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Preparating Quantum Entanglement Between Microtoroidal-Resonator–Mediated Nitrogen-Vacancy Centers in Diamond

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Abstract

We propose a scheme to prepare entangled states between solid-state nitrogen-vacancy (NV) centers. In our scheme, NV centers are coupled to the whispering-gallery modes of microtoroidal resonators (MTRs). With a protocaled polarized photon input and measurement of single-photon detector, NV centers will be projected on entangled states with the assistance of the special input-output process of polarized photon in MTRs. More importantly, both Bell and W state can be generated with our optical system. This scheme provides a physical feasibility for entangling NV centers, and maybe pave the way towards NV center-based quantum information processing (QIP).

Keywords Entanglement · NV centers · Bell state · W state · Microtoroidal resonator

1 Introduction

Quantum entanglement is the simultaneous occurrence of superposition and correlation in composite systems and gives rise to some counterintuitive phenomena of quantum mechanics. Entanglement has become one of the symbols of quantum mechanics for its nonlocal connotations [1]. In recent years, however, it has been shown that entanglement represents an unique quantum resource whose production is a sort of elementary prerequisite for quantum communication [2] and quantum information processing (QIP) [3, 4], even for quantum machine learning [5]. There are lots of efforts in order to create entanglement in theoretical and experimental research. Creating entanglement has been possible in many physical systems, such as optical systems [6–8], atomic ensembles [9], trapped atomic ions [10] and solid-state system [11, 12].

In this paper, we consider another possible solid-state system, the nitrogen vacancy (NV) centers in diamond, to generate bi- and multipartite entangled states. We propose a scheme to prepare quantum entanglement between NV centers coupled to microtoroidal resonators

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(MTRs). This paper is organized as follows: Section 2 describes the physical structure of NV centers and MTRs as well as explaining the input-output process of photon in the coupling system of NV centers and MTRs. In Section 3, we put forward a scheme to generate Bell state and W state between NV centers based on the optical absorption-emission process of NV centers and the input-output relation of MTRs. Finally, we give some remarks and draw a conclusion in Section 4.

2 NV Centers and MTRs

In recent years, the negatively charged nitrogen-vacancy (NV) center in diamond has received considerable attention [13–16]. NV center is a point defect in diamond with C_{3v} symmetry consisting of a substitutional nitrogen-lattice vacancy pair orientated along the crystalline direction, which provides a spin system composed of electron spins and nuclear spins. It is considered to be a very promising system for quantum information processing due to its stable electronic state and long coherence time even at room temperature. NV centers have a very complex energy-level structure due to the coupling of eletron and nuclear and particular optical transition [17]. The ground state (GS) of a NV center is an orbit-singlet $|E_0\rangle$ coupling an electronic spin-triplet with a 2.87-GHz zero-field spliting between the magnetic sublevel $|0\rangle$ and $|\pm 1\rangle$ states. Although the two states $|\pm 1\rangle$ cannot be mutually transformed directly, strong correlation is possible through an orbital excited state (ES) $|A_2\rangle$. The state $|A_2\rangle$, which is an entangled Bell state, raises from the spin-orbit and spin-spin interaction denoted as $|A_2\rangle = (|E_+\rangle|-1\rangle + |E_-\rangle|+1\rangle)/\sqrt{2}$, where $|E_\pm\rangle$ are obital states with angular momentum ± 1 along the NV axis. Owing to the zero magnetic moment of the electron spin in the $|A_2\rangle$ and total angular momentum conservation, the GS $|+1\rangle$ can be excited to the state $|A_2\rangle$ through absorption of a photon with right-hand circular polarization $|R\rangle$, whereas the $|-1\rangle$ state couples the state $|A_2\rangle$ through absorping a left-handed circularly polarized photon $|L\rangle$. Moreover, the ES $|A_2\rangle$ decays with equal probability to the $|+1\rangle$ ground state through $|R\rangle$ polarized radiation and to $|-1\rangle$ through $|L\rangle$ radiation. Hence, a NV center in diamond offers an ideally degenerate three-level Λ system under a zero field, as shown in Fig. 1.

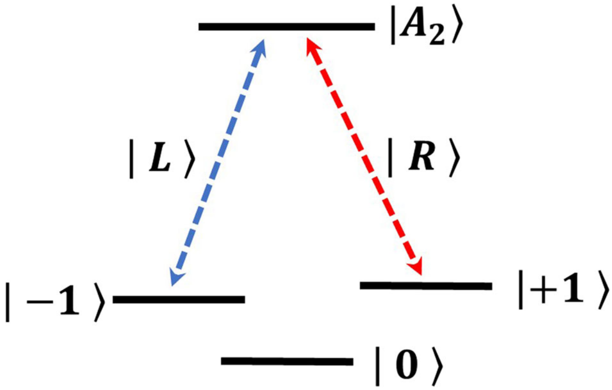

Fig. 1 Λ configuration within the NV center-level structure depicting excitation with two circular polarized photons $|L\rangle$ and $|R\rangle$ from ground states $|\pm 1\rangle$ to the excited state $|A_2\rangle$

One of the remaining challenges on the way towards diamond-based QIP is the establishment of a scalable architecture allowing for the coherent coupling and entanglement between NV spins. A NV center could be indirectly linked or entangled together with other NV centers by transferring information via photons, and coupling NV centers to a high-quality-factor, low-mode-volume optical microcavity offers a medium to dramatically improve the interactions between the photons and NV spins. Many experiments have involved coupling relationships between NV centers and different optical resonators, such as GaP microdisks [18], and Si microspheres [19]. In 2012, a scheme was proposed to generate GHZ state with distant NV centers confined in separated photonic crystal nanocavities [20]. The microtoroidal resonators (MTRs) with a quantized whispering-gallery mode (WGM) was firstly introduced in 2008 to achieve strong interaction between single atom in cavity and incident photons [21]. The strong coupling between NV center and WGM has been demonstrated in some experiments [22–25]. As a kind of optical microcavities, MTR has a high quality factor Q, a small size and relatively convenient preparation process. Therefore, the NV-MTR system has gained widespread attention in the field of quantum information because of the long-coherence time and the single-photon input-output process [26–30]. In our scheme, a NV center is fixed on the surface of MTR and coupled to the WGM of MTR, and the polarized photon can be input to and output from MTRs through fibers, as shown in Fig. 2.

If a NV center in the GS $|-1\rangle$ in the NV-MTR system is input a polarized photon $|L\rangle$ in resonance with the $|A_2\rangle$ transition, the GS will be excited to the state $|A_2\rangle$ through absorption of the photon, and the output photon from subsequent emission will differ from the input photon in phase by a factor of $e^{i\phi}$. Similarly, if an incident photon $|R\rangle$ is coupled to the GS $|+1\rangle$, the absorption-emission process of NV-MTR system will emit a photon with phase shift ϕ. In contrast, if the NV center is initially prepared in $|+1\rangle$, the input single photon $|L\rangle$ would sense a bare MTR due to the poliarization mismatch, yielding a phase shift ϕ_0. A input photon $|R\rangle$ is always reflected by a bare cavity with $|-1\rangle$ NV center and is shifted a phase ϕ_0 [27].

Assuming that some coupling parameters of the NV-MTR system are appropriately modulated and the threshold condition is met, we achieve the phase shifts $\phi = 0$ and $\phi_0 = \pi$ between the input and output photon. Then, we have the following relations

$$\begin{aligned} |R\rangle|+1\rangle &\rightarrow |R\rangle|+1\rangle, \\ |R\rangle|-1\rangle &\rightarrow -|R\rangle|-1\rangle, \\ |L\rangle|+1\rangle &\rightarrow -|L\rangle|+1\rangle, \\ |L\rangle|-1\rangle &\rightarrow |L\rangle|-1\rangle. \end{aligned} \tag{1}$$

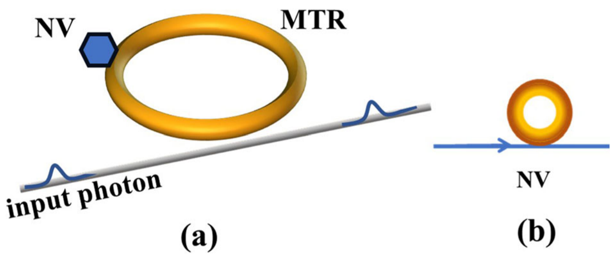

Fig. 2 **a** Diagram illustrates the basic model of the coupling system of NV center and MTR. NV center is isolated and coupled to the WGM of the MTR cavity. A single-photon pulse is imported to interact with the MTR-mediated NV center. **b** Simplified diagram of the NV-MTR system

In this work, we encode quantum information in the superposition of two degenerate GSs, i.e., $|\pm\rangle = \frac{1}{\sqrt{2}}(|+1\rangle \pm |-1\rangle)$. Obviously, we may then, follwing the relation above, obtain input-output relations as

$$|R\rangle|\pm\rangle \rightarrow |R\rangle|\mp\rangle, |L\rangle|\pm\rangle \rightarrow -|L\rangle|\mp\rangle \quad (2)$$

3 Bell State and W State Generation Based on NV Center and MTR Coupling System

In the section, we will demonstrate how to generate quantum entangled states with the coupling systems of NV centers and MTRs. The key framework of our scheme is illustrated in Fig. 3.

In the scheme, a photon with exact polarization state is inported into the proposed optical circuit, and the results of detection at the output ports reveals whether entanglement between NV centers is successly generated or not The polarization states of photons are represented in terms of two bases: $\{|L\rangle,|R\rangle\}$ and $\{|H\rangle,|V\rangle\}$, where $|H\rangle$ and $|V\rangle$ indicate horizontal and vertical polarization, respectively. The basic building elements of the optical circuit are polarizing beam splitter (PBS) and half- and quarter-wave plates. The PBS, which is sensitive to the polarization of the photon, transmits $|H\rangle$ polarized photon while reflecting $|V\rangle$ polarized photon. A half-wave plate (HWP) oriented at $\pi/4$ along its optical axis performs a NOT operation on the state of polarization, which changes a state of polarization to orthogonal one, i.e., $|H\rangle \Longleftrightarrow |V\rangle$. But if a HWP is oriented at $\pi/8$, a Hadmard operation will be perform on the ingoing polarized photon, i.e., $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, and $|V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. A quarter-wave plate (QWP) converts a linear-polarized photon into a circular-polarized one, i.e., $|H\rangle \rightarrow |L\rangle$, and $|V\rangle \rightarrow |R\rangle$.

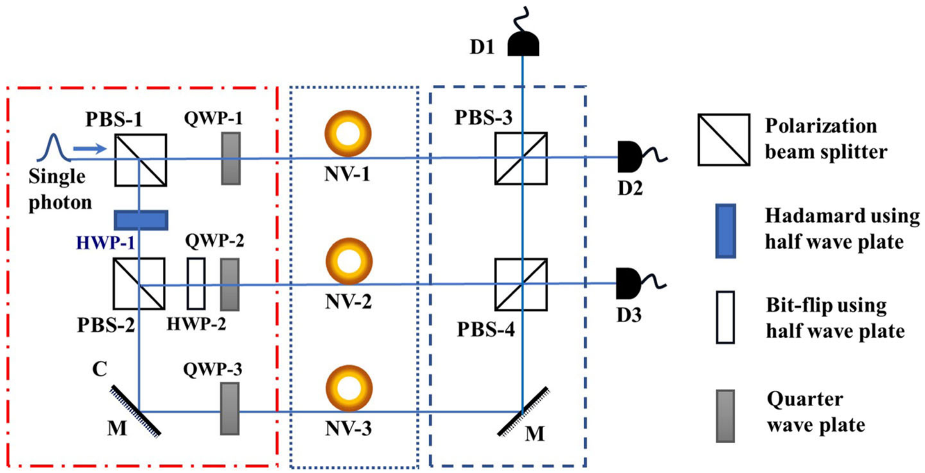

Fig. 3 Schematic representation illustrating the module and the entanglement generation scheme. The circuit can be divided into three parts: input module(in dotted line), coupling module (in short dashed line) and output module (in long dashed line). Every NV center is isolated and coupled to a MTR cavity. A polarized single-photon is imported the NV-MTR system, and the output photon is detected by the detecter D1, D2 and D3. PBS,the polarization beam splitter; HWP, the half wave plate; QWP, the quarter wave plate; M,the mirror; D, detector

The optical circuit can be divided into three parts according to their functions: input, coupling and output module. In the input module, a input photon initially prepared in the polarization state $\frac{1}{\sqrt{3}}(|H\rangle + \sqrt{2}|V\rangle)$ enters the first polarization beam splitter PBS-1, the polarization of the photon, then, is divided into two components travelling in different ways. The polarization component $|H\rangle$ is directly transmitted to QWP-1 with probability 1/3, and the component $|V\rangle$ is reflected to HWP-1 with probability 2/3. Firstly, the polaried photon $|H\rangle$, if it exists in the path, will hit the quarter-wave plate QWP-1 and become to left-circular-polarization state $|L\rangle$, then interact with the NV-MTR system, i.e., NV-1 in the setup. The $|V\rangle$ component probabilitically reflected from PSB-1 is successively injected into HWP-1 oriented at $\pi/8$ and PBS-2, and then divided into $|V\rangle$ passing through HWP-2 and QWP-2 and $|H\rangle$ through M (mirror) and QWP-3. Obviously, by using linear optical components, the input polarized photon is equiprobably divided into three spatial paths with left-rotation polarization $|L\rangle$, then interacts with one of NV-MTR systems. In other words, with the input module consisting of linear optical elements, an incoming photon with protocaled poliarizition can be transformed into a which-way photon in three or more spatical paths.

To begin the entanglement protocol, we initialize three NV spins (denoted as NV-1, NV-2 and NV-3) in the coupling module in the state $|+\rangle$, and equiprobably interact them with the ingoing polarized photon $|L\rangle$. Now, the state of the combined system of the incoming which-way photon and NV-centers can be expressed as

$$\left(|L\rangle_1|+\rangle_1|+\rangle_2|+\rangle_3 + |L\rangle_2|+\rangle_1|+\rangle_2|+\rangle_3 + |L\rangle_3|+\rangle_1|+\rangle_2|+\rangle_3\right)/\sqrt{3}. \tag{3}$$

After the input-output process of NV-MTR coupled system with $|L\rangle$ photons, the outcoming state of the coupling module is given by

$$\left(|L\rangle_1|-\rangle_1|+\rangle_2|+\rangle_3 + |L\rangle_2|+\rangle_1|-\rangle_2|+\rangle_3 + |L\rangle_3|+\rangle_1|+\rangle_2|-\rangle_3\right)/\sqrt{3}. \tag{4}$$

That is to say, a polarized photon $|L\rangle$ ($|L\rangle_1$ in (4)), emitted from NV-1 with probability 1/3 directly enters PBS-3 in the output module , or from NV-2 with the same probability enters PBS-4, or from NV-3 be reflecd by the mirror M to PSB-4. Now, we express the left-hand circular polariztion $|L\rangle$ in the basis of linearly polarization $\{|H\rangle,|V\rangle\}$, i.e., $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$. Considering the function of PBS for horizontal and vertical polarization, the polarized states of photon are possibly detected by every detectors, as listed in Table 1.

As seen in Table 1, when the detector D1 clicks, for instance, one photon from path 1 or path 3 with the same probability has been detected, but the state ($|V\rangle_1$ and $|H\rangle_3$) can not be distinguished by the detector. This means that the path information of the photon from NV-1 or NV-3 have been erased. Therefore, on detection of a photon by D1, NV centers in path 1 and path 3 are projected to an entangled state, viz.,

$$\frac{1}{\sqrt{2}}|+\rangle_2\left(|-\rangle_1|+\rangle_3 + |+\rangle_1|-\rangle_3\right). \tag{5}$$

Similarly, the response of detector D2 will project NV-1 and NV-2 on entangled Bell state, and D3 project NV-2 and NV-3 entangled. Here, we assume that every optical device has

Table 1 The polarized state of output photons detected with centain probability by every single-photon detector

Detector	polarization detected
D1	$\|V\rangle_1, \|H\rangle_3$
D2	$\|H\rangle_1, \|V\rangle_2$
D3	$\|H\rangle_2, \|V\rangle_3$

efficiency 100% as if there were no technical limit. With a protocaled polarized photon input and one detector responds, consequently, two-NV centers Bell states will be deterministicly generated. It should be noted that the word ”deterministicly” here emphasizes the one-to-one correspondence between the response of dector and generation of Bell state, but which two NV centers are entangled or which detector cliks is probabilitic.

Besides heralded preparing the Bell entangled state, we can furtherly prepare a tripartite entangled state with a supplementary 50-50 beam splitter (BS), as depicted in Fig. 4.

A 50-50 BS, which is insensitive to the photon polarization states, reflects the photon and transmits the photon with half probability respectively. Evidently, the introduction of the BS furtherly mixes the path information of outcoming photon. Both the detectors D2 and D3 record polarized state $|H\rangle_1$, $|H\rangle_2$, $|V\rangle_2$ and $|V\rangle_3$, but give no path information of the photon. Therefore, once detector D2 or D3 clicks, the so-called W state of NV spins has been prepared, that in the particular case of 3 partites takes the form

$$\frac{1}{\sqrt{3}}(|-\rangle_1 |+\rangle_2 |+\rangle_3 + |+\rangle_1 |-\rangle_2 |+\rangle_3 + |+\rangle_1 |+\rangle_2 |-\rangle_3). \tag{6}$$

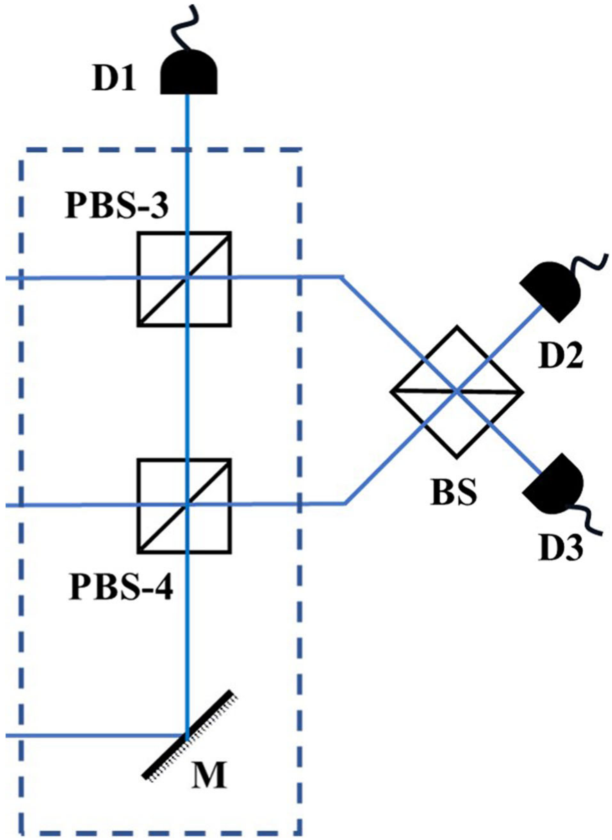

Fig. 4 Schematic of the optical detection with a 50-50 beam splitter.The BS mixes the path information of the outcoming photon from PBS-3 or PBS-4

Namely, the success of generation of W state is signaled by the response of D2 and D3. Considering the case of D1 response with probability 1/3, the W state is, accordingly, successfully prepared with probability 2/3. Moreover, the response of detectors just indicates a outcoming photon but give no information about which way photon comes from. Thus, the entanglement of NV centers is derived from path information of the which-way photon.

By now, we implement the entanglement of NV centers in the optical scheme proposed, which entangles any two specific NV centers in Bell state with successful possibility 1/3 and all three of NV centers in W state with probability 2/3.

4 Remarks and Conclusion

As we have seen, an optical coupling system of NV centers and MTRs have been proposed to generate quantum entanglement among three NV centers. In order to entangle two or more NV centers without direct interaction, we inject the path information of a which-way polarized photon into three NV centers based on the special optical absorption and emission of NV centers. The introduction of MTR cavity enhances the coupling of NV centers and incident photons, and the input-output relation of photon in the NV-MTR system makes it possible to entangle NV spins. In the scheme, except for the polarization transformations of lossless optical elements and measurement of single-photon detectors, no directly operations are performed on NV centers. Therefore, the key to our architecture is the coupling coefficient of NV-MTR systems implementing an efficient input-output process of polarized photon. Because of the long decoherence time at room temperature and the ultra-high quality factor of the MTR cavity, physical requirements of our proposal have been greatly reduced, and technological developments might further help meet these requirements.

In summary, we have proposed a theoretical scheme to generate entangled Bell and W states in solid-state NV centers with an optical circuit. The scheme is pratical and feasible with the present technology. Moreover, the optical system presented here can be regarded as a black box which users do not need to understand its architecture. To meet different need for entanglement, we can adjust some detective devices and perform some local operations to achieve a desired entangled state. From this view of point, our scheme maybe provides an effective way to realize large-scale quantum computing networks.

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References

1. Einstein, A., Podolsky, B., Rosen, N.: Phys. Rev. **47**, 777 (1935)
2. Wu, F.Z., Yang, G.J., Wang, H.B., Xiong, J., Alzahrani, F., Hobiny, A., Deng, F.G.: Sci. Chin. Phys. Mech. **60**(12), 120313 (2017)
3. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2011)
4. Deng, F.G., Ren, B.C., Li, X.H.: Sci. Bull. **62**(1), 46–68 (2017)
5. Sheng, Y.B., Zhou, L.: Sci. Bull. **62**(14), 1025–1029 (2017)
6. Kwiat, P.G., Mattle, K., Weinfurter, H., Zeilinger, A., Sergienko, A.V., Shih, Y.: Phys. Rev. Lett. **75**(24), 4337 (1995)
7. Bouwmeester, D., Pan, J.W., Daniell, M., Weinfurter, H., Zeilinger, A.: Phys. Rev. Lett. **82**, 1345 (1999)
8. Yi, N., Wang, Q., Lei, Z., Liu, X., Peng, J., Zhou, B.: Opt. Commun. **238**(1), 45 (2004)
9. Duan, L.M., Monroe, C.: Adv. At. Mol. Opt. Phys. **55**(07), 419 (2008)

10. Blinov, B., Moehring, D.L., Duan, L.M., Monroe, C.: Nature **428** (6979), 153 (2004)
11. Simmons, S., Brown, R., Riemann, H., Abrosimov, N., Becker, P., Pohl, H.J., Thewalt, M., Itoh, K.: J. Morton, Nature **470**, 69 (2011)
12. Neumann, P., Mizuochi, N., Rempp, F., Hemmer, P., Watanabe, H., Yamasaki, S., Jacques, V., Gaebel, T., Jelezko, F.: J. Wrachtrup, Science **320**(5881), 1326 (2008)
13. Davies, G., Hamer, M.F.: Proc. R. Soc. Lond. A Math. Phys. Sci. **348**(1653), 285 (1976)
14. Harley, R.T., Henderson, M.J., Macfarlane, R.M.: J. Phys. C Solid State Phys. **17**(8), L233 (1984)
15. van Oort, E., Manson, N.B., Glasbeek, M.: J. Phys. C Solid State Phys. **21**(23), 4385 (1988)
16. Doherty, M.W., Manson, N.B., Delaney, P., Jelezko, F., Wrachtrup, J., Hollenberg, L.C.L.: Phys. Rep. **528**(1), 1 (2013)
17. Manson, N.B., Harrison, J.P., Sellars, M.J.: Phys. Rev. B **74**, 104303 (2006)
18. Mccutcheon, M.W., Marko, L.: Opt. Express **16**(23), 19136 (2008)
19. Hijlkema, M., Weber, B., Specht, H.P., Webster, S.C., Kuhn, A., Rempe, G.: .. In: European Conference on Lasers and Electro-Optics and the International Quantum Electronics Conference, pp. 1–1 (2007). https://doi.org/10.1109/CLEOE-IQEC.2007.4386786
20. Zheng, A., Li, J., Yu, R., Lü, X.Y., Wu, Y.: Opt. Express **20**(15), 16902 (2012)
21. Barak, D., Parkins, A.S., Takao, A., Ostby, E.P., Vahala, K.J., Kimble, H.J.: Science **319**(5866), 1062 (2008)
22. Park, Y.S., Cook, A.K., Wang, H.: Nano Lett. **6**(9), 2075 (2006). PMID: 16968028
23. Larsson, M., Dinyari, K.N., Wang, H.: Nano Lett. **9**(4), 1447 (2009)
24. Barclay, P.E., Fu, K.M.C., Santori, C., Beausoleil, R.G.: In: Lasers and Electro-Optics, pp. 1–2 (2010)
25. Barbour, R.J., Dinyari, K.N., Wang, H.: Opt. Express **18**(18), 18968 (2010)
26. Cheng, L.Y., Wang, H.F., Zhang, S., Yeon, K.H.: Opt. Express **21**(5), 5988 (2013)
27. Chen, Q., Yang, W., Feng, M., Du, J.: Phys. Rev. A **83**(5), 10997 (2011)
28. Tong, X., Wang, C., Cao, C., He, L.Y., Zhang, R.: Opt. Comm. **310**(1), 166 (2014)
29. Wei, H.R., Long, G.L.: Sci. Rep. **5**, 12918 (2015)
30. Cao, C., Wang, T.J., Zhang, R., Wang, C.: Laser Phys. Lett. **12**(3), 036001 (2015)