, larger values of β do not favor the system to enhance the QE with the N.

Figure 4 illustrates the GQD and VNE behavior for a moving two TLS (N=2) with the Stark shift. The Stark shift parameter β is studied with various values to examine the dynamic behavior. In this case of two TLS (N=2), we have taken the number of photons and the average number of photons for the Fock and the coherent field as $n = 49$ and $|\alpha|^2 = 49$ respectively. The dynamical behavior is studied with the initial mixed atomic state for $\eta = 1$, $\theta = 3\pi/4$, $p = 0.5$. For the Fock field, we observe periodic oscillations of the quantifiers for different β values. The period of oscillations remains the same for all the values of β but the oscillations get more rapid as we increase the value of β. The amplitude of the oscillation decreases with the increase of β. For the coherent states, the magnitude of the quantifiers decrease with β. Furthermore, the collapses and revivals are observed for each value of the β. The amplitude of the revivals decreases with the increase of β but the number of main revivals per unit scaled time remains the same for the whole range of β. For $\beta = 1$, the collapses are more sustained as compared to $\beta = 0.3$ and $\beta = 3$. For $\beta = 3$, we observe further revivals along with the main revivals. These revivals occur in the middle of the main revivals and have a smaller amplitude as compared to the main revivals that correspond to $\beta = 3$. These results

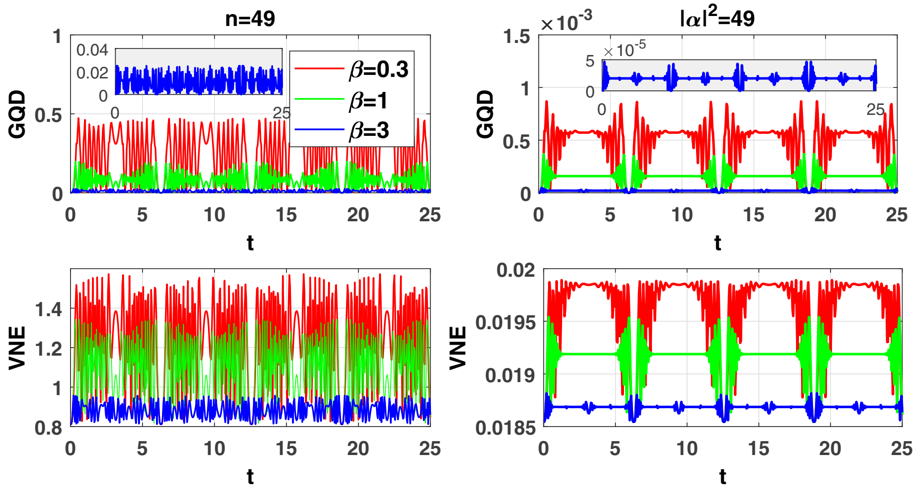

Fig. 4 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a moving two TLS (N=2) are plotted with varying β values. The parameters used throughout the data are $\eta = 1$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

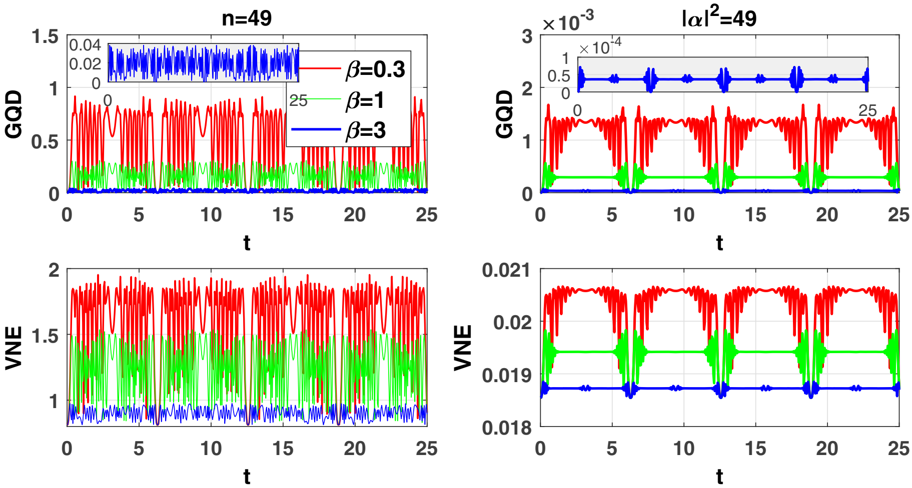

Fig. 5 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a moving three TLS (N=3) are plotted with varying β values. The parameters used throughout the data are $\eta = 1$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

suggest that the QE is sustained and maintained for the system with the atomic motion, for both the Fock and the coherent field in the presence of the Stark shift.

The dynamical behavior of the moving three (N=3) and four (N=4) TLS for the Fock and coherent field is shown in Figs. 5 and 6. It is seen that both quantifiers show periodic oscillations for the Fock field. In the case of the coherent states, both GQD and VNE show collapses and revivals for all the values of the β. For the three (N=3) and four (N=4) TLS, it is seen that the magnitude of the GQD and VNE are suppressed by the increase of β and the

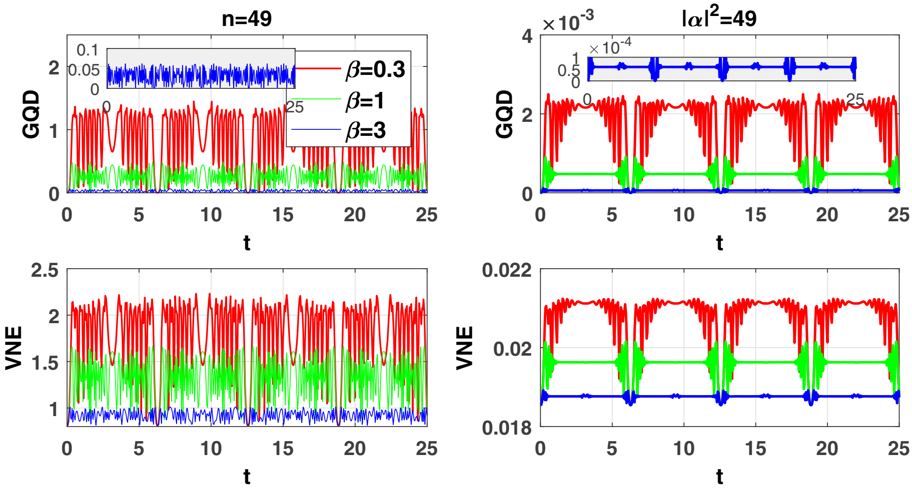

Fig. 6 (color online) For two different field states with $n = 49$ and $|\alpha|^2 = 49$, the quantifiers' dynamics of a moving four TLS (N=4) are plotted with varying β values. The parameters used throughout the data are $\eta = 1$, $\theta = 3\pi/4$, $p = 0.5$. The quantifiers for $\beta = 3$ are magnified in the insets

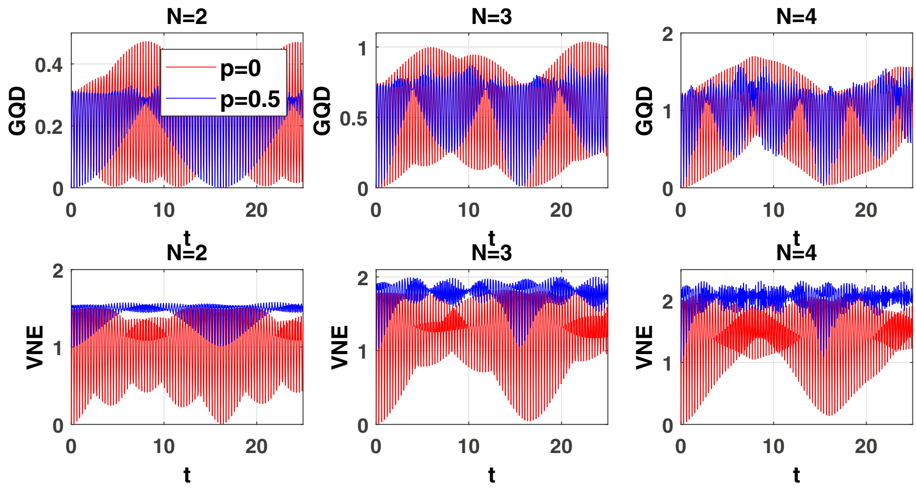

Fig. 7 (color online) The dynamics of the quantifiers for the pure and mixed initial atomic states for the Fock field with the parameters $n = 49$, $\beta = 0.3$, $\theta = 0$, $\eta = 0$

rapid oscillations increase with its increase. The number of periodic oscillations and revivals for the GQD and VNE, per unit scaled time, remain the same for both Fock and coherent field. The magnitude of the quantifiers increase by N increase for the Fock states, while for the coherent state this increase is only prominent for $\beta = 0.3$ and $\beta = 1$. The atomic motion does not affect the period of the quantifiers' oscillations as the number of atoms N is increased. In the case of the Fock field, the period of oscillation remains the same with the increase of the β. Furthermore, for the Fock state, with the increase of β, rapid oscillations of the system increase, and it remain the same for the different number of the atoms N. Atomic motion has an impact on the coherent field, causing revivals to happen at the same scaled time for all values of β. With the increase in N, the quantifiers' magnitude increases. When compared to the Fock field, the coherent field has decreased GQD and VNE due to atomic motion. In comparison for both cases of with and without atomic motion, the nonclassical fields and Stark shift parameter β have some interesting effects on the dynamical behavior of the GQD and VNE of the system. For the case of the Fock field, atomic motion does not affect the magnitude of the GQD for different values of β and the number of atoms N. In this case, the atomic motion changes the shape of the oscillations of the GQD and VNE and change the period of these oscillations as compared to the case when the atomic motion is not present. This change of the period of oscillations is different for the different values of β. The number of revivals per unit scaled time of the GQD and VNE differs depending on the value of β when compared to the case η. Compared to the case where $\eta = 0$, the case of $\beta = 0.3$ results in a higher number of revivals per unit scaled time. In the case of $\beta = 1$ and $\beta = 3$, the number of revivals per unit scaled time remains unchanged. The oscillations of the GQD and VNE thus show the sustained behavior of the QE with atomic motion in the presence of the Stark effect.

The initial atomic state effects on the dynamics of the GQD and VNE for the Fock field and coherent field is shown in Figs. 7 and 8. The dynamics of GQD and VNE are studied with the two different initial atomic states corresponding to ($p = 0$) and ($p = 0.5$) for $\theta = 0$. Both the Fock and Coherent fields, Stark shift parameter is fixed at $\beta = 0.3$. For the Fock field, as depicted in Fig. 7, the pure states have marginally higher GQD than the mixed state.

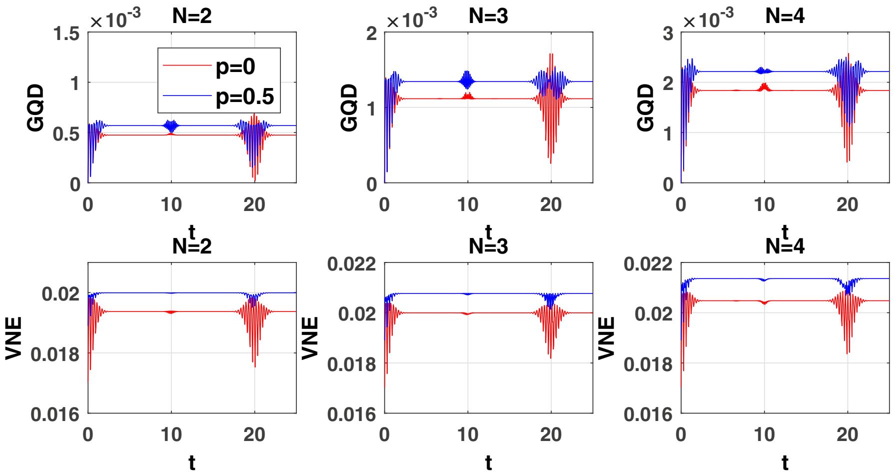

Fig. 8 (color online) The dynamics of the quantifiers for the pure and mixed initial atomic states for the coherent field with $|\alpha|^2 = 49$ is shown for the system composed of N TLS with $\beta = 0.3$, $\theta = 0$, $\eta = 0$

However, in the higher N systems, this discrepancy is diminished and the GQD of both the initial pure and mixed states is nearly equal. Furthermore, for N=2, the GQD has an almost equal number of periodic oscillations with the time evolution. For the higher N values, it is observed that the mixed state shows more periodic oscillations as compared to the pure state. On the other hand, the initially mixed state shows slightly more VNE as compared to the pure state for all values of N (number of atoms). The initially mixed state has a non-zero VNE value showing that the system has sustained the QE. In the scenario of the coherent field, Fig. (8) illustrates the impact of initial atomic states on the GQD and VNE for the system. GQD and VNE collapses and revivals are seen for initial mixed and pure states. In comparison to the initial pure state, the mixed state has slightly more GQD and VNE with time evolution. Both states have non-zero VNE values, indicating that the QE is present in both pure and mixed states for all values of N (number of atoms).

4 Conclusions

In the presence of the Stark shift, we investigated the dynamics of the GQD and VNE of moving 2, 3, and 4 TLS interacting with the Fock and coherent field. It was seen that the Stark effect affected the quantifiers over time for N TLS. The Stark effect was observed to play a significant role in the time evolution of the quantum system for the GQD and VNE. The magnitude of the GQD and VNE changed when the Stark shift was present, and this suggested that quantum correlations were decreased as β increased for both the Fock and coherent fields. The system lost quantum correlations more rapidly in the presence of the coherent field as compared to the Fock field with an increasing value of β. It was seen that in the presence of the Stark shift, the GQD was decreased more for the coherent field as compared to the Fock field due to the increase of the Stark shift parameter β, and we observed that the Fock field sustain the quantum correlations in the system. The maximum value of the QE was decreased more rapidly for the Fock field with the increase of β as compared to the coherent field. The quantifiers were increased with the presence Stark shift for large

N systems. It is important to mention that varying the Stark shift parameter β affected the correlations of the N TLS. The higher N systems were more prone to increasing values of β and the magnitude of the GQD was more suppressed as compared to the case of $N = 2$. In comparison, the system had increased GQD for the Fock field as compared to the coherent field and the GQD was rather more suppressed with the increase of β for the coherent field as compared to the Fock field. The dynamics of the VNE show that the QE decreased with the increase of β and the QE and non-classical correlations were affected in a different way for the Fock and the Coherent field. The GQD increased with the number of atoms N for both the Fock and the coherent field while the VNE increased only for the Fock field. In the case of the coherent field, larger values of β did not favor the system to increase the QE with N. The QE was sustained and maintained for the system for both the Fock and the coherent field in the presence of the Stark shift. It was observed that for the higher N atomic systems, the atomic motion did not affect the period of oscillations of the system. The oscillations of the quantifiers showed the sustained behavior of the QE with atomic motion in the presence of the Stark effect. For initial mixed and pure states, collapses and revivals were observed for the quantifiers for the different β values for both the Fock and coherent fields. The mixed state had slightly more GQD and VNE with time evolution as compared to the initial pure state. Both states had non-zero VNE values describing that QE was present for all values of N (number of atoms).

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Declarations

Competing interests The authors declare no competing interests.

References

1. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935)
2. Greenberger, D.M., Horne, M.A., Zeilinger, A.: Bell's theorem without inequalities. Am. J. Phys. **58**, 1131 (1990)
3. Pan, J.W., et al.: Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement. Nature (London) **403**, 515 (2000)
4. Bennett, C.H., et al.: Teleportation of quantum states. Phys. Rev. Lett. **70**, 1895 (1993)
5. Jaynes, E.T., Cummings, F.W.: Comparison of quantum and semiclassical radiation theories with application to the beam maser. Proc IEEE **51**, 89 (1963)
6. Cardimona, D.A.: Effect of atomic-state coherence and spontaneous emission on three-level dynamics. Phys Rev A **41**, 5016 (1990)
7. Faghihi, M.J., Tavassoly, M.K., Hatami, M.: Dynamics of entanglement of a three-level atom in motion interacting with two coupled modes including parametric down conversion. Physica A **407**, 100 (2014)
8. Baghshahi, H.R., Tavassoly, M.K.: Entanglement, quantum statistics and squeezing of two Ξ-type three-level atoms interacting nonlinearly with a single-mode field. Phys. Scr. **89**, 075101 (2014)

9. Baghshahi, H.R., Tavassoly, M.K., Behjat, A.: Entropy squeezing and atomic inversion in the k-photon Jaynes-Cummings model in the presence of the Stark shift and a Kerr medium: A full nonlinear approach. Chin. Phys. B **23**, 074203 (2014)
10. Sukumar, C.V., Buck, B.: Multi-phonon generalisation of the Jaynes-Cummings model. Phys. Lett. A **83**, 211 (1981)
11. Faghihi, M.J., Tavassoly, M.K.: Quantum entanglement and position-momentum entropic squeezing of a moving Lambda-type three-level atom interacting with a single-mode quantized field with intensity-dependent coupling. J. Phys. B **46**, 145506 (2013)
12. Alsing, P., Guo, D.-S., Carmichael, H.J.: Dynamic Stark effect for the Jaynes-Cummings system. Phys. Rev. A **45**, 5135 (1992)
13. Biswas, A., Agarwal, G.S.: Quantum logic gates using Stark-shifted Raman transitions in a cavity. Phys. Rev. A **69**, 062306 (2004)
14. Dasgupta, S., Biswas, A., Agarwal, G.S.: Implementing Deutsch-Jozsa algorithm using light shifts and atomic ensembles. Phys. Rev. A **71**, 012333 (2005)
15. Zou, X.-B., Xiao, Y.-F., Li, S.-B., Guo, G.-C.: Quantum phase gate through a dispersive atom-field interaction. Phys. Rev. A **75**, 064301 (2007)
16. Fernee, M.J., -Dunlop, H.R., Milburn, G.J.: Improving single-photon sources with Stark tuning. Phys. Rev. A **75**, 043815 (2007)
17. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
18. Bennett, C.H., Brassard, G.: Conf. on Computers, Systems and Signal Processing (Bangalore, India) (New York: IEEE) p 175 (1984)
19. Bennett, C.H.: Quantum cryptography using any two nonorthogonal states. Phys. Rev. Lett. **68**, 3121 (1992)
20. Knill, E., Laflamme, R.: Power of one bit of quantum information. Phys. Rev. Lett. **81**, 5672 (1998)
21. Datta, A., Flammia, S.T., Caves, C.M.: Entanglement and the power of one qubit. Phys. Rev. A. **72**, 042316 (2005)
22. Datta, A., Vidral, G.: Role of entanglement and correlations in mixed-state quantum computation. Phys. Rev. A. **75**, 042310 (2007)
23. Ollivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. **88**, 017109 (2001)
24. Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A **34**, 6899 (2001)
25. Rulli, C.C., Sarandy, M.S.: Global quantum discord in multipartite systems. Phys. Rev. A. **84**, 042109 (2011)
26. Xu, J.: Analytical expressions of global quantum discord for two classes of multi-qubit states. Phys. Lett. A **377**, 238 (2013)
27. Majtey, A.P., Plastino, A.R., Plastino, A.: New features of quantum discord uncovered by q-entropies. Physica A **391**, 2491 (2012)
28. Abdel-Aty, M., Sebawe, A.M., Obada, A.S.F.: Uncertainty relation and information entropy of a time-dependent bimodal two-level system. J. Phys. B **35**, 4773 (2002)
29. Abdel-Khalek, S., Obada, A.-S.F.: The atomic Wehrl entropy of a V-type three-level atom interacting with two-mode squeezed vacuum state. J. Russ. Laser Res. **30**, 146 (2009)
30. Eleuch, H., Guerin, S., Jauslin, H.R.: Effects of an environment on a cavity-quantum-electrodynamics system controlled by bichromatic adiabatic passage. Phys. Rev. A **85**, 013830 (2012)
31. Abdel-Khalek, S., Nofal, T.A.: Correlation and entanglement of a three-level atom inside a dissipative cavity. Physica A **390**, 2626 (2011)
32. Golkar, S., Tavassoly, M.K., Nourmandipour, A.: Entanglement dynamics of moving qubits in a common environment. J. of the Opt. Soc. of Am. B **37**, 400 (2020)
33. Anwar, S.J., Ramzan, M., Khan, M.K.: Effect of Stark- and Kerr-like medium on the entanglement dynamics of two three-level atomic systems. Quantum Inf. Process. **18**, 192 (2019)
34. Faghihi, M.J., Tavassoly, M.K.: Quantum entanglement and position-momentum entropic squeezing of a moving Lambda-type three-level atom interacting with a single-mode quantized field with intensity-dependent coupling. J. Phys. B: At. Mol. Opt. Phys. **46**, 145506 (2013)
35. Anwar, S.J., Ramzan, M., Usman, M., Khan, M.K.: Quantum fisher information of two-level atomic system under the influence of thermal field. Intrinsic Decoherence, Stark Effect and Kerr-Like Medium, JQIS **11**, 24 (2021)
36. Anwar, S.J., Ramzan, M., Usman, M., Khan, M.K.: Stark and Kerr effects on the dynamics of moving N-level atomic system. JQIS **9**, 22 (2019)
37. Anwar, S.J., Usman, M., Ramzan, M., Khan, M.K.: Quantum fisher information and entanglement of moving two two-level atoms under the influence of environmental effects. Physics **1**, 131 (2019)

38. Tavis, M., Cummings, F.W.: Exact solution for an N-Molecule–Radiation-Field Hamiltonian. Phys. Rev. **170**, 379 (1968)
39. Moya-Cessa, H., Bužek, V., Knight, P.L.: Power broadening and shifts of micromaser lineshapes. Optics Commun. **85**, 267 (1991)
40. Schlicher, R.R.: Jaynes-Cummings model with atomic motion. Opt. Commun. **70**, 97 (1989)
41. Milburn, G.J.: Intrinsic decoherence in quantum mechanics. Phys. Rev. A **44**, 5401 (1991)
42. Campbell, S., Mazzola, L., De Chiara, G., Apollaro, T.J.G., Plastina, F., Busch, Th., Paternostro, M.: Global quantum correlations in finite-size spin chains. New J. Phys. **15**, 043033 (2013)
43. von Neumann, J.: Mathematische Grundlagen der Quantenmechanik, 2nd edn. Springer, Berlin (1996)

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