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A Novel Scheme for Quantum Teleportation of N-Particle Generalized Bell-Type States Using Minimal Entanglement Resources

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Abstract
In this paper, we present an efficient scheme for the teleportation of an arbitrary N-particle generalized Bell-type state using a single Bell pair as entanglement channel. For odd and even N-particle states the scheme slightly differs and it is discussed in detail for the simplest cases, 3 and 4 particle states. Further, we have compared the quantum cost of our scheme with other teleportation schemes which use a single Bell pair as entanglement channel.

Keywords Teleportation · Bell-type state · Bell pair · Quantum cost

1 Introduction

Quantum teleportation is an essential tool in quantum information processing and quantum cryptography. In this process, an unknown quantum state is transferred to a particle at a remote location with the help of an entanglement channel shared between the users and a classical communication channel. The discovery of this phenomenon [1] led to many single particle state teleportation [2, 3], bipartite [4–10] and multipartite entangled state teleportation schemes [11, 12] using different entanglement channels.

In 1999, Gorbachev and Trubilko [13] proposed a scheme for teleportation of EPR-nplet or N-particle GHZ state via N + 1 maximally entangled GHZ state and achieved it through mutual cooperation of N users. Later, Li et al. [14] proposed a scheme which requires N + 1 W-class state as entanglement channel. But the requirement of multipartite entanglement channels is an experimental challenge in the implementation of these schemes. Later many alternative schemes were proposed to perform the same quantum task with lesser

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entanglement resources. The more feasible scheme proposed by Wang et al. [15], its modified version by Wang Xin-Wen et al. [16] (WXW) and a simplified scheme proposed by Yan and Gao [17] (YG) all requires only a single Bell pair as entanglement channel. Later, Pathak and Banerjee [18] (PB) proposed a scheme for teleportation of the generalized Bell-type states [19] using minimal entanglement resources. The scheme proposed by Yan and Gao [17] is a special case of this scheme.

In this paper, we propose a novel technique to teleport generalized Bell-type states using minimal entanglement resources. The technique used in this work significantly reduces the depth of the quantum circuit required for the task. It is known that for any practical application, the quantum circuits must be designed to complete its execution well within the coherence time of qubits and the gate operation time [20]. Unlike the schemes proposed by YG [17] and PB [18] the generalized schemes of Wang [15], WXW [16] and our scheme reduces the depth of the circuit. The scheme also facilitates distributed-quantum state teleportation. i.e., if the state is initially split among different users, it can still be teleported by any one of them to the receiver with cooperation from others. For example, if we consider the odd 'N' case, where an unknown N-particle generalized Bell-type state gets distributed among $\frac{N+1}{2}$ users, then any one of these users can teleport the unknown state by sharing a single Bell pair with the receiver and mutual cooperation from other users. In the case of an even number of qubits N, the state can be distributed to a maximum of $\frac{N+2}{2}$ users. To achieve teleportation, the users with two qubits performs Bell measurements and those with only one qubit performs Hadamard operation followed by measurement in Z-basis. Here, we note that by performing a Bell measurement between any two qubits of a generalized Bell-type state we can reduce the total depth of the circuit.

This paper is organized as follows. In Section 2, we present our scheme for the teleportation of 3 and 4-particle generalized Bell-type state and also evaluate the quantum costs for both the cases. A comparison of the quantum cost and efficiency of our scheme with similar schemes is also given here. In Section 3, we generalize our teleportation scheme to odd and even N-particle generalized Bell-type state. In the final section, we present our conclusions.

2 Teleportation of 3-Particle and 4-Particle Generalized Bell-Type State

We explain our scheme for the simplest case of 3-particle generalized Bell-type state where the state to be teleported is

$$|\Psi\rangle_{123} = (\alpha|x_1x_2x_3\rangle + \beta|\overline{x}_1\overline{x}_2\overline{x}_3\rangle)_{123} \tag{1}$$

Here, x_1, x_2, x_3 and the conjugates $\overline{x}_1, \overline{x}_2, \overline{x}_3$ can take values from {0,1} and α and β are the unknown coefficients satisfying the condition $|\alpha|^2 + |\beta|^2 = 1$. Initially, Alice the sender and Bob the receiver share one particle each from the entanglement channel $|\Psi\rangle_{AB}$.

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB} \tag{2}$$

Alice is in possession of particles (1, 2, 3, A) and Bob with particle B, ancilla particles (4,5). The total wavefunction is

$$\begin{aligned}|\Psi\rangle_{tot} &= |\Psi\rangle_{123} \otimes |\Psi\rangle_{AB} \\ &= \frac{1}{\sqrt{2}}\{\alpha|x_1x_2x_300\rangle + \alpha|x_1x_2x_311\rangle \\ &\quad +\beta|\overline{x}_1\overline{x}_2\overline{x}_300\rangle + \beta|\overline{x}_1\overline{x}_2\overline{x}_311\rangle\}_{123AB}\end{aligned}$$

We propose the following four steps for teleportation of the given state:

Step 1: Alice performs Bell measurement on particles (1, A) by applying $CNOT(1, A)$, Hadamard gate (H_1) on particle 1 and finally measure them both in Z-basis. The CNOT operation between any two qubits $|ij\rangle$ is given by $\mathrm{CNOT}|\mathrm{ij}\rangle = |i(i \oplus j)\rangle$ (where, $\oplus$ denotes addition modulo 2) and the Hadamard operation on any qubit $|i\rangle$ is given by $H|i\rangle = \frac{(-1)^i|i\rangle + |\bar{\mathrm{i}}\rangle}{\sqrt{2}}$. Then the total wavefunction becomes,

$$\begin{aligned}|\Psi\rangle_{tot} = & \frac{1}{2}\{\alpha(-1)^{x_1}|x_1x_2x_3x_10\rangle + \alpha|\overline{x}_1x_2x_3x_10\rangle \\ & +\alpha(-1)^{x_1}|x_1x_2x_3\overline{x}_11\rangle + \alpha|\overline{x}_1x_2x_3\overline{x}_11\rangle \\ & \beta(-1)^{\overline{x}_1}|\overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_10\rangle + \beta|x_1\overline{x}_2\overline{x}_3\overline{x}_10\rangle \\ & +\beta(-1)^{\overline{x}_1}|\overline{x}_1\overline{x}_2\overline{x}_3x_11\rangle + \beta|x_1\overline{x}_2\overline{x}_3x_11\rangle\}_{123AB}\end{aligned}$$

Step 2: On the remaining particles (2, 3) Alice performs a second Bell measurement. This measurement collapses them to one of the two possible states $|x_2(x_2 \oplus x_3)\rangle_{23}$ and $|\overline{x}_2(x_2 \oplus x_3)\rangle_{23}$, which can be communicated via one classical bit. Now, all the measurement results can be communicated to Bob via 3 bits of classical information. On rearranging the total wavefunction accordingly, we get

$$\begin{aligned}|\Psi\rangle_{tot} = & \frac{1}{2\sqrt{2}}\{|x_1x_1\rangle_{1A}|x_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_1+x_2}\alpha|0\rangle + \beta|1\rangle_B \\ & +|x_1x_1\rangle_{1A}|\overline{x}_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_1}\alpha|0\rangle + (-1)^{\overline{x}_2}\beta|1\rangle_B \\ & +|\overline{x}_1x_1\rangle_{1A}|x_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_2}\alpha|0\rangle + (-1)^{\overline{x}_1}\beta|1\rangle_B \\ & +|\overline{x}_1x_1\rangle_{1A}|\overline{x}_2(x_2 \oplus x_3)\rangle_{23}\alpha|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta|1\rangle_B \\ & +|x_1\overline{x}_1\rangle_{1A}|x_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_1+x_2}\alpha|1\rangle + \beta|0\rangle_B \\ & +|x_1\overline{x}_1\rangle_{1A}|\overline{x}_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_1}\alpha|1\rangle + (-1)^{\overline{x}_2}\beta|0\rangle_B \\ & +|\overline{x}_1\overline{x}_1\rangle_{1A}|x_2(x_2 \oplus x_3)\rangle_{23}(-1)^{x_2}\alpha|1\rangle + (-1)^{\overline{x}_1}\beta|0\rangle_B \\ & +|\overline{x}_1\overline{x}_1\rangle_{1A}|\overline{x}_2(x_2 \oplus x_3)\rangle_{23}\alpha|1\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta|0\rangle_B\}\end{aligned}$$

The state of particle B at receiving end is in superposition of eight possible states which is given in Table 1. To convert the state of qubit B in to $(\alpha|x_1\rangle + \beta|\overline{x}_1\rangle)_B$, Bob relies on the measurement results communicated by Alice.

Step 3: Depending on the Bell measurement result of particles 1 and A, Bob applies one of the four unitary operations $\{I,\sigma_x,\sigma_y,\sigma_z\}$ on qubit B as shown in Table 2. If the measurement result of particle 2 is $|0\rangle_2$ then no unitary operation is performed on B. If it is $|1\rangle_2$ then σ_z operation is performed on B. Here, it should be noted that the state of particle B

Table 1 Outcome of Alice's measurement and state of the qubit B

Bell measurement Result of (1,A)	Measurement result of (2, 3)	State of particle B
$\|x_1x_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_1+x_2}\alpha\|0\rangle + \beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_1}\alpha\|0\rangle + (-1)^{\overline{x}_2}\beta\|1\rangle$
$\|\overline{x}_1x_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_2}\alpha\|0\rangle + (-1)^{\overline{x}_1}\beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta\|1\rangle$
$\|x_1\overline{x}_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_1+x_2}\alpha\|1\rangle + \beta\|0\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_1}\alpha\|1\rangle + (-1)^{\overline{x}_2}\beta\|0\rangle$
$\|\overline{x}_1\overline{x}_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$(-1)^{x_2}\alpha\|1\rangle + (-1)^{\overline{x}_1}\beta\|0\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\alpha\|1\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta\|0\rangle$

Table 2 Alice's measurement on (1, A) and Unitary operation performed by Bob on B

Bell measurement result of (1,A)	Corresponding unitary operation on B
$\|00\rangle_{1A}$	I
$\|01\rangle_{1A}$	X
$\|10\rangle_{1A}$	Z
$\|11\rangle_{1A}$	iY

does not depend on the value of x_3 (after the Bell measurement on (2,3)) and also the overall phase factors that occurs for different values of x_1, x_2, x_3 has no physical significance. The above operations changes the state of B to the desired state $(\alpha|x_1\rangle + \beta|\overline{x}_1\rangle)_B$.

Step 4: Based on the information obtained from Alice prior to the teleportation process, Bob prepares two ancilla bits in the state $|(x_1 \oplus x_2)(x_1 \oplus x_3)\rangle_{45}$. Now, Bob adds these two qubits to particle B, and applies two CNOT operations with particle B as control qubit and the two ancillary qubits as target qubits to recover the teleported state.

The equivalent quantum circuit representation of our scheme is given in the Fig. 1. In this circuit, the measuring devices are removed and the recovery operations performed by the receiver based on classical information are replaced by equivalent unitary operations. We assume that the bit values of x_2 and x_3 are known to Bob via the classical information sent by Alice. For comparison, we have also given the equivalent circuit of Wang's scheme for the special case of teleportation of three particle GHZ states in Fig. 2.

In order to calculate the quantum cost, we use the method prescribed by Perkowski et al. [21], according to which, all two-qubit gates costs one unit and one-qubit gates costs nothing if they succeeds or precedes a two qubit gate. Using this definition the quantum cost of our circuit is found to be 10, which is equivalent to the cost of Wang et al. [15] and Pathak and Banerjee [18].

However, there are subtle differences in the way these schemes work. In Pathak's scheme, the state is completely disentangled into an unknown single particle state at the sender's end by performing $N-1$ CNOT operations with respect to one of the qubit as control and others as target qubits. Later this state is teleported via Bell pair and finally, it is reconstructed at the receiver's end with the help of the measurement results of other qubits. It is important to note that this method demands $N-1$ units of time from the sender to perform those

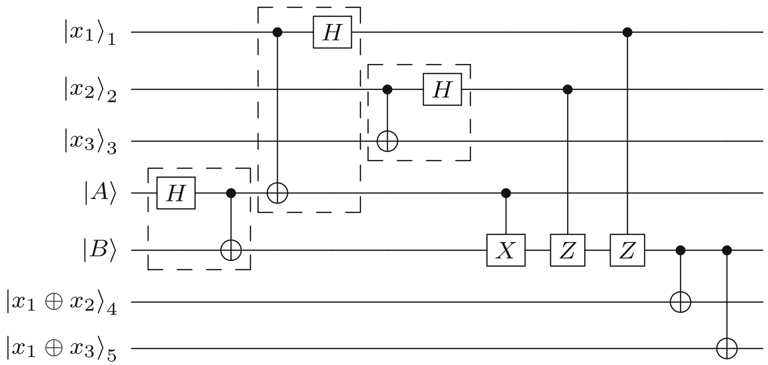

Fig. 1 The proposed equivalent circuit for teleportation of 3-particle Bell-type state using single Bell pair as entanglement channel. Note that the Bell measurements between qubits (1, A) and (2, 3) can be performed simultaneously, thereby reducing the depth of the circuit. The quantum cost of the circuit inside dashed boxes are of one unit

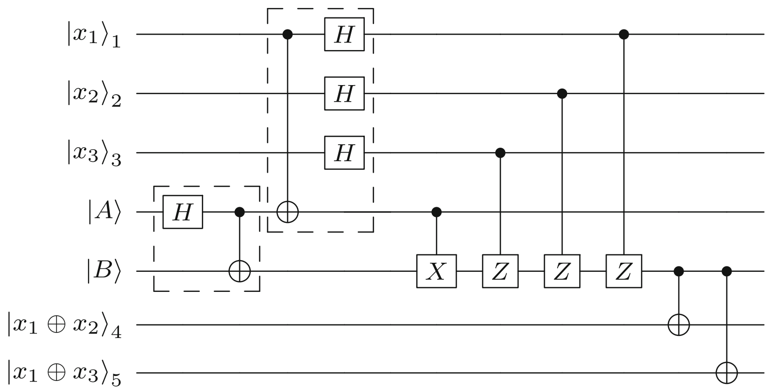

Fig. 2 The equivalent circuit of Wang et al., for teleportation of 3-particle entangled state using single Bell pair as entanglement channel. The simultaneous application of Hadamard gates on qubits 2 and 3 reduces the depth of the circuit, but this scheme uses more number of gates than the proposed scheme

operations, which increases the depth of this circuit. However, in Wang's scheme (where GHZ state is teleported) and our scheme the sender can perform concurrent operations and measurements on the qubits, allowing the sender to transfer the quantum state in a single unit of time. This reduction in the circuit depth clearly shows the advantage of these schemes.

On comparing our scheme with Wang [15], we find that our scheme requires lesser number gates and classical bits for teleportation of N-particle generalized Bell-type state. For instance, in the case of $N = 3$, our scheme requires 11 quantum gate operation and five bits of classical information as against 12 gates and 6 bits of classical information used in Wang's.

Similarly, for teleporting a generalized Bell-type state with N = 4, we follow the same procedure as discussed for the case of N = 3 with slight modification. The scheme is explicitly described below. The unknown four-particle Bell-type state is given by

$$|\Psi\rangle_{1234} = (\alpha|x_1x_2x_3x_4\rangle + \beta|\overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4\rangle)_{1234} \tag{3}$$

where, x_1, x_2, x_3 and x_4 can take values {0,1}, α and β are the unknown coefficients and $|\alpha|^2 + |\beta|^2 = 1$. The entanglement channel shared between the two users is given by

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB} \tag{4}$$

Now, the combined wave function of the system becomes,

$$\begin{aligned}|\Psi\rangle_{tot} &= |\Psi\rangle_{1234} \otimes |\Psi\rangle_{AB} \\ &= \frac{1}{\sqrt{2}}\{\alpha|x_1x_2x_3x_4 00\rangle + \alpha|x_1x_2x_3x_4 11\rangle \\ &\quad +\beta|\overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4 00\rangle + \beta|\overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4 11\rangle\}_{1234AB}\end{aligned}$$

Alice is in possession of particles (1, 2, 3, 4, A) and Bob has particles B and ancilla bits (5, 6, 7).

The sender adopts following steps to teleport the given state:

Step 1: Initially, Alice performs Bell measurements on particle pairs (1, A) and (2, 3) as discussed in the previous case. But, one may observe that after the Bell measurements the particles (4, B) gets entangled. We notice this by rearranging the total wavefunction,

$$
\begin{aligned}
|\Psi\rangle_{tot} = \frac{1}{2\sqrt{2}} \{ & |x_1 x_1\rangle_{1A} |x_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_1+x_2} \alpha |x_4 0\rangle + \beta |x_4 1\rangle_B \\
& + |x_1 x_1\rangle_{1A} |\overline{x}_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_1} \alpha |x_4 0\rangle + (-1)^{\overline{x}_2} \beta |x_4 1\rangle_B \\
& + |\overline{x}_1 x_1\rangle_{1A} |x_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_2} \alpha |x_4 0\rangle + (-1)^{\overline{x}_1} \beta |x_4 1\rangle_B \\
& + |\overline{x}_1 x_1\rangle_{1A} |\overline{x}_2(x_2 \oplus x_3)\rangle_{23} \alpha |x_4 0\rangle + (-1)^{\overline{x}_1+\overline{x}_2} \beta |x_4 1\rangle_B \\
& + |x_1 \overline{x}_1\rangle_{1A} |x_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_1+x_2} \alpha |x_4 1\rangle + \beta |x_4 0\rangle_B \\
& + |x_1 \overline{x}_1\rangle_{1A} |\overline{x}_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_1} \alpha |x_4 1\rangle + (-1)^{\overline{x}_2} \beta |x_4 0\rangle_B \\
& + |\overline{x}_1 \overline{x}_1\rangle_{1A} |x_2(x_2 \oplus x_3)\rangle_{23} (-1)^{x_2} \alpha |x_4 1\rangle + (-1)^{\overline{x}_1} \beta |x_4 0\rangle_B \\
& + |\overline{x}_1 \overline{x}_1\rangle_{1A} |\overline{x}_2(x_2 \oplus x_3)\rangle_{23} \alpha |x_4 1\rangle + (-1)^{\overline{x}_1+\overline{x}_2} \beta |x_4 0\rangle_B \}
\end{aligned}
\tag{5}
$$

Step 2: Alice disentangles particle 4 from B by performing a Hadamard operation on particle 4 and measuring it in the computational basis. Now, Alice publishes all Bell measurement results and single particle measurement result through four bits of classical information. Table 3 shows the collapsed state of particle B with respect to 16 possible results.

Step 3: Depending on Alice's measurement result Bob applies one of four unitary operations $\{I,\sigma_x,\sigma_y,\sigma_z\}$ on particle B as shown in Table 2. If the measurement result of particle 2 is $|0\rangle_2$, identity operation is performed on B and if it is $|1\rangle_2$ then Z operation is performed. Finally for particle 4, if single particle measurement result is $|0\rangle_4$ then I is performed and for result $|1\rangle_4$, operation Z acts on B. Also, note that the result of qubit 3 does not affect the state of qubits 4 and B. These operations change the state of B to $(\alpha|x_1\rangle + \beta|\overline{x}_1\rangle)_B$.

Step 4: Finally, depending on the classical information obtained from Alice, Bob adds three ancilla bits (5, 6, 7) in the state $|(x_1 \oplus x_2)(x_1 \oplus x_3)(x_1 \oplus x_4)\rangle_{567}$ to particle B and applies three CNOT gates with qubit B acting as control qubit and each ancilla qubit as target bits to recover the teleported state. The equivalent quantum circuit representation of

Table 3 Outcome of Alice's measurement and resulting state of particle B

Result (1,A)	Result (2,3)	Z-basis measurement	state of particle B
$\|x_1x_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_1+x_2+x_4}\alpha\|0\rangle + \beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_1+x_2}\alpha\|0\rangle + (-1)^{\overline{x}_4}\beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_1+x_4}\alpha\|0\rangle + (-1)^{\overline{x}_2}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_1}\alpha\|0\rangle + (-1)^{\overline{x}_2+\overline{x}_4}\beta\|1\rangle$
$\|\overline{x}_1x_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_2+x_4}\alpha\|0\rangle + (-1)^{\overline{x}_1}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_2}\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_4}\beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_4}\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2+\overline{x}_4}\beta\|1\rangle$
$\|x_1\overline{x}_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_1+x_2+x_4}\alpha\|0\rangle + \beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_1+x_2}\alpha\|0\rangle + (-1)^{\overline{x}_4}\beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_1+x_4}\alpha\|0\rangle + (-1)^{\overline{x}_2}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_1}\alpha\|0\rangle + (-1)^{\overline{x}_2+\overline{x}_4}\beta\|1\rangle$
$\|\overline{x}_1\overline{x}_1\rangle$	$\|x_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_2+x_4}\alpha\|0\rangle + (-1)^{\overline{x}_1}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$(-1)^{x_2}\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_4}\beta\|1\rangle$
	$\|\overline{x}_2(x_2 \oplus x_3)\rangle$	$\|x_4\rangle$	$(-1)^{x_4}\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2}\beta\|1\rangle$
		$\|\overline{x}_4\rangle$	$\alpha\|0\rangle + (-1)^{\overline{x}_1+\overline{x}_2+\overline{x}_4}\beta\|1\rangle$

our scheme is given in Fig. 3. The quantum cost of this circuit is found to be 13, which is the same as that of Wang's and Pathak's scheme.

3 Generalization of Scheme to N Particle Generalized Bell-Type state

The schemes given above can be generalized to teleport N-particle generalized Bell-type states. From the two examples given in section 2, it may be observed that the schemes differ slightly for odd and even N.

The generalized odd N particle Bell-type state is given by

$$|\Psi\rangle_{123\ldots N} = (\alpha|x_1x_2x_3\ldots x_N\rangle + \beta|\overline{x}_1\overline{x}_2\overline{x}_3\ldots\overline{x}_N\rangle)_{123\ldots N} \tag{6}$$

where x_1, x_2, x_3,... x_N and their conjugates $\overline{x}_1$, $\overline{x}_2$, $\overline{x}_3$,... $\overline{x}_N$ can take values {0,1}. The unknown coefficients α and β satisfies the relation $|\alpha|^2 + |\beta|^2 = 1$.

Step 1: Alice performs a Bell measurement between particle pair $(1, A)$.

Step 2: She continues to make $\frac{N+1}{2}$ such Bell measurements among the remaining pairs i.e. (x_2, x_3) (x_4, x_5)...(x_{N-1}, x_N). The measurement results of the first particle of each pair chosen for Bell measurement is communicated to Bob via classical information.

Step 3: Bob performs one of the four unitary operations $\{I, \sigma_x, \sigma_y, \sigma_z\}$, given in Table 2, on qubit B with respect to the measurement result of (1, A). For rest of the Bell measurement results, Bob performs Z operation on B if the measurement result of the first particle in each pair chosen for Bell measurement is $|1\rangle$ and otherwise does nothing. This process is continued for all pairs up to (x_{N-1}, x_N). This converts the state of qubit B to $(\alpha|x_1\rangle + \beta|\overline{x}_1\rangle)_B$.

Step 4: Finally, Bob adds $(N-1)$ ancilla bits prepared in states based on the $(N-1)$ bits of information obtained from Alice prior to the teleportation process and performs $(N-1)$ CNOT operations with particle B as control bit and other ancilla bits as target bits to recover the teleported state.

To teleport even N-particle generalized Bell-type state, the first two steps are followed as in the odd particle case and Alice makes $\frac{N}{2}$ Bell measurements. After the last Bell measurement between particles (x_{N-2}, x_{N-1}), particles x_N and B get entangled. To disentangle

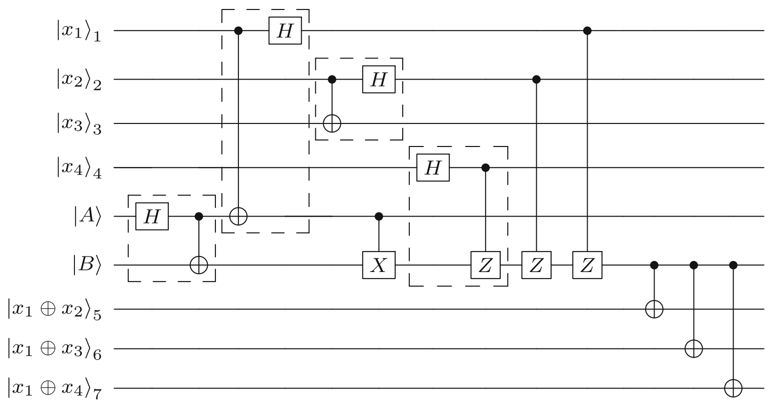

Fig. 3 Proposed equivalent circuit for teleportation of 4-particle Bell-type state using single Bell pair as entanglement channel

x_N from B a Hadamard operation is performed on x_N and it is then measured in the computational basis. If the measurement result of x_N is $|1\rangle_N$, then a Z operation is performed on B and otherwise do nothing. Now by following the steps 3 and 4 as in odd particle case, the even N particle state can also be teleported.

In the schemes of YG [17] and PB [18] the unknown N particle state is initially disentangled by performing $(N-1)$ CNOT operations consuming $(N-1)$ units of time. Our scheme introduces a new method to quicken the teleportation process at the sender's end. Here, the sender is able to perform all $\frac{N+1}{2}$ or $\frac{N}{2}$ bell measurements concurrently in a single unit time, which clearly shows a significant reduction in the circuit depth. Later, the teleported state is reconstructed at the receiving end by performing unitary operations based on the received information as in the other similar schemes [15–18].

We note that corresponding to every Bell measurement in step 2, a unitary operation is performed on particle B at the receiving end i.e. $\frac{N-1}{2}$ operations for odd and $\frac{N-2}{2}$ operations for even case. We find that the quantum cost of our scheme is $3N+1$ for both odd and even particle case. This value indicates that the cost depends directly on the number of particles to be teleported. Further, it easy to see that the scheme allows successful teleportation even if the given quantum state is split among different users in the initial stage.

4 Conclusion

Our scheme relies only on Bell measurements for odd N and also on one single-particle measurement for even N case, for the teleportation of unknown N-particle generalized Bell-type state. A detailed survey of the experimental challenges faced in different quantum systems in achieving complete Bell detection has been given by Pirandola et al. [22]. Based on this detection efficiency, the scheme turns into either probabilistic or deterministic teleportation. Therefore, the implementation of the proposed scheme requires a quantum system with 100% Bell detection efficiency. We compared our scheme with already existing schemes[15, 18] and showed that they are technically different from each other. Even though the quantum cost of all these schemes are found to be the same, our scheme is shown to be experimentally more feasible than others due to its simple, straightforward technique and the possible speed up it allows from the reduction in the circuit depth. We also found that the quantum cost of our scheme for all N-particle generalized Bell-type state is $3N+1$.

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References

1. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
2. Cao, H.J., Song, H.S.: Teleportation of a single qubit state via unique w state. Int. J. Theor. Phys. **46**, 1636 (2007)
3. Li, M.L., Ye, L., Yang, J.: Teleportation of a product state of an arbitrary single-particle state. J. At. Mol. Sci. **3**, 64 (2012)

4. Rigolin, G.: Quantum teleportation of an arbitrary two-qubit state and its relation to multipartite entanglement. Phys. Rev. A **71**, 032303 (2005)
5. Yeo, Y., Chua, W.K.: Teleportation and dense coding with genuine multipartite entanglement. Phys. Rev. Lett. **96**, 060502 (2006)
6. Marinatto, L., Weber, T.: Which kind of two-particle states can be teleported through a three-particle quantum channel? Found. Phys. Lett. **13**, 119 (2000)
7. Cola, M.M., Paris, M.G.: Teleportation of bipartite states using a single entangled pair. Phys. Lett. A **337**, 10–16 (2005)
8. Cao, Z.L., Song, W.: Teleportation of a two-particle entangled state via W class states. Physica A **347**, 177–183 (2005)
9. Chuang, L.D., Liang, C.Z.: Teleportation of two-particle entangled state via cluster state. Commun. Theor. Phys. **47**, 464 (2007)
10. Nandi, K., Mazumdar, C.: Quantum teleportation of a two qubit state using GHZ-like state. Int. J. Theor. Phys. **53**, 1322–1324 (2014)
11. Liu, J.M., Guo, G.C.: Quantum teleportation of a three-particle entangled state. Chin. Phys. Lett. **19**, 456 (2002)
12. Yin, X.F., Liu, Y.M., Zhang, Z.Y., Zhang, W., Zhang, Z.J.: Perfect teleportation of an arbitrary three-qubit state with the highly entangled six-qubit genuine state. Sci. China Phys. Mech. Astron **53**(11), 2059–2063 (2010)
13. Gorbachev, V.N., Trubilko, A.I.: Quantum teleportation of an Einstein-Podolsy-Rosen pair using an entangled three-particle state. J. Exp. Theor. Phys. **91**, 894–898 (2000)
14. Shang-Guan, L.Y., et al.: Perfect teleportation, superdense coding via a kind of W-Class state. Int. J. Quantum Inf. **8**, 1411 (2010)
15. Wang, Y.H., Yu, C.S., Song, H.S.: Teleportation of an arbitrary multipartite GHZ-class state by one EPR pair. Chin. Phys. Lett. **23**, 3142 (2006)
16. Wang, X.W., Wang, Z.Y., Xia, L.X.: Scheme for teleportation of a multipartite quantum state by using a single entangled pair as quantum channel. Commun. Theor. Phys. (Beijing China) **47**, 257 (2007)
17. Yan, L.H., Gao, Y.F.: Simplified scheme for teleportation of a multipartite quantum state using a single entangled pair. Commun. Theor. Phys. (Beijing China) **51**, 235–238 (2009)
18. Pathak, A., Banerjee, A.: Efficient quantum circuits for perfect and controlled teleportation of n-qubit non-maximally entangled states of generalized Bell-type. Int. J. Quantum Inf. **9**, 389 (2011)
19. Gupta, M., Pathak, A., Srikanth, R., Panigrahi, P.: Circuits for distributing quantum measurement. AIP Conf. Proc. **864**, 197 (2006)
20. Abdessaied, N., Wille, R., Soeken, M., Drechsler, R.: Reducing the depth of quantum circuits using additional circuit lines. In: Dueck, G.W., Miller, D. (eds.) Reversible Computation. RC 2013. Lecture Notes in Computer Science, vol. 7948. Springer, Berlin, Heidelberg (2013)
21. Perkowski, M., et al.: A hierarchical approach to computer-aided design of quantum circuits. In the 6th International Symposium on Representations and Methodology of Future Computing Technology, 201–209 (2003)
22. Pirandola, S., Eisert, J., Weedbrook, C., Furusawa, A., Braunstein, S.L.: Advances in quantum teleportation. Nat. Photonics **9**, 641–652 (2015)