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The Population Inversion and the Entropy of a Moving Two-Level Atom in Interaction with a Quantized Field

D. A. M. Abo-Kahla[1,2] · M. Abdel-Aty[3,4] · A. Farouk[5]

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Abstract An atom with only two energy eigenvalues is described by a two-dimensional state space spanned by the two energy eigenstates is called a two-level atom. We consider the interaction between a two-level atom system with a constant velocity. An analytic solution of the systems which interacts with a quantized field is provided. Furthermore, the significant effect of the temperature on the atomic inversion, the purity and the information entropy are discussed in case of the initial state either an exited state or a maximally mixed state. Additionally, the effect of the half wavelengths number of the field-mode is investigated.

Keywords Quantum information · Thermal state · Entropy

1 Introduction

The fundamental idea of the quantum computing and communications has been introduced by Stephen Wiesner, when he introduced the principles of numerous developments of quantum computing and cryptography field in 1970. She presented the fundamental concept of the no-cloning theory which states that the quantum information can't be replicated [1].

✉ A. Farouk
dr.ahmedfarouk85@yahoo.com

1 Department of Mathematics, Faculty of Science, Taibah University, Madina, Saudi Arabia

2 Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt

3 Deanship of Scientific Research and Graduate Studies, Applied Science University, Sitra, Bahrain

4 Sohag University, Sohag, Egypt

5 Department of Physics and Computer Science, Faculty of Science, Wilfrid Laurier University, Waterloo, ON, Canada

Contrasting to the classical computer, the quantum computer can enhance the substantial processing ability and achieve many tasks by applying all prospective transformations simultaneously. Furthermore, the architecture of quantum computers is contradictory from the classical one which is controlled by a number of transistors. The data can be represented as bits (0 or 1) in the classical environment. On the other hand, the quantum environment represented the data represented as quantum bits and in a superposition state. The superposition state denotes that the quantum state can be 0, 1 or in both states simultaneously. Quantum computers reveal hypothetical relationships with nondeterministic and probabilistic computers [2–4]. By way of 2017, the improvement and growth of a real quantum computer is still in early stages, but much practical and theoretical experimentation were implemented by many research groups [5–27].

The most important principles of quantum computing are entanglement and entropy. Quantum entanglement is a constructive area which can be used to measure the operation of the quantum communication as Shor's algorithm [28–30], Quantum teleportation [31–34], and super dense coding [35–38]. Entanglement is one of the distinctive physical singularity that defines the way of how the particles can be correlated to each other regardless of the distance have been widely studied in [39–55]. A detection of new dynamical features of two coupled spins with an antiferromagnetic environment at finite temperature in the thermodynamics limit models are presented in [39]. In [40] relation between information entropy approach and von Neumann entropy of multi-qubit Rabi system is introduced by utilizing different measurement schemes. In [42, 43] information-theoretic aspects of quantum inseparability of mixed states are investigated in terms of α entropy inequalities and teleportation fidelity. A framework for a quantum mechanical information theory that is based entirely on density operators, and gives rise to a unified description of classical correlation and quantum entanglement is introduced in [44, 45]. The entangled maximum entropy states compatible with data coming from nonentangled (separable) states are produced by applying the Jaynes principle to the compound quantum system as shown in [46]. In [47] a new measure of information in quantum mechanics is proposed which takes into account that for quantum systems the only features known before an experiment is performed are the probabilities for various events to occur. The problem of quantum-state inference and the concept of quantum entanglement are studied using a non-additive measure in the form of the Tsallis entropy indexed by the positive parameter q is introduced in [48]. A generalized approach of the von Neumann mutual information in the context of Tsallis' nonextensive statistics is presented in [49]. A generalized non-additive entropies and quantum entanglement by examining the inference of quantum density operators from incomplete information by means of the maximization of general non-additive entropic forms are introduced in [53].

The information entropy has been developed during the past few decades for representing the central concept in both classical and quantum systems. Information entropy measures the uncertainty and the information content in the physical system state. The time evolution of a dynamic quantum system has been interested for its various applications [56]. The important characteristics for the dynamic system are the atomic inversion and the purity. The atomic population inversion appears when a system exists in a state in which more elements of the system are in higher excited states than in lower ground states. The range of the purity is between zero and $\frac{1}{k}$, (here, k is the dimension of the density matrix), which the first indicates a completely pure state and the later to a completely mixed state. One of the most important problems in quantum optics is the interaction between two or more quantum systems, like the field-atom interactions [56–63], or field-field interactions [64–72], or atom-atom interactions [73–76]. In this paper, we study the system of a two-level atom with a constant velocity which interacts with a quantized field. Furthermore, we focus on the effect of the temperature, the half wavelengths number of the field-mode and the initial state of a two-level atom. This article is organized as; in Section 1, The Hamiltonian and the solution of our model in a particular case under the rotating-wave approximation are discussed.Additionally, the time dependent reduced density operator of both a two-level atom with a quantized field and the density operator of the whole system are discussed. In Section 2, the atomic population inversion of the system is calculated. In Section 3, the purity and the information entropy of the system is calculated. Subsequently, the results of the whole system with different properties are discussed. Finally, Section 4 concludes the paper.

2 The Model and Solution

The nonlinearity effect has been discussed in [77, 78]. In [77] it is shown that the motion of a two-level atom through a spatially varying single-mode field gives rise to nonlinear transient effects which are similar to self-induced transparency and adiabatic following. The effect of atomic motion on Rydberg atoms undergoing two-photon transitions in a lossless cavity is examined in [78]. For our proposed model, we consider a system of a two-level atom with a constant velocity which interacts with a quantized field under the effect of one-photon transition. We study the system in the resonance case as discussed in [77, 78] . The Hamiltonian model under the rotating-wave approximation can be written as in (1)

$$H = \omega a^\dagger a + \omega\sigma_z + gR(z)(\sigma_+ a + a^\dagger\sigma_-), \tag{1}$$

where a and $a^\dagger$ are the annihilation and the creation operators of the single-mode cavity field with the frequency of the field ω. σ_+ and σ_- represent the raising and the lowering operators of the two-level atom respectively, and g is the field-atom coupling constant. We study the motion of the two-level atom along the Z −direction; therefore, the motion function depends only on Z. The motion of the atom will be combined with the shape function of the cavity field mode $R(z)$ as discussed in [79] and represented by (2).

$$R(z) \longrightarrow R(vt). \tag{2}$$

Where, v represents the velocity of the atomic motion; therefore, the cavity field-mode can be defined as shown by (3).

$$R(vt) = \cos\left(\frac{\pi P vt}{L}\right). \tag{3}$$

Where, P represents the half wavelengths number of the field-mode inside a cavity of length L. The time evolution operator of a two-level atom interacting resonantly with a single-mode radiation field can be obtained by exploiting the standard techniques as shown by (4).

$$\grave{H}U = i\frac{\partial U}{\partial t}. \tag{4}$$

Where, $\grave{H}$ is the Hamiltonian in the interaction picture. By applying successive calculations, the time evolution operator can be written as in (5).

$$\begin{aligned} U &= \cos(g\phi(t)\sqrt{a^\dagger a})\left|-\right\rangle\left\langle-\right| + \cos(g\phi(t)\sqrt{aa^\dagger})\left|+\right\rangle\left\langle+\right| \\ &-\frac{ia^\dagger}{\sqrt{aa^\dagger}}\sin(g\phi(t)\sqrt{aa^\dagger})\left|-\right\rangle\left\langle+\right| - \frac{ia}{\sqrt{a^\dagger a}}\sin(g\phi(t)\sqrt{a^\dagger a})\left|+\right\rangle\left\langle-\right|. \end{aligned} \tag{5}$$

Where $|+\rangle$ and $|-\rangle$ represent both the excited and the ground state of a two-level atom, respectively. $\phi(t)$ correspond to the subsequent time-dependent function $\phi(t) = \int R(vt)dt$. To simplify the subsequent time-dependent function, For simplicity, the velocity of the atomic motion is chosen to be $v = \frac{gL}{\pi}$, so $\phi(t)$ can be written as in (6).

$$\begin{aligned}\phi(t) &= \int R(vt)dt \\ &= \int_0^t \cos\left(\frac{\pi P v\grave{t}}{L}\right) d\grave{t} \\ &= \frac{\sin(gPt)}{gP}.\end{aligned} \tag{6}$$

By assuming that the initial density operator of a two-level atom is given by (7).

$$\rho_A(0) = B_e |+\rangle \langle +| + B_g |-\rangle \langle -|. \tag{7}$$

Where $0 \leq B_e \leq 1$, $B_g = 1 - B_e$. While B_e has three different values which are 1, 0 and 0.5. These values correspond to the atom in the excited state, the ground state and the maximally mixed state, respectively. Moreover, the cavity field which is initially presented by the single-mode thermal state can be written as in (8).

$$\rho_F(0) = \sum_{n=0}^{\infty} P_n |n\rangle \langle n|, \; P_n = \frac{m^n}{(m+1)^{n+1}}, \; m = \left(\exp\left(\frac{\omega}{T}\right) - 1\right)^{-1}. \tag{8}$$

Where, m corresponds to the mean photon number of the cavity field for thermal equilibrium, T is a certain temperature. The initial density operator of the whole system can be stated by (9).

$$\begin{aligned}\rho_{AF}(0) &= \rho_F(0) \otimes \rho_A(0) \\ &= B_e \sum_{n=0}^{\infty} P_n |n,+\rangle \langle +,n| + B_g \sum_{n=0}^{\infty} P_n |n,-\rangle \langle -,n|.\end{aligned} \tag{9}$$

But, we know that the time dependent density operator can be given by (10).

$$\rho_{AF}(t) = U \rho_{AF}(0) U^{\dagger}. \tag{10}$$

So, the time dependent density operator of the whole system can be described by (11).

$$\rho_{AF}(t) = \sum_n P_n (B_e \rho_e(t) + B_g \rho_g(t)). \tag{11}$$

Where,

$$\begin{aligned}\rho_e(t) &= \cos^2(g\phi(t)\sqrt{n+1}) |n,+\rangle \langle +,n| \\ &+ \sin^2(g\phi(t)\sqrt{n+1}) |n+1,-\rangle \langle -,n+1| \\ &+ i \cos(g\phi(t)\sqrt{n+1}) \sin(g\phi(t)\sqrt{n+1}) |n,+\rangle \langle -,n+1| \\ &- i \cos(g\phi(t)\sqrt{n+1}) \sin(g\phi(t)\sqrt{n+1}) |n+1,-\rangle \langle +,n|,\end{aligned} \tag{12}$$

$$\begin{aligned}\rho_g(t) &= \sin^2(g\phi(t)\sqrt{n-1}) |n-1,+\rangle \langle +,n-1| \\ &+ \cos^2(g\phi(t)\sqrt{n}) |n,-\rangle \langle -,n| \\ &- i \cos(g\phi(t)\sqrt{n}) \sin(g\phi(t)\sqrt{n}) |n-1,+\rangle \langle -,n| \\ &+ i \cos(g\phi(t)\sqrt{n}) \sin(g\phi(t)\sqrt{n}) |n,-\rangle \langle +,n-1|.\end{aligned} \tag{13}$$

So, the time dependent reduced density operator of a two-level atom is given by (14).

$$\begin{aligned}\rho_A(t) &= Tr_F(\rho_{AF}(t)) \\ &= \mu_1(t)\,|+\rangle\,\langle+| + \mu_2(t)\,|-\rangle\,\langle-|\,,\end{aligned} \tag{14}$$

Furthermore, the time dependent reduced density operator of the cavity field is given by (15).

$$\begin{aligned}\rho_F(t) &= Tr_A(\rho_{AF}(t)) \\ &= (\mu_1(t) + \mu_2(t))\,|n\rangle\,\langle n|\,.\end{aligned} \tag{15}$$

Where,

$$\begin{aligned}\mu_1(t) &= B_e \sum_n P_n \cos^2(g\phi(t)\sqrt{n+1}) + B_g \sum_n P_{n+1} \sin^2(g\phi(t)\sqrt{n+1}), \\ \mu_2(t) &= B_e \sum_n P_{n-1} \sin^2(g\phi(t)\sqrt{n}) + B_g \sum_n P_n \cos^2(g\phi(t)\sqrt{n}).\end{aligned} \tag{16}$$

3 Results and Discussion

The atomic population inversion is considered as one of the quantities which is simple and important. It is defined as the difference between the probabilities of finding the atom in the exited state $|+\rangle$ and in the ground state $|-\rangle$. It can be presented as in (17 and 18).

$$\langle\sigma_z\rangle = \rho_{++} - \rho_{--} \tag{17}$$

$$= \mu_1(t) - \mu_2(t) \tag{18}$$

The purity evolution $P_A(t)$ is given by (19).

$$\begin{aligned}P_A(t) &= Tr_A(\rho_A^2(t)) \\ &= \rho_{++}^2 + 2\,|\rho_{+-}|^2 + \rho_{--}^2 \\ &= \mu_1(t)^2 + \mu_2(t)^2\end{aligned} \tag{19}$$

Where, $\rho_A(t)$ is the reduced density matrix of a two-level atom.

The information entropy of the system is defined as in [80, 81] and given by (20).

$$H(\sigma_z) = \sum_{i=1}^{N} P_i(\sigma_z) \ln P_i(\sigma_z), \tag{20}$$

The information entropy of the atomic operator σ_z in case of $N = 2$ (two-level atom) and applying the atomic reduced density operator $\rho_A(t)$ can be written in the form of (21).

$$H(\sigma_z) = -\left(\frac{1}{2} + \langle\sigma_z\rangle\right) \ln\left(\frac{1}{2} + \langle\sigma_z\rangle\right) - \left(\frac{1}{2} - \langle\sigma_z\rangle\right) \ln\left(\frac{1}{2} - \langle\sigma_z\rangle\right), \tag{21}$$

Therefore,

$$H(\sigma_z) = -\mu_1(t)\ln(\mu_1(t)) - \mu_2(t)\ln(\mu_2(t)). \tag{22}$$

The effect of the temperature T on the atomic inversion $\langle\sigma_z\rangle$, the purity $P_A(t)$ and the information entropy $H(\sigma_z)$ in case of the half wavelengths number of the field-mode $P = 1$ and $\rho_A(0) = |+\rangle\,\langle+|$, (the two-level atom is initially in exited state) is studied in Fig. 1. The atomic inversion $\langle\sigma_z\rangle$, the purity $P_A(t)$ and the information entropy $H(\sigma_z)$ have regular and periodic oscillations. Furthermore, when the atomic inversion $\langle\sigma_z\rangle$ and the information entropy $H(\sigma_z)$ start from their minimum value until they reach the maximum value , we observed that while the temperature increases, the maximum value and the amplitude of both the oscillations of the atomic inversion $\langle\sigma_z\rangle$ and the information entropy $H(\sigma_z)$ increase. On the contrary, when the purity $P_A(t)$ starts from its maximum value until it reaches the minimum value, we observed that while the temperature increases, the minimum value decreases, but the amplitude of the oscillations increases. Therefore, we can conclude that the temperature has a clear effect when the initial state is an exited state

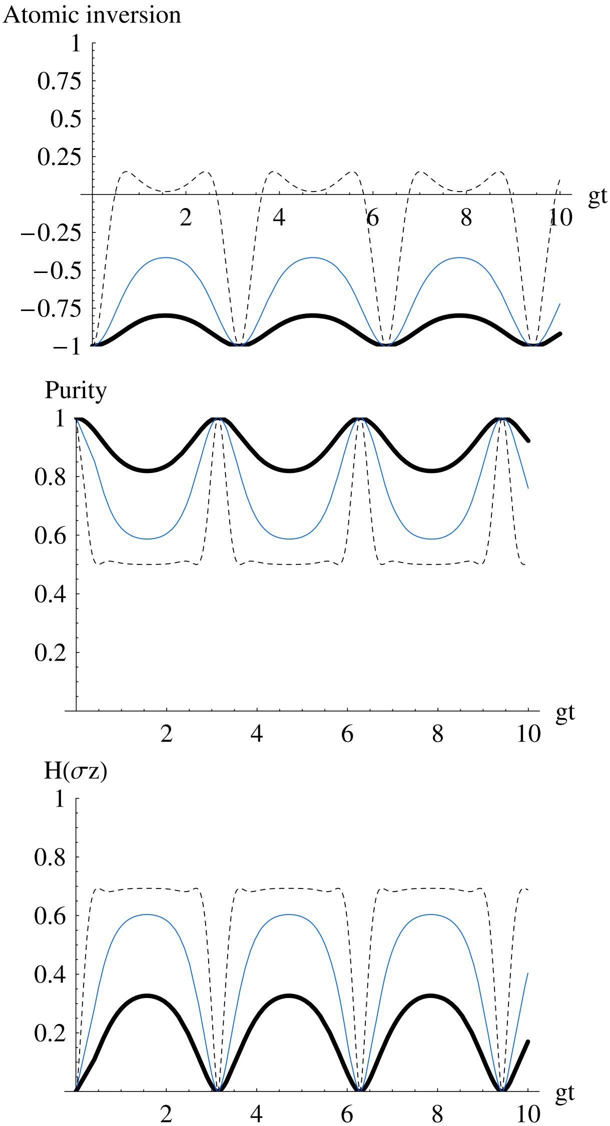

Fig. 1 Show the case in which $\omega = 1$, $g = 1$, $P = 1$, $B_g = 0$, $B_e = 1$, where solid, blue and dot curves correspond, respectively, to $T = 5$, $T = 1$ and $T = 0.5$

In Fig. 2, We discuss the effect of the temperature T in case of the two-level atom is initially in exited state and the half wavelengths number of the field-mode $P = 2$. The effect is similar to the one discussed in Fig. 1 except the number of oscillations. From Fig. 2, we observed that the number of oscillations is increased apparently compared to the result of Fig. 1. This difference is a consequence of the effect of the half wavelengths number of the field-mode P.

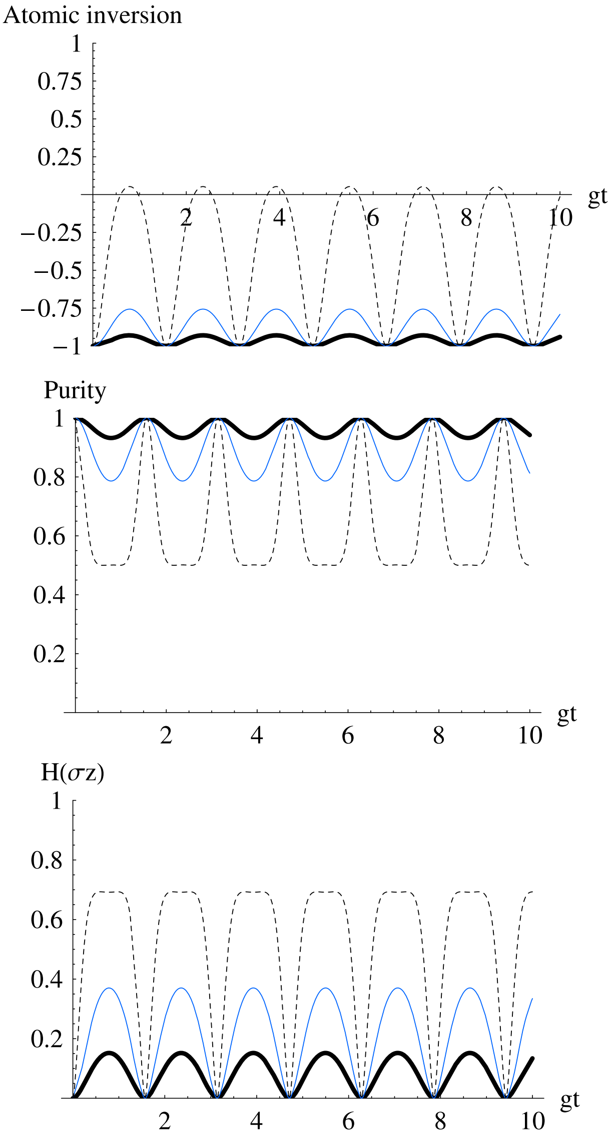

Fig. 2 Show the case in which $\omega = 1$, $g = 1$, $P = 2$, $B_g = 0$, $B_e = 1$, where solid, blue and dot curves correspond, respectively, to $T = 5$, $T = 1$ and $T = 0.5$

The effect of the temperature T on the atomic inversion $\langle\sigma_z\rangle$, the purity $P_A(t)$ and the information entropy $H(\sigma_z)$ in case of the half wavelengths number of the field-mode $P=1$ and $\rho_A(0)=\frac{1}{2}\left|+\right\rangle\left\langle+\right|+\frac{1}{2}\left|-\right\rangle\left\langle-\right|$, (the two-level atom is initially in the maximally mixed state) is studied in Fig. 3. When the atomic inversion $\langle\sigma_z\rangle$ and the information entropy $H(\sigma_z)$ start from their maximum value until they reach to the minimum value , we observed that while the temperature increases, the minimum value of both atomic inversion $\langle\sigma_z\rangle$ and the information entropy $H(\sigma_z)$ increase but the amplitude of the oscillations decrease. On the

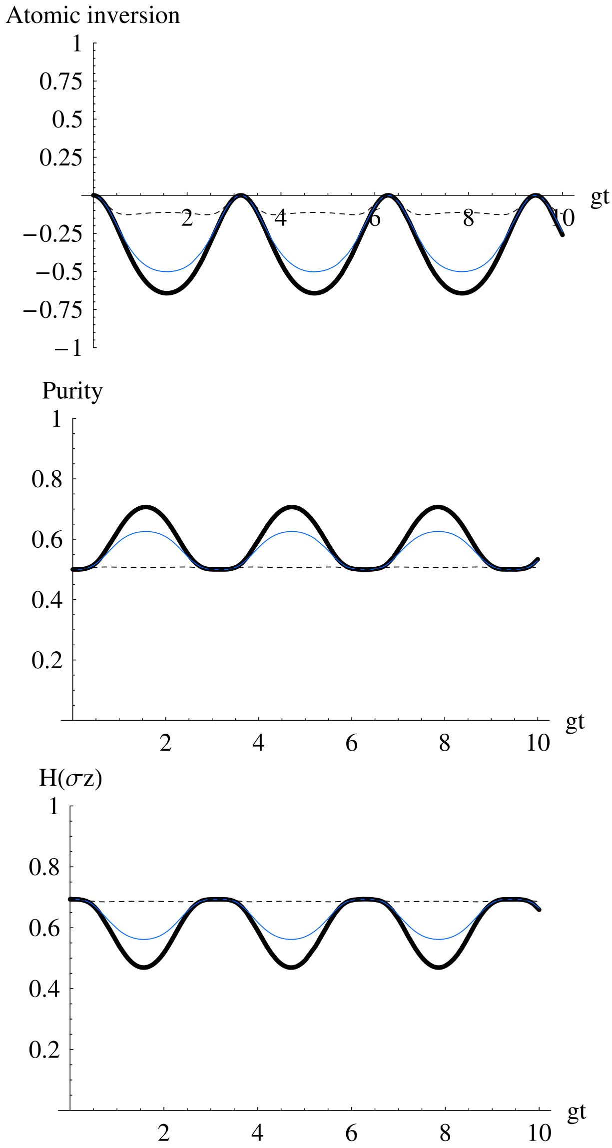

Fig. 3 Show the case in which $\omega=1$, $g=1$, $P=1$, $B_g=\frac{1}{2}$ and $B_e=\frac{1}{2}$, where solid, blue and dot curves correspond, respectively, to $T=5$, $T=1$ and $T=0.5$

contrary, when the purity $P_A(t)$ starts from its minimum value until it reaches the maximum value, we observed that while the temperature increases, the maximum value the amplitude of the oscillations of the purity $P_A(t)$ decrease. Therefore, we can conclude that the temperature has also a clear effect when the initial state is a maximally mixed state.

4 Conclusion

In this paper, we studied the system of a two-level atom with a constant velocity which interacts with a quantized field under the effect of one-photon transition. Moreover, it studied in the resonance case and the rotating-wave approximation. We focus on the effect of the temperature, the half wavelengths number of the field-mode and the initial state of a two-level atom for atomic inversion, the purity and the information entropy. We can conclude that the temperature has a clear effect when the initial state is either an exited state or a maximally mixed state. Additionally, the number of oscillations is changed accordingly to the effect of the half wavelengths number of the field-mode.

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