



Influence of an External Classical Field on the Interaction Between a Field and an Atom in Presence of Intrinsic Damping

A.-S. F. Obada¹ · E. M. Khalil^{1,2} · M. M. A. Ahmed¹ ·
M. M. Y. Elmalky¹ 

Received: 10 January 2018 / Accepted: 7 June 2018 / Published online: 21 June 2018
© Springer Science+Business Media, LLC, part of Springer Nature 2018

Abstract The effect of intrinsic damping on the interaction between a two-level atom and a multi-photon cavity field in the presence of an external classical field is studied. Under certain conditions and use of a transformation, the system is transformed to a generalized Jaynes Cummings model, with the influence of classical field included in the detuning parameter. The temporal evolution of some statistical aspects such as, the atomic inversion, the squeezing phenomena and linear entropy are obtained. In addition, we present the effects of the intrinsic damping and detuning parameters on the above mentioned quantities, for one and two photons. The entropy is used as a measure of the degree of entanglement, and consequently discussed.

Keywords External classical field · Atomic inversion · Damping · Linear entropy · Variance squeezing · Entropy squeezing

1 Introduction

Intrinsic damping is one of the methods to study damping in quantum mechanics [1]. In general, a system suffers a damping rate due to coupling to some sufficiently large external reservoir. The intrinsic damping approach has been investigated in the framework of several models, in particular Jaynes -Cummings model(JCM) [2]. Furthermore, JCM was solved for intrinsic damping in resonance case [3]. The collapse and revival phenomenon and the photon-number distribution for multiphoton JCM in off-resonance case process were studied [4]. The influence of intrinsic damping on the growth of entropy and the entanglement

✉ M. M. Y. Elmalky
m.elmalky@azhar.edu.eg

¹ Mathematics Department, Faculty of Science, Al-Azhar University, Nasr City 11884, Cairo, Egypt

² Mathematics Department, Faculty of Science, Taif University, Taif, Saudi Arabia

for multiphoton JCM was studied analytically [5]. Another work studied the effect of intrinsic damping on the entropy squeezing of coupled field-superconducting charge qubit [6], a general solution for the total correlation function between any two charge qubits initially prepared in a mixed state in presence of intrinsic damping was explicitly investigated [7]. The phenomena of entanglement decay, sudden rebirth, and sudden death as well as for the local quantum uncertainty, uncertainty-induced quantum non-locality under influence of intrinsic decoherence have been discussed [8]. The model of two charged qubits, in a SC-cavity field is studied, it is shown that the entanglement depend on the phase decoherence rate [9]. The Negativity and the atomic population for the interaction between su(1,1) and a two two-level in non-resonance case is obtained [10]. The quantum Fisher information and von Neumann entropy are investigated numerically for N-level atomic system in the presence of intrinsic decoherence, where it is found that the local maximum value of entanglement decrease [11]. The information about degree of entanglement a JCM with k-photon process is obtained, by comparing the results for the atomic Wehrl entropy and negativity with the analytical results [12]. The degree of entanglement was also discussed for JCM by using quantum mutual entropy [13]. The interaction between a system of N two-level atoms in a cavity of quantized field with a strong classical field has been studied and a method of generating entanglement has been proposed [14]. The effect of damping on atomic variable squeezing or squeezing in the entropy leads to degradation of squeezing [15]. The influence of the external classical field on the interaction between a two level atom and a quantized cavity field has been studied [16] and the existence of the classical field leads to a variation in the atomic inversion. It has a great effect on the phenomenon of squeezing and the linear entropy, as well as to improve the amount of entanglement [17].

In this paper, we study the influence of an external classical field on a two-level atom interacting with a quantized field in presence of the intrinsic damping. In Section 2 we apply the rotating wave approximation(RWA) and some canonical transformations for the excited state and ground state to transform the system into a JCM with the classical field term included in the detuning parameter. By using the exact solution of Milburn equation we get the density operator for this system to investigate some statistical aspects. In Section 3, we discuss the collapses and revivals of the atomic inversion for different values of classical field and intrinsic damping parameters. Section 4 is devoted to he phenomenon of squeezing by using the variance and entropy squeezing. Section 5 is devoted to examine influence of classical field and damping parameters on the degree of entanglement by using the linear entropy for the field. Finally, we conclude our work in Section 6.

2 The Model

The interaction between an electromagnetic quantized field via k-photon and a two-level atom in the presence of an external field can be described by the Hamiltonian ($\hbar = 1$):

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_0}{2} \hat{\sigma}_z + i\lambda_1 (\hat{a}^k + \hat{a}^{\dagger k}) (\hat{\sigma}_+ - \hat{\sigma}_-) + \lambda_2 (\hat{\sigma}_+ e^{-i\omega_c t} + \hat{\sigma}_- e^{i\omega_c t}), \quad (1)$$

where ω , ω_0 and ω_c are frequencies of the cavity field, the atomic transition and the classical field respectively, $\hat{\sigma}_z$, $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are the Pauli spin operators; they are defined by ground state ($|g\rangle$) and excited state ($|e\rangle$) of the atom as $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, and $\hat{\sigma}_- = |g\rangle\langle e|$, such that $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$, and $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$. While \hat{a} , and \hat{a}^\dagger are annihilation and creation operators of the cavity field which satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = I$,

λ_1 and λ_2 are the coupling constants of the interaction of the atom with the cavity field and with the classical field respectively.

By using the transformation $\hat{U}_1 = e^{-i\omega_c \hat{\sigma}_z t/2}$ to transform the Hamiltonian (1) to \hat{H}_1 where.

$$\begin{aligned} \hat{H}_1 &= \hat{U}_1^\dagger \hat{H} \hat{U}_1 - i\hat{U}_1^\dagger \frac{\partial \hat{U}_1}{\partial t}, \\ &= \omega \hat{a}^\dagger \hat{a} + \frac{\omega_0 - \omega_c}{2} \hat{\sigma}_z + i\lambda_1 (\hat{a}^k + \hat{a}^{\dagger k}) (\hat{\sigma}_+ e^{i\omega_c t} - \hat{\sigma}_- e^{-i\omega_c t}) + \lambda_2 (\hat{\sigma}_+ + \hat{\sigma}_-). \end{aligned} \tag{2}$$

If we define a new excited (ground) state as $|+\rangle$ ($|-\rangle$), and the operators \hat{S}_s where $\hat{S}_z = |+\rangle\langle +| - |-\rangle\langle -|$, $\hat{S}_+ = |+\rangle\langle -|$ and $\hat{S}_- = |-\rangle\langle +|$ which satisfy commutation relations $[\hat{S}_z, \hat{S}_\pm] = \pm 2\hat{S}_\pm$ and $[\hat{S}_+, \hat{S}_-] = \hat{S}_z$ which are related to the states $|e\rangle$ and $|g\rangle$ by the following form:

$$\begin{aligned} |+\rangle &= \cos\xi |e\rangle + \sin\xi |g\rangle, \quad |-\rangle = \cos\xi |g\rangle - \sin\xi |e\rangle, \\ \xi &= \frac{1}{2} \tan^{-1} \left(\frac{2\lambda_2}{\omega_0 - \omega_c} \right), \quad -\frac{\pi}{2} < \xi < \frac{\pi}{2}. \end{aligned} \tag{3}$$

So, we can write the \hat{S}_s operators in terms of the $\hat{\sigma}_s$ operators as following:

$$\begin{bmatrix} \hat{S}_z \\ \hat{S}_+ \\ \hat{S}_- \end{bmatrix} = \begin{bmatrix} \cos 2\xi & \sin 2\xi & \sin 2\xi \\ -\frac{1}{2} \sin 2\xi & \cos 2\xi & -\sin 2\xi \\ -\frac{1}{2} \sin 2\xi & -\sin 2\xi & \cos 2\xi \end{bmatrix} \begin{bmatrix} \hat{\sigma}_z \\ \hat{\sigma}_+ \\ \hat{\sigma}_- \end{bmatrix} \tag{4}$$

At $\lambda_2 = 0$ i.e. $\xi = 0$ it is easy to see that $|+\rangle = |e\rangle$ and $|-\rangle = |g\rangle$. After applying this transformation and using the rotating wave approximation, the Hamiltonian (2) can be cast in the following form:

$$\hat{H}_2 = \omega \hat{a}^\dagger \hat{a} + \frac{\Delta}{2} \hat{S}_z + i\lambda_1 \cos^2 \xi (\hat{a}^k \hat{S}_+ e^{i\omega_c t} - \hat{a}^{\dagger k} \hat{S}_- e^{-i\omega_c t}), \tag{5}$$

where $\Delta = \sqrt{(\omega_0 - \omega_c)^2 + 4\lambda_2^2}$, we note that the classical field parameter is now included in Δ as well as in the new coupling $\lambda_1 \cos^2 \xi$. Now, we use the transformation $\hat{U}_2 = e^{i\omega_c \hat{S}_z t/2}$ to transform the Hamiltonian (5) to $\hat{\mathcal{H}} = \hat{U}_2^\dagger \hat{H}_2 \hat{U}_2 - i\hat{U}_2^\dagger \frac{\partial \hat{U}_2}{\partial t}$.

Then, we can rewrite it in the rotating reference frame as follows:

$$\hat{\mathcal{H}} = \omega \hat{a}^\dagger \hat{a} + \left(\frac{\Delta + \omega_c}{2} \right) \hat{S}_z + i\lambda_1 \cos^2 \xi (\hat{a}^k \hat{S}_+ - \hat{a}^{\dagger k} \hat{S}_-), \tag{6}$$

which is a general form of the JCM with the atomic frequency shifted to $\Delta + \omega_c$.

Milburn assumed that [1, 3], the system changes stochastically in the presence of sufficiently short time steps. Based on this assumption, he obtained the rate of change of density operator $\hat{\rho}(t)$ which alters the Schrödinger dynamics as follows:

$$\frac{d\hat{\rho}}{dt} = \gamma \left(e^{-\frac{i}{\gamma} \hat{\mathcal{H}}} \hat{\rho}(t) e^{\frac{i}{\gamma} \hat{\mathcal{H}}} - \hat{\rho}(t) \right), \tag{7}$$

where γ is the decay rate and corresponds to a sufficiently short time step. The system develops gradually with a probability $\mathcal{P}(\tau) = \gamma\tau$. Also, the von Neumann equation for the density matrix is recovered in the limit $\gamma \rightarrow \infty$ (time step equal zero). For the first order in γ^{-1} the time evolution density operator is obtained:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{\mathcal{H}}, \hat{\rho}] - \frac{1}{2\gamma} [\hat{\mathcal{H}}, [\hat{\mathcal{H}}, \hat{\rho}]]. \tag{8}$$

We express the solution for the density operator $\hat{\rho}(t)$ in (8) by using superoperator technique [18] and applying it on the Hamiltonian (6) thus we have:

$$\hat{\rho}(t) = e^{\hat{R}t} e^{\hat{S}t} e^{\hat{T}t} \hat{\rho}(0), \tag{9}$$

where the three superoperators are defined as $\hat{R}\hat{\rho} = \frac{1}{\gamma} \hat{\mathcal{H}} \hat{\rho} \hat{\mathcal{H}}^*$, $\hat{S}\hat{\rho} = -i[\hat{\mathcal{H}}, \hat{\rho}]$ and $\hat{T}\hat{\rho} = -\frac{1}{2\gamma} \{ \hat{\mathcal{H}}^2, \hat{\rho} \}$ so that:

$$\begin{aligned} e^{\hat{R}t} \hat{\rho}(t) &= \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{t}{\gamma}\right)^m \hat{\mathcal{H}}^m \hat{\rho}(t) \hat{\mathcal{H}}^{*m}, \\ e^{\hat{S}t} \hat{\rho}(t) &= e^{-i\hat{\mathcal{H}}t} \hat{\rho}(t) e^{i\hat{\mathcal{H}}t}, \\ e^{\hat{T}t} \hat{\rho}(t) &= e^{-\frac{t}{2\gamma} \hat{\mathcal{H}}^2} \hat{\rho}(t) e^{-\frac{t}{2\gamma} \hat{\mathcal{H}}^2}. \end{aligned} \tag{10}$$

Also, $\hat{\rho}(0)$ is the initial density operator of the system (atom-field). To assume the effect of intrinsic damping on the system, we consider $\hat{\rho}(0) = |+, \alpha\rangle\langle+, \alpha|$, where the atom is prepared initially in its new excited state $|+\rangle$ and the field in the coherent state.

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} (\alpha^n / \sqrt{n!}) |n\rangle. \tag{11}$$

The Hamiltonian (6) can be written as $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1$, where $[\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_1] = 0$. By using $\lambda = \lambda_1 \cos^2 \xi$, then $\hat{\mathcal{H}}_0$, and $\hat{\mathcal{H}}_1$ can be written as:

$$\hat{\mathcal{H}}_0 = \omega \left(\hat{a}^\dagger \hat{a} + \frac{k}{2} \hat{S}_z \right), \quad \hat{\mathcal{H}}_1 = \frac{\delta}{2} \hat{S}_z + i\lambda (\hat{a}^k \hat{S}_+ - \hat{a}^{\dagger k} \hat{S}_-), \tag{12}$$

where $\delta = \Delta + \omega_c - k\omega$.

Also, $\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_1$ can be written in the matrix representation as follows:

$$\hat{\mathcal{H}}_0 = \begin{bmatrix} \omega \left(\hat{n} + \frac{k}{2} \right) & 0 \\ 0 & \omega \left(\hat{n} - \frac{k}{2} \right) \end{bmatrix}, \quad \hat{\mathcal{H}}_1 = \begin{bmatrix} \frac{\delta}{2} & i\lambda \hat{a}^k \\ -i\lambda \hat{a}^{\dagger k} & -\frac{\delta}{2} \end{bmatrix}. \tag{13}$$

The square of the Hamiltonian (6) takes the form $\hat{\mathcal{H}}^2 = \hat{A} + \hat{B}$, where $[\hat{A}, \hat{B}] = 0$ so \hat{A} and \hat{B} take the following forms:

$$\hat{A} = \begin{bmatrix} \omega^2 \left(\hat{n} + \frac{k}{2} \right)^2 + \hat{M}_1^2 & 0 \\ 0 & \omega^2 \left(\hat{n} - \frac{k}{2} \right)^2 + \hat{M}_2^2 \end{bmatrix}, \tag{14}$$

$$\hat{B} = 2 \begin{bmatrix} \frac{\omega\delta}{2} \left(\hat{n} + \frac{k}{2} \right) & i\lambda\omega \left(\hat{n} + \frac{k}{2} \right) \hat{a}^k \\ -i\lambda\omega \left(\hat{n} - \frac{k}{2} \right) \hat{a}^{\dagger k} & -\frac{\delta\omega}{2} \left(\hat{n} - \frac{k}{2} \right) \end{bmatrix}, \tag{15}$$

where $\hat{M}_1 = \sqrt{\left(\frac{\delta}{2}\right)^2 + \lambda^2 \hat{a}^k \hat{a}^{\dagger k}}$ and $\hat{M}_2 = \sqrt{\left(\frac{\delta}{2}\right)^2 + \lambda^2 \hat{a}^{\dagger k} \hat{a}^k}$.

By using the definition (10) we can write:

$$\begin{aligned} \hat{\rho}_2(t) &= e^{\hat{S}t} e^{\hat{T}t} \hat{\rho}(0) = e^{-i\hat{\mathcal{H}}_1 t} e^{-\frac{t}{2\gamma} \hat{B}} \hat{\rho}_1(t) e^{-\frac{t}{2\gamma} \hat{B}^*} e^{i\hat{\mathcal{H}}_1 t}, \\ \hat{\rho}_1(t) &= |\psi(t), +\rangle\langle+, \psi(t)|, \end{aligned} \tag{16}$$

where $|\psi(t)\rangle = e^{-\frac{t}{2\gamma} \hat{A}} |\alpha\rangle = \exp\left(-\frac{t}{2\gamma} \left(\omega^2 \left(\hat{n} + \frac{k}{2}\right)^2 + \hat{M}_1^2\right)\right) |\alpha\rangle$.

We can write the operator $\exp\left(-\frac{t}{2\gamma}\hat{B}\right)$ as:

$$e^{-\frac{t}{2\gamma}\hat{B}} = \begin{bmatrix} \hat{X}_+(n+k, t) & \hat{X}_3(n+k, t)\hat{a}^k \\ -\hat{X}_3(n, t)\hat{a}^{\dagger k} & \hat{X}_-(n, t) \end{bmatrix}, \tag{17}$$

where $\hat{X}_+(n+k, t) = \cosh\left(\frac{t}{\gamma}\hat{q}_+\right) - \frac{\delta}{2\hat{M}_1}\sinh\left(\frac{t}{\gamma}\hat{q}_+\right)$, $\hat{X}_-(n, t) = \cosh\left(\frac{t}{\gamma}\hat{q}_-\right) + \frac{\delta}{2\hat{M}_2}\sinh\left(\frac{t}{\gamma}\hat{q}_-\right)$, $\hat{X}_3(n+k, t) = \frac{-i\lambda}{\hat{M}_1}\sinh\left(\frac{t}{\gamma}\hat{q}_+\right)$, $q_+ = \omega\left(\hat{n} + \frac{k}{2}\right)\hat{M}_1$ and $q_- = \omega\left(\hat{n} - \frac{k}{2}\right)\hat{M}_2$.

Similarly, the operator $\exp(-i\hat{\mathcal{H}}_1 t)$ can be written in the form:

$$e^{-i\hat{\mathcal{H}}_1 t} = \begin{bmatrix} \hat{R}_+(n+k, t) & \hat{R}_{12}(n+k, t)\hat{a}^k \\ \hat{R}_{21}(n, t)\hat{a}^{\dagger k} & \hat{R}_-(n, t) \end{bmatrix}, \tag{18}$$

where $\hat{R}_+(n+k, t) = \cos(\hat{M}_1 t) - \frac{i\delta}{2\hat{M}_1}\sin(\hat{M}_1 t)$, $\hat{R}_-(n, t) = \cos(\hat{M}_2 t) + \frac{i\delta}{2\hat{M}_1}\sin(\hat{M}_2 t)$, $\hat{R}_{12}(n+k, t) = \frac{\lambda}{\hat{M}_1}\sin(\hat{M}_1 t)$, and $\hat{R}_{21}(n, t) = \frac{-\lambda}{\hat{M}_2}\sin(\hat{M}_2 t)$.

The m -th power of the Hamiltonian (6) can be also written in the form:

$$\hat{\mathcal{H}}^m = \begin{bmatrix} \hat{f}_+^{(m)} & i\lambda\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k \\ -i\lambda\frac{\hat{\phi}_+^{(m)}}{\hat{M}_2}\hat{a}^{\dagger k} & \hat{f}_-^{(m)} \end{bmatrix}, \tag{19}$$

where $\hat{f}_+^{(m)} = \hat{\phi}_+^{(m)} + \frac{\delta}{2\hat{M}_1}\hat{\phi}_-^{(m)}$, $\hat{f}_-^{(m)} = \hat{\Pi}_+^{(m)} - \frac{\delta}{2\hat{M}_2}\hat{\Pi}_-^{(m)}$, and the operators $\hat{\phi}_\pm^{(m)}$ and $\hat{\Pi}_\pm^{(m)}$ are defined by: $\hat{\phi}_\pm^{(m)} = \frac{1}{2}((\hat{s}_\pm^{(m)}) \pm \hat{s}_\pm^{(m)})$, $\hat{\Pi}_\pm^{(m)} = \frac{1}{2}((\hat{v}_\pm^{(m)}) \pm \hat{v}_\pm^{(m)})$, also $\hat{s}_\pm = \omega\left(\hat{n} + \frac{k}{2}\right) \pm \hat{M}_1$ and $\hat{v}_\pm = \omega\left(\hat{n} - \frac{k}{2}\right) \pm \hat{M}_2$.

Finally, we obtain the exact solution of the master (8) for $t > 0$ as follows:

$$\hat{\rho}(t) = \hat{\rho}_{11}(t)|+\rangle\langle+| + \hat{\rho}_{12}(t)|+\rangle\langle-| + \hat{\rho}_{21}(t)|-\rangle\langle+| + \hat{\rho}_{22}(t)|-\rangle\langle-|, \tag{20}$$

where the elements of the density operator are $\hat{\rho}_{ij}(t) = \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{t}{\gamma}\right)^m \hat{\Gamma}_{ij}^{(m)}$, and we define $\hat{\Gamma}^{(m)} = \hat{\mathcal{H}}^m \hat{\rho}_2(t) \hat{\mathcal{H}}^m$, so we can write:

$$\begin{aligned} \hat{\Gamma}_{11}^{(m)} &= \hat{f}_+^{(m)}\psi_{11}\hat{f}_+^{(m)} - \lambda^2\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k\psi_{22}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^{\dagger k} \\ &\quad + i\lambda\hat{f}_+^{(m)}\psi_{12}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^{\dagger k} + i\lambda\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k\psi_{21}\hat{f}_+^{(m)}, \end{aligned} \tag{21}$$

$$\begin{aligned} \hat{\Gamma}_{22}^{(m)} &= -\lambda^2\hat{a}^{\dagger k}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\psi_{11}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k - \hat{f}_-^{(m)}\psi_{22}\hat{f}_-^{(m)} \\ &\quad + i\lambda\hat{a}^{\dagger k}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\psi_{12}\hat{f}_+^{(m)} + i\lambda\hat{f}_-^{(m)}\psi_{21}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k, \end{aligned} \tag{22}$$

$$\begin{aligned} \hat{\Gamma}_{12}^{(m)} &= i\lambda\hat{f}_+^{(m)}\psi_{11}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k - i\lambda\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k\psi_{22}\hat{f}_-^{(m)} \\ &\quad - \hat{f}_+^{(m)}\psi_{12}\hat{f}_-^{(m)} - \lambda^2\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^k\psi_{21}\frac{\hat{\phi}_-^{(m)}}{\hat{M}_1}\hat{a}^{\dagger k}, \end{aligned} \tag{23}$$

and

$$\hat{\Gamma}_{21}^{(m)} = (\hat{\Gamma}_{12}^{(m)})^\dagger. \tag{24}$$

By using (3), the solution of the master (8) takes the following form using the original atomic states ($|e\rangle, |g\rangle$):

$$\begin{aligned} \hat{\rho}(t) &= [\hat{\rho}_{11}(t) \cos^2(\xi) + \hat{\rho}_{22}(t) \sin^2(\xi) - \sin(\xi) \cos(\xi)(\hat{\rho}_{12}(t) + \hat{\rho}_{21}(t))] |e\rangle\langle e| \\ &\quad + [\hat{\rho}_{11}(t) \sin^2(\xi) + \hat{\rho}_{22}(t) \cos^2(\xi) + \sin(\xi) \cos(\xi)(\hat{\rho}_{12}(t) + \hat{\rho}_{21}(t))] |g\rangle\langle g| \\ &\quad + [\hat{\rho}_{12}(t) \cos^2(\xi) - \hat{\rho}_{21}(t) \sin^2(\xi) + \sin(\xi) \cos(\xi)(\hat{\rho}_{11}(t) - \hat{\rho}_{22}(t))] |e\rangle\langle g| \\ &\quad + [-\hat{\rho}_{12}(t) \sin^2(\xi) + \hat{\rho}_{21}(t) \cos^2(\xi) + \sin(\xi) \cos(\xi)(\hat{\rho}_{11}(t) - \hat{\rho}_{22}(t))] |g\rangle\langle e| \\ &= \sum_{r,s=e,g} \hat{\rho}_{rs} |r\rangle\langle s|. \end{aligned} \tag{25}$$

Now, we use these results to study the influence of the intrinsic damping as well as the classical field on the dynamics of the system and discuss some aspects for different values of damping and detuning parameter. Also, we compare previous results with the present system.

3 Collapses and Revivals of the Atomic Inversion

In this section, we use the analytic expression of the atomic inversion for this system to study this phenomenon. The atomic inversion gives us information about the atom during the scaled time of the interaction and probabilities of the atom being found in its excited state or ground state. By using the solution (25), it is easy to write the atomic inversion of the system as:

$$\langle \hat{\sigma}_z(t) \rangle = \sum_{n=0}^{\infty} (\langle n | \hat{\rho}_{ee}(t) | n \rangle - \langle n | \hat{\rho}_{gg}(t) | n \rangle). \tag{26}$$

We discuss the effect of intrinsic damping and the classical field terms on the atomic population inversion for one photon and two photon interactions. In Fig. 1, we plot the atomic inversion for one photon ($k = 1$) against the scaled time $\lambda_1 t$ with different values of the decay parameter γ and the coupling constant for classical field λ_2 , where the atom initially in the excited state and the cavity field is prepared initially in a coherent state with $|\alpha|^2 = 25$, and $\omega_0 = \omega = \lambda_1$. In Fig. 1a, if we take $\gamma = 10^6$ and $\lambda_2 = 0 (\Delta = 0)$, the system shows behavior similar to that of the atomic inversion in the JCM as would be expected [19]. When we consider $\gamma = 5 * 10^4$ and $\lambda_2 = 0$, we note that the amplitude of the function decreases as the time develops and consequently tend to zero that is evident in Fig. 1b [20]. When we take the effect of classical field into account by setting $\lambda_2 = 3\lambda_1$ in Fig. 1c and the decay parameter $\gamma = 10^6$, we observe a decrease in its amplitude of revivals and the atomic inversion has periodic oscillatory envelope oscillates around zero, where collapses and revivals are similar to the case in [16]. When we take the decay factor $\gamma = 5 * 10^4$ and classical field parameter $\lambda_2 = 3\lambda_1$ in Fig. 1d, we see that change in envelope of the amplitude of collapse and revivals while it oscillates around zero but as the time increases the amplitudes of the revivals tend to zero and mixed state is obtained.

In Fig. 2, we plot the atomic inversion for two photon ($k = 2$) with $w = \lambda_1, \omega_0 = 3\lambda_1$ under the same condition on the atom and the cavity field. In Fig. 2a, if we take $\lambda_2 = 0$ and $\gamma = 10^6$, we find that the atomic population inversion agrees with the two-photon JCM and the revival time is periodic after every 2π [21]. When we take the decay parameter

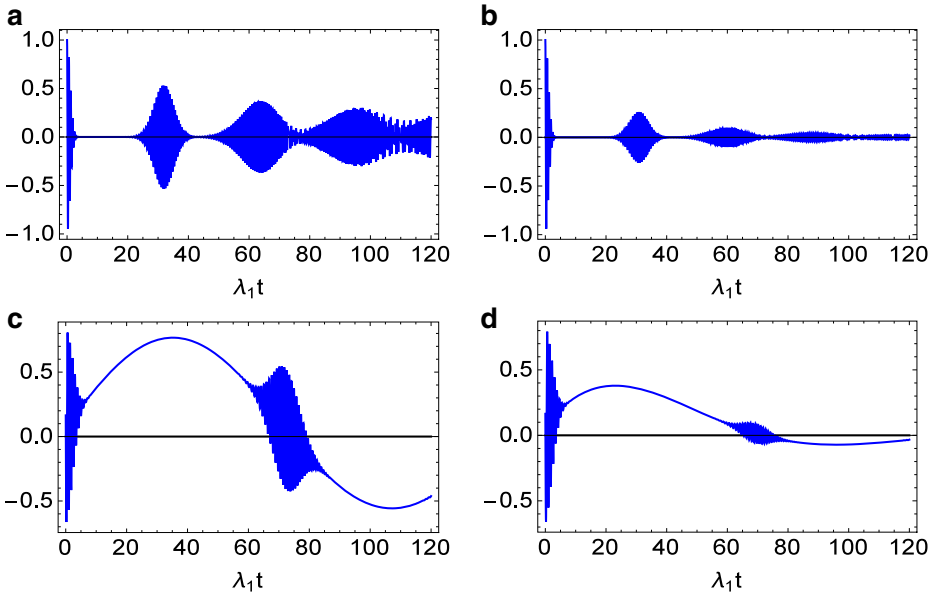


Fig. 1 The atomic inversion $\langle \hat{\sigma}_z(t) \rangle$ for $k=1$ against the scaled time $\lambda_1 t$ with $\alpha = 5$ where **a** $\gamma = 10^6$, $\lambda_2 = 0$, **b** $\gamma = 5 * 10^4$, $\lambda_2 = 0$, **c** $\gamma = 10^6$, $\lambda_2 = 3\lambda_1$ and **d** $\gamma = 5 * 10^4$, $\lambda_2 = 3\lambda_1$

($\gamma = 10^4$) and keeping $\lambda_2 = 0$, we observe that amplitudes of revivals tend to zero with time increasing see Fig. 2b. The effect of classical field on the population inversion is shown in Fig. 2c, where we see that the oscillatory envelope of collapses and elongation of the

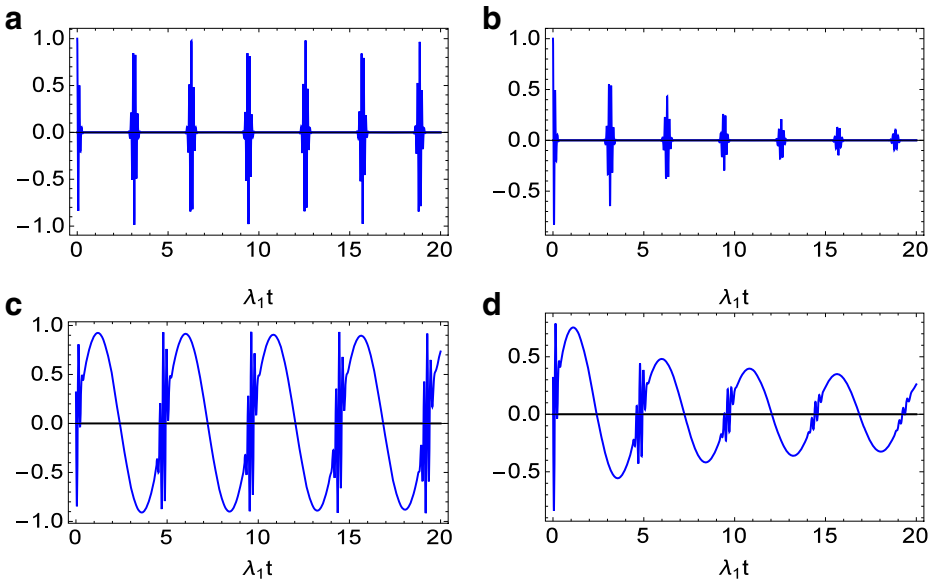


Fig. 2 The atomic inversion $\langle \hat{\sigma}_z(t) \rangle$ for $k=2$ against the scaled time at $\lambda_1 t$ with $\alpha = 5$ where **a** $\gamma = 10^6$, $\lambda_2 = 0$, **b** $\gamma = 10^4$, $\lambda_2 = 0$, **c** $\gamma = 10^6$, $\lambda_2 = 3\lambda_1$ and **d** $\gamma = 10^4$, $\lambda_2 = 3\lambda_1$

collapse period. When we take $\gamma = 10^4$ with classical field $\lambda_2 = 3\lambda_1$, we note that the envelope of the function is oscillates around zero and tends to zero as the time develops (see Fig. 2d), for more information we may refer to [16, 22]. Generally, the classical field parameter plays two roles firstly, it weakens the interaction between the cavity field and the atom and secondly, it delays the decay. Furthermore, the decay parameter plays the role of making the state of the atom tends to a mixed state.

4 Variance and Entropy Squeezing

In this section, we shall employ the uncertainty relation to discuss the entropy and variance squeezing for k-photon proses especially $k = 1$ and $k = 2$. Also, we display the effect of classical field and decay parameter on the phenomenon of squeezing. The uncertainty relation for a two-level atom characterized by Pauli operators $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ is defined by:

$$\Delta\hat{\sigma}_x\Delta\hat{\sigma}_y \geq \frac{1}{2}|\langle\hat{\sigma}_z\rangle|, \tag{27}$$

where $\Delta\hat{\sigma}_i = \sqrt{\langle\hat{\sigma}_i^2\rangle - \langle\hat{\sigma}_i\rangle^2}$ and $i = x, y$, while $\langle\hat{\sigma}_i\rangle^2$ for this system are given by:

$$\langle\hat{\sigma}_x\rangle = 2Re\langle\hat{\rho}_{eg}(t)\rangle, \quad \langle\hat{\sigma}_y\rangle = 2Im\langle\hat{\rho}_{eg}(t)\rangle, \quad \text{and} \quad \langle\hat{\sigma}_z\rangle = \langle\hat{\rho}_{ee}(t) - \hat{\rho}_{gg}(t)\rangle. \tag{28}$$

For the atomic variance squeezing for the following condition has to be satisfied.

$$V_i(t) = \left(\Delta\hat{\sigma}_i(t) - \sqrt{\frac{|\langle\hat{\sigma}_z(t)\rangle|}{2}} \right) < 0 \quad , \quad i = x, y \tag{29}$$

To investigate the information entropy of the variable σ_k in an even N -dimensional Hilbert space for sets of $N + 1$ complementary observable with the non-degenerate eigenvalues, we use the inequality [23–25].

$$\sum_{k=1}^N H(\hat{\sigma}_k) \geq \left(\frac{N}{2} \ln \frac{N}{2} + \left(1 + \frac{N}{2}\right) \ln\left(1 + \frac{N}{2}\right) \right). \tag{30}$$

Entropies of atomic operators $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ for a two-level atom by use the reduced atomic density operator can be written as:

$$H(\hat{\sigma}_l) = - \left[\frac{1}{2} + \langle\hat{\sigma}_l\rangle \right] \ln \left[\frac{1}{2} + \langle\hat{\sigma}_l\rangle \right] - \left[\frac{1}{2} - \langle\hat{\sigma}_l\rangle \right] \ln \left[\frac{1}{2} - \langle\hat{\sigma}_l\rangle \right],$$

where $l = x, y, z$. But if we know for $N = 2$, we have $0 \leq H(\hat{\sigma}_i) \leq \ln 2$ and these entropies satisfy the inequality.

$$H(\hat{\sigma}_x) + H(\hat{\sigma}_y) + H(\hat{\sigma}_z) \geq 2\ln 2. \tag{31}$$

When we set $\delta H(\hat{\sigma}_i) = \exp(H(\hat{\sigma}_i))$, then we can write [26, 27].

$$\delta H(\hat{\sigma}_x)\delta H(\hat{\sigma}_y)\delta H(\hat{\sigma}_z) \geq 4. \tag{32}$$

Now, we state that the system has the entropy squeezing, or the fluctuation in component $\hat{\sigma}_j, j = x, y$ of the atomic dipole, if entropies $H(\hat{\sigma}_j)$ satisfy the condition:

$$E(\hat{\sigma}_j) = \left(\delta H(\hat{\sigma}_j) - \frac{2}{\sqrt{\delta H(\hat{\sigma}_z)}} \right) < 0, \quad j = x \quad \text{or} \quad y, \tag{33}$$

where $E(\hat{\sigma}_j)$ is the entropy squeezing parameter. Now, we discuss results for variances squeezing and entropy squeezing.

In Fig. 3, we have plotted V_x (solid curve) and V_y (dash curve) against the scaled time $\lambda_1 t$, when we take $k = 1$ with the same parameters as in Fig. 1. At first we restrict our examination for $\lambda_2 = 0$ and $\gamma = 10^6$, where the system turns to almost JCM in resonance case. In this case, we see that the phenomenon of squeezing can be only observed in the first quadrature V_x but no squeezing in V_y as observed in Fig. 3a. On the other hand, when we choose $\lambda_2 = 0$ and $\gamma = 5 * 10^4$, we can see that there is no squeezing in either quadrature V_x or V_y except after the onset of the interaction in V_x , see Fig. 3b. Different behavior can be seen when we take the classical external field into account and adjust $\lambda_2 = 3\lambda_1$ and $\gamma = 10^6$ Fig. 3c. We note that the squeezing occurs in both quadratures V_x and V_y after the start of the interaction which agrees with the (A.3) at $\lambda_2 = 3\lambda_1$, while the variance squeezing takes the value $\simeq -0.2$ at $t = 0$, for more details see Appendix. In Fig. 3d, we display the phenomenon of squeezing at $\lambda_2 = 3\lambda_1$ and $\gamma = 5 * 10^4$ where we observe that V_x and V_y almost coincide as time grows but here in an extremely short period at the beginning of the interaction, we note that there is squeezing for V_y and V_x .

Now, we discuss the phenomenon of the variance squeezing for two photon ($k = 2$) and for the same conditions of the system. In Fig. 4, we have plotted V_x and V_y with the same values of the parameters in Fig. 2. We note that in Fig. 4a, a squeezing V_x occurs only at the revival time $\lambda_1 t = n\pi$, $n = 0, 1, 2, \dots$ but squeezing disappears in the quadrature V_y [21]. In Fig. 4b, we can see that there is no squeezing in quadrature V_y while V_x has a short period of squeezing just after the onset of the interaction then there is no squeezing. In Fig. 4c, V_y reaches to 1 at some point and has nearly a similar behavior of V_x . However, only V_x

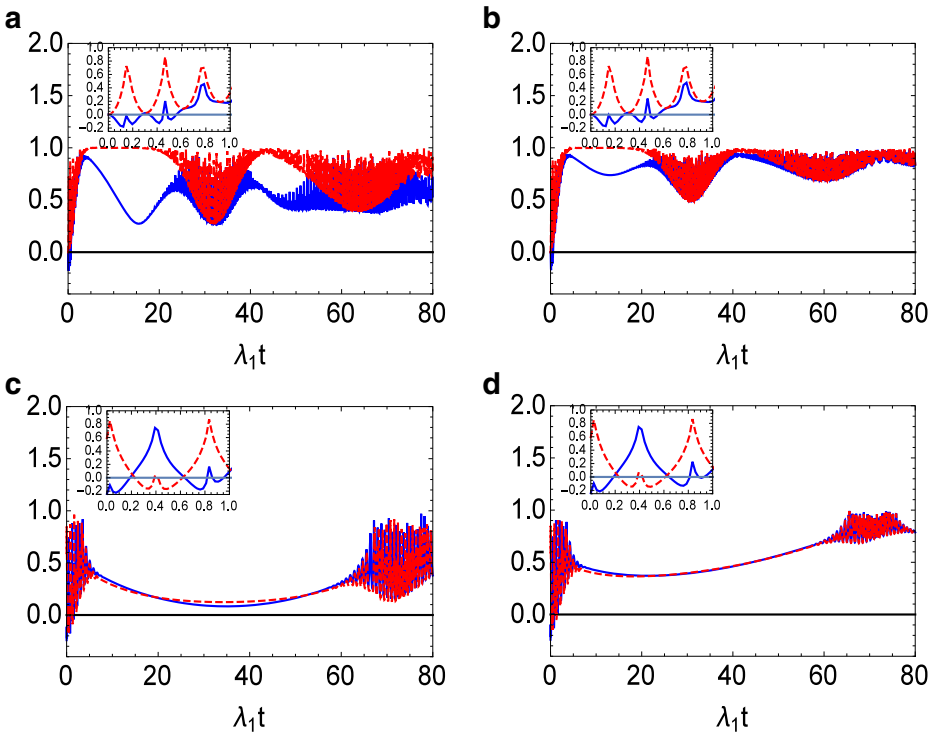


Fig. 3 Variance squeezing V_x (solid curve), V_y (dashed curve) for $k = 1$ against the scaled time with the same parameters in Fig. 1

has a squeezing at the some points, also the starting point has negative value as observed in Fig. 4c. In Fig. 4d, V_x and V_y have almost the same behavior, we note that the squeezing occurs for V_x just after the onset of interaction.

Now, we examine the behavior of the entropy squeezing $E(\hat{\sigma}_x)$ (solid line) and $E(\hat{\sigma}_y)$ (dashed line) for one and two photon against the scaled time $\lambda_1 t$, we considered $\alpha = 5$ and the same value of ω'^s as in Fig. 1, these are shown in Figs. 5, 6 for one-photon ($k = 1$) and two-photon ($k = 2$) processes respectively. We note that when we take the parameters $\lambda_2 = 0$ and $\gamma = 10^6$, we get a usual JCM for one photon [24] and for two photon [21]. While we plot Figs. 5a, 6a, to see that the squeezing occurs at several periods of time in the first quadrature of the entropy squeezing $E(\hat{\sigma}_x)$, but it is absent from $E(\hat{\sigma}_y)$. When we take the damping into consideration ($\lambda_2 = 0$ and $\gamma = 5 * 10^4$ for $k = 1$ or $\gamma = 10^4$ for $k = 2$), there is no squeezing in both $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ except just after the onset of interaction in $E(\hat{\sigma}_x)$, see Figs. 5b, 6b. We note a different behavior when we consider $\lambda_2 = 3\lambda_1$ and $\gamma = 10^6$, then both $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ have squeezing at the beginning see Figs. 5c, 6c. But for $k = 2$ the squeezing is absent from the second quadrature $E(\hat{\sigma}_y)$. Finally, we can see that the function of the entropy squeezing, when we take $\lambda_2 = 3\lambda_1$ and $\gamma = 5 * 10^4$ one photon has almost the same behavior for both $E(\hat{\sigma}_x)$ and $E(\hat{\sigma}_y)$ (see Fig. 5d). At $k = 2$ there is no squeezing for $E(\hat{\sigma}_y)$ at $\lambda_2 = 3\lambda_1$ and $\gamma = 10^4$, while the behavior $E(\hat{\sigma}_x)$ is different in this case (see Fig. 6d).

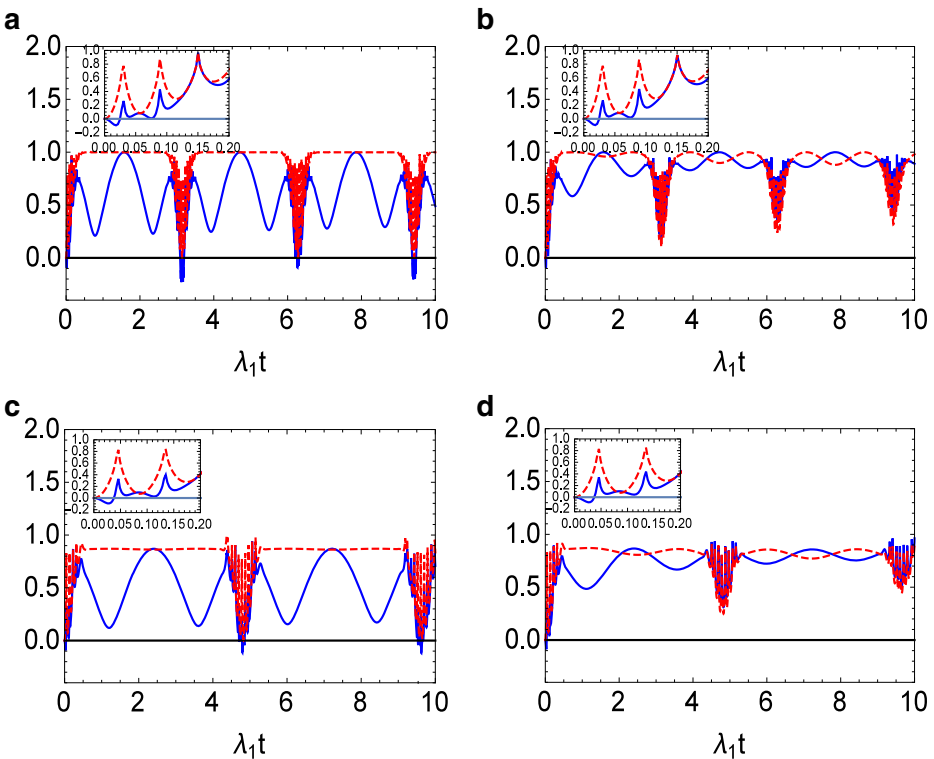


Fig. 4 Variance squeezing V_x (solid curve), V_y (dashed curve) for $k=2$ against the scaled time where the parameters are the same as those of Fig. 2

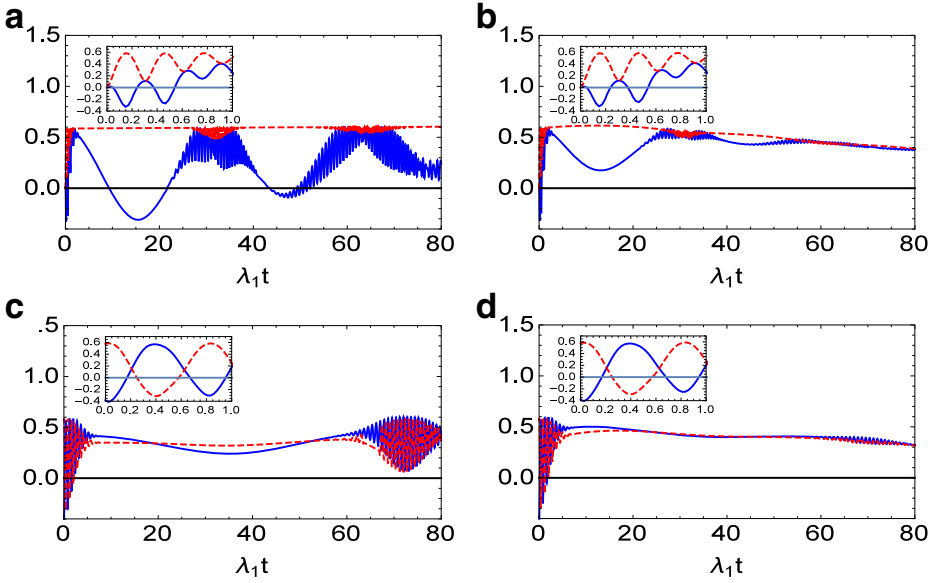


Fig. 5 Entropy squeezing E_x (solid line), E_y (dashed line) for $k=1$ against the scaled time with the same parameters in Fig. 1

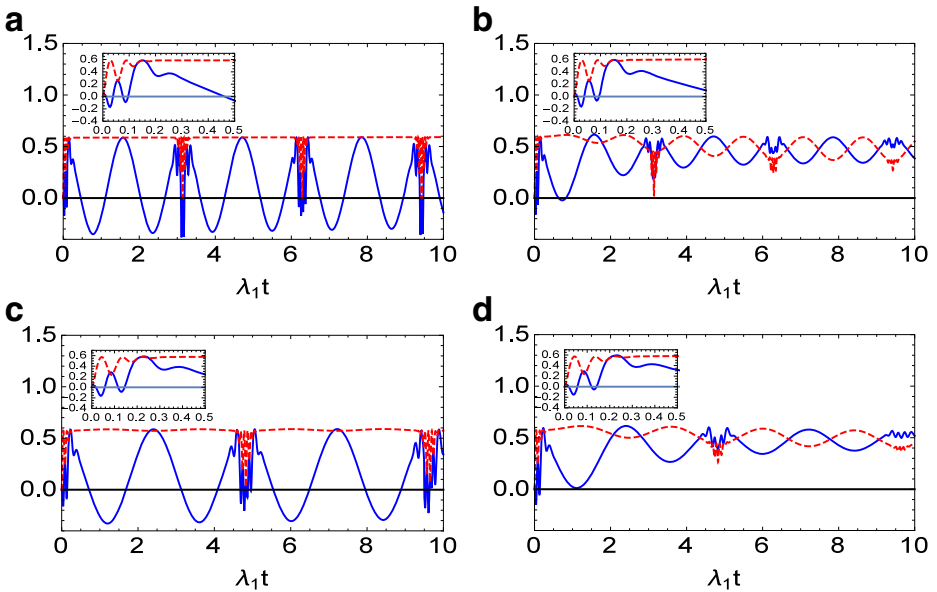


Fig. 6 Entropy squeezing E_x (solid line), E_y (dashed line) for $k=2$ against the scaled time with the same parameters in Fig. 2

5 Linear Entropy

Finally, we discuss the purity of the system by using linear entropy as an indicator. Further, in atomic states preparation through interacting quantum systems to determine cavity field through the disentanglement of the two quantum systems [28, 29]. On the other hand, for the system consisting of two subsystems and being prepared in a pure state the condition $Tr(\hat{\rho}_a^2) = 1$ is satisfied, but if the system is in a mixed state then $Tr(\hat{\rho}_a^2) < 1$, the maximum value of the mixed state is $Tr(\hat{\rho}_a^2) = 0.5$. In general, the linear entropy is much simpler to compute than the von Neumann entropy [30] because there is no need to diagonalize the density matrix. We start with definition of the linear entropy [29], which is given by:

$$P(t) = \frac{1}{2} \left(1 - \sqrt{\langle \hat{\sigma}_x \rangle^2 + \langle \hat{\sigma}_y \rangle^2 + \langle \hat{\sigma}_z \rangle^2} \right), \tag{34}$$

where $\langle \hat{\sigma}_i \rangle, i = x, y, z$ are defined in (28), we draw the linear entropy to discuss entanglement. In Fig. 7, we plot the function $P(t)$ against the scaled time $\lambda_1 t$ for $k = 1$, where we consider the atom in the excited state, $|\alpha|^2 = 25$ and $\omega = \omega_0 = \lambda_1$ with different values of the coupling parameter of the classical field λ_2 and the decay parameter γ . In Fig. 7a, we observe that when we take $\gamma = 10^6$ and $\lambda_2 = 0$ the function shows almost the JCM behavior for which the system shows periods of entanglement. In Fig. 7b, the influence of damping $\gamma = 5 * 10^4$ without classical external field is shown, where we can see that increases in the maximum value of the entanglement and it is sustained for larger values of $\lambda_1 t$. But we

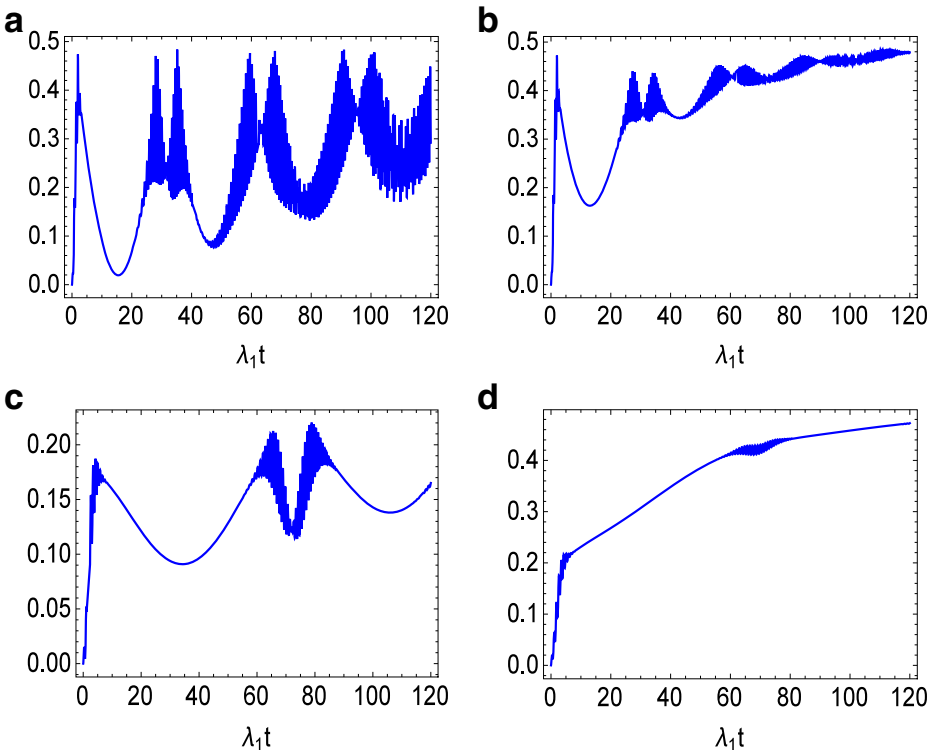


Fig. 7 The Linear entropy $P(t)$ for $k = 1$ against the scaled time with the same parameters in Fig. 1

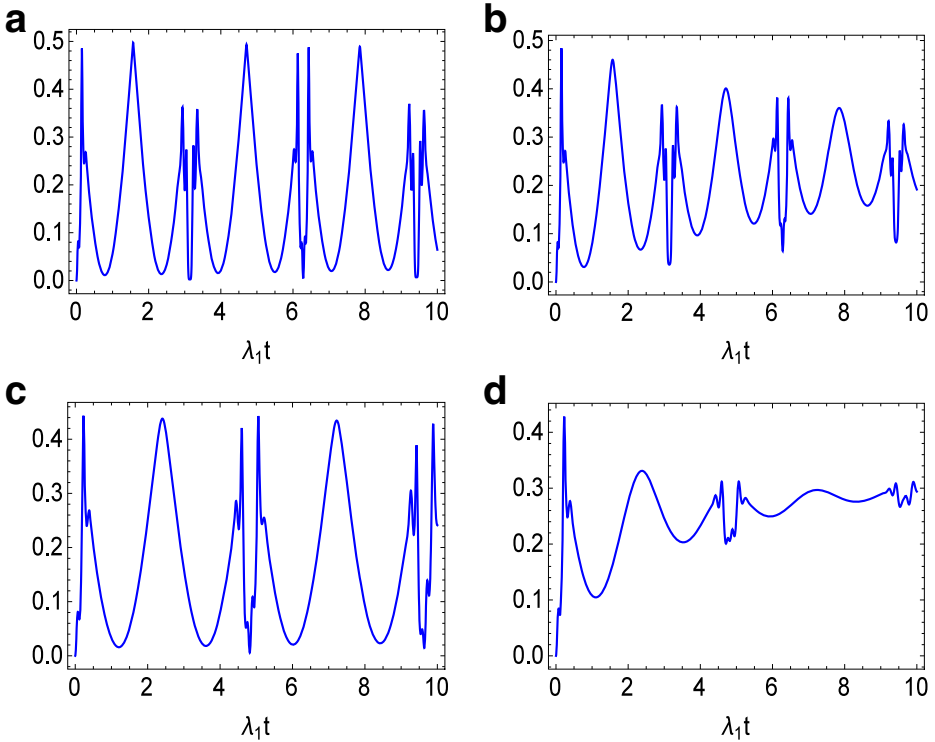


Fig. 8 The Linear entropy $P(t)$ for $k = 2$ against the scaled time with the same parameters in Fig. 2

note decreases in entanglement when we add the influence of classical field $\lambda_2 = 3\lambda_1$ with damping $\gamma = 10^6$, which is shown in Fig. 7c. While in Fig. 7d, the influence of the classical field $\lambda_2 = 3\lambda_1$ with an increases of damping parameter $\gamma = 5 \times 10^4$, where we note that the entanglement is increasing with time, and it is sustained for larger values of the scaled time.

In Fig. 8, the function $P(t)$ is studied against the scaled time $\lambda_1 t$ with $k = 2$ for the same values of parameters in Fig. 2. In Fig. 8a, we see that the maximum value of the function $P(t) = 0.5$, which reaches to a pure state every $n\pi$, $n = 0, 1, 2, \dots$, but under influence of damping the maximum value decreases as time developed, see Fig. 8b. The classical field $\lambda_2 = 3\lambda_1$ leads to a decrease in the maximum value of function $P(t)$ to 0.45, see Fig. 8c. In Fig. 8d, we note that $P(t)$ reaches to zero at the onset interaction, and the maximum value decreases as the time increases. It is to be noted that once damping is taken into consideration, then the system never reaches a pure state. In the case of the two photon interaction, pure state can be reached even when classical field is present, provided damping is being absent.

6 Conclusion

In this paper, we have investigated the effect of classical field, and multiphoton processes in the presence of intrinsic damping. By using some transformations we managed to cast the problem in a generalized JCM and by applying the Milburn’s equation for intrinsic damping

we obtained the density operator. Then, we used the results to investigate some statistical aspects. We studied the atomic inversion for the system with various values of damping and classical field parameters, we found that the classical field plays important roles on shifting and elongation of the collapse periods. The damping on the other hand decreases the amplitude of revivals as time increases and results in the atom end up in a mixed state. Furthermore, we discussed the effect of damping and classical field on the phenomenon of squeezing. It is shown that the damping plays a role in washing out the squeezing phenomenon, while the classical field increases the squeezing for the quadrature V_y . Finally, the results show that the damping parameter increases the entanglement and system never reaches a pure state with time grows. While, in the presence of the classical field and absence of the damping the system shows partial entanglement with rapid fluctuations. The pure state can be reached periodically for the case of two photon interaction, even with classical field being present.

Appendix

We derive the formula of the variance squeezing at the initial time. From (4) we get:

$$\begin{aligned}\langle \hat{\sigma}_x \rangle &= \langle \hat{S}_x \rangle \cos 2\xi + \langle \hat{S}_z \rangle \sin 2\xi \\ \langle \hat{\sigma}_y \rangle &= \langle \hat{S}_y \rangle \\ \langle \hat{\sigma}_z \rangle &= \langle \hat{S}_z \rangle \cos 2\xi - \langle \hat{S}_x \rangle \sin 2\xi\end{aligned}\quad (\text{A.1})$$

then

$$\langle \hat{\sigma}_x^2 \rangle = 1 \quad , \quad \langle \hat{\sigma}_x \rangle = \sin 2\xi \quad \text{and} \quad \langle \hat{\sigma}_z \rangle = \cos 2\xi \quad (\text{A.2})$$

by using (29) then :

$$V_x = \cos 2\xi - \sqrt{|\cos 2\xi|} \quad (\text{A.3})$$

Similarly, we can calculate V_y and determine starting point.

References

1. Milburn, G.J.: Phys. Rev. A **44**, 5401 (1991)
2. Jaynes, E.T., Cummings, F.W.: Proc. IEEE **51**, 89 (1963)
3. Moya-Cessa, H., Bužek, V., Kim, M.S., Knight, P.L.: Phys. Rev. A **48**, 3900 (1993)
4. Kuang, L.M., Chen, X., Chen, G.H., Ge, M.L.: Phys. Rev. A **56**, 3139 (1997)
5. Obada, A.S., Hessian, H.A.: JOSA B **21**, 1535 (2004)
6. Xue-Qun, Y., Bin, S., Jian, Z.: Commun. Theor. Phys. **48**, 63 (2007)
7. Abdel-Aty, M.: Phys. Lett. A **372**, 3719 (2008)
8. Mohamed, A.B., Metwally, N.: Ann. Phys. **381**, 137 (2017)
9. Mohamed, A.B.A.: Eur. Phys. J. D **71**(10), 261 (2017)
10. Alqannas, H.S., Khalil, E.: Phys. A **489**, 1 (2018)
11. Anwar, S.J., Ramzan, M., Khan, M.K.: Quantum Inf. Process. **16**(6), 142 (2017)
12. Abdel-Khalek, S., Zidan, N., Abdel-Aty, M.: Phys. E **44**, 6 (2011)
13. Furuichi, S., Ohya, M.: Lett. Math. Phys. **49**, 279 (1999)
14. Solano, E., Agarwal, G.S., Walther, H.: Phys. Rev. Lett. **90**, 027903, 4p (2003)
15. Mohamed, A.B.A., Abdalla, M.S., Obada, A.S.F.: Eur. Phys. J. D **71**(9), 223 (2017)
16. Abdalla, M.S., Khalil, E., Obada, A.S.: Ann. Phys. **326**, 2486 (2011)
17. Khalil, E.: Int. J. Theor. Phys. **52**, 1122 (2013)
18. Moya-Cessa, H.: Phys. Rep. **432**, 1 (2006)
19. Shore, B.W., Knight, P.L.: J. Mod. Opt. **40**, 1195 (1993)
20. Obada, A.S.F., Hessian, H., Mohamed, A.B.: J. Phys. B **41**, 135503, 7pp (2008)

21. Abdalla, M.S., Khalil, E., Obada, A.S.: *Ann. Phys.* **322**, 2554 (2007)
22. Abdalla, M.S., Obada, A.S., Mohamed, A.B., Khalil, E.: *Int. J. Theor. Phys.* **53**, 1325 (2014)
23. Sánchez-Ruiz, J.: *Phys. Lett. A* **201**, 125 (1995)
24. Fang, M., Zhou, P., Swain, S.: *J. Mod. Opt.* **47**, 1043 (2000)
25. Zurek, W.H., Habib, S., Paz, J.P.: *Phys. Rev. Lett.* **70**, 1187 (1993)
26. Abdel-Aty, M., Abdalla, M.S., Obada, A.S.F.: *J Phys. A* **34**, 9129 (2001)
27. Abdel-Aty, M., Abdalla, M.S., Obada, A.S.F.: *J. Opt. B* **4**, 134 (2002)
28. Phoenix, S., Knight, P.: *Ann. Phys.* **186**, 381 (1988)
29. Phoenix, S.J.D., Knight, P.L.: *Phys. Rev. A* **44**, 6023 (1991)
30. Von Neumann, J.: *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, Princeton (1955)