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Cyclic Controlled Teleportation by Using a Seven-Qubit Entangled State

Zhi-wen Sang[1]

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Abstract

We propose a scheme of cyclic controlled teleportation for three arbitrary single-qubit states by using a seven-qubit entangled state. In our scheme, Alice can teleport her single-qubit state of qubit a to Bob, Bob can teleport his single-qubit state of qubit b to Charlie and Charlie can also teleport his single-qubit state of qubit c to Alice via the control of the supervisor David.

Keywords Quantum information · Cyclic controlled teleportation · Single-qubit state

1 Introduction

Entangled states are one of the most important resources in quantum information processing tasks, which has been widely applied in various fields such as quantum teleportation [1–3], remote state preparation [4–6], photonic quantum interface [7–9], quantum information splitting [10–12], quantum key distribution [13–15], bidirectional controlled quantum teleportation [16–18], quantum nonlinear optics [19–21], quantum secret sharing [22], quantum communication [23–25] and quantum dense coding [26–28]. Recently, a scheme for cyclic quantum teleportation of three arbitrary single-qubit states was proposed by using a six-qubit entangled state [29], and it is shown that this scheme can realize perfect quantum teleportation in quantum information networks with $N(N \geq 3)$ observers in different directions.

In this work, we demonstrate that a seven-qubit entangled state can be used to realize the perfect cyclic controlled teleportation for three arbitrary single-qubit states. In our scheme, Alice can teleport her single-qubit state of qubit a to Bob, Bob can teleport his single-qubit state of qubit b to Charlie and Charlie can also teleport his single-qubit state of qubit c to Alice via the control of the supervisor David. The senders perform a Bell-state measurement (BSM) respectively, and the success probability of this cyclic controlled teleportation scheme is 100%.

✉ Zhi-wen Sang
zhiwensang@163.com

[1] School of Physics and Electronic Information, Shangrao Normal University, Shangrao 334001, China

2 Cyclic Controlled Teleportation

Let us consider there are three observers Alice, Bob and Charlie, and each of them has an arbitrary single qubit a, b, c in arbitrary single-qubit state, which are given by

$$|\psi\rangle_a = a_0|0\rangle + a_1|1\rangle, \tag{1}$$

$$|\psi\rangle_b = b_0|0\rangle + b_1|1\rangle, \tag{2}$$

$$|\psi\rangle_c = c_0|0\rangle + c_1|1\rangle, \tag{3}$$

where $a_0, a_1, b_0, b_1, c_0, c_1$ are complex numbers and satisfy that $|a_0|^2 + |a_1|^2 = 1$, $|b_0|^2 + |b_1|^2 = 1$, $|c_0|^2 + |c_1|^2 = 1$. Now Alice wants to teleport her single-qubit state of qubit a to Bob, Bob wants to teleport his single-qubit state of qubit b to Charlie and Charlie wants to teleport his single-qubit state of qubit c to Alice. Suppose that Alice, Bob, Charlie and David share a seven-qubit entangled state, which is given by

$$\begin{aligned}|\varphi\rangle_{1234567} = \frac{1}{2\sqrt{2}}(&|0001110\rangle + |0011100\rangle + |0101010\rangle + |0111000\rangle \\ &+|1000111\rangle + |1010101\rangle + |1100011\rangle + |1110001\rangle)_{1234567},\end{aligned} \tag{4}$$

where the qubits 1 and 6 belong to Alice, the qubits 2 and 4 belong to Bob, the qubits 3 and 5 belong to Charlie, the qubit 7 belongs to David, respectively. Qubits a, b, $c$and 1, 2, 3, 4, 5, 6, 7 are in a pure product state, which is expressed as

$$|\xi\rangle_{abc1234567} = |\psi\rangle_a \otimes |\psi\rangle_b \otimes |\psi\rangle_c \otimes |\varphi\rangle_{1234567}. \tag{5}$$

In order to realize the cyclic controlled teleportation, Alice must apply a complete measurement of BSM on her qubits a and 1, and the measurement result of BSM is given by

$$|\Phi^+\rangle_{a1} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{a1}, \tag{6}$$

$$|\Phi^-\rangle_{a1} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{a1}, \tag{7}$$

$$|\Psi^+\rangle_{a1} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{a1}, \tag{8}$$

$$|\Psi^-\rangle_{a1} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{a1}. \tag{9}$$

If Alice's BSM result is $|\Phi^+\rangle_{a1}$, the other qubits b, c and 2, 3, 4, 5, 6, 7 are collapsed into the product state

$$\begin{aligned}|\mu\rangle_{bc234567} = &(b_0|0\rangle + b_1|1\rangle)_b \otimes (c_0|0\rangle + c_1|1\rangle)_c \\ &\otimes\frac{1}{2}(a_0|001110\rangle + a_0|011100\rangle + a_0|101010\rangle + a_0|111000\rangle \\ &+a_1|000111\rangle + a_1|010101\rangle + a_1|100011\rangle + a_1|110001\rangle)_{234567}.\end{aligned} \tag{10}$$

Subsequently, Bob performs a BSM on his own qubits b and 2, and one has

$$|\Phi^+\rangle_{b2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{b2}, \tag{11}$$

$$|\Phi^-\rangle_{b2} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{b2}, \tag{12}$$

$$|\Psi^+\rangle_{b2} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{b2}, \tag{13}$$

$$|\Psi^-\rangle_{b2} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{b2}. \tag{14}$$

If Bob's BSM result is $|\Phi^+\rangle_{b2}$, the other qubits c and 3, 4, 5, 6, 7 are collapsed into the following product state

$$\begin{aligned}|\zeta\rangle_{c34567} = {} & (c_0|0\rangle + c_1|1\rangle)_c \otimes \frac{1}{\sqrt{2}}(a_0b_0|01110\rangle + a_0b_0|11100\rangle + a_0b_1|01010\rangle \\ & + a_0b_1|11000\rangle + a_1b_0|00111\rangle + a_1b_0|10101\rangle + a_1b_1|00011\rangle + a_1b_1|10001\rangle)_{34567}.\end{aligned} \tag{15}$$

Thirdly, Charlie perform a BSM on his own qubits c and 3, and

$$|\Phi^+\rangle_{c3} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{c3}, \tag{16}$$

$$|\Phi^-\rangle_{c3} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{c3}, \tag{17}$$

$$|\Psi^+\rangle_{c3} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{c3}, \tag{18}$$

$$|\Psi^-\rangle_{c3} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{c3}. \tag{19}$$

If Charlie's BSM result is $|\Phi^+\rangle_{c3}$, then the other qubits 4, 5, 6 and 7 are collapsed into the following entangled state

$$\begin{aligned}|\upsilon\rangle_{4567} = {} & (a_0b_0c_0|1110\rangle + a_0b_0c_1|1100\rangle + a_0b_1c_0|1010\rangle + a_0b_1c_1|1000\rangle \\ & + a_1b_0c_0|0111\rangle + a_1b_0c_1|0101\rangle + a_1b_1c_0|0011\rangle + a_1b_1c_1|0001\rangle)_{4567}.\end{aligned} \tag{20}$$

Finally, a single-qubit measurement is performed by David with measurement base $\left\{\frac{\sqrt{2}}{2}(|0\rangle + |1\rangle)_7, \frac{\sqrt{2}}{2}(|0\rangle - |1\rangle)_7\right\}$. If David's single-qubit measured result is $\frac{\sqrt{2}}{2}(|0\rangle + |1\rangle)_7$, then the qubits 4, 5 and 6 will collapse into the following product state

$$\begin{aligned}|\chi\rangle_{456} = {} & (a_0b_0c_0|111\rangle + a_0b_0c_1|110\rangle + a_0b_1c_0|101\rangle + a_0b_1c_1|100\rangle \\ & + a_1b_0c_0|011\rangle + a_1b_0c_1|010\rangle + a_1b_1c_0|001\rangle + a_1b_1c_1|000\rangle)_{456} \\ = {} & (a_0|1\rangle + a_1|0\rangle)_4 \otimes (b_0|1\rangle + b_1|0\rangle)_5 \otimes (c_0|1\rangle + c_1|0\rangle)_6.\end{aligned} \tag{21}$$

Then Alice, Bob, Charlie perform a local unitary operation $\sigma_4^x \otimes \sigma_5^x \otimes \sigma_6^x$ on qubits 4, 5 and 6, thus the desired state can be reconstructed. Therefore, the cyclic controlled teleportation is successfully realized.

For other 127 measurement results, Alice, Bob, Charlie can perform an appropriate unitary operation on their own qubit according to the BSM results and the single-qubit measurement result, the cyclic controlled teleportation is realized easily.

3 Conclusions

In summary, we have demonstrated that a seven-qubit entangled state can be used to realize the cyclic controlled teleportation. In this work, Alice can teleport her single-qubit state of qubit a to Bob, Bob can teleport his single-qubit state of qubit b to Charlie and Charlie can also teleport his single-qubit state of qubit c to Alice via the control of the supervisor David. Alice, Bob and Charlie can operate an appropriate unitary operation to obtain the desired state. In addition, Alice, Bob and Charlie cannot obtain the original state without the help of David.

References

1. Zeilinger, A.: Nat. Phys. **14**, 3 (2018)
2. Ren, J.G., Xu, P., Yong, H.L., et al.: Nature **549**, 70 (2017)
3. Li, Y.H., Li, X.L., Nie, L.P., et al.: Int. J. Theor. Phys. **55**, 1820 (2016)
4. Sang, M.H., Nie, Y.Y.: Int. J. Theor. Phys. **56**, 3002 (2017)
5. Bennett, C.H., DiVincenzo, D.P., Shor, P.W., et al.: Phys. Rev. Lett. **077902**, 87 (2001)
6. Ye, M.Y., Zhang, Y.S., Guo, G.C.: Phys. Rev. A **022310**, 69 (2004)
7. Li, Y.H., Xiang, T., Nie, Y.Y., et al.: Sci. Rep. **7**, 43494 (2017)
8. Tanzilli, S., Tittel, W., Halder, M., et al.: Nature **437**, 116 (2005)
9. Li, Y.H., Xiang, T., Nie, Y.Y., et al.: Photon. Res. **5**, 324 (2017)
10. Muralidharan, S., Panigrahi, P.K.: Phys. Rev. A **062333**, 78 (2008)
11. Nie, Y.Y., Li, Y.H., Liu, J.C., et al.: Quantum Inf. Process. **10**, 297 (2011)
12. Zheng, S.B.: Phys. Rev. A **054303**, 74 (2006)
13. Zhang, Z., Shi, R., Zeng, G., et al.: Quantum Inf. Process. **17**, 133 (2018)
14. Long, G.L., Liu, X.S.: Phys. Rev. A **032302**, 65 (2002)
15. Poppe, A., Fedrizzi, A., Ursin, R., et al.: Opt. Express **12**, 3865 (2004)
16. Ma, P.C., Chen, G.B., Li, X.W., et al.: Int. J. Theor. Phys. **57**, 2233 (2018)
17. Zha, X.W., Zou, Z.C., Qi, J.X., et al.: Int. J. Theor. Phys. **52**, 1740 (2013)
18. Hong, W.: Int. J. Theor. Phys. **55**, 384 (2016)
19. Zhang, Q., Cheng, X., Chen, H., et al.: Photon. Res. **6**, 162 (2018)
20. Xiang, T., Sun, Q.C., Li, Y.H., et al.: Phys. Rev. A **063810**, 97 (2018)
21. Wang, J., Guo, Y., Liu, H., et al.: Photon. Res. **6**, 647 (2018)
22. Yang, X., Wei, K., Ma, H., et al.: Sci. Rep. **8**, 5728 (2018)
23. Macchiavello, C., Sacchi, M.F.: Phys. Rev. A **012303**, 97 (2018)
24. Li, Y.H., Li, X.L., Sang, M.H., et al.: Quantum Inf. Process. **12**, 3835 (2013)
25. Li, Y.H., Jin, X.M.: Quantum Inf. Process. **15**, 929 (2016)
26. Tian, M.B., Zhang, G.F.: Quantum Inf. Process. **17**, 19 (2018)
27. Oh, C., Kim, H., Jeong, K., et al.: Sci. Rep. **7**, 3765 (2017)
28. Yang, X., Bai, M., Mo, Z.: Int. J. Theor. Phys. **56**, 3525 (2017)
29. Chen, Y.X., Du, J., Liu, S.Y., et al.: Quantum Inf. Process. **16**, 201 (2017)