CrossMark

A Novel Quantum Image Steganography Scheme Based on LSB

Ri-Gui Zhou[1] · Jia Luo[1] (iD) · XingAo Liu[1] ·
Changming Zhu[1] · Lai Wei[1] · Xiafen Zhang[1]

© Springer Science+Business Media, LLC, part of Springer Nature 2018

Abstract Based on the NEQR representation of quantum images and least significant bit (LSB) scheme, a novel quantum image steganography scheme is proposed. The sizes of the cover image and the original information image are assumed to be $4n \times 4n$ and $n \times n$, respectively. Firstly, the bit-plane scrambling method is used to scramble the original information image. Then the scrambled information image is expanded to the same size of the cover image by using the key only known to the operator. The expanded image is scrambled to be a meaningless image with the Arnold scrambling. The embedding procedure and extracting procedure are carried out by K_1 and K_2 which are under control of the operator. For validation of the presented scheme, the peak-signal-to-noise ratio (PSNR), the capacity, the security of the images and the circuit complexity are analyzed.

1 Introduction

With the rapid development of quantum computation and quantum information, the research of quantum image processing is springing up. It can be divided into two directions: representations of quantum images and quantum image processing algorithms. The first direction, representations of quantum images, is the foundation of the quantum image processing. The task in this direction is the construction of a pattern to store the images on quantum computers. In recent years, lots of quantum image representations were studied, i.e., Qubit

---

Ri-Gui Zhou and Jia Luo are both first authors

✉ Jia Luo
luojia@stu.shmtu.edu.cn

Ri-Gui Zhou
rgzhou@shmtu.edu.cn

[1] College of Information Engineering, Shanghai Maritime University, Shanghai, 201306, China

Lattice [1], Real Ket [2], Entangled Image [3], a flexible representation for quantum images (FRQI) [4], multi-channel representation of quantum image (MCRQI) [5], a normal arbitrary quantum superposition state (NAQSS) [6], and a novel enhanced quantum representation (NEQR) [7]. Of these quantum representation models, two of them are most commonly used: one is FRQI using an angle to encode color, and the other is NEQR using a qubit sequence to store the color value of every pixel.

Many algorithms have been proposed in the second direction as well, such as Geometric transformation [8, 9], Image scaling [10, 28], Color transformation [7], Image scrambling [11, 12], Feature extraction [13], Quantum image encryption [14–18], Quantum image information hiding [19–27], and so on.

The quantum image information hiding includes quantum watermarking and quantum steganography of which have many similarities. For example, the goal of both methods is to hide the information into the carrier. Moreover, they both include embedding and extracting procedures.

Due to quantum information hiding is important for hiding information into other images and protecting the image copyright. The quantum image information hiding has a wide range of uses. In recent years, researchers had been committed to study in it. In 2012 A.M. Iliyasu, et al. proposed a quantum image watermarking algorithm based on restricted geometric transformations [19]. Later on, in 2013, watermark image was embedded into the Fourier coefficients of the quantum carrier image in the protocol proposed by W. W. Zhang et al., in which the carrier images would not be visually affected by the watermark images [20]. Then X.H. Song et al. proposed a dynamic watermarking scheme for quantum images based on Hadamard transform [21]. Recently, a blind LSB steganography in the form of quantum circuits had been proposed by N. Jiang et al. [22]. In 2016, S. Heidari, et al. proposed a novel LSB based quantum watermarking protocol [23]. Before this, S. Miyake proposed a quantum watermarking scheme using simple and small-scale quantum circuits [24]. The proposed scheme took a gray scale image as the watermark image by extending a $n \times n$ sized image with eight bits of gray scale to a $2n \times 2n$ sized image with two bits of gray scale. In 2017, B. Abd-El-Atty et al. proposed the New Quantum Image Steganography Scheme with Hadamard Transformation [26], in which the quantum text message was hided into the carrier image.

In order to enhance the security of the information image, a novel quantum steganography scheme based on LSB is proposed in this paper. The outline of this paper is structured as follows. Section 2 introduces the preliminary knowledge of quantum image model NEQR, LSB scheme, Bit-plane scrambling and Arnold scrambling methods. The proposed scheme is explained in Section 3 in detail. Section 4 presents the simulation and analysis of the scheme. Finally, a brief conclusion is drawn in Section 5.

2 Background

2.1 Novel Enhanced Quantum Representation for Quantum Images

A Novel Enhanced Quantum Representation of digital images (NEQR) was proposed in 2013 [7]. The expression form of a quantum image described by the NEQR model is:

$$|I\rangle = \frac{1}{2^n} \sum_{Y=0}^{2^n-1} \sum_{X=0}^{2^n-1} |C_{YX}\rangle |Y\rangle |X\rangle = \frac{1}{2^n} \sum_{YX=0}^{2^{2n}-1} \overset{q-1}{\underset{i=0}{\otimes}} C_{YX}^i \otimes |YX\rangle \qquad (1)$$
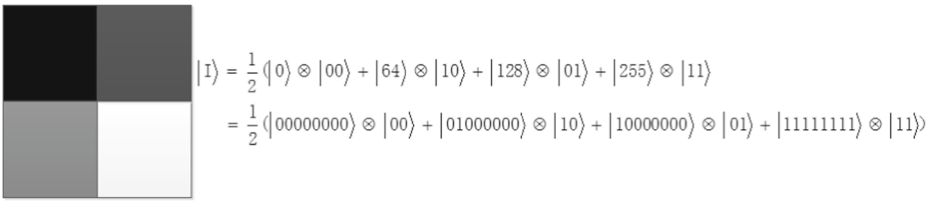
Fig. 1 A 2×2 NEQR image representation

where $|YX\rangle$ stands for position information and $|C_{YX}^i\rangle$ is the color information.

$$|YX\rangle = |Y\rangle|X\rangle = |y_0 y_1 ... y_{n-1}\rangle|x_0 x_1 ... x_{n-1}\rangle, \; y_i, x_i \in \{0, 1\}, \; i = 0, 1, ..., n-1$$
$$|C_{YX}\rangle = \left|C_{YX}^0 C_{YX}^1 ... C_{YX}^{q-1}\right\rangle, \; C_{YX}^i \in \{0, 1\}, \; i = 0, 1, ..., q-1 \qquad (2)$$

Thus, there are $q + 2n$ qubits to represent a quantum image of size $2^n \times 2^n$ with the gray range of 2^q. Figure 1 shows an example of an image of 2×2 and the corresponding NEQR representation is on the right of the image.

2.2 The Least Significant Bit (LSB)

The least significant bit is one of the pixel bits shown in Fig. 2. If the control bit is "1", we only need to change the least significant bit from "1" to "0", i.e., color value 227 is changed into 226. Simply, the change of pixel value is only one gray level, so the visual effect is not affected basically.

2.3 Bit-Plane Scrambling

The pixel values in the image are represented by its corresponding binary values. And then, every single bit of all the pixels will constitute a binary image, it is called bit-plane. For example, if the gray range is between 0 and 2^8, the image will be separated into 8 bit-planes. Figure 3 shows the result of bit-planes of Lena image of size $2^8 \times 2^8$.

From above, it can be known that different bit-planes carry different visual information. If these bit-planes are measured, the information will become disordered. By scrambling the bit-plane, the image will be scrambled.
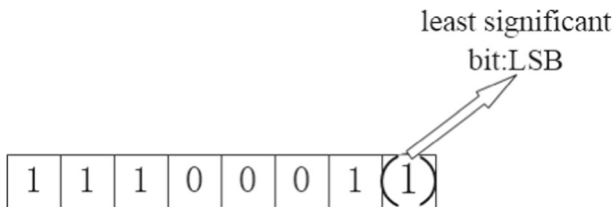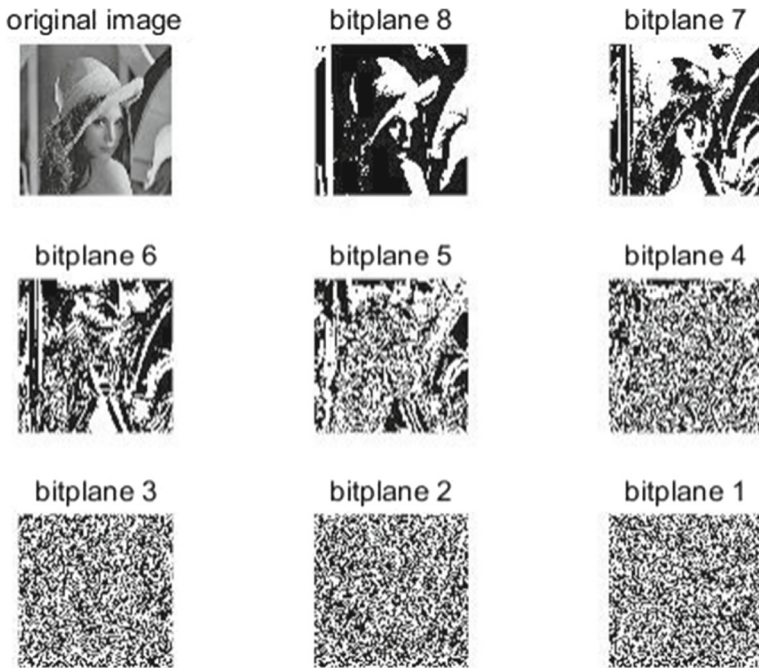

Fig. 2 An example of LSB

Fig. 3 Bit-planes of Lena

2.4 Arnold Scrambling

Arnold scrambling is used to change the pixel horizontal and vertical coordinates to disrupt the original image information. The Arnold scrambling for a digital image can be illustrated specifically as following.

Supposing that there is an original image with size of $2^n \times 2^n$, the pixel coordinates of the original image and the scrambled image are (X, Y) and (X_A, Y_A), respectively. Two-dimensional Arnold scrambling process is expressed by (3) and its inverse operation is shown in (4).

$$\begin{bmatrix} X_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \bmod 2^n$$
$$X_A = (X + Y) \bmod 2^n , \ Y_A = (X + 2Y) \bmod 2^n \tag{3}$$

The inverse operation is:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} X_A \\ Y_A \end{bmatrix} \bmod 2^n = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \end{bmatrix} \bmod 2^n$$
$$X = (2X_A - Y_A) \bmod 2^n , \ Y = (Y_A - X_A) \bmod 2^n \tag{4}$$

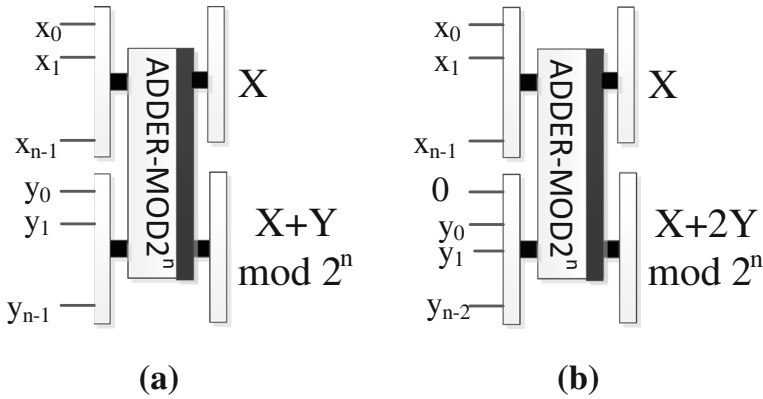According to literature [11], the circuits of Arnold scrambling and inverse-scrambling are shown in Figs. 4 and 5.

Fig. 4 The quantum circuit of Arnold image scrambling: **a** $|x_A\rangle$ circuit **b** $|y_A\rangle$ circuit

3 The Proposed Quantum Image Steganography Scheme

The proposed quantum image steganography scheme hides a gray scale image (secret information image) into the other gray scale image (cover image), and the size of two images is $n \times n$ and $4n \times 4n$, respectively. The embedding and extracting procedures are shown in Fig. 6, where Fig. 6a is embedding procedure and Fig. 6b is extracting procedure. The specific scheme is described as follows.

3.1 Preparation Procedure

In this section the original information image $|I\rangle$ should be scrambled by the bit-plane scrambling. Figure 7 shows an example of the circuit that is used for processing an image with gray scale of 2^8. Firstly, there are four SWAP gates and four CNOT gates utilized in this circuit. The lowest bit $|C_7\rangle$ swaps with the highest bit $|C_0\rangle$, the bit $|C_6\rangle$ swaps with the bit $|C_1\rangle$, the bit $|C_5\rangle$ swaps with the bit $|C_2\rangle$ and the bit $|C_4\rangle$ swaps with the bit $|C_3\rangle$. Then, four low bits will be the control bits and apply the results to four high bits. Therefore, the scrambled information image $|I_1\rangle$ is prepared.

3.2 Embedding Procedure

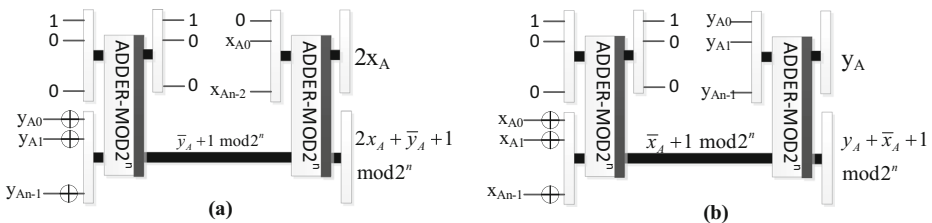The embedding procedure consists of four steps that illustrate as follows:

Fig. 5 The quantum circuit of inverse Arnold image scrambling: **a** $|x\rangle$ circuit **b** $|y\rangle$ circuit
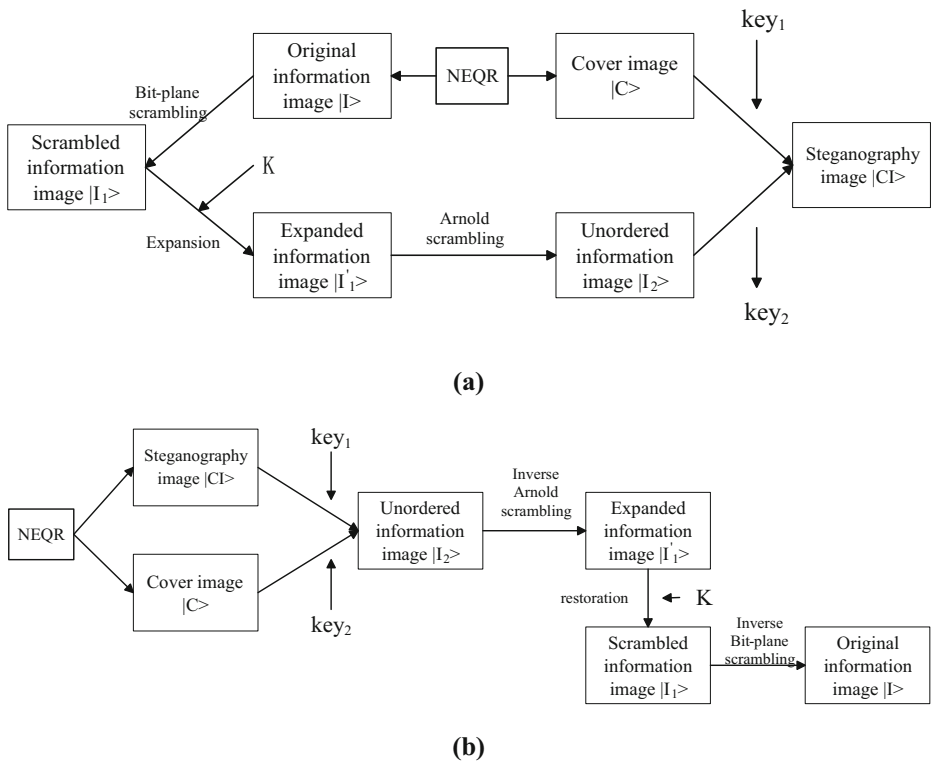
(a)

(b)

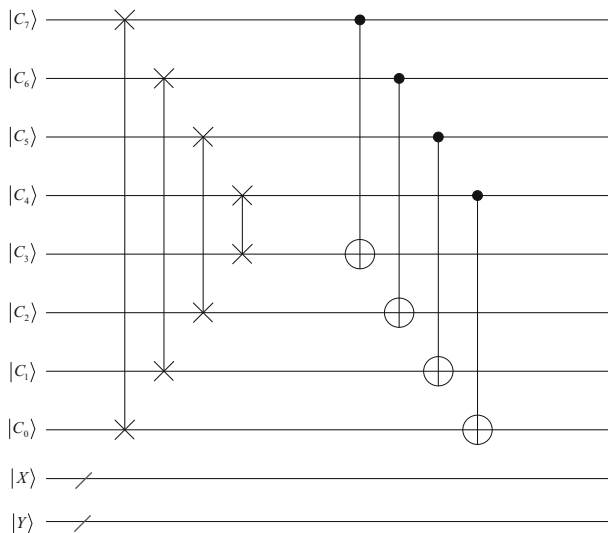Fig. 6 Quantum image steganography scheme: **a** embedding procedure **b** extracting procedure

Fig. 7 Bit-plane scrambling for a NEQR quantum image with gray scale of 2^8

Step 1　The aim of this step expands the scrambled image to the expanded image which has same size with the cover image. An example, as shown in Fig. 8a, both of the key image $|K\rangle$ that only known to the operator and the gray scale image $|I_1\rangle$ are size of 1×1, in which $|K\rangle$ is a binary image and $|I_1\rangle$ will be expanded to a 4×4 binary image $|I_1'\rangle$. Figure 8b gives the concrete quantum circuit realization of expanding the scrambled image to the expanded image which has same size with the cover image, where the key image $|K\rangle$ and the gray scale image $|I_1\rangle$ both are size of $2^n \times 2^n$, respectively. If the color value $|K^i\rangle$ of key $|K\rangle$ is equal to 1, an 8 bits string is divided into 8 single bits whose positions are $(4i, 4j + 1)$, $(4i, 4j + 2)$, $(4i + 1, 4j)$, $(4i + 1, 4j + 1)$, $(4i + 1, 4j + 2)$, $(4i + 2, 4j)$, $(4i + 2, 4j + 1)$ and $(4i + 2, 4j + 2)$, respectively. The value of the position $(4i, 4j)$ is equal to 1 as well. If the key $|K^i\rangle$ is equal to 0, an 8 bits string will be divided into 8 single bits by reverse permutation. Similarly, the value of the position $(4i, 4j)$ is equal to 0, and the rest of the expanded image is covered by the value 1.

Step 2　The Arnold scrambling is used for scrambling the expanded image $|I_1'\rangle$ to an unordered image $|I_2\rangle$.

Step 3　The unordered information image $|I_2\rangle$, the key $|K_1\rangle$ and the cover image $|C\rangle$ have been prepared. Their NEQR representations are:

$$|I_2\rangle = \frac{1}{2^n} \sum_{j=0}^{2^{2n}-1} |I_2^j\rangle \otimes |YX\rangle,\ I_2^j \in \{0, 1\} \tag{5}$$

$$|K_1\rangle = \frac{1}{2^n} \sum_{j=0}^{2^{2n}-1} |K_1^j\rangle \otimes |YX\rangle,\ K_1^j \in \{0, 1\} \tag{6}$$

$$|C\rangle = \frac{1}{2^n} \sum_{YX=0}^{2^{2n}-1} \overset{q-1}{\underset{i=0}{\otimes}} C_{YX}^i \otimes |YX\rangle,\ |C_{YX}\rangle = |C_{YX}^0 C_{YX}^1 \dots C_{YX}^{q-1}\rangle,\ C_{YX}^i \in \{0, 1\},\ i = 0, 1, ..., q-1 \tag{7}$$

where $|YX\rangle = |y_{n-1}y_{n-2}...y_0\rangle|x_{n-1}x_{n-2}...x_0\rangle$, $y_i, x_i \in \{0, 1\}$, $i = 0, 1, ..., n-1$

Each position in the unordered information image $|I_2\rangle$ will be compared with the position in the cover image $|C\rangle$. Literature [25] proposed a quantum equal circuit presented in Fig. 9. The comparator compares $|YX\rangle$ and $|AB\rangle$, where $|YX\rangle = |Y\rangle|X\rangle = |y_{n-1}...y_0\rangle|x_{n-1}...x_0\rangle$ and $|AB\rangle = |A\rangle|B\rangle = |a_{n-1}...a_0\rangle|b_{n-1}...b_0\rangle$, $y_i, x_i, a_i, b_i \in 0, 1, i = n-1, ..., 0$. Qubit $|c\rangle$ is output. If $|c\rangle = |1\rangle$, $|YX\rangle = |AB\rangle$, otherwise, $|YX\rangle \neq |AB\rangle$.

When the unordered information image is embedded, the steganography image and the binary image key $|K_2\rangle$ are obtained. The quantum steganography embedding circuit is shown in Fig. 10. If the position is the same, then implement the embedding procedure as follows:
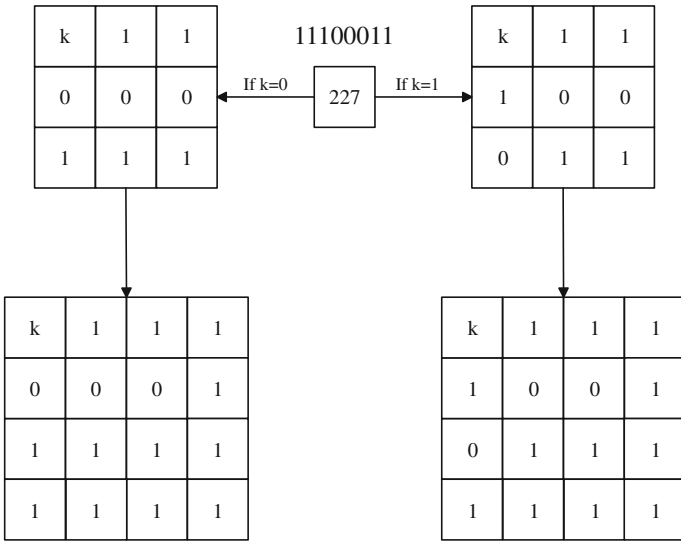
> For $i = 0$ to $2^{2n} - 1$
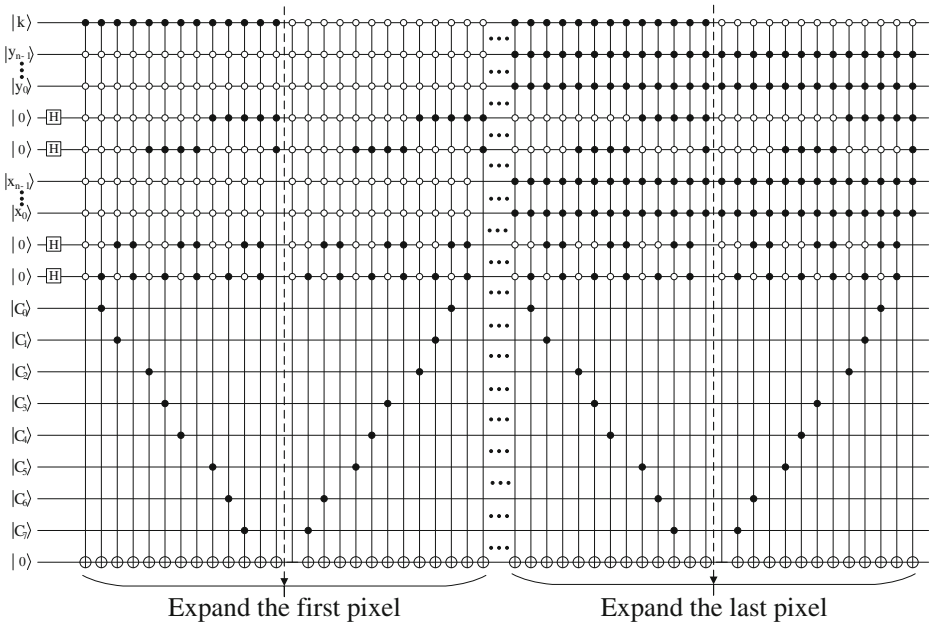> > If ($|I_2^i\rangle == |0\rangle$) then
> > > If ($|K_1^i\rangle == |1\rangle$) then $|CI_i^0\rangle = |C_i^0\rangle \oplus |1\rangle$
> > > Else if ($|K_1^i\rangle == |0\rangle$) then $|K_2^i\rangle == |1\rangle$
> End

(a)

Expand the first pixel   Expand the last pixel

(b)

Fig. 8 **a** An example of 1 × 1 image expands to 4 × 4 image **b** The expanding circuit for a $2^n \times 2^n$ NEQR image
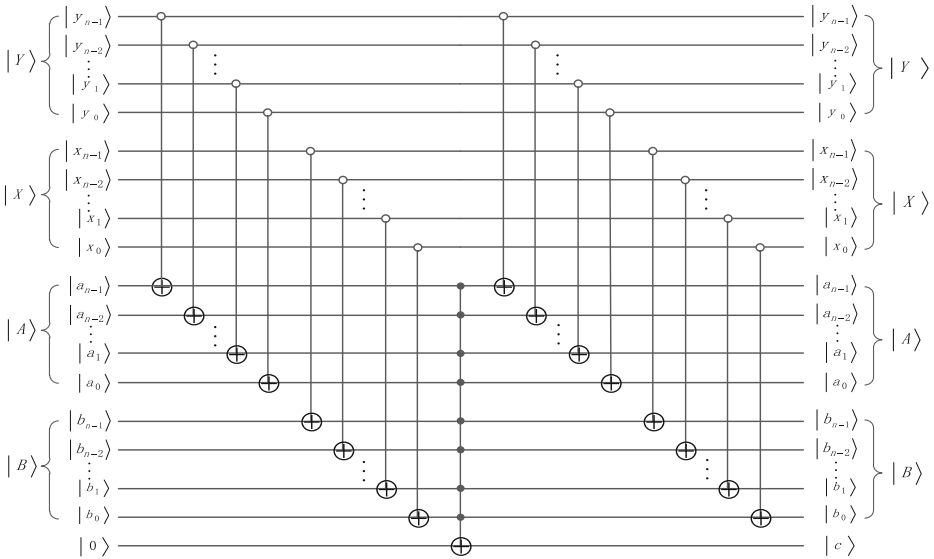
Fig. 9 Quantum Equal circuit (QE)

3.3 Extracting Procedure

Considering the security of the unordered information image, the extracting procedure is more complex than embedding procedure.

The first step extracts the information from the steganography image $|CI\rangle$ and the key $|K_2\rangle$. It is necessary to have the cover image $|C\rangle$, the steganography image $|CI\rangle$ and
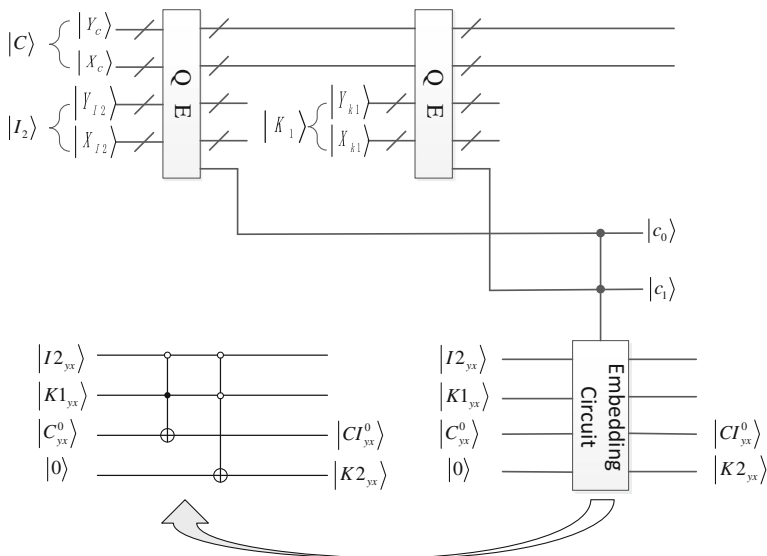

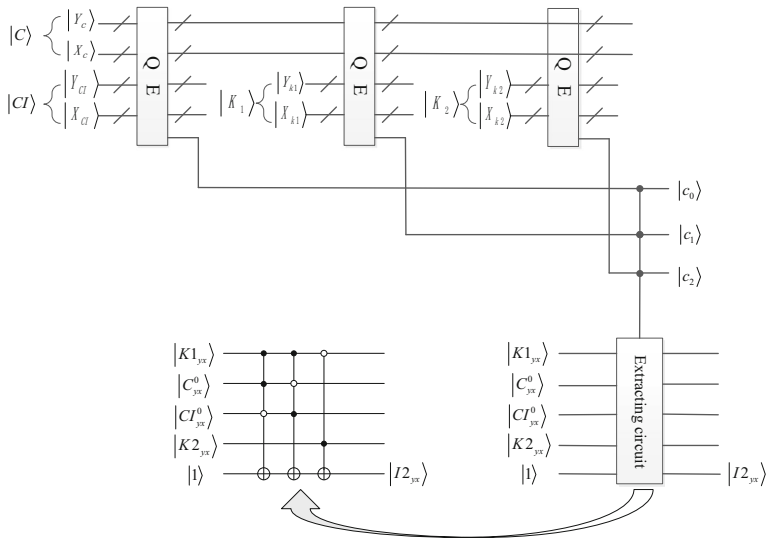**Fig. 10** Circuit realization of quantum steganography embedding

Fig. 11 Circuit realization of quantum steganography extracting

two keys $|K_1\rangle$ and $|K_2\rangle$ extracted from the unordered information image. The quantum steganography extracting circuit is shown in Fig. 11.

If the position is the same, the corresponding color value will be compared. The extracting procedure can be done in the following steps:

For $i = 0$ to $2^{2n} - 1$

    If $|K_1^i\rangle == |1\rangle$ then

        If $|C_i^0\rangle \oplus |CI_i^0\rangle == |1\rangle$ then $|I_2^i\rangle == |0\rangle$

    Else if $|K_1^i\rangle == |0\rangle$ then

        If $|K_2^i\rangle == |1\rangle$ then $|I_2^i\rangle == |0\rangle$

End

The second step of extracting procedure is recovering the unordered information image $|I_2\rangle$ to the expanded image $|I_1'\rangle$ by the inverse operation of the Arnold scrambling.

The third step is restoring the expanded image $|I_1'\rangle$ to the scrambled information image $|I_1\rangle$. To judge the value of the position (i, j) in key $|K\rangle$, where i and j are less than n, if the value equals to 1, the 8 bits string of the gray scale of the pixel (i, j) in image $|I_1\rangle$ equals to the value of these positions $[(4i, 4j + 1), (4i, 4j + 2), (4i + 1, 4j), (4i + 1, 4j + 1), (4i + 1, 4j + 2), (4i + 2, 4j), (4i + 2, 4j + 1), (4i + 2, 4j + 2)]$ in image $|I_1'\rangle$; if the value equals to 0, the 8 bits string is in the opposite order. The restoring circuit is shown in Fig. 12.

The final step is the inverse bit-plane scrambling operation to the scrambled information image that is importing the color value in the right side of the circuit like Fig. 7.
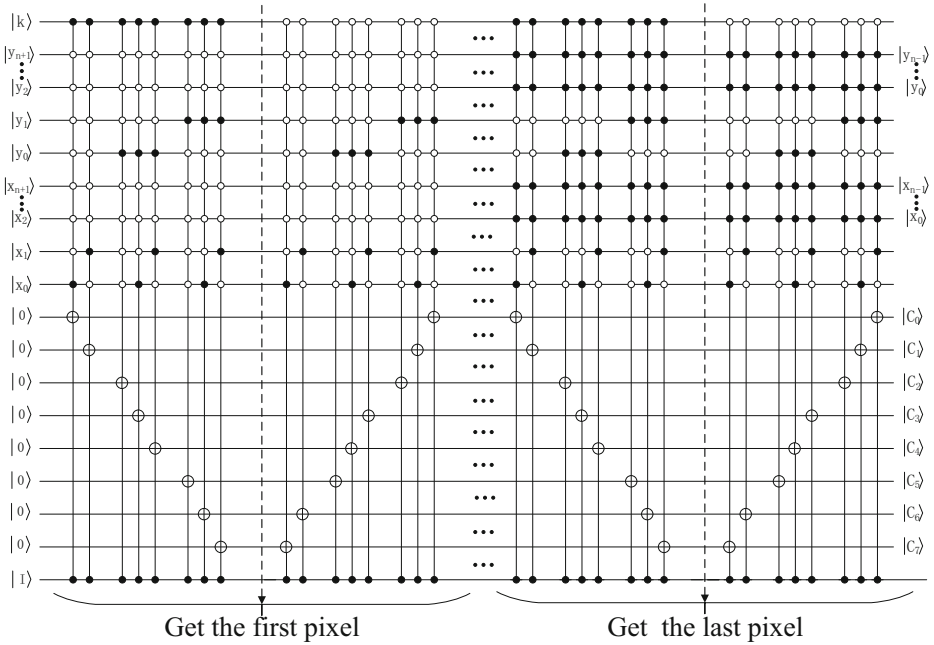
Fig. 12 The restoring circuit

4 Simulation and Analysis

Since there is no enough physical quantum hardware to implement the proposed scheme, the simulation of quantum operations is performed with MATLAB on the classical computer. Six familiar images ("Mandrill", "Cameraman", "Airplane", "Pepper", "Rice" and "Lena")
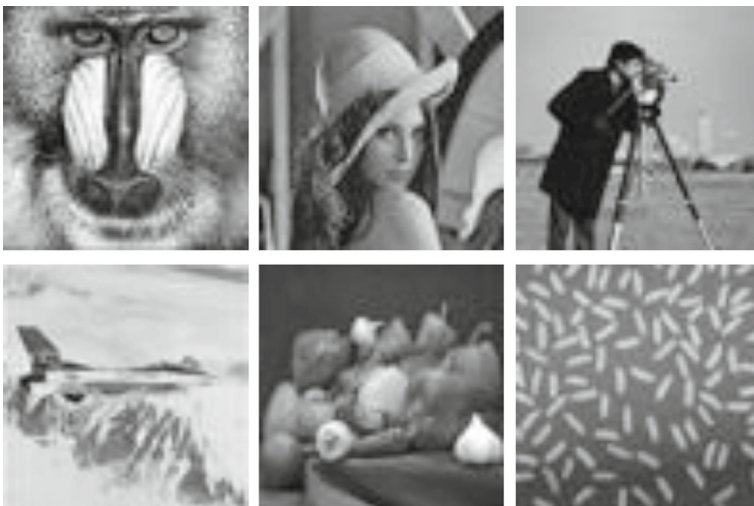

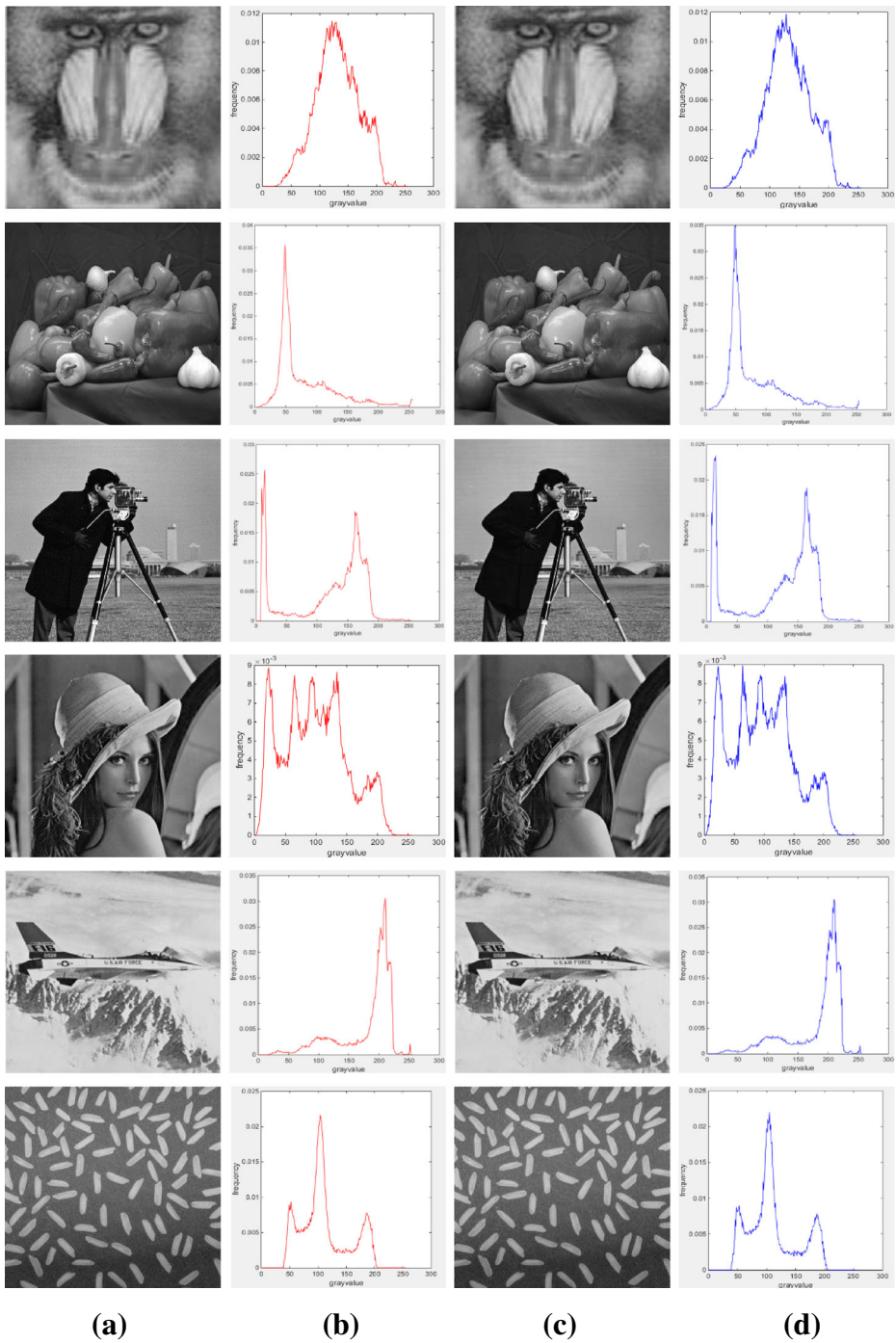**Fig. 13** Six information images with size 64×64

Fig. 14 a the cover images, **b** their corresponding histograms **c** the steganography images (which embeds information images of Mandrill, Pepper, Cameraman, Lena, Airplane, Rice, respectively). **d** their corresponding histograms

Table 1 PSNR values in proposed scheme

Cover image	Information image	PSNR
Mandrill	Lena	50.22
Pepper	Airplane	50.23
Cameraman	Pepper	50.19
Lena	Rice	50.21
Airplane	Cameraman	50.28
Rice	Mandrill	50.23

are used. And the sizes of cover image and information image are 256×256 and 64×64, respectively. The information images are shown in Fig. 13.

4.1 Visual Quality

There are quantities of techniques to measure the difference of pixels between the steganography image and the cover image, where the peak-signal-to-noise ratio (PSNR) is one of the most employed. It is defined as the mean squared error (MSE) for two m×n images P and Q.

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [(P(i,j) - Q(i,j))^2] \tag{8}$$

PSNR is defined as:

$$PSNR = 20 \log_{10} \left(\frac{MAX_P}{\sqrt{MSE}} \right) \tag{9}$$

where MAX_P is the maximum pixel value of the cover image C. In our scheme, P is associated with the steganography image and Q is associated with the cover image.

Obviously, human eyes can't identify the difference between the cover image and the steganography image, which is shown in Fig. 14. Compared Table 1 with Table 2, it can be obtained that the values of PSNR of our proposed scheme are higher than literature [24] which is also use the gray scale image as the information image.

4.2 Capacity

The steganography capacity can be stated as the ratio between the number of embedded bits of information image and the number of pixels of cover image. The proposed scheme's capacity is given as follows:

$$C = \frac{the\ num.\ of\ information\ bits}{the\ num.\ of\ cover\ image\ pixels} = \frac{8 \times 2^{n-2} \times 2^{n-2}}{2^{2n}} = \frac{1}{2} (bit/pixel) \tag{10}$$

In order to enhance the confidentiality, half of the pixels are not to carry information.

Table 2 PSNR values in [24]

Cover image	Information image	PSNR
Cameraman	Lena	44.17
Cameraman	Mandrill	44.05
Lena	Cameraman	43.88
Lena	Mandrill	44.05

4.3 Security Analysis

In our proposed scheme, we put the security in a significant status. First, the original image is scrambled to a disordered image. And then the meaningless image consists of half of the information pixels and half of blank pixels will be hiding in the cover image. Furthermore, the extracting operation needs the cover image, the steganography image and two keys, which are all indispensable. In all, the security of the proposed scheme is excellent.

4.4 Circuit Complexity

In this section, the circuit complexity of the proposed scheme is discussed. We assume that the sizes of cover image and original information image are $2^n \times 2^n$ and $2^{n-2} \times 2^{n-2}$, respectively. Literature [10] pointed out that circuit complexity of a n-CNOT gate ($n \geqslant 3$) is about $12n-9$ and each swap gate can be realized by three CNOT gates.

a.   The Time Complexity of Embedding Procedure

The embedding procedure is shown in Fig. 6a and the bit-plane scrambling circuit is shown in Fig. 7 of which complexity is $(4 \times 3 + 4)$. According to [11], the complexities of Arnold scrambling circuit $|x_A\rangle$ and $|y_A\rangle$ are $28n$ and $28n$, respectively. The expanding circuit is shown in Fig. 8b, supposing it expands $2^{n-2} \times 2^{n-2}$ pixels, and every pixel needs twenty-six $(2n+2)$-CNOT gates (In order to facilitate the calculation, all CNOT gates are considered to have the same number of control bits). Thus, the complexity of the expanding circuit is $2^{2n-4} \times 26 \times [12(2n+2)-9]$. Additionally, Fig. 9 shows the complexity of single QE circuit is $4n + (12 \times 2n - 9)$. Therefore, the complexity of the embedding circuit is $2 \times [4n + (12 \times 2n - 9)] + 2 \times 6$. Consequently, the time complexity of the whole embedding procedure is:

$$O\left(\begin{array}{c} 4 \times 3 + 4 + 28n + 28n + 2^{2n-4} \times 26 \times [12(2n+2) - 9] \\ +2 \times [4n + (12 \times 2n - 9)] + 2 \times 6 \end{array} \right)$$

$$= O\left(16 + 56n + 2^{2n-4} \times 26(24n + 15) + 56n - 6 \right)$$

$$= O\left(2^{2n-4} \times (624n + 390) + 112n + 10 \right) \approx O\left(624n \cdot 2^{2n-4} \right) \quad (11)$$

b.   The Time Complexity of Extracting Procedure

The extracting procedure is shown in Fig. 6b. Figure 11 shows extracting circuit in which the complexity is $3 \times [4n + (12 \times 2n - 9)] + 2 \times (12 \times 3 - 9) + 6$. The complexities of inverse Arnold scrambling $|x\rangle$, $|y\rangle$ and bit-plane scrambling are $56n$, $56n$ and $(4 \times 3 + 4)$, respectively. Finally, the complexity of restoring circuit is $2^{2n-4} \times 16 \times [12(2n+2) - 9]$. In summary, the time complexity of the whole extracting procedure is:

$$O\left(\begin{array}{c} 4 \times 3 + 4 + 56n + 56n + 2^{2n-4} \times 16 \times [12(2n+2) - 9] \\ +3 \times [4n + (12 \times 2n - 9)] + 2 \times (12 \times 3 - 9) + 6 \end{array} \right)$$

$$= O\left(16 + 112n + 2^{2n-4} \times 16(24n + 15) + 84n + 33 \right)$$

$$= O\left(2^{2n-4} \times (384n + 240) + 196n + 49 \right)$$

$$\approx O\left(384n \cdot 2^{2n-4} \right) \quad (12)$$

According to the above analysis, the circuit complexity of embedding and extracting procedures of the proposed scheme is higher than [24]. But this paper designed the expanding and restoring circuit realization while other researchers have not mentioned it in their gray scale image steganography scheme.

5 Conclusion

Based on LSB scheme, a novel quantum image steganography scheme is proposed, where the NEQR representation is utilized. Therein, the image sizes for cover and information are $4n \times 4n$ and $n \times n$, respectively. The information hiding part contains two procedures. The first procedure is the preparation of scrambling the original information image. And the other procedure is embedding work which concludes three steps: The first step is expanding the scrambled information image to the same size as the cover image; the second step is to scramble the expanded image to a meaningless image; and the third step is embedding the unordered information image into the cover image. In the information extracting part, two keys that are only possessed by the operator are used for extracting the unordered information image from the steganography image and then the inverse operation of information hiding part will be executed. The experiments show that the scheme has a good visual quality than another scheme. Furthermore, the security of the scheme is excellent according to the analysis.

References

1. Venegas-Andraca, S.E., Bose, S.: Storing, processing and retrieving an image using quantum mechanics. Proc. SPIE Conf. Quantum Inf. Comput. **5105**, 137–147 (2003)
2. Latorre, J.I.: Image compression and entanglement. arXiv:quant-ph/0510031 (2005)
3. Venegas-Andraca, S.E., Ball, J.L., Burnett, K., Bose, S.: Processing images in entangled quantum systems. Quantum Inf. Process. **9**, 1–11 (2010)
4. Le, P.Q., Dong, F., Hirota, K.: A flexible representation of quantum images for polynomial preparation, image compression and processing operations. Quantum Inf. Process. **10**(1), 63–84 (2010)
5. Sun, B., Le, P.Q., Iliyasu, A.M.: A multi-channel representation for images on qunatum computers using the $RGB\alpha$ color space. In: IEEE 7th International Symposium on Intelligent Signal Processing, Floriana, Malta, pp. 1–6 (2011)
6. Li, H.S., Zhu, Q., Zhou, R.G., Song, L., Yang, X.J.: Multi-dimensional color image storage and retrieval for a normal arbitrary quantum superposition state. Quantum Inf. Process. **13**(4), 991–1011 (2014)
7. Zhang, Y., Lu, K., Gao, Y.H., Xu, K.: A novel quantum representation for log-polar images. Quantum Inf. Process. **12**(9), 3103–3126 (2013)
8. Le, P.Q., Iliyasu, A.M., Dong, F.Y., Hirota, K.: Fast geometric transformation on quantum images. IAENG Int. J. Appl. Math. **40**(3), 113–123 (2010)
9. Zhou, R.-G., Tan, C.Y., Hou, I.: Global and local translation designs of quantum image based on FRQI. Int. J. Theor. Phys. **56**(4), 1382–1398 (2017)
10. Jiang, N., Wang, J., Mu, Y.: Quantum image scaling up based on nearest-neighbor interpolation with integer scaling ratio. Quantum Inf. Process. **14**(11), 4001–4026 (2015)
11. Jiang, N., Wang, L.: Analysis and improvement of the quantum Arnold image scrambling. Quantum Inf. Process. **13**(7), 1545–1551 (2014)

12. Jiang, N., Wang, L., Wu, W.Y.: Quantum Hilbert image scrambling. Int. J. Theor. Phys. **53**(7), 2463–2484 (2014)
13. Zhang, Y., Lu, K., Xu, K., Gao, Y.H., Wilson, R.: Local feature point extraction for quantum images. Quantum Inf. Process. **14**(5), 1573–1588 (2015)
14. Zhou, R.-G., Wu, Q., Zhang, M.-Q., et al.: Quantum image encryption and decryption algorithms based on quantum image geometric transformations. Int. J. Theor. Phys. **52**, 1802–1817 (2013)
15. Zhou, R.-G., Sun, Y.-J., Fan, P.: Quantum image Gray-code and bit-plane scrambling. Quantum Inf. Process. **14**, 1717–1734 (2015)
16. Wang, S., Song, X.H., Niu, X.M.: A novel encryption algorithm for quantum images based on quantum wavelet transform and diffusion. Adv. Intell. Syst. Comput. **298**, 243–250 (2014)
17. Hua, T., Chen, J., Pei, D., et al.: Quantum image encryption algorithm based on image correlation decomposition. Int. J. Theor. Phys. **54**(2), 526–537 (2015)
18. Gong, L.-H., He, X.-T., Cheng, S., et al.: Quantum Image Encryption Algorithm Based On Quantum Image XOR operations. Int. J. Theor. Phys. **55**, 3234–3250 (2016)
19. Iliyasu, A.M., Le, P.Q., Dong, F., Hirota, K.: Watermarking and authentication of quantum images based on restricted geometric transformations. Inf. Sci. **186**, 126–149 (2012)
20. Zhang, W.W., Gao, F., Liu, B., et al.: A watermark strategy for quantum images based on quantum Fourier transform. Quantum Inf. Process. **12**(4), 793–803 (2013)
21. Song, X.H., Wang, S., Liu, S., Abd El-Latif, A.A., Niu, X.M.: Dynamic watermarking scheme for quantum images based on Hadamard transform. Multimed. Syst. **20**(4), 379–388 (2014)
22. Jiang, N., Zhao, N., Wang, L.: LSB based quantum image steganography algorithm. Int. J. Theor. Phys. **55**, 107–123 (2015)
23. Heidari, S., Naseri, M.: A novel LSB based quantum watermarking. Int. J. Theor. Phys. **55**, 4205–4218 (2016)
24. Miyake, S., Nakamae, K.: A quantum watermarking scheme using simple and small-scale quantum circuits. Quantum Inf. Process. **15**, 1849–1864 (2016)
25. Zhou, R.-G., Hu, W.W., Fan, P.: Quantum watermarking scheme through Arnold scrambling and LSB steganography. Quantum Inf. Process. **16**(9), 212 (2017)
26. Abd-El-Atty, B., Abd El-Latif, A.A., Amin, M.: New quantum image steganography scheme with hadamard transformation. Adv. Intell. Syst. Comput. **533**, 342–352 (2017)
27. Li, P., Zhao, Y., Xiao, H., et al.: An improved quantum watermarking scheme using small-scale quantum circuits and color scrambling. Quantum Inf. Process. **16**(5), 127 (2017)
28. Zhou, R.-G., Hu, W.W., Fan, P., Hou, I.: Quantum realization of the bilinear interpolation method for NEQR. Sci. Rep. **7**(1), 2511 (2017)