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Bidirectional Quantum Teleportation of a Class of n-Qubit States by Using $(2n + 2)$-Qubit Entangled States as Quantum Channel

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Abstract A new protocol of bidirectional quantum teleportation (BQT) is proposed in which the users can transmit a class of n-qubit state to each other simultaneously, by using $(2n + 2)$-qubit entangled states as quantum channel. The state of the art approaches can only transmit two-qubit states in each round. This scheme is based on control-not operation, single-qubit measurements and appropriate single-qubit unitary operations. It is shown that the protocol is secure in preparation phase.

Keywords Bidirectional quantum teleportation · n-qubit state · $(2n + 2)$-qubit channel · Entanglement · Security

1 Introduction

Quantum science has been attracting much attention in recent years. Quantum teleportation (QT) [1] is one of the most important protocols of quantum information theory. QT is a type of quantum communication, where by using entanglement and classical information, one of the users from one place teleport an unknown quantum state to the other user in the other place. This kind of protocol was presented in 1993. Bennett et al. [2] proposed the first protocol of QT by using Einstein-Podolsky-Rosen (EPR) pair. Since 1993, several QT schemes have been proposed so far [3–11] by using other entangled states as a quantum channel including Greenberger—Horne—Zeilinger (GHZ) state, W state, GHZ-like state, and etc, where the only difference among these protocols is the quantum channel.

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A different kind of QT protocols was proposed in 1998 by Karlsson and Bourennane [12]. This type of QT is a controlled quantum teleportation (CQT), in which the sender transmits an unknown state to a remote receiver under the control of the supervisor. After that, several CQT protocols have been proposed [13–17] by using one or more than one controller. In these protocols the third person controls the qubits of users by applying measurement on his qubits.

Several experiments have been performed for realizing quantum teleportation. In pioneering work the teleportation protocol was implemented with single photons [18] over lab-scale distances and later also over km-scale distances in free space [19–22].

One of the most important subfields of QT is bidirectional quantum teleportation (BQT). In BQT, two users can teleport qubits to each other and two users play sender role and receiver role, simultaneously. In 2013, the first CBQT was proposed by Zha et al. [23]. After that, several BCQT protocols have been proposed [24–28]. Yan [24] and Sun et al. [25] proposed two protocols of BQCT by using six-qubit cluster state and six-qubit maximally-entangled state as a quantum channel, respectively. Li and Nie [26] presented a protocol of BQCT via five-qubit composite GHZ-Bell state as a quantum channel but Shukla et al. [27] showed that the Li's protocol [26] is not a BQCT scheme. Then, Li et al. [28] proposed a protocol via five-qubit entangled state. All of those protocols were proposed in 2013.

In 2014, Fu et al. [29] presented a BQT scheme by using a four-qubit cluster state as a quantum channel. In this scheme, users can simultaneously exchange their single-qubit states by applying Hadamard operation. In the same year, Chen [30], Duan et al. [31] and Duan and Zha [32], proposed new schemes of BCQT by using five-qubit entangled state, seven-qubit entangled state, and six-qubit entangled state, as a quantum channel, respectively. In Duan et al.'s scheme [31], Charlie improves the security of the protocol by performing single-qubit measurements three times. In the next study, Duan and Zha [32] improved the security of their protocol by applying two single-qubit measurements.

In 2015, different schemes of BCQT by using six-qubit genuine state [33], GHZ-type state [34], eight-qubit entangled state [35], and six-qubit entangled state [36] as a quantum channel were proposed. The scheme in [35] is better than the previous ones in terms of quantum resource consumptions. In Hassanpour's scheme [36], the quantum channel is easier to prepare.

In all mentioned protocols, Alice and Bob can only teleport an arbitrary single-qubit state to each other. In 2016, two different protocols of BCQT by using seven-qubit entangled state as a quantum channel were proposed, that Alice can teleport an arbitrary single-qubit state to Bob and Bob can teleport an arbitrary two-qubit state to Alice [37, 38]. The same year in February, a BQT protocol by using six-qubit entangled state as a quantum channel was presented [39]. In this protocol, Alice and Bob can teleport a pure EPR state to each other. Then a BCQT protocol was proposed by using nine-qubit entangled state as a quantum channel, where Alice and Bob can teleport an arbitrary two-qubit state to each other [40]. In June 2016, a BCQT protocol by using six-qubit cluster state as a quantum channel was proposed [41]. In this protocol, Alice can teleport an arbitrary two-qubit state to Bob and Bob can teleport an arbitrary single-qubit state to Alice. In [42], a new scheme of bidirectional quantum teleportation (BQT) making use of an eight-qubit entangled state as the quantum channel is presented, where Alice and Bob can teleport two-qubit states to each other simultaneously.

In this study, for the first time, a BQT protocol is presented by which n-qubit states (as (1) and (2) shows) can be teleported simultaneously. This protocol utilizes $(2n + 2)$-qubit

entangled state as a channel. In this protocol, each user applies one CNOT gate and $(n + 1)$ single-qubit unitary gates as well as performing n measurements in X basis.

The rest of the paper is organized as follows. In Section 2, the proposed protocol is explained. Section 3, includes summary and analysis of previous research. Conclusion and future work are presented in Section 4.

2 Description of the Presented Protocol

The proposed scheme is a BQT protocol, where Alice and Bob as two legitimate users want to teleport an n-qubit state as (1) and (2) to each other. (It should be noted that EPR and GHZ states are examples of states in (1) and (2).)

$$|\emptyset\rangle_{A_1 A_2 \ldots A_n} = \alpha_0 |q_1 q_2 \ldots q_n\rangle + \alpha_1 |\bar{q}_1 \bar{q}_2 \ldots \bar{q}_n\rangle, \tag{1}$$

$$|\emptyset\rangle_{B_1 B_2 \ldots B_n} = \beta_0 |q_1 q_2 \ldots q_n\rangle + \beta_1 |\bar{q}_1 \bar{q}_2 \ldots \bar{q}_n\rangle, \tag{2}$$

where $|\alpha_0|^2 + |\alpha_1|^2 = 1$, $|\beta_0|^2 + |\beta_1|^2 = 1$ and $q_1 q_2 \ldots q_n \in \{0, 1\}$ and q bar is the flipped state of q.

2.1 Preparation Phase

In this protocol, two users teleport their qubits by using $(2n + 2)$-qubit entangled state as a quantum channel,

$$
\begin{aligned}
|G\rangle_{a_1 b_1 b_2 \ldots b_n a_2 a_3 \ldots a_{n+1} b_{n+1}} = \frac{1}{2} \big[& |0 q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n 0\rangle + |0 q_1 q_2 \ldots q_n \bar{q}_1 \bar{q}_2 \ldots \bar{q}_n 1\rangle \\
& + |1 \bar{q}_1 \bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n 0\rangle + |1 \bar{q}_1 \bar{q}_2 \ldots \bar{q}_n \bar{q}_1 \bar{q}_2 \ldots \bar{q}_n 1\rangle \big],
\end{aligned}
\tag{3}
$$

Alice generates the N states described in (3). She keeps qubits $a_1 \ldots a_{n+1}$ for herself and sends the qubits $b_1 \ldots b_{n+1}$ to Bob.

2.2 Security Checking Phase

For making sure that qubits of (3) are shared without eavesdropping, Alice selects randomly enough states of sequence as checking qubits. The remaining qubits are used as teleporting qubits. To measure the checking qubits, she randomly selects one of the Z or X bases for each checking qubits. Then she announces the orders and the bases of the checking qubits to her Bob. Bob measures the corresponding qubits under the same measuring bases. Then, he announces their measurement results to Alice.

If there is no Eve interference and the channel is secure, the measurement results must be correlated. As an example suppose $q_1 = q_2 = \cdots = q_n = 0$.

Table 1 shows the correlation of Alice and Bob measurement results where n_- indicates the number of the qubits that are collapsed to $|-\rangle$ when measured in X basis. It should be noted that:

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}, |1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

Table 1 Measurement bases and measurement results

Measurement basis	Measurement results on $a_1b_1b_2\ldots b_n$	Measurement results on $a_2a_3\ldots a_{n+1}b_{n+1}$	Probability
X	n_- is even	n_- is even	100%
X	n_- is even	n_- is odd	0%
X	n_- is odd	n_- is even	0%
X	n_- is odd	n_- is odd	0%
Z	$0\ldots0$	$0\ldots0$	25%
Z	$0\ldots0$	$1\ldots1$	25%
Z	$1\ldots1$	$0\ldots0$	25%
Z	$1\ldots1$	$1\ldots1$	25%
Z	Other	Other	0%

If the error rate is more than the threshold, they discard and abort the communication; otherwise they continue the communication and execute the next phase.

2.3 Main Protocol

In this subsection the steps of the protocol is described. Figure 1 illustrates the protocol.

Step 1.   After Alice and Bob share a $(2n + 2)$-qubit entangled state as (3), as a quantum channel, the initial state of the whole system can be expressed as (4),

$$|\varphi\rangle_{a_1b_1\ldots b_na_2\ldots a_{n+1}b_{n+1}A_1\ldots A_nB_1\ldots B_n} = |G\rangle_{a_1b_1\ldots b_na_2\ldots a_{n+1}b_{n+1}} \otimes |\emptyset\rangle_{A_1\ldots A_n} \otimes |\emptyset\rangle_{B_1\ldots B_n}, \tag{4}$$

where the qubits $a_1\ldots a_{n+1}$ and $A_1\ldots A_n$ belong to Alice and qubits $b_1\ldots b_{n+1}$ and $B_1\ldots B_n$ belong to Bob, respectively.
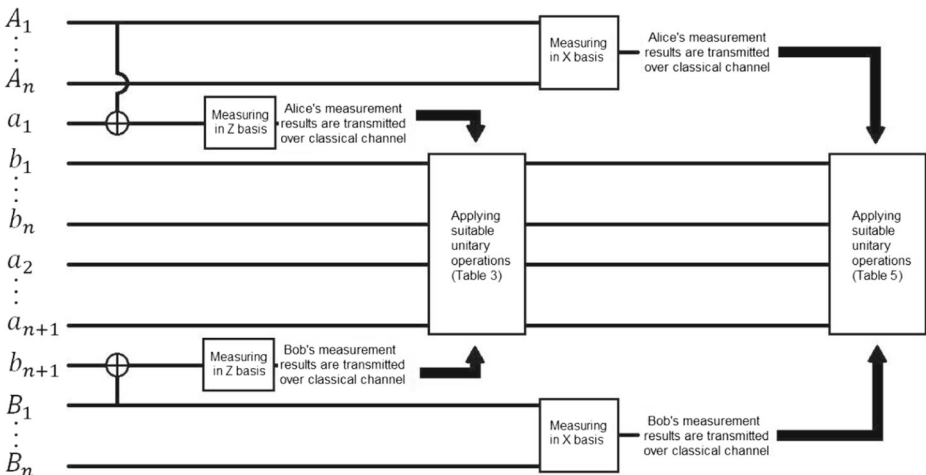
Fig. 1 Steps of the protocol

Step 2. In this step, Alice and Bob apply a controlled-not operation with A_1 and B_1 as control qubits and qubits a_1 and b_{n+1} as target respectively. Now the state will be in the form of (5) (without loss of generality suppose $q_1 = 0$).

$$
\begin{aligned}
&|\varphi_1\rangle_{a_1b_1\ldots b_na_2\ldots a_{n+1}b_{n+1}A_1\ldots A_nB_1\ldots B_n} \\
&= \frac{1}{2}\Big[\alpha_0\beta_0 \,|0q_2\ldots q_nq_1q_2\ldots q_n0\rangle\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_0\beta_0\,|0q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n1\rangle\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_0\beta_0\,|1\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n0\rangle\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_0\beta_0\,|1\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n1\rangle\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_0\beta_1\,|0q_2\ldots q_nq_1q_2\ldots q_n1\rangle\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_0\beta_1\,|0q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n0\rangle\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_0\beta_1\,|1\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n1\rangle\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_0\beta_1\,|1\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n0\rangle\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_1\beta_0\,|1q_2\ldots q_nq_1q_2\ldots q_n0\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_1\beta_0\,|1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n1\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_1\beta_0\,|0\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n0\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_1\beta_0\,|0\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n1\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_1\beta_1\,|1q_2\ldots q_nq_1q_2\ldots q_n1\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_1\beta_1\,|1q_2\ldots q_nq_1q_2\ldots q_n0\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_1\beta_1\,|0\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n1\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_1\beta_1\,|0\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n0\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle\Big].
\end{aligned}
\tag{5}
$$

Step 3. In this step, Alice and Bob perform a single-qubit measurement in the Z-basis on qubits a_1 and b_{n+1} respectively. According to Table 2, the remaining particles may collapse into one of the 4 possible states with 25% probability.

Step 4. After Alice and Bob tell their measurement results to each other, they apply suitable unitary operation as is shown in Table 3.

After users applied unitary operation on their qubits, all of the 4 states will be in the form of (6).

$$
\begin{aligned}
&(\alpha_0\beta_0\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle\,|q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_0\beta_1\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle\,|q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle \\
&\quad+\alpha_1\beta_0\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_nq_1q_2\ldots q_n\rangle \\
&\quad+\alpha_1\beta_1\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle\,|\overline{q}_1\overline{q}_2\ldots\overline{q}_n\overline{q}_1\overline{q}_2\ldots\overline{q}_n\rangle)_{b_1\ldots b_na_2\ldots a_{n+1}A_1\ldots A_nB_1\ldots B_n}.
\end{aligned}
\tag{6}
$$

Step 5. In this step, Alice and Bob perform n single-qubit measurement in the X-basis on qubits $A_1\ldots A_n$ and $B_1\ldots B_n$ respectively. The remaining particles may collapse into one of the $2n$ possible states with the same probability as described in Table 4, where n_- and m_- are the numbers of $|-\rangle$ in the collapsed state after the Alice's measurement and Bob's measurement respectively.

Step 6. In this step, Alice and Bob can reconstruct the n-qubit state by applying suitable unitary operation as shown in Table 5.

Table 2 The (Z-basis) measurement results of users and the corresponding collapsed state

Alice's result	Bob's result	The collapsed state of qubits $b_1 \ldots b_n a_2 \ldots a_{n+1} A_1 \ldots A_n B_1 \ldots B_n$
0	0	$\alpha_0\beta_0 \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$ $+\alpha_1\beta_0 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$
0	1	$\alpha_0\beta_0 \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$ $+\alpha_1\beta_0 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle \lvert \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n$
1	0	$\alpha_0\beta_0 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$ $+\alpha_1\beta_0 \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$
1	1	$\alpha_0\beta_0 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$ $+\alpha_1\beta_0 \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle \lvert \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle$

After applying the unitary operators in Table 5, the state will be as below:

$$
\alpha_0\beta_0 \lvert q_1 q_2 \ldots q_n q_1 q_2 \ldots q_n\rangle + \alpha_0\beta_1 \lvert q_1 q_2 \ldots q_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle
$$
$$
+\alpha_1\beta_0 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n q_1 q_2 \ldots q_n\rangle + \alpha_1\beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle
$$
$$
= \left(\beta_0 \lvert q_1 q_2 \ldots q_n\rangle + \beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle\right)_{a_2 \ldots a_{n+1}} \otimes \left(\alpha_0 \lvert q_1 q_2 \ldots q_n\rangle + \alpha_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle\right)_{b_1 \ldots b_n} ,
\tag{7}
$$

Now, Alice and Bob reconstruct the n-qubit state, and the bidirectional teleportation is successfully finished which is shown in (8) and (9).

$$
\lvert \emptyset\rangle_{a_2 \ldots a_{n+1}} = \beta_0 \lvert q_1 q_2 \ldots q_n\rangle + \beta_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle
\tag{8}
$$

$$
\lvert \emptyset\rangle_{b_1 \ldots b_n} = \alpha_0 \lvert q_1 q_2 \ldots q_n\rangle + \alpha_1 \lvert \bar{q}_1\bar{q}_2 \ldots \bar{q}_n\rangle
\tag{9}
$$

Considering that the protocol is based on quantum teleportation and no qubit is transmitted in the channel, the protocol is secure against attacks such as intercept-resend.

Table 3 Applying suitale unitary operations

Alice's results	Bob's results	Bob's operation on qubits $b_1 \ldots b_n$	Alice's operation on qubits $a_2 \ldots a_{n+1}$
0	0	$I^{\otimes n}$	$I^{\otimes n}$
0	1	$I^{\otimes n}$	$X^{\otimes n}$
1	0	$X^{\otimes n}$	$I^{\otimes n}$
1	1	$X^{\otimes n}$	$X^{\otimes n}$

Table 4 The (X-Basis) measurement results of users and the corresponding collapsed state

n_-	m_-	The collapsed state of qubits $b_1 \ldots b_n a_2 \ldots a_{n+1}$	Alice's state	Bob's state
Even	Even	$\alpha_0\beta_0 \lvert q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$ $+\alpha_1\beta_0 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_nq_1q_2\ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$	$\beta_0 \lvert q_1q_2\ldots q_n\rangle$ $+\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle)$	$\alpha_0 \lvert q_1q_2\ldots q_n\rangle$ $+\alpha_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$
Even	Odd	$\alpha_0\beta_0 \lvert q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle$ $-\alpha_0\beta_1 \lvert q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$ $+\alpha_1\beta_0 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_nq_1q_2\ldots q_n\rangle$ $-\alpha_1\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$	$\beta_0 \lvert q_1q_2\ldots q_n\rangle$ $-\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle)$	$\alpha_0 \lvert q_1q_2\ldots q_n\rangle$ $+\alpha_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$
Odd	Even	$\alpha_0\beta_0 \lvert q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle$ $+\alpha_0\beta_1 \lvert q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$ $-\alpha_1\beta_0 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_nq_1q_2\ldots q_n\rangle$ $-\alpha_1\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$	$\beta_0 \lvert q_1q_2\ldots q_n\rangle$ $+\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle)$	$\alpha_0 \lvert q_1q_2\ldots q_n\rangle$ $-\alpha_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$
Odd	Odd	$\alpha_0\beta_0 \lvert q_1q_2\ldots q_nq_1q_2\ldots q_n\rangle$ $-\alpha_0\beta_1 \lvert q_1q_2\ldots q_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$ $-\alpha_1\beta_0 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_nq_1q_2\ldots q_n\rangle$ $+\alpha_1\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$	$\beta_0 \lvert q_1q_2\ldots q_n\rangle$ $-\beta_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle)$	$\alpha_0 \lvert q_1q_2\ldots q_n\rangle$ $-\alpha_1 \lvert \overline{q}_1\overline{q}_2\ldots \overline{q}_n\rangle$

3 Comparison

The most important advantage of the proposed protocol is that Alice and Bob can simultaneously teleport an n-qubit state in the form of (1) and (2) to each other. In the best approach previously proposed [40], the users can transmit only two-qubit states while in the proposed protocol in this paper, the users can teleport as many qubits as they desire through the bidirectional quantum teleportation.

Consider m_u the number of transmitted qubits, q_k the number of used qubits in the channel and b_k the number of transmitted bits, the efficiency is calculated as the following relation:

$$\eta = \frac{m_u}{q_k + b_k}. \tag{10}$$

By setting the values of these parameters, we will have:

$$m_u = 2n, \; q_k = 2n+2, \; b_k = 16, \; \eta = \frac{m_u}{q_k + b_k} = \frac{2n}{2n+18}, \tag{11}$$

which implies that by increasing the value of n, the efficiency of this protocol gets closer to one.

Table 5 Required unitary operations

n_-	m_-	Alice's operation on qubits $a_2 \ldots a_{n+1}$	Bob's operation on qubits $b_1 \ldots b_n$
Even	Even	$I^{\otimes n}$	$I^{\otimes n}$
Even	Odd	$ZI^{\otimes(n-1)}$	$I^{\otimes n}$
Odd	Even	$I^{\otimes n}$	$ZI^{\otimes(n-1)}$
Odd	Odd	$ZI^{\otimes(n-1)}$	$ZI^{\otimes(n-1)}$

4 Conclusions

The advantage of this protocol is that users can simultaneously teleport an n-qubit state to each other. In the best case of previous protocols the users can only teleport a two-qubit state simultaneously. In this protocol, only single-qubit measurements are required so it is more simple and feasible in experiment. As a future work, this protocol can be extended to transmitting an *arbitrary n*-qubit state. Further, the effects of the amplitude-damping noise and the phase-damping noise for BQT process could be investigated. The other direction for a future work is to include a third party as a controller in the protocol, where without controller's cooperation Alice and Bob cannot recover the teleported quantum states.

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