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Improving the Teleportation Scheme of Five-Qubit State with a Seven-Qubit Quantum Channel

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Abstract

Recently, Min Li et al. (Int. J. Theor. Phys. **56**, 2710, 2017) proposed an improved quantum teleportation scheme for one five-qubit unknown state with a seven-qubit quantum channel. In this paper, we present an improved protocol with only single-qubit measurements and the same seven-qubit quantum channel. Compared with previous scheme proposed, our scheme has obvious advantages of requiring fewer classical resources, possessing higher intrinsic efficiency and lower operation complexity that bring a better flexibility.

Keywords Quantum teleportation · Cluster state · Five-qubit state

1 Introduction

In recent years, quantum teleportation (QT) [1–3] has received extensive attention and developed rapidly. It is used to transmit unknown states by use of shared entanglement and classical channels between the sender A (conventionally named Alice) and the receiver B (conventionally named Bob). In 1993, Bennett et al. [4] first presented a quantum teleportation scheme in which an arbitrary unknown single qubit state could be teleported from a sender to a distant receiver with the aid of an Einstein-Podolsky-Rosen (EPR) pair [5–7]. This protocol was experimentally verified soon in 1997 [8]. Since the original work, due to its important applications in quantum communication, quantum teleportation has attracted widespread attention, and has been widely studied in past years both theoretically and experimentally. In 1998, Karlsson and Bourennane [9] presented a protocol for teleporting a two-qubit state with GHZ states instead of an EPR pair. In 2002, B.S. Shi et al. [10] proposed a teleportation protocol via W state, which could teleport an unknown state probabilistically. In 2005, Deng et al. [11] introduced a symmetric multiparty controlled teleportation scheme for an arbitrary two-particle entangled state [12–16]. In 2013, Zha et al. [17] firstly presented the original bidirectional quantum controlled teleportation protocol via a five-qubit cluster state.

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In the past few years, we have witnessed rapid development of quantum teleportation where various quantum states such as Bell states [18], GHZ states [19] and W states have served as different quantum channels [20] to satisfy a variety of different quantum communication scenarios. Besides quantum teleportation, there are some other important quantum communication protocols, such as quantum key distribution (QKD) [21–23], quantum secret sharing (QSS) [24–26], quantum secure direct communication (QSDC) [27–29], joint remote state preparation (JRSP) [30–32] and so on. Recently, Min Li et al. [33] proposed an improved quantum teleportation scheme for one five-qubit unknown state with a seven-qubit quantum channel.

In this paper, we put forward an improved scheme with single-qubit measurements only. In this way, our improved scheme possesses higher intrinsic efficiency and requires fewer classical resources and lower technical complexity. The organization of the rest of this paper is as follows. In the next section, we review the scheme proposed by Min Li et al. [33]. In Section 3, the improvement of protocol is studied in details. The comparison is given in Section 4. Finally, a summary is given in Section 5.

2 The Protocol of Min Li et al.

In this section, first of all, we review the idea of Min Li et al. [33]. In their scheme, Alice aims to transmit an unknown five-qubit entangled state to a distant receiver Bob. The five-qubit unknown state they take into account is described by

$$|\chi\rangle_{abcde} = (\alpha|00000\rangle + \beta|00011\rangle + \gamma|11100\rangle + \delta|11111\rangle)_{abcde} \tag{1}$$

where α, β, γ and δ are unknown coefficients satisfying the relation $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. In the beginning, Alice and Bob share a seven-qubit cluster state

$$|\phi\rangle_{1234567} = \frac{1}{2}(|0000000\rangle + |0001101\rangle + |1110010\rangle + |1111111\rangle)_{1234567} \tag{2}$$

The qubits a, b, c, d, e, 6 and 7 belong to Alice and qubits 1, 2, 3, 4 and 5 belong to Bob respectively. Now the state of the whole system composed of the unknown five-qubit state and the quantum channels is

$$|\tau\rangle_{abcde1234567} = |\chi\rangle_{abcde} \otimes |\phi\rangle_{1234567} \tag{3}$$

To achieve teleportation, Alice performs a seven-qubit joint measurement on her qubits a, b, c, d, e, 6 and 7. However, in their scheme, the calculation procedure seems a little complicated. In particular, a seven-qubit joint measurement is a huge technical challenge in terms of current experimental technology. Considering the operational difficulty of their scheme, we propose an improved scheme with single-qubit measurements only instead of a seven-qubit joint measurement.

3 The Improvement

We first recall two important gates, Hadamard gate and CNOT gate.

Hadamard gate is performed onto a single qubit mapping the basis state $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|1\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. It's operation corresponds to the following transformation matrix:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{4}$$

CNOT gate can be represented by the following matrix

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

It operates on a quantum register consisting of 2 qubits. One is the controlling qubit and the other is the target qubit. If the state of the controlling qubit is 1, the state of the target bit is flipped. Otherwise, the state of the target bit remains the same.

Now, let's present our improved scheme.

In the first step as shown in Fig. 1(I). Alice performs CNOT gates on her qubits a, b, c, d, e. Firstly, qubit a serves as a controlling qubit and both qubits b and c act as two target qubits. Meanwhile, qubit d works as the controlling qubit and qubit e acts as the target qubit. After that, the following state can be obtained as

$$|\chi\rangle_{abcde} = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{ad}|000\rangle_{bce} \tag{6}$$

Now the quantum state of the system composed of qubits a, d, 1, 2, 3, 4, 5, 6, 7 is written as

$$\begin{aligned} |\tau\rangle_{ab1234567} = \frac{1}{2}&(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle_{ad} \otimes |0000000\rangle + |0001101\rangle \\ &+|1110010\rangle + |1111111\rangle)_{1234567} \end{aligned} \tag{7}$$

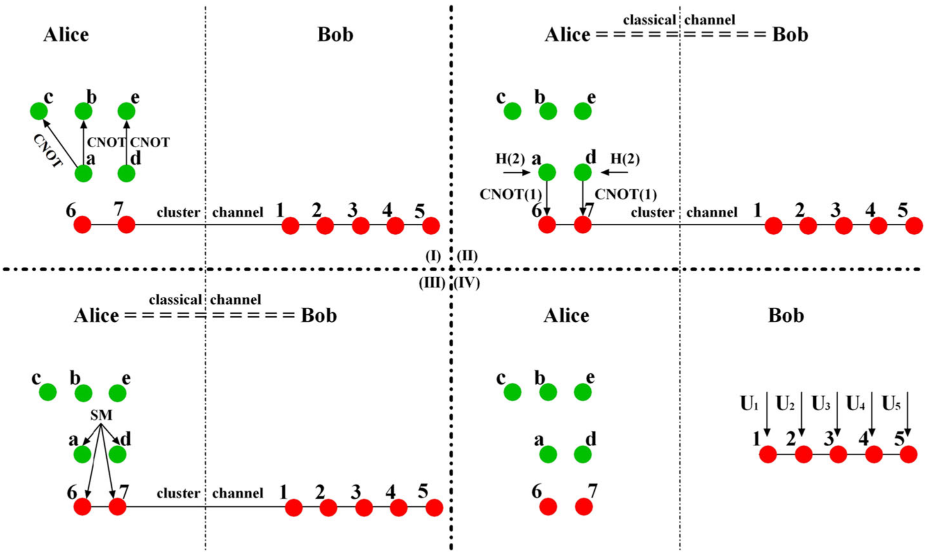

Fig. 1 Sketch of our improved teleportation scheme of five-qubit state with seven-qubit quantum channels. Here H, SM and U denote the Hadamard gate operation, the single-qubit measurement and single-qubit unitary operation, respectively

In the second step, as illustrated in Fig. 1(II), Alice performs a CNOT operation on qubits (a,6) and qubits (d,7) where qubits a and d act as controlling qubits and qubits 6 and 7 serve as target qubits, respectively. Now the above state $|\tau\rangle_{ad1234567}$ evolve to

$$\begin{aligned}|\tau'\rangle_{ad1234567} = \frac{1}{2}[&\alpha|00\rangle_{ad} \otimes (|0000000\rangle + |0100011\rangle + |1011100\rangle + |1111111\rangle)_{6712345}\\ &+\beta|01\rangle_{ad} \otimes (|0100000\rangle + |0000011\rangle + |1111100\rangle + |1011111\rangle)_{6712345}\\ &+\gamma|10\rangle_{ad} \otimes (|1000000\rangle + |1100011\rangle + |0011100\rangle + |0111111\rangle)_{6712345}\\ &+\delta|11\rangle_{ad} \otimes (|1100000\rangle + |1000011\rangle + |0111100\rangle + |0011111\rangle)_{6712345}]\end{aligned} \tag{8}$$

Alice performs a Hadamard operation on her qubits a and d, which transforms the state $|\tau'\rangle_{ad1234567}$ into the following form

$$\begin{aligned}|\tau''\rangle_{ad1234567} &= \frac{1}{4}[\alpha(|0\rangle + |1\rangle)_a(|0\rangle + |1\rangle)_d \otimes (|0000000\rangle + |0100011\rangle\\ &\quad +|1011100\rangle + |1111111\rangle)_{6712345}\\ &\quad +\beta(|0\rangle + |1\rangle)_a(|0\rangle - |1\rangle)_d \otimes (|0100000\rangle + |0000011\rangle\\ &\quad +|1111100\rangle + |1011111\rangle)_{6712345}\\ &\quad +\gamma(|0\rangle - |1\rangle)_a(|0\rangle + |1\rangle)_d \otimes (|1000000\rangle + |1100011\rangle\\ &\quad +|0011100\rangle + |0111111\rangle)_{6712345}\\ &\quad +\delta(|0\rangle - |1\rangle)_a(|0\rangle - |1\rangle)_d \otimes (|1100000\rangle + |1000011\rangle\\ &\quad +|0111100\rangle + |0011111\rangle)_{6712345}]\\ &= \frac{1}{4}[|0000\rangle_{ad67}(\alpha|00000\rangle + \beta|00011\rangle + \gamma|11100\rangle + \delta|11111\rangle)_{12345}\\ &\quad +|0001\rangle_{ad67}(\alpha|00011\rangle + \beta|00000\rangle + \gamma|11111\rangle + \delta|11100\rangle)_{12345}\\ &\quad +|0010\rangle_{ad67}(\alpha|11100\rangle + \beta|11111\rangle + \gamma|00000\rangle + \delta|00011\rangle)_{12345}\\ &\quad +|0011\rangle_{ad67}(\alpha|11111\rangle + \beta|11100\rangle + \gamma|00011\rangle + \delta|00000\rangle)_{12345}\\ &\quad +|0100\rangle_{ad67}(\alpha|00000\rangle - \beta|00011\rangle + \gamma|11100\rangle - \delta|11111\rangle)_{12345}\\ &\quad +|0101\rangle_{ad67}(\alpha|00011\rangle - \beta|00000\rangle + \gamma|11111\rangle - \delta|11100\rangle)_{12345}\\ &\quad +|0110\rangle_{ad67}(\alpha|11100\rangle - \beta|11111\rangle + \gamma|00000\rangle - \delta|00011\rangle)_{12345}\\ &\quad +|0111\rangle_{ad67}(\alpha|11111\rangle - \beta|11100\rangle + \gamma|00011\rangle - \delta|00000\rangle)_{12345}\\ &\quad +|1000\rangle_{ad67}(\alpha|00000\rangle + \beta|00011\rangle - \gamma|11100\rangle - \delta|11111\rangle)_{12345}\\ &\quad +|1001\rangle_{ad67}(\alpha|00011\rangle + \beta|00000\rangle - \gamma|11111\rangle - \delta|11100\rangle)_{12345}\\ &\quad +|1010\rangle_{ad67}(\alpha|11100\rangle + \beta|11111\rangle - \gamma|00000\rangle - \delta|00011\rangle)_{12345}\\ &\quad +|1011\rangle_{ad67}(\alpha|11111\rangle + \beta|11100\rangle - \gamma|00011\rangle - \delta|00000\rangle)_{12345}\\ &\quad +|1110\rangle_{ad67}(\alpha|00000\rangle - \beta|00011\rangle - \gamma|11100\rangle + \delta|11111\rangle)_{12345}\\ &\quad +|1101\rangle_{ad67}(\alpha|00011\rangle - \beta|00000\rangle - \gamma|11111\rangle + \delta|11100\rangle)_{12345}\\ &\quad +|1110\rangle_{ad67}(\alpha|11100\rangle - \beta|11111\rangle - \gamma|00000\rangle + \delta|00011\rangle)_{12345}\\ &\quad +|1111\rangle_{ad67}(\alpha|11111\rangle - \beta|11100\rangle - \gamma|00011\rangle + \delta|00000\rangle)_{12345}\end{aligned} \tag{9}$$

In the third step as shown in Fig. 1(III), Alice performs a single-qubit measurement with measurement basis $\{|0\rangle, |1\rangle\}$ which is performed on her system of qubits a, d, 6 and 7 and sends her measurement results to Bob via classical channels.

In the final step as depicted in Fig. 1(IV), according to Alice's measurement outcomes, Bob implements an appropriate unitary operation on his five-qubits 1, 2, 3, 4, 5 to restore the state $(\alpha|0000\rangle+\beta|00011\rangle+\gamma|11100\rangle+\delta|11111\rangle)_{abcde}$. The detail relationship between Alice's measurement result and Bob's unitary operation with Pauli matrices $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and the identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are respectively given in Table 1.

For example, if Bob receives the measurement outcome $|1111\rangle$, he needs to apply five single-qubit unitary operations $(\sigma_y)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_y)_4 \otimes (\sigma_x)_5$ on his qubits 1, 2, 3, 4, 5 to recover the initial input state $(\alpha|00000\rangle + \beta|00011\rangle + \gamma|11100\rangle + \delta|11111\rangle)_{12345}$.

In this way, finally Bob could obtain the original state $|\chi\rangle_{abcde}$ in (1) with success possibility 100%.

4 Comparison

A comparison is made between our scheme and Ref. [33] from the following four aspects: the cost of quantum resource (QR), the cost of classical resource (CR), intrinsic efficiency of the scheme and the difficulty of necessary operation (NO).

First, both our scheme and Ref. [33] can be implemented with a seven-qubit cluster state acting as the quantum channel. Thus the cost of QR for both schemes is the same, i.e., a seven-qubit cluster state.

Second, in Ref. [33], since Alice performs a seven-qubit joint measurement on qubits a, b, c, d, e, 6 and 7 and her measurement results correspond to seven bits. Therefore, the cost of CR in their scheme is 7. However, in our scheme Alice utilizes a single-qubit measurement on qubits a, d, 6 and 7 and her measurement results is denoted by four bits. This means that

Table 1 The corresponding relation between Alices's measurement results (ARMs) and Bob's operation U_{12345}

ARMs	U_{12345}
$\|0000\rangle$	$I_1 \otimes I_2 \otimes I_3 \otimes I_4 \otimes I_5$
$\|0001\rangle$	$I_1 \otimes I_2 \otimes I_3 \otimes (\sigma_x)_4 \otimes (\sigma_x)_5$
$\|0010\rangle$	$(\sigma_x)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes I_4 \otimes I_5$
$\|0011\rangle$	$(\sigma_x)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_x)_4 \otimes (\sigma_x)_5$
$\|0100\rangle$	$I_1 \otimes I_2 \otimes I_3 \otimes (\sigma_z)_4 \otimes I_5$
$\|0101\rangle$	$I_1 \otimes I_2 \otimes I_3 \otimes (\sigma_y)_4 \otimes (\sigma_x)_5$
$\|0110\rangle$	$(\sigma_x)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_Z)_4 \otimes I_5$
$\|0111\rangle$	$(\sigma_x)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_y)_4 \otimes (\sigma_x)_5$
$\|1000\rangle$	$(\sigma_z)_1 \otimes I_2 \otimes I_3 \otimes I_4 \otimes I_5$
$\|1001\rangle$	$(\sigma_z)_1 \otimes I_2 \otimes I_3 \otimes (\sigma_x)_4 \otimes (\sigma_x)_5$
$\|1010\rangle$	$(\sigma_y)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes I_4 \otimes I_5$
$\|1011\rangle$	$(\sigma_y)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_x)_4 \otimes (\sigma_x)_5$
$\|1100\rangle$	$(\sigma_z)_1 \otimes I_2 \otimes I_3 \otimes (\sigma_z)_4 \otimes I_5$
$\|1101\rangle$	$(\sigma_z)_1 \otimes I_2 \otimes I_3 \otimes (\sigma_y)_4(\sigma_x)_5$
$\|1110\rangle$	$(\sigma_y)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_z)_4 \otimes I_5$
$\|1111\rangle$	$(\sigma_y)_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3 \otimes (\sigma_y)_4 \otimes (\sigma_x)_5$

in our scheme the cost of CR is 4. By comparing the two schemes, we show that our scheme has lower cost of CR than the scheme in Ref. [33].

Third, efficiency is a main factor that can be utilized for comparing the performance of schemes. In quantum communication schemes, the efficiency can be defined as [34]

$$\eta = \frac{q_t}{q_s + b_t} \tag{10}$$

where q_t is the number of qubits that consist of the quantum information to be exchanged, q_s is the number of the qubits that are used as the quantum channel and b_t is the classical bits exchanged between participants. Therefore, in our scheme, for purpose of transmitting an unknown five-qubit entangled state to a distant receiver Bob ($q_t = 5$), Alice should prepare a seven-qubit cluster state ($q_s = 7$) and then informs her measurement results to Bob via a classical channel ($b_t = 4$). Without considering the quantum resources and the classical resources which are used for identity authentication and security checks, qubit efficiency is calculated as $\eta = 5/11$. Similarly, according to the definition of qubit efficiency, the qubit efficiency of Ref. [33] is calculated as $\eta_1 = 5/14$. Consequently, our scheme has higher intrinsic efficiency.

Finally, let us show the necessary operations of our scheme. Here some simple operations such as CNOT, and single-qubit operations in our scheme are utilized. All these operations involved in our scheme could be achieved by present technologies which makes it more feasible.

Therefore, compared with previous study for one five-qubit unknown state with a seven-qubit quantum channel schemes, our scheme has the advantages of consuming fewer classical resources, possessing higher intrinsic efficiency and lessening the difficulty of necessary operations.

5 Summary

In summary, we have revisited and improved the scheme proposed by Min Li et al. for teleporting a five-qubit state using seven-qubit quantum channels with a seven-qubit joint measurement. Considering the feasibility of this scheme, we only employ some simple operations such as single-qubit quantum logic gates, single-qubit measurements and CNOT [35] gates. All these operations are feasible in experiments and our scheme demonstrates a higher efficiency than before. Hence, we improve the previous scheme in a relatively simple way. With the development of quantum communication technology, we hope that it will be actually implemented in labs in the future.

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References

1. Vaidman, L.: Teleporting of quantum states. Phys. Rev. A **49**, 1473 (1994)
2. Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A **58**, 4394 (1998)
3. Pirandola, S., Eisert, J., Weedbrook, C., Furusawa, A., Braunstein, S.L.: Advances in quantum teleportation. Nat. Photonics **9**, 641–652 (2015)

4. Bennett, C.H., Brassard, G., Crépeau, C., et al.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
5. Hagley, E., Maitre, X., Nogues, G.: Generation of Einstein-Podolsky-Rosen pairs of atoms. Phys. Rev. Lett. **79**, 1 (1997)
6. Gneiting, C., Hornberger, K.: Nonlocal Young tests with Einstein-Podolsky-Rosen-correlated particle pairs. Phys. Rev. A **88**, 013610 (2013)
7. Deng, F.G., Li, X.H., et al.: Quantum secure direct communication network with Einstein-Podolsky-Rosen pairs. Phys. Lett. A **359**, 359–365 (2006)
8. Bouwmeester, D., Pan, J.W., Mattle, K., Eibl, M., Weinfurter, H., Zeilinger, A.: Experimental quantum teleportation. Nature **390**, 575–579 (1997)
9. Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A **58**, 4394–4400 (1998)
10. Bao, S.S., Tomita, A.: Teleportation of an unknown state by W state. Phys. Lett. A **296**, 161–164 (2002)
11. Deng, F.G., Li, C.Y., Li, Y.S., Zhou, H.Y., Wang, Y.: Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. Phys. Rev. A **72**, 022338 (2005)
12. Yeo, Y., Chua, W.K.: Teleportation and dense coding with genuine multipartite entanglement. Phys. Rev. Lett. **96**, 060502 (2006)
13. Roa, L., Delgado, A., Fuentes-Guridi, I.: Optimal conclusive teleportation of quantum states. Phys. Rev. A **68**, 022310 (2003)
14. Wang, M.Y., Yan, F.L.: Chain teleportation via partially entangled states. Eur. Phys. J. D **54**, 111–114 (2009)
15. Li, H.Q., Xu, S.M., Xu, X.L., et al.: The construction of the generalized continuously variable two-mode entangled state and its application. Sci. China. Ser. G-Phys. Mech. Astron. **52**, 1932–1937 (2009)
16. Zhang, X.H., Yang, Z.Y., Xu, P.P.: Teleporting N-qubit unknown atomic state by utilizing the V-type three-level atom. Sci China Ser G-Phys Mech. Astron. **52**, 1034–1038 (2009)
17. Zha, X.W., Zou, Z.C., Qi, J.X., Song, H.Y.: Bidirectional quantum controlled teleportation via five-qubit cluster state. Int. J. Theor. Phys. **52**, 1740–1744 (2013)
18. Wang, W.H., Cao, H.X.: An improved multiparty quantum secret sharing with bell states and bell measurement. Int. J. Theor. Phys. **52**, 2099–2111 (2013)
19. Gao, T.: Controlled and secure direct communication using GHZ state and teleportation. Z. Naturforsch. A **59**, 597–601 (2004)
20. Masashi, B.: Correlation and information in quantum channels. Int. J. Theor. Phys. **43**, 323–339 (2004)
21. Renner, R.: Security of quantum key distribution. Int. J. Quantum Inform. **6**, 1–127 (2008)
22. Degiovanni, I.P., Berchera, I.R.: Quantum dense key distribution. Phys. Rev. A **69**, 032310 (2004)
23. Kurtsiefer, C., Zarda, P.: A step towards global key distribution. Nature **419**, 450 (2002)
24. Hillery, M., Buzek, V., Berthiaume, A.: Quantum secret sharing. Phys. Rev. A **59**, 1829 (1999)
25. Deng, F.G., Li, X.H.: Improving the security of multiparty quantum secret sharing against Trojan horse attack. Phys. Rev. A **72**, 044302 (2005)
26. Gottesman, D.: Theory of quantum secret sharing. Phys. Rev. A **61**, 042311 (2000)
27. Bostrom, K., Felbinger, T.: Deterministic secure direct communication using entanglement. Phys. Rev. Lett. **89**, 187902 (2002)
28. Lin, S., Wen, Q.Y., et al.: Quantum secure direct communication with chi-type entangled states. Phys. Rev. A **78**, 064304 (2008)
29. Jin, X.R., Ji, X., et al.: Three-party quantum secure direct communication based on GHZ states. Phys. Lett. A **354**, 67–70 (2006)
30. Luo, M.X., Chen, X.B., et al.: Joint remote preparation of an arbitrary three-qubit state. Opt. Commun. **283**, 4796–4801 (2010)
31. Chen, Q.Q., Xia, Y., et al.: Joint remote state preparation of a W-type state via W-type states. Phys. Lett. A **374**, 4483–4487 (2010)
32. An, N.B., Bich, C.T., et al.: Deterministic joint remote state preparation. Phys. Lett. A **375**, 3570–3573 (2011)
33. Li, M., Zhao, N., et al.: Quantum teleportation of five-qubit state. Int. J. Theor. Phys. **56**, 2710–2715 (2017)
34. Yuan, H., Liu, Y.M., Zhang, W., et al.: Optimizing resource consumption, operation complexity and efficiency in quantum-state sharing. J. Phys. B: At. Mol. Opt. Phys. **41**, 145506 (2008)
35. Gao, W.B., Xu, P., Yao, X.C., et al.: Experimental realization of a controlled-NOT gate with four-photon six-qubit cluster states. Phys. Rev. Lett. **104**, 020501 (2010)