International Journal of Theoretical Physics (2021) 60:2790–2803
https://doi.org/10.1007/s10773-021-04853-6

New Higher-Order Generalized Uncertainty Principle: Applications

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Received: 16 February 2021 / Accepted: 28 May 2021 / Published online: 30 June 2021

Abstract
Recently Chung and Hassanabadi proposed a higher order general uncertainty principle (GUP*) that predicts a minimal length as well as possesses a upper bound momentum limit. In this article, we have discussed an ideal gas system and its thermal properties using that deformed canonical algebra introduced by them. Moreover, we examined blackbody radiation spectrum and the cosmological constant in the presence of the GUP*. After a comparison with the existing literature, we concluded that the given formalism of Chung and Hassanabadi yields more accurate results.

Keywords A new higher order GUP · Ideal Gas · Blackbody · The cosmological constant

1 Introduction

In quantum mechanics, there is the concept of wave-particle duality, which has no counterpart in classical mechanics. According to this concept, it is argued that every quantum quantity has a particle and wave property. For some pairs of physical observables of a point-like considered particle, such as its position and momentum, a fundamental limit exists on the measurement of them simultaneously, which is governed with the Heisenberg uncertainty principle (HUP). However, the uncertainty values of any of these observables can be arbitrarily small. Contrarily, researches based on the string theory [1–6], background independent [7, 8] and dependent [9–16] quantum theory of gravity, doubly special relativity (DSR) theories [17–19], non commutative space-time [20–23] and field theory [24], black hole physics [25–35], and gedanken experiments [36–38], etc. predict a finite lower limit

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value for these measurements [38, 39]. This lower bound is called the minimum length (ML) and it is assumed to be on the Planck scale. In this context, an ML can be regarded as the fuzziness of space-time [40]. Technically, in order to derive an ML scale out of the canonical commutation relations (CCR), some small correction terms are implemented [41]. Thus, the HUP is modified to a more general form of it, namely the generalized uncertainty principle (GUP).

Kempf et al., in [42, 43], showed that the formulation of the generalization can not be uniquely expressed [44]. For instance, in one approach [21], Kempf with Mangano and Mann (KMM) deformed the CCR in one dimension as

$$[X, P] = i\hbar(1 + \beta P^2), \tag{1}$$

where β is the deformation parameter and it is $\beta \sim 10^{19}$ GeV. Then, they obtained the GUP in the form of

$$(\Delta X)(\Delta P) \geq \frac{\hbar}{2}\left[1 + \beta^2 (\Delta P)^2 + \beta\langle P^2\rangle\right], \tag{2}$$

and derived the ML in positions as $(\Delta x_0)_{KMM} = \hbar\sqrt{\beta}$. However, the lower bound limit in position does not predict a maximal observable momentum value [45]. To embody an upper bound limit to the observable momentum, Nouicer [46] and Ali et al. [47–49] proposed two different deformation to the CCR. Within the perturbative approximation, they obtained uncertainty in position and momentum proportional and inversely proportional to the deformation parameter, respectively. It is worth noting that these results are valid only for small deformation parameters. Moreover, the findings are not appropriate to the DSR theories since maximal momentum differs from the maximal momentum uncertainty [17–19, 50]. To overcome these objections, a higher-order of GUP (GUP*) is considered [51–56]. For example, Pedram, in 2012, proposed a higher-order GUP* [50, 51] with the following deformed CCR

$$[X, P] = \frac{i\hbar}{1 - \beta^2 P^2}. \tag{3}$$

Since the deformed CCR is singular at a certain momentum value, i.e. $P = 1/\beta$, the particle's momentum can not exceed a certain maximal value. Thus, the objections due to the DSR theories are eliminated. In this higher-order GUP scenario, the smallest observable uncertainty in positions reduces to $(\Delta x_0)_{Pedram} = \frac{3\sqrt{3}}{4}\hbar\sqrt{\beta}$. In a comparison with other scenarios, Pedram showed that $(\Delta x_0)_{KMM} < (\Delta x_0)_{Noucier} < (\Delta x_0)_{Pedram}$. Moreover, he extended the GUP formalism to D-dimension and discussed several applications such as hydrogen atom, harmonic oscillator and free particle's solutions, a confined particle in a box, cosmological constant, and black body radiation etc.. [50, 51, 57]. However, in this formalism, the energy spectrum of the position operator's eigenfunctions has a divergency problem.

Very recently, Chung and Hassanabadi proposed a new higher-order GUP (GUP*) formalism [56] that has an upper bound momentum value in the following form:

$$[X, P] = \frac{i\hbar}{1 - \alpha |P|}, \quad \alpha > 0, \tag{4}$$

where $|P| = \sqrt{|P^2|}$. They examined the algebraic structure of the GUP* formalism and showed that it is different than Pedram's approach. After a detailed examination they concluded that in the latter scenario the divergency problem of the Pedram's formalism does not arise [56].

In very recent studies, i.e. El-Nabulsi [58], Bensalem et al. [59], authors presented very interesting discussions on the statistical mechanics of ideal gas according to different

GUP scenarios. For some other interesting works, we refer readers to look at [60–69]. In this paper, we are motivated to investigate a classical ideal gas system within a statistical mechanics context by means of the thermodynamic functions according to the GUP*. Furthermore, we revisit the blackbody radiation and cosmological constant problems according to the new framework. We construct the paper as follows: In the next section, we review the GUP*. In Section 3, we discuss how to construct a partition function with the deformed formalism. Then, we consider an ideal gas that is constituted from a monatomic atom in a canonical ensemble and derive the thermodynamic functions. We analyze the results with numerical values. In Section 4, we employ the new formalism in two other quantum phenomena, namely the blackbody radiation and cosmological constant problems. Finally we end our paper, with a brief conclusion section.

2 A New Higher Order GUP

In this contribution, we use the GUP* formalism which is given by Chung and Hassanabadi [56]. In one dimension, they presented the deformed CCR between the position X and momentum P operators as written in (4). Here, the deformation parameter $\alpha = \alpha_0/\left(m_P c\right)$, where m_P is the Planck mass and α_0 is of the order of the unity. It is worth noting that, alike the Pedram's work, the CCR contains a singularity at $|P| = 1/\alpha$, which means that the momentum of the particle cannot surpass $1/\alpha$, thus, it is compatible with the DSR theory [17–19, 50, 56]. Furthermore, the physical observables such as momentum and the energy are not only nonsingular, but also they are bounded from above. The uncertainty relation that appears in the GUP* approach is found in the form of

$$\begin{aligned}(\Delta X)\,(\Delta P) \;&\geq\; \frac{\hbar}{2}\left\langle\frac{1}{1-\alpha\,|P|}\right\rangle,\\ &=\frac{\hbar}{2}\left[\frac{1}{1-\alpha\,(\Delta P)}-\alpha\,(\Delta P)\right].\end{aligned} \tag{5}$$

For $(\Delta P) = 1/(2\alpha)$, the ML uncertainty reaches its minimum value

$$(\Delta X)_{\min} = \frac{3\hbar\alpha}{2}. \tag{6}$$

In the momentum space we employ the following realization of the position and momentum operators

$$X = \frac{i\hbar}{1-\alpha\,|p|}\frac{d}{dp}; \quad P = p. \tag{7}$$

which satisfy the deformed CRR given in (4). In this case the scalar product is not the usual one, instead it is defined with

$$\langle\phi|\,\psi\rangle = \int_{-1/\alpha}^{1/\alpha} dp\,(1-\alpha\,|p|)\,\phi^*(p)\,\psi(p)\,. \tag{8}$$

This definition preserve the hermiticity of the position and momentum operators. It is worth noting that the measure of the momentum space is modified, i.e.,

$$dp \to dp\,(1-\alpha\,|p|)\,. \tag{9}$$

In the classical domain, the commutator in quantum mechanics is replaced poisson bracket as,

$$\frac{1}{i\hbar}\,[X, P] \to \{X, P\}\,. \tag{10}$$

Consequently the classical limit of (4) give

$$\{X, P\} = \frac{1}{1-\alpha|P|}. \tag{11}$$

3 Ideal Gas and its Thermal Quantities

In this section, in the framework of the GUP* we investigate the thermal quantities of an ideal gas. In this context, we first discuss the construction of a partition function in the deformed algebra. Then, we assume that an ideal gas in canonical ensemble and examine its thermal quantities by obtaining the Helmholtz free energy, internal energy, entropy and specific heat functions. Finally, we assign numerical values and present the effect of the deformation parameter on the thermal quantities.

3.1 Construction of the Partition Function

There are two possibilities to establish a partition function of a single particle in a canonical ensemble in the presence of the GUP* [51, 70–74]:

1. Taking into account the commutation relations in the presence of the GUP* which are simultaneous to the undeformed Hamiltonian.
2. Employing the standard commutation relation, however considering the deformed Hamiltonian.

In this paper, we prefer to use the first method. Therefore, for a system with one particles, we construct the partition function in the presence of GUP* with

$$Z = \frac{1}{h^3}\int\int \frac{d^3X d^3P}{J} e^{-\beta H}, \tag{12}$$

where $\beta = (K_B T)^{-1}$, K_B is the Boltzmann constant. Furthermore, T represents the thermodynamic temperature and J is the Jacobian of the transformation

$$J = \prod_{j=1}^{3}\left\{X_j, P_j\right\} = (1-\alpha|P|)^3. \tag{13}$$

The Hamiltonian of classical ideal gas is only a function of momenta, i.e. $H = \frac{P^2}{2m}$. For simplicity, we work in one dimension, thus, the partition function for a single particle reads as

$$Z = \frac{1}{h}\int\int \frac{dXdP}{J} e^{-\beta H} = \frac{L}{h}\int_{-1/\alpha}^{1/\alpha} (1-\alpha|P|)\, e^{-\frac{\beta P^2}{2m}} dP. \tag{14}$$

The integral has an exact solution. After simple algebra, we get

$$Z = \frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}}\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1 - e^{-\frac{\beta}{2\alpha^2 m}}\right)\right], \tag{15}$$

where $\mathrm{erf}\,(x)$ is the “error function”. As α is supposed to be a small parameter, one can expand (15) up to first order of α, this yields

$$Z \simeq \frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}}\left[1 - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\right]. \tag{16}$$

The first term of (16) is the conventional partition function of the ideal gas, while the second term represents the quantum gravity correction.

3.2 Thermal Quantities

In this subsection, based on the obtained partition function we derive some thermodynamic functions. We start with the Helmholtz free energy,

$$F=-\frac{1}{\beta}\ln Z=-\frac{1}{\beta}\ln\frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}}-\frac{1}{\beta}\ln\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right)-2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]. \tag{17}$$

Analogously, for very small deformation parameter the Helmholtz free energy reduces to the following form

$$F\simeq-\frac{1}{\beta}\ln\frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}}+\frac{2\alpha}{\beta}\sqrt{\frac{m}{2\pi\beta}}. \tag{18}$$

Here, the first term is the ordinary Helmholtz free energy for the one dimensional classical ideal gas and the second term is the modification due to the GUP* effects. For the internal energy, we utilize the following formula

$$U=-\frac{\partial}{\partial\beta}\ln Z. \tag{19}$$

Inserting (15) into(19) yields:

$$U=\frac{1}{2\beta}-\frac{\frac{\alpha^2 m}{\beta}}{\sqrt{2\pi m\alpha^2\beta}}\frac{1-e^{-\frac{\beta}{2m\alpha^2}}}{\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right)-2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]}. \tag{20}$$

In the limit of $\alpha\ll 1$, up to the first order of α we arrive at

$$U\simeq\frac{1}{2\beta}-\frac{\alpha}{\beta}\sqrt{\frac{m}{2\pi\beta}}. \tag{21}$$

Next, we take the mass of the monatomic particle, $m=10^{-27}\ kg$, as the ideal gas system into consideration and we calculate the ratio of the internal energy shift to the internal energy.

$$\frac{\Delta U}{U_{\alpha=0}}\simeq\alpha\,(K_B T)^{\frac{1}{2}}\times 10^{-14}. \tag{22}$$

We see that the GUP* induced correction term is very small to be measured in low temperature. Therefore, we conclude that the quantum gravity effect modifies the thermal quantities only at very high temperature limits. Then, we study another important thermodynamic function, namely the entropy. We use the well-known definition of the reduced entropy function

$$\frac{S}{K_B}=\beta^2\frac{\partial}{\partial\beta}F. \tag{23}$$

Substituting (17) into (23), we find

$$\frac{S}{K_B} = \frac{1}{2} + \ln \frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}} - \frac{\frac{\alpha m}{\sqrt{2\pi m\beta}}\left(1-e^{-\frac{\beta}{2m\alpha^2}}\right)}{\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]}$$
$$+ \ln\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]. \quad (24)$$

In the limit where $\alpha \ll 1$, one finds the reduced entropy function up to the first order of the deformation parameter as

$$\frac{S}{K_B} \simeq \frac{1}{2} + \ln \frac{L}{\hbar}\sqrt{\frac{m}{2\pi\beta}} - \alpha\sqrt{\frac{m}{2\pi\beta}}. \quad (25)$$

We see that the GUP* ends with a negative correction term, which conducts to a decrease on the reduced entropy of the system. It is noteworthy that for $\alpha = 0$, the ordinary entropy is obtained. Finally, we explore the reduced specific heat function of the system by employing the following formula:

$$\frac{C}{K_B} = -\beta^2 \frac{\partial U}{\partial \beta}. \quad (26)$$

By substituting (20) into (26), we arrive at

$$\frac{C}{K_B} = \frac{1}{2} - \frac{3}{2}\frac{\alpha m}{\sqrt{2\pi m\beta}} \frac{1-e^{-\frac{\beta}{2m\alpha^2}}}{\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]}$$
$$+ \frac{\alpha m\beta}{\sqrt{2\pi m\beta}} \frac{\frac{1}{2m\alpha^2}e^{-\frac{\beta}{2m\alpha^2}}\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right] - \frac{m\alpha}{\beta\sqrt{2\pi m\beta}}\left(1-e^{-\frac{\beta}{2m\alpha^2}}\right)^2}{\left[\mathrm{erf}\left(\sqrt{\frac{\beta}{2\alpha^2 m}}\right) - 2\alpha\sqrt{\frac{m}{2\pi\beta}}\left(1-e^{-\frac{\beta}{2\alpha^2 m}}\right)\right]^2}. \quad (27)$$

Considering a very small deformation parameter, we expand (27) up to the first order in α. We find

$$\frac{C}{K_B} \simeq \frac{1}{2} - \frac{3\alpha}{2}\sqrt{\frac{m}{2\pi\beta}}. \quad (28)$$

The first term, which has a constant value, is the conventional reduced specific heat function of an ideal gas in one dimension. The second term is the correction term. In order to predict at which temperature the correction term would have a significant effect, we examine the ratio of the reduced specific shift to the reduced specific heat function.

$$\frac{\Delta C}{C_{\alpha=0}} = 3\alpha\sqrt{\frac{m}{2\pi\beta}}, \quad (29)$$

For a hydrogen atom, which has a mass $m = 9.11 \times 10^{-31} kg$, at room temperature $T = 300K$, the evaluation of the ratio gives

$$\frac{\Delta C}{C_{\alpha=0}} \simeq \alpha \times 7.3518 \times 10^{-26}. \quad (30)$$

The accuracy concerning the measurement of the specific heat is about 10^{-7} [75]. Considering this precision, we get the following upper bound for the minimal length as:

$$(\Delta X)_{\min} \simeq 0.2\,056\,4 \ \text{ fm}. \tag{31}$$

This bound is consistent with [76–78].

3.2.1 Numerical Results and Discussions

In this subsection, we use numerical values to examine our findings graphically. We consider a non-interacting monatomic ideal gas. We take $m = 10^{-27}\ kg$, $L = 1\ m$ and use the Boltzmann and the reduced Planck constants to plot the thermodynamic functions. In all graphs, we employ three non-zero deformation parameters to demonstrate the effect of the GUP*. In the figures, we employ the black and solid lines for presenting the non deformed case.

At first, we plot the Helmholtz free energy versus temperature in Fig. 1. We see that at low temperatures the GUP* is not observable. In very high temperatures, the Helmholtz free energy function has a shift upwards. This result is in a complete agreement with (18). For higher values of the deformation parameter, the shift becomes larger.

We schematically depict the behavior of the internal energy function versus temperature in Fig. 2. At low temperatures, the internal energy has a linear increase and the effect of the GUP* does not arise. However at very high temperatures, the internal energy function takes lower values with the higher deformation parameter. This behavior confirms (21).

Next, we present the behavior of the reduced entropy function versus temperature in Fig. 3. We do not observe the effect of the GUP* at low temperatures, therefore, in the plot, we do not focus on that region. At very high temperatures the entropy function has smaller values via the higher values of the deformation parameter. Equation 25 has a negative valued expression which is linearly proportional to the deformation parameter, so that, we conclude that the graphical representations of the reduced entropy are in an agreement with our findings.

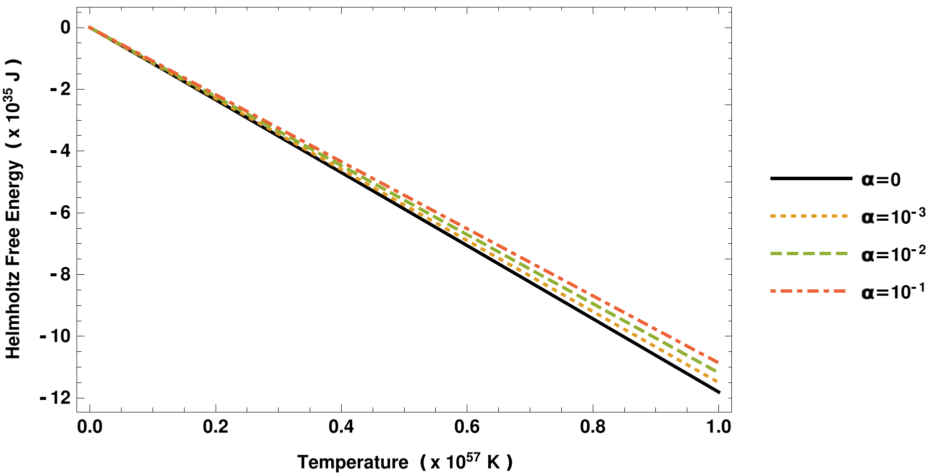

Fig. 1 Helmholtz free energy versus temperature

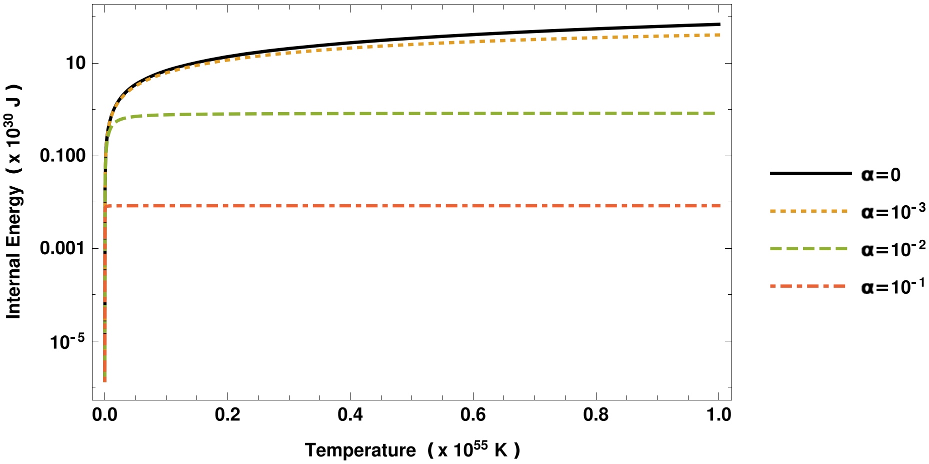

Fig. 2 Internal energy versus temperature

Finally, we plot the reduced specific heat function versus temperature in Fig. 4. We see that in the ordinary case the specific heat function is a constant. In the GUP* framework at low temperatures we do not observe a modification. However, in very high temperatures we see a decrease in the reduced specific heat function. These changes differ via the values of the deformation parameter. Equation (28) approves such behaviors.

All these figures prove that the thermal properties of the ideal gas are affected by the deformation in the GUP* approach. We observe the effects become significant only in the very high-temperatures. It is worth noting that at low-temperature values the thermal properties are the same, thus, they do not observed.

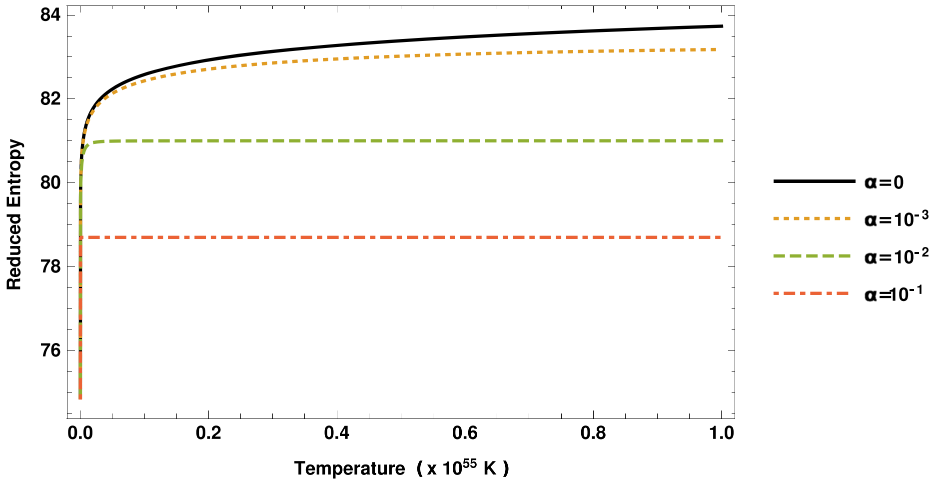

Fig. 3 Reduced entropy function versus temperature

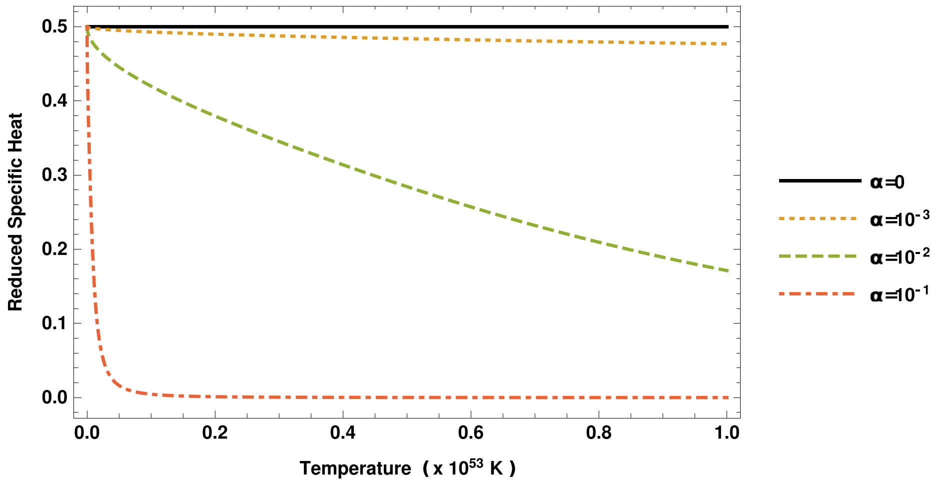

Fig. 4 Reduced specific heat function versus temperature

4 Other Applications

In this paper, we consider two more applications which are frequently discussed in the contents of the works of deformed algebra: The blackbody radiation spectrum and the cosmological constant.

4.1 The Blackbody Radiation Spectrum

In this subsection, we investigate the influence of GUP* on the spectrum of the black body radiation. According to (13), the number of quantum states per momentum space volume is modified by the weight factor that is given in 3-dimensions as $J = 1/(1-\alpha|P|)^3$. So that, we express the energy density of the electromagnetic field per unit volume at a finite temperature by [44, 50]

$$\begin{aligned} E &= 2\int \frac{d^3k}{(2\pi)^3}(1-\alpha\hbar|k|)^3 \frac{\hbar kc}{e^{\frac{\hbar kc}{K_BT}}-1}, \\ &= 8\pi\int_0^{1/\hbar\alpha} \frac{k^2dk}{(2\pi)^3}(1-\alpha\hbar k)^3 \frac{\hbar kc}{e^{\frac{\hbar kc}{K_BT}}-1}, \\ &= \int_0^{\nu_\alpha} d\nu \mathcal{I}_\alpha(\nu,\nu_\alpha,T,T_\alpha), \end{aligned} \tag{32}$$

where

$$\mathcal{I}_\alpha(\nu,\nu_\alpha,T,T_\alpha) = \left(1-\frac{\nu}{\nu_\alpha}\right)^3 \mathcal{I}_0(\nu,T), \tag{33a}$$

$$\nu_\alpha = \frac{c}{h\alpha}, \tag{33b}$$

$$T_\alpha = \frac{c}{K_B\alpha}, \tag{33c}$$

and

$$\mathcal{I}_0\left(\nu,\nu_\alpha,T,T_\alpha\right)=\frac{8\pi}{c^3}\frac{h\nu^3}{e^{\frac{\nu T_\alpha}{\nu_\alpha T}}-1}. \tag{34}$$

Here, $\mathcal{I}_\alpha\left(\nu,\nu_\alpha,T,T_\alpha\right)$ and $\mathcal{I}_0\left(\nu,\nu_\alpha,T,T_\alpha\right)$ are the modified and regular spectral functions, respectively. To observe the effect of GUP* on the shape of the spectral function we plot the functions

$$\mathcal{F}_\alpha\left(\frac{\nu}{\nu_\alpha},\frac{T_\alpha}{T}\right)=\left(1-\frac{\nu}{\nu_\alpha}\right)^3\mathcal{F}_0, \tag{35}$$

and

$$\mathcal{F}_0\left(\frac{\nu}{\nu_\alpha},\frac{T_\alpha}{T}\right)=\frac{\left(\frac{\nu}{\nu_\alpha}\right)^3}{e^{\frac{\nu T_\alpha}{\nu_\alpha T}}-1}, \tag{36}$$

versus the frequency ratio, $(\nu/\nu_\alpha)\in[0,1]$, at three different values of temperatures. When the deformation parameter is very small the distortion in the spectral functions is undetectable in low and high frequencies as we present in Fig. 5 .

For a higher value of deformation parameter spectral functions closely coincide with each others only in very small frequencies as we demonstrate in Fig. 6. In such a case the spectral function reaches its maximal value at a relatively lower frequency value. Moreover, in the GUP* framework, the peak value of the spectral functions, hence, the average energy is relatively small.

In Fig 7, we reveal the spectral functions at $T=T_\alpha$. We observe that the deviations increase while the average energy decreases. We conclude that at high frequencies the blackbody radiations are modified.

Our findings approve the studies of Chang et al. [44] and Pedram [50]. The distortion in the blackbody radiation can not be observed until the temperature is high enough.

4.2 The Cosmological Constant

Finally, we discuss the cosmological constant in the framework of the GUP*. It is well-known that the cosmological constant can be achieved by summing over the momentum

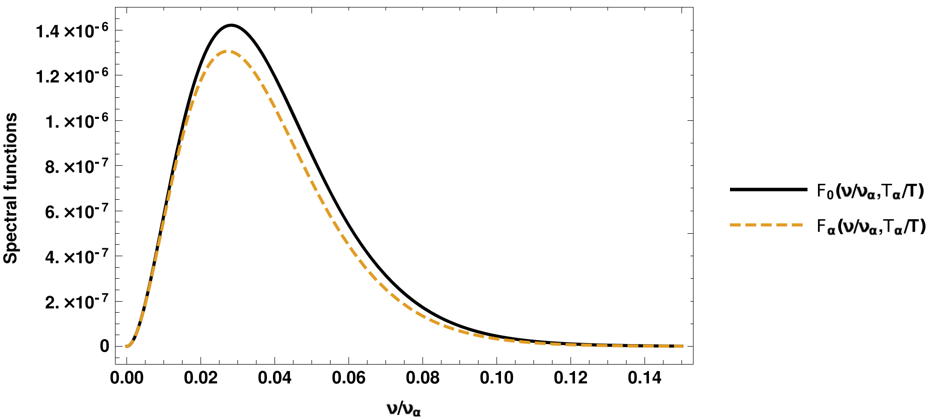

Fig. 5 The blackbody radiation spectrum versus frequency in the GUP* framework at temperature $T=0.01T_\alpha$

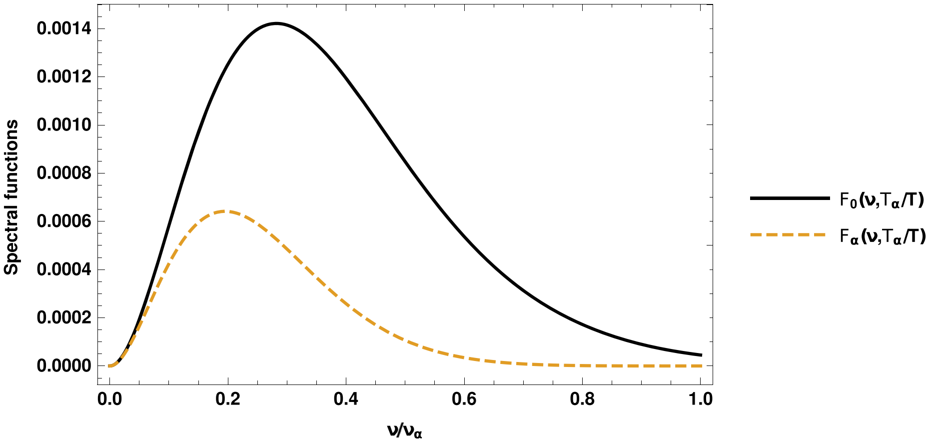

Fig. 6 The blackbody radiation spectrum versus frequency in the GUP* framework at temperature $T = 0.1T_\alpha$

states of the zero-point energies of the harmonic oscillator [79, 80]. So that, we employ the standard form of the zero-point energy of each oscillator of mass m

$$\frac{\hbar\omega}{2} = \frac{1}{2}\sqrt{p^2 + m^2}, \tag{37}$$

to sum over all momentum states per unit volume. We get

$$\begin{aligned}\Lambda\,(m) &= \frac{1}{2}\int d^3p\,(1-\alpha\,|p|)^3\,\sqrt{p^2+m^2},\\ &= 2\pi\int_0^{1/\alpha} dp\,(1-\alpha p)^3\,p^2\sqrt{p^2+m^2}.\end{aligned} \tag{38}$$

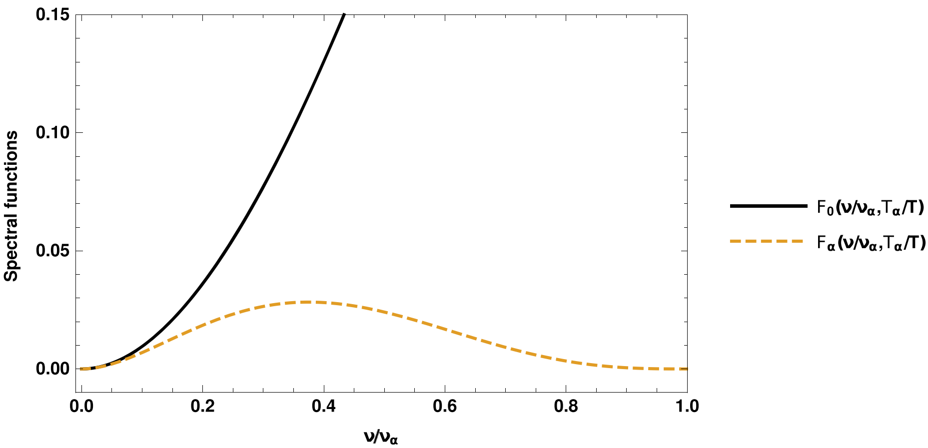

Fig. 7 The blackbody radiation spectrum versus frequency in the GUP* framework at temperature

We perform the substitution $k = \frac{p}{m}$, and then we find

$$\Lambda\left(m\right) = 2\pi m^4 \int_0^a dk \left(1 - \frac{k}{a}\right)^3 k^2 \sqrt{1 + k^2}, \tag{39}$$

where $a = 1/\left(\alpha m\right)$. After we take the integrals, we arrive at

$$\begin{aligned} \Lambda\left(m\right) \ = \ 2\pi m^4 \Bigg\{ & \left[\frac{a\left(a^2+1\right)^{\frac{3}{2}}}{4} - \frac{a\sqrt{a^2+1}}{8} - \frac{\text{arcsinh}a}{8} \right] - \frac{3}{a}\left[\frac{2}{15} + \frac{a^2\left(a^2+1\right)^{\frac{3}{2}}}{5} - \frac{2}{15}\left(a^2+1\right)^{\frac{3}{2}} \right] \\ & + \frac{3}{a^2}\left[\frac{a^3\left(a^2+1\right)^{\frac{3}{2}}}{6} - \frac{a\left(a^2+1\right)^{\frac{3}{2}}}{8} + \frac{a\sqrt{a^2+1}}{16} + \frac{\text{arcsinh}a}{16} \right] \\ & - \frac{1}{a^3}\left[-\frac{8}{105} + \frac{a^4\left(a^2+1\right)^{\frac{3}{2}}}{7} - \frac{4}{35}a^2\left(a^2+1\right)^{\frac{3}{2}} + \frac{8}{105}\left(a^2+1\right)^{\frac{3}{2}} \right] \Bigg\}. \end{aligned} \tag{40}$$

In the massless case it reduces into the form of

$$\Lambda\left(0\right) = \Lambda_{GUP^*} = \frac{\pi}{70\alpha^4} = \frac{\pi}{70}\left(\frac{\hbar}{\ell_P}\right)^4, \tag{41}$$

where ℓ_P is the Planck length. Note that due to the GUP* influence, the cosmological constant is automatically rendered finite acting effectively as the UV cutoff.

At this point to end the paper with a comparison, we recall the cosmological constant predictions that exist in the literature [21, 44, 51].

$$\Lambda_{KMM} = \frac{\pi}{2}\left(\frac{\hbar}{\ell_P}\right)^4, \tag{42}$$

$$\Lambda_{Pedram} = \frac{\pi 64}{3645}\left(\frac{\hbar}{\ell_P}\right)^4. \tag{43}$$

Then, we compare our finding, (41), with the others (42) and (43). We find

$$\Lambda_{GUP^*} < \Lambda_{Pedram} < \Lambda_{KMM}, \tag{44}$$

$$\frac{\Lambda_{GUP^*}}{\Lambda_{Pedram}} = 0.8; \qquad \frac{\Lambda_{GUP^*}}{\Lambda_{KMM}} = 0.028. \tag{45}$$

This means that the model proposed by Chung and Hassanabadi is interesting since it gives the smallest amount among these massless cosmological constants. Therefore, we conclude that this approximation is more appropriate than the others.

5 Conclusion

In this paper, we used a higher order generalized uncertainty principle that is presented by Chung and Hassanabadi which predicts a minimal length uncertainty and a maximal observable momentum value. After a brief introduction of the new deformed formalism, we studied a one dimensional classical ideal gas in a canonical ensemble. We constructed the partition function by taking the deformed commutation relations into account. We derived the internal and Helmhotz free energy, reduced entropy and specific heat functions. Then, we have depicted them by employing numerical values. We discussed the effect of the deformation parameters on the thermal quantities. We observed that, one can observe these effects only in very high temperatures. Then, we discussed the blackbody radiation and cosmological constant problems according to the new approach. We showed that the latter formalism gave more accurate results after a comparison with the existing ones in the literature.

Acknowledgment The authors thank to the referees for a thorough reading of our manuscript and for constructive suggestion.

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