

Ghost Dark Energy with Sign-changeable Interaction Term

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Abstract Regarding the Veneziano ghost of QCD and its generalized form, we consider a Friedmann-Robertson-Walker (FRW) universe filled by a pressureless matter and a dark energy component interacting with each other through a mutual sign-changeable interaction of positive coupling constant. Our study shows that, at the late time, for the deceleration parameter we have $q \rightarrow -1$, while the equation of state parameter of the interacting ghost dark energy (GDE) does not cross the phantom line, namely $\omega_D \ge -1$. We also extend our study to the generalized ghost dark energy (GGDE) model and show that, at late time, the equation of state parameter of the interacting GGDE also respects the phantom line in both flat and non-flat universes. Moreover, we find out that, unlike the non-flat universe, we have $q \rightarrow -1$ at late time for flat FRW universe. In order to make the behavior of the underlying models more clear, the deceleration parameter q as well as the equation of state parameter w_D for flat and closed universes have been plotted against the redshift parameter, z. All of the studied cases admit a transition in the expansion history of universe from a deceleration phase to an accelerated one around $z \approx 0.6$.

Keywords Ghost \cdot Dark energy \cdot Sign-changeable \cdot Interaction

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1 Introduction

The cause of the accelerated expansion of universe, predicted by the observations of type Ia supernova [1–4], is the backbone of a big challenge in the modern physics. This phase of the universe expansion has been confirmed by observing the anisotropies of Cosmic Microwave Background (CMB) [5, 6]. The CMB observation can be considered as a signal to the universe flatness and claims that the energy density of the cosmic fluid is very close to the critical density [7]. Large-Scale Structure (LSS) [8–11], Baryon Acoustic Oscillations (BAO) in the Sloan Sky Digital Survey (SSDS) luminous galaxy sample [12, 13], and Plank data [14] are other observations supporting an accelerated universe.

Since the cosmic fluid, supporting the current accelerating universe, does not interact with light, it is called "dark energy"(DE), an oddity with negative pressure and negative equation of state parameter (EoS) $\omega < -1/3$. In general relativity (GR), there is a very simple model for describing the above mentioned picture called cosmological constant model. According to this model, there is an isotropic and homogeneous fluid with constant positive energy density and constant negative pressure with EoS parameter $\omega_{\Lambda} = -1$. Although the cosmological constant model of DE helps us in providing a well initial picture for the current accelerating phase, it suffers from some problems such as the fine-tuning and the coincidence problems [15].

In order to find a more realistic model of DE, various fluids with time varying EoS parameter have been introduced which are supported and constrained by the observational data [16–19]. Quintessence [20, 21], phantom (ghost) field [22–24], K-essence [25–27], Chaplygin gas [28, 29], holographic dark energy which originates from quantum gravity [30–37] and agegraphic DE [38–50] are some examples of DE models with time varying EoS parameter. On the other hand, in another approach, some physicists try to solve the DE problem by modifying the field equations of GR in such a way that the phase of acceleration is reproduced without including any new kind of energy [51–54]. Indeed, in the modified gravity approach, one may consider a new degree(s) of freedom leading to many unknown features and thus one should investigate their nature and new consequences in the universe meaning that this approach adds more complexity to the system. Therefore, it is impressive and economic if we can explain DE without entering the new degrees of freedom.

GDE is a model for DE wherein we do not need to introduce new degrees of freedom or modify gravity. This model is based on the Veneziano ghost field used in order to solve the so-called U(1) problem in QCD theory [55–59]. Although there is not any observable consequence from the ghost field in a Minkowskian spacetime, it produces a small vacuum energy density proportional to $\rho_D \sim \Lambda_{QCD}^3 H \sim (3 \times 10^{-3} eV)^4$, which solves the finetuning problem [60], in curved spacetime. Here, $\Lambda_{QCD} \sim 100 MeV$ and $H \sim 10^{-33} eV$ are QCD mass scale and Hubble parameter, respectively [60]. Different features of GDE have been studied in ample details [61–70]. It has been found that the contribution of the Veneziano QCD ghost field to the vacuum energy is not exactly of order of H and there is also a second order term proportional to H^2 which contributes to the vacuum energy density [71]. Adding the H^2 correction term to the GDE model, one may study the GGDE model in which the energy density is taken as $\rho_D = \alpha H + \beta H^2$ [72–74].

Based on the cosmological principle, the universe is homogeneous and isotropic in scales larger than 100-Mpc and it can be open, flat or closed denoted by the curvature constant k = -1, 0, 1, respectively [15]. It is useful to mention here that although some observations indicate a flat universe, the nonflat case is not completely rejected by observations [15, 75–85]. In addition, there are also several observations which indicate a mutual interaction between DE and dark matter (DM) [86–94]. The initial simple models of the mutual interaction between DE and DM are linear functions of ρ_D and ρ_m [95–104], where ρ_m is the energy density of DM.

Moreover, investigations confirm that the sign of the mutual interaction between DM and DE is changed during the history of universe [105]. In this regards, Wei [106, 107]. proposed a sign-changeable interaction term in the form $Q = q(\alpha \dot{\rho} + 3\beta H\rho)$, where α and β are dimensionless constant and q is the deceleration parameter. It is obvious that the sign of Q is changed whenever the universe expansion phase is changed from a deceleration phase (q > 0) to an acceleration one (q < 0). It is also worth mentioning that, from the dimensional point of view, one may consider $\alpha = 0$ and discard the $\alpha \dot{\rho}$ term [107–109]. In fact, the sign-changeable interaction has attracted a lot of attentions [110–119]. For example, the Chaplygin gas model of DE with sign-changeable interaction has been investigated widely in the literatures [110–116]. The agegraphic and new agegraphic models of DE with the sign-changeable interaction have also been explored, respectively, in [117] and [118]. Very recently, we have studied the holographic DE model with the sign-changeable interaction term with various IR cutoffs [119].

In the present paper, we are interested in studying the effects of considering a mutual sign-changeable interaction between DM and the DE candidates, including GDE and GGDE, on the evolution history of universe. Indeed, we are going to investigate how a sign-changeable interaction affects the description of GDE and GGDE models of DE about the current phase of the cosmic expansion. We also investigate the evolution of the system parameters, such as the equation of state (EoS) parameter as well as the deceleration and dimensionless density parameters, during the cosmic evolution from the matter dominated era to the current accelerating epoch. In order to present our work, we organize the paper according to the following sections. In Section 2, we study GDE with the sign-changeable interacting GGDE in both the flat and nonflat universes in Section 3 and investigate the cosmological implications of the model. In Section 4, we compare the EoS parameter of the sign-changeable interaction GDE and the standard GDE model. We summarize our results in Section 5.

2 GDE with the Sign-changeable Interaction

In this section, we study the GDE in the presence of the sign-changeable interaction term in both flat and nonflat universe.

2.1 Flat Universe

The first Friedmann equation in a flat homogeneous and isotropic FRW universe is written as [15]

$$H^{2} = \frac{8\pi G}{3}(\rho_{m} + \rho_{D}),$$
(1)

where ρ_D is the GDE density and ρ_m is the energy density of DM. For the GDE density we have [60]

$$\rho_D = \alpha H,\tag{2}$$

where α is a constant of order Λ^3_{QCD} and Λ_{QCD} is the QCD mass scale [60]. The fractional energy density parameters and the energy density ratio are defined as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G\rho_m}{3H^2}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{8\pi G\alpha}{3H}, \quad (3)$$

and

$$r = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = \frac{1 - \Omega_D}{\Omega_D}.$$
(4)

For an interacting universe in which there is a mutual interaction between dark sectors of cosmos, the energy-momentum conservation law can be written as

$$\dot{\rho}_m + 3H\rho_m = Q,\tag{5}$$

$$\dot{\rho}_D + 3H(1+\omega_D)\rho_D = -Q. \tag{6}$$

In the above equations, Q denotes the interaction term between DE and DM. Here, we consider the interaction term as [105, 107]

$$Q = 3\beta Hq(\rho_D + \rho_m),\tag{7}$$

where β is the coupling constant of interaction Q, and q is the deceleration parameter defined as

$$q = -1 - \frac{H}{H^2}.$$
(8)

Let us note that although some negative values are allowed for the coupling constant β , we only focus on the $\beta = b^2 > 0$ case [105, 107]. Taking the time derivative of relation (2) and considering (1), we obtain

$$\dot{\rho}_D = \rho_D \frac{\dot{H}}{H} = -4\pi G \alpha \rho_D (1 + r + \omega_D).$$
(9)

Substituting (9) and (7) into (6) and bearing (4) in mind, one reaches at

$$\omega_D = -\frac{1}{2 - \Omega_D} \left(1 + \frac{2b^2 q}{\Omega_D} \right). \tag{10}$$

If we set q = 1 in (7) and (10), then Q and ω_D are reduced to relations obtained in Ref. [64]. In Fig. 1, considering the initial condition $\Omega_D(z=0) = 0.72$, the evolution of ω_D is plotted against the redshift parameter z. Intersetingly, the EoS parameter of the sign-changeable interacting GDE cannot cross the phantom divide ($\omega_D = -1$) at the late time where $\Omega_D \rightarrow$ 1. This is due to the fact that at the late time q becomes negative and hence $w_D = -(1 + 2b^2q) \ge -1$. This is in contrast to the case of standard interacting GDE, where in the late time the EoS parameter of interacting GDE necessary crosses the phantom line, namely, $w_D = -(1+2b^2) < -1$ independent of the value of coupling constant b^2 [64]. For example, taking $\Omega_D = 0.72$ for the present time, the phantom crossing take places provided $b^2 > 0.1$ [64]. Using (8) and (9), we find

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\Omega_D(1+r+\omega_D),$$
 (11)

which can be combined with (10) and (8) to reach at

$$q = \left(\frac{1}{2} - \frac{3\Omega_D}{2(2 - \Omega_D)}\right) \left[\frac{2 - \Omega_D}{2 - \Omega_D + 3b^2}\right].$$
 (12)

Considering $\Omega_D(z = 0) = 0.72$ for the initial condition, we have plotted q against the redshift parameter in Fig. 2. As it is obvious, there is a transition from the deceleration phase to the acceleration one at $z \approx 0.6$.



Fig. 1 The evolution of ω_D versus redshift parameter z for the sign-changeable interacting GDE in flat universe

Taking the derivative with regard to time from $\Omega_D = (8\pi G\rho_D)/3H^2$ and combining the result with (6) and (25), one can find

$$\frac{d\Omega_D}{d\ln a} = 3\Omega_D \left[\frac{1 - \Omega_D}{2 - \Omega_D} \left(1 + \frac{2b^2 q}{\Omega_D} \right) - \frac{b^2 q}{\Omega_D} \right] = \frac{3}{2}\Omega_D \left[1 - \frac{\Omega_D}{2 - \Omega_D} \left(1 + \frac{2b^2 q}{\Omega_D} \right) \right].$$
(13)



Fig. 2 The evolution of q versus redshift parameter z for the sign-changeable interacting GDE in flat universe



Fig. 3 The evolution of Ω_D versus redshift parameter z for the sign-changeable interacting GDE in flat universe when $\Omega_D(z=0) = 0.72$

We have plotted the dynamics of dimensionless GDE density in Fig. 3. We observe that at the early time $\Omega_D \rightarrow 0$ and at the late time $\Omega_D \rightarrow 1$, as expected. It is easy to check that, as previous, the result of Ref. [64] are obtainable when q = 1. In summary, for the sign-changeable interacting GDE in flat universe, at the late time where $z \rightarrow 0$, we have $q \rightarrow -1$ and $\omega_D \geq -1$.



Fig. 4 The evolution of ω_D versus redshift parameter z for GDE in a nonflat universe when $\Omega_D(z=0) = 0.72$ and k = 1

2.2 Nonflat Universe

Here we consider the sign-changeable interacting GDE in a nonflat universe. It has been argued that the flatness is not a necessary consequence of inflation if the number of e-folding is not very large [120]. The spatial curvature made a contribution to the energy components of cosmos which is constrained as $-0.0175 < \Omega_k < 0.0085$ with 95% confidence level by current observations [121]. The first Friedmann equation in a nonflat homogeneous and isotropic FRW universe is

$$H^{2} + \frac{k}{a^{2}} = \frac{8\pi G}{3}(\rho_{m} + \rho_{D}), \qquad (14)$$

where k = -1, 0, 1 is the curvature parameter corresponding to open, flat, and closed universes, respectively. The curvature fractional density parameter is defined as $\Omega_k = k/(a^2H^2)$, and thus the Friedmann equation can be rewritten in the following form

$$\Omega_m + \Omega_D = 1 + \Omega_k,\tag{15}$$

which also yields

$$r = \frac{\Omega_m}{\Omega_D} = \frac{1 + \Omega_k - \Omega_D}{\Omega_D},\tag{16}$$

for the energy density ratio. Combining the time derivative of (14) with (15), we obtain

$$\frac{\dot{H}}{H^2} = \Omega_k - \frac{3}{2}\Omega_D(1+r+\omega_D).$$
(17)

Inserting the above relation into (6) and using (7) and (9), we reach at

$$\omega_D = -\frac{1}{2 - \Omega_D} \left(1 - \frac{\Omega_k}{3} + \frac{2qb^2}{\Omega_D} (1 + \Omega_k) \right),\tag{18}$$



Fig. 5 The evolution of q versus redshift parameter z for GDE in a nonflat universe. Here we have taken $\Omega_D^0 = 0.72$ and k = 1

for the EoS parameter of sign-changeable interacting GDE in a nonflat universe. Substituting (18) and (17) into (8), the deceleration parameter in a nonflat background is obtained as

$$q = \left[\frac{1+\Omega_k}{2} - \frac{3\Omega_D}{2(2-\Omega_D)}\left(1-\frac{\Omega_k}{3}\right)\right] \left(\frac{2-\Omega_D}{2-\Omega_D+3b^2(1+\Omega_k)}\right).$$
(19)

We plot the evolution of ω_D and q against the redshift parameter (z) for GDE in the closed universe in Figs. 4 and 5, respectively. Again, we see that the universe has a phase transition from deceleration to an acceleration around $z \approx 0.6$.

It is a matter of calculation to show that

$$\frac{d\Omega_D}{d\ln a} = \frac{3\Omega_D}{2} \left[1 + \frac{\Omega_k}{3} - \frac{\Omega_D}{2 - \Omega_D} \left(1 - \frac{\Omega_k}{3} + \frac{2qb^2}{\Omega_D} (1 + \Omega_k) \right) \right],\tag{20}$$

where we used (17) and (6) to get the above equation. It is worthwhile to mention here that the results of flat case, obtained in previous subsection, are covered by setting $\Omega_k = 0$. The dynamics of GDE in terms of the redshift parameter is plotted in Fig. 6. Clearly, at the early time it shows $\Omega_D \rightarrow 0$ and at the late time the DE dominates. In the following we can have $q \rightarrow -1$ and $\omega_D \ge -1$ at the late time where $z \rightarrow 0$.

3 GGDE with the Sign-changeable Interaction

In the previous section, we have assumed the energy density of GDE as $\rho_D = \alpha H$, while, in general, the vacuum energy of the Veneziano ghost field in QCD is of the form $H + O(H^2)$ [71]. Motivated by the argument given in [122], one may expect that the subleading term H^2 in the GDE model might play a crucial role in the early evolution of the universe, acting as the early DE. It was shown [72–74] that taking the second term into account can give better agreement with observational data compared to the usual GDE. This mode is usually called



Fig. 6 The evolution of Ω_D versus redshift parameter z for the sign-changeable interacting GDE in nonflat universe when $\Omega_D(z=0) = 0.72$ and k = 1

the generalized ghost dark energy (GGDE) and our main task in this section is to investigate the properties of this model in the presence of the sign-changeable interacting term. Again, we first consider a flat universe and then generalize our study to the nonflat case.

3.1 Flat Universe

For the energy density of GGDE we have

$$\rho_D = (\alpha H + \beta H^2), \tag{21}$$

where β is a constant [71, 72]. The fractional energy density parameters also take the below forms

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G\rho_m}{3H^2}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{8\pi G(\alpha + \beta H)}{3H}.$$
 (22)

Here, $\rho_{cr} = \frac{3H^2}{8\pi G}$ denotes again the critical density. Finally, use (22) and (21) to obtain

$$\frac{4\pi G}{3H}(\alpha + 2\beta H) = \frac{\Omega_D}{2} + \frac{4\pi G\beta}{3}.$$
(23)

Taking the time derivative of (21), one can find

$$\dot{\rho}_D = \dot{H}(\alpha + 2\beta H),\tag{24}$$

combined with (22) to reach at

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\Omega_D(1+r+\omega_D),$$
 (25)

finally leading to

$$\dot{H} = -4\pi G\rho_D (1 + r + \omega_D), \qquad (26)$$



Fig. 7 The evolution of ω_D versus redshift parameter z for the sign-changeable interacting GGDE in flat universe when $\Omega_D(z=0) = 0.72$, $\zeta = 0.1$

where r is the energy density ratio (4). Substituting (24) and (7) into (6) and using (23), (4) and (26), we find out

$$\omega_D = -\frac{1}{2 - \Omega_D - \zeta} \left(1 + \frac{2b^2 q}{\Omega_D} - \frac{\zeta}{\Omega_D} \right). \tag{27}$$

Here, $\zeta = \frac{8\pi G\beta}{3}$. It is obvious that, as the flat case, this equation is reduced to the result of Ref. [73] in the q = 1 limit. The evolution of ω_D has been plotted against the redshift parameter (z) for GGDE in Fig. 7.

As the flat case, the EoS of sign-changeable interaction GGDE cannot cross the phantom division ($\omega_D \ge -1$). Let us note that at the late time where the universe is in the accelerated phase, q becomes negative and considering the fact that $\zeta = .1$, we arrive at $\omega_D = -\left(1 + \frac{2b^2q}{\Omega_D} - \frac{\zeta}{\Omega_D}\right) \ge -1$. Taking q = 1, we have $\omega_D = -\left(1 + \frac{2b^2}{\Omega_D} - \frac{\zeta}{\Omega_D}\right) < -1$, and the result of Ref. [73] is restored.

Substituting (25) in (8) and using (27), one can also obtain

$$q = \frac{1 - 2\Omega_D + \zeta}{2 - \Omega_D - \zeta + 3b^2}.$$
(28)

It is easy to verify that the result of Ref. [73] is covered when b = 0. Moreover, for b = 0 and $\zeta = 0$, we have $q = \frac{1-2\Omega_D}{2-\Omega_D} = \frac{1}{2} - \frac{3}{2} \frac{\Omega_D}{2-\Omega_D}$ [63]. The behavior of q has also been plotted in Fig. 8, addressing a transition from the deceler-

The behavior of q has also been plotted in Fig. 8, addressing a transition from the deceleration phase to the acceleration one at $z \approx 0.6$. Finally, taking the time derivative of relation $\Omega_D = \frac{8\pi G\rho_D}{3H^2}$ and using (6) and (25), we find

$$\Omega'_D = 3\Omega_D \left[\frac{1 - \Omega_D}{2 - \Omega_D - \zeta} \left(1 + \frac{2b^2 q}{\Omega_D} - \frac{\zeta}{\Omega_D} \right) - \frac{b^2 q}{\Omega_D} \right].$$
(29)



Fig. 8 The evolution of q versus redshift parameter z for the sign-changeable interacting GGDE in flat universe when $\Omega_D(z=0) = 0.72$, $\zeta = 0.1$

0.5

1.5

2



Fig. 9 The evolution of Ω_D versus redshift parameter z for the sign-changeable interacting GGDE in flat universe when $\Omega_D(z=0) = 0.72$, $\zeta = .1$

2.5

3

1+Z

3.5

4

4.5

It is also easy to check that the results of Refs. [63, 73] are obtainable from the above relations.

We have plotted the dynamics of density parameter in Fig. 9, and the behavior is similar to the previous case; at the early time $\Omega_D \rightarrow 0$, while at the late time $\Omega_D \rightarrow 1$.

3.2 Nonflat Universe

In order to find the EoS parameter of sign-changeable interacting GGDE in the non-flat universe, inserting (26) into (24) and combining the result with (6) and (16), we get

$$\omega_D = -\frac{1}{2 - \Omega_D - \zeta} \left[2 - \left(1 + \frac{\zeta}{\Omega_D} \right) \left(1 + \frac{\Omega_k}{3} \right) + \frac{2b^2 q}{\Omega_D} \left(1 + \Omega_k \right) \right].$$
(30)

As one can see the EoS parameter cannot cross the phantom divide at the late time, because at this epoch we have $\Omega_D \rightarrow 1$ and q becomes negative, therefore $\omega_D = -\left(2 - (1+\zeta)(1+\frac{\Omega_k}{3}) + 2b^2q(1+\Omega_k)\right) \geq -1$ (note that we have chosen $\zeta = .1$ and $\Omega_k = .01$). If we set q = 1 we get $\omega_D = -\left(2 - (1+\zeta)(1+\frac{\Omega_k}{3}) + 2b^2(1+\Omega_k)\right) < -1$, which is the result of Ref. [73]. Thus in contrast to the EoS parameter of the usual interacting GGDE which the phantom regime can be achieved, in case of sign-changeable interaction term the EoS parameter of GGDE is always $\omega_D \geq -1$.

Combining (30) with (17) and (8), one arrives at

$$q = \left(\frac{1+\Omega_k}{2} - \frac{3\Omega_D}{2(2-\Omega_D - \zeta)}(1-\frac{\Omega_k}{3})\right) \left[\frac{2-\Omega_D}{2-\Omega_D + 3b^2(1+\Omega_k)}\right],$$
 (31)



Fig. 10 The evolution of ω_D versus redshift parameter z for the sign-changeable interacting GGDE in nonflat universe when $\Omega_D(z=0) = 0.72$, $\zeta = 0.1$ and k = 1

for the deceleration parameter. One can finally use (22), (6) and (17) in order to obtain

$$\Omega'_D = 3\Omega_D \left[\frac{\Omega_k}{3} + \frac{1 - \Omega_D}{2 - \Omega_D - \zeta} \left(2 - (1 + \frac{\zeta}{\Omega_D})(1 + \frac{\Omega_k}{3}) + \frac{2b^2q}{\Omega_D}(1 + \Omega_k) \right) - \frac{b^2q}{\Omega_D}(1 + \Omega_k) \right].$$
(32)



Fig. 11 The evolution of q versus redshift parameter z for the sign-changeable interacting GGDE in nonflat universe when $\Omega_D(z=0) = 0.72$, $\zeta = 0.1$ and k = 1



Fig. 12 The evolution of Ω_D versus redshift parameter z for the sign-changeable interacting GGDE in nonflat universe when $\Omega_D(z=0) = 0.72$, $\zeta = 0.1$ and k = 1

It is worth mentioning that in the limit of $\Omega_k = 0$, all the obtained relations in this subsection restore their respective expressions in the previous subsections for flat universe. The behaviors of ω_D and q against the redshift parameter for GGDE in the closed universe have also been plotted in Figs. 10 and 11. The main results of this figures are: (*i*) at late time, we have $\omega_D \ge -1$ and q < -1. (*ii*) there is a transition from the deceleration phase to the accelerated one around $z \simeq 0.6$. We have also plotted the evolutionary of the GGDE density in Fig. 12.



Fig. 13 The evolution of ω_D versus redshift parameter z for GDE in a flat and nonflat universe when $b^2 = .1, .04, \Omega_D(z=0) = 0.72$ and k = 1



Fig. 14 The evolution of ω_D versus redshift parameter z for GGDE in a flat and nonflat universe when $b^2 = .1, .04, \Omega_D(z = 0) = 0.72, \zeta = 0.1$ and k = 1

4 Comparison of EoS Parameter of Usual Interacting GDE and Sign-changeable Model

Finally, we compare the original interating GDE model with the sign-changeable interacting GDE model. For this purpose, we plot the evolution of ω_D versus redshift parameter *z* in Figs. 13 and 14 for both of models GDE and GGDE in a flat and nonflat universe. The long-dash and dash-dot lines show the evolution of ω_D for the sign-changeable interacting GDE model and the solid and dashed lines show the usual interacting GDE model with interaction term $Q = 3b^2 H(\rho_D + \rho_m)$. From these figures, we observe that the EoS parameter of both GDE and GGDE with sign-changeable interaction term cannot cross the phantom divide $\omega_D = -1$ and we always have $\omega_D \geq -1$ at the late time. In contrast, the EoS parameter of the usual interacting GDE and GGDE can cross the phantom line, namely $\omega_D < -1$ at the late time.

5 Closing Remarks

The DE puzzle is undoubtedly one of the most important challenges of modern cosmology [123, 124]. In this paper, we considered a flat FRW universe filled by a DM and GDE interacting with each other through a sign-changeable interaction term. The generalization to the nonflat case is also investigated, which shows that, for a closed universe, although $\omega_D \ge -1$ at late time, we have q < -1 for the deceleration parameter. Our studies show that, at the late time, we have $q \rightarrow -1$ and $\omega_D \ge -1$ meaning that this model does not cross the phantom line, a result which is consistent with the cosmological constant model of DE.

The values of the model parameters can be estimated by fitting the model with observational data. The observational data for coefficient β in original interaction model, $Q = 3\beta H(\rho_D + \rho_m)$, implies a positive value ($\beta > 0$), hence we consider β to be positive and can be rewritten $\beta = b^2 > 0$. We found out that if we select sign-changeable interaction model, $Q = 3b^2qH(\rho_D + \rho_m)$, because q at the late time should have a negative value, we cannot have crossing phantom. Our studies here show that with the sign-changeable interaction term, only if coefficient β in Q is chosen as a negative value, we can reach the

phantom regime. All of the studied cases indicate a transition from the deceleration phase to an accelerated one which take places around $z \approx 0.6$.

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References

- Riess, A.G., et al.: Observational evidence from supernovae for an accelerating Universe and a cosmological constant. Astron. J 116, 1009 (1998). arXiv:astro-ph/9805201
- Perlmutter, S., et al.: Measurements of omega and lambda from 42 high-redshift supernovae. Astrophys. J 517, 565 (1999). arXiv:astro-ph/9812133
- deBernardis, P., et al.: A flat Universe from high-resolution maps of the cosmic microwave background radiation. Nature 404, 955 (2000). arXiv:astro-ph/0004404
- 4. Perlmutter, S., et al.: New constraints on Ω_M , Ω_Λ , and w from an independent set of 11 highredshift supernovae observed with the hubble space telescope. Astrophys. J **598**, 102 (2003). arXiv:astro-ph/0309368
- Hanany, S., et al.: MAXIMA-1: A measurement of the cosmic microwave background anisotropy on angular scales of 10 arcminutes to 5 degrees. Astrophys. J. Lett. 545, L5 (2000). arXiv:astro-ph/0005123
- Netterfield, C.B., et al.: A measurement by BOOMERANG of multiple peaks in the angular power spectrum of the cosmic microwave background. Astrophys. J 571, 604 (2002). arXiv:astro-ph/0104460
- Spergel, D.N., et al.: First year Wilkinson microwave anisotropy probe (WMAP) observations: determination of cosmological parameters. Astrophys. J. Suppl 148, 175 (2003). arXiv:astro-ph/0302209
- Colless, M., et al.: The 2dF galaxy redshift survey: Spectra and redshifts. Mon. Not. R. Astron. Soc 328, 1039 (2001). arXiv:astro-ph/0106498
- Tegmark, M., et al.: Cosmological parameters from SDSS and WMAP. Phys. Rev. D 69, 103501 (2004). arXiv:astro-ph/0310723
- Cole, S., et al.: The 2dF Galaxy redshift survey: Power-spectrum analysis of the final dataset and cosmological implications. Mon. Not. R. Astron. Soc 362, 505 (2005). arXiv:astro-ph/0501174
- Springel, V., Frenk, C.S., White, S.M.D.: The large-scale structure of the Universe. Nature(London) 440, 1137 (2006). arXiv:astro-ph/0604561
- Tegmark, M., et al.: The three-dimensional power spectrum of galaxies from the sloan digital sky survey. Astrophys. J 606, 702 (2004)
- Tegmark, M., et al.: Cosmological parameters from SDSS and WMAP. Phys. Rev. D 69, 103501 (2004). arXiv:astro-ph/0310723
- Ade, P.A.R., et al.: Planck 2013 results. XVI. Cosmological parameters. Astron. Astrophys. 571, A16 (2014). arXiv:1303.5076
- 15. Roos, M.: Introduction to Cosmology. Wiley, UK (2003)
- Alam, U., Sahni, V., Starobinsky, A.: The case for dynamical dark energy revisited. JCAP 0406, 008 (2004). arXiv:astro-ph/0403687
- Gong, Y.G.: Model-independent analysis of dark energy: Supernova fitting result. Class Quant. Grav. 22, 2121 (2005). arXiv:astro-ph/0405446
- Alam, U., Sahni, V., Saini, T., Starobinsky, A.: Is there supernova evidence for dark energy metamorphosis? Mon. Not. Roy. Astron. Soc 354, 275 (2004). arXiv:astro-ph/0311364
- Choudhury, T., Padmanabhan, T.: Cosmological parameters from supernova observations: A critical comparison of three data sets. Astron. Astrophys. 429, 807 (2005). arXiv:astro-ph/0311622
- 20. Wetterich, C.: Cosmology and the fate of dilatation symmetry. Nucl. Phys. B 302, 668 (1988)
- Ratra, B., Peebles, J.: Cosmological consequences of a rolling homogeneous scalar field. Phys. Rev. D 37, 3406 (1988)
- 22. Caldwell, R.R.: A phantom menace? Phys. Lett. B 545, 23 (2002). arXiv:astro-ph/9908168
- Caldwell, R.R., Kamionkowski, M., Weinberg, N.N.: Phantom energy and cosmic doomsday. Phys. Rev. Lett. 91, 071301 (2003). arXiv:astro-ph/0302506
- Nojiri, S., Odintsov, S.D.: Quantum de Sitter cosmology and phantom matter. Phys. Lett. B 562, 147 (2003). arXiv:hep-th/0303117
- Chiba, T., Okabe, T., Yamaguchi, M.: Kinetically driven quintessence. Phys. Rev. D 62, 023511 (2000). arXiv:astro-ph/9912463

- Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration. Phys. Rev. Lett., 854438 (2000). arXiv:astro-ph/0004134
- Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Essentials of k-essence. Phys. Rev. D 63, 103510 (2001). arXiv:astro-ph/0006373
- Kamenshchik, A.Y., Moschella, U., Pasquier, V.: An alternative to quintessence. Phys. Lett. B 511, 265 (2001). arXiv:gr-qc/0103004
- Bento, M.C., Bertolami, O., Sen, A.A.: Generalized Chaplygin gas, accelerated expansion and dark energy-matter unification. Phys. Rev. D 66, 043507 (2002). arXiv:gr-qc/0202064
- 30. Li, M.: A model of holographic dark energy. Phys. Lett. B 603, 1 (2004). arXiv:hep-th/0403127
- Pavon, D., Zimdahl, W.: Holographic dark energy and cosmic coincidence. Phys. Lett. B 628, 206 (2005). arXiv:gr-qc/0505020
- Wang, B., Gong, Y., Abdalla, E.: Thermodynamics of an accelerated expanding universe. Phys. Lett. B 624, 141 (2005). arXiv:gr-qc/0511051
- Sheykhi, A.: Thermodynamics of interacting holographic dark energy with the apparent horizon as an IR cutoff. Class. Quantum Gravit., 27025007 (2010). arXiv:0910.0510
- Sheykhi, A.: Holographic scalar field models of dark energy. Phys. Rev. D 84, 107302 (2011). arXiv:1106.5697
- Sheykhi, A.: Interacting holographic dark energy in Brans-Dicke theory. Phys. Lett. B 681, 205 (2009). arXiv:1609.01477
- Nojiri, S., Odintsov, S.D.: Unifying phantom inflation with late-time acceleration: Scalar phantom-nonphantom transition model and generalized holographic dark energy. Gen. Rel. Grav. 38, 1285 (2006). arXiv:hep-th/0506212
- Nojiri, S., Odintsov, S.D.: Covariant Generalized Holographic Dark Energy and Accelerating Universe. arXiv:1703.06372
- Cai, R.G.: A dark energy model characterized by the age of the Universe. Phys. Lett. B 657, 228 (2007). arXiv:0707.4049
- Wei, H., Cai, R.G.: A new model of agegraphic dark energy. Phys. Lett. B 660, 113 (2008). arXiv:0708.0884 [astro-ph]
- Kim, K.Y., Lee, H.W., Myung, Y.S.: Instability of agegraphic dark energy models. Phys. Lett. B 660, 118 (2008). arXiv:0709.274
- Wei, H., Cai, R.G.: Cosmological constraints on new agegraphic dark energy. Phys. Lett. B 663, 1 (2008). arXiv:0708.1894
- Wu, J.P., Ma, D.Z., Ling, Y.: Quintessence reconstruction of the new agegraphic dark energy model. Phys. Lett. B 663, 152 (2008). arXiv:0805.0546 [hep-th]
- Zhang, J., Zhang, X., Liu, H.: Agegraphic dark energy as a quintessence. Eur. Phys. J. C 54, 303 (2008). arXiv:0801.2809
- 44. Wei, H., Cai, R.G.: Interacting agegraphic dark energy. Eur. Phys. J. C 59, 99 (2009). arXiv:0707.4052
- 45. Neupane, I.P.: A note on agegraphic dark energy. Phys. Lett. B 673, 111 (2009). arXiv:0708.2910
- Sheykhi, A.: Interacting agegraphic dark energy models in non-flat universe. Phys. Lett. B 680, 113 (2009). arXiv:0907.5144 [hep-th]
- Sheykhi, A.: Interacting agegraphic tachyon model of dark energy. Phys. Lett. B 682, 329 (2010). arXiv:0907.2491 [hep-th]
- Sheykhi, A.: Interacting new agegraphic dark energy in non-flat Brans-Dicke cosmology. Phys. Rev. D 81, 023525 (2010). arXiv:0908.0606 [gr-qc]
- Sheykhi, A., Jamil, M.: Interacting HDE and NADE in Brans-Dicke chameleon cosmology. Phys. Lett. B 694, 284 (2011). arXiv:1010.0385 [hep-th]
- Sheykhi, A., Setare, M.R.: Interacting new agegraphic viscous dark energy with varying G. Int. J. Theor. Phys. 49, 2777 (2010). arXiv:1003.1109 [physics.gen-ph]
- Capozziello, S., Carloni, S., Troisi, A.: Quintessence without scalar fields. Recent Res. Dev. Astron. Astrophys 1, 625 (2003). arXiv:astro-ph/0303041
- Capozziello, S., Cardone, V.F., Carloni, S., Troisi, A.: Curvature quintessence matched with observational data. Int. J. Mod. Phys. D., 121969 (2003). arXiv:astro-ph/0307018
- Carroll, S.M., Duvvuri, V., Trodden, M., Turner, M.S.: Is cosmic speed-up due to new gravitational physics? Phys. Rev. D 70, 043528 (2004). arXiv:astro-ph/0306438
- Nojiri, S., Odintsov, S.D.: Modified gravity with negative and positive powers of the curvature: Unification of the inflation and of the cosmic acceleration. Phys. Rev. D 68, 123512 (2003). arXiv:hep-th/0307288
- 55. Kawarabayashi, K., Ohta, N.: The problem of η in the large N limit: Effective Lagrangian approach. Nucl. Phys. B **175**, 477 (1980)

- 56. Witten, E.: Current algebra theorems for the U (1)Goldstone boson. Nucl. Phys. B 156, 269 (1979)
- 57. Veneziano, G.: U(1) without instantons. Nucl. Phys. B 159, 213 (1979)
- Rosenzweig, C., Schechter, J., Trahern, C.G.: Is the effective Lagrangian for quantum chromodynamics a σ model? Phys. Rev. D, 213388 (1980)
- 59. Nath, P., Arnowitt, R.L.: U (1) problem: Current algebra and the vacuum. Phys. Rev D., 23473 (1981)
- 60. Ohta, N.: Dark energy and QCD ghost. Phys. Lett. B 695, 41 (2011). arXiv:1010.1339
- Sheykhi, A., Bagheri, A.: Quintessence ghost dark energy model. Europhys. Lett 95, 39001 (2011). arXiv:1104.5271
- Ebrahimi, E., Sheykhi, A.: Interacting ghost dark energy in Brans-Dicke theory. Phys. Lett. B 705, 19 (2011). arXiv:1105.5680
- Ebrahimi, E., Sheykhi, A.: Instability of QCD ghost dark energy model. Int. J. Mod. Phys. D 20, 2369 (2011). arXiv:1106.3504
- Sheykhi, A., Sadegh Movahed, M.: Interacting ghost dark energy in non-flat Universe. Gen. Relativ. Gravit. 44, 449 (2012). arXiv:1104.4713
- Sheykhi, A., Tavayef, M.: Ghost dark energy in a cyclic universe Iran. J. Sci. Technol. Trans. Sci. https://doi.org/10.1007/s40995-016-0083-y
- Feng, C.-J., Li, X.-Z., Xi, P.: Global behavior of cosmological dynamics with interacting Veneziano ghost. JHEP 1205, 046 (2012). arXiv:1204.4055
- Feng, C.-J., Li, X.-Z., Shen, X.-Y.: Latest observational constraints to the ghost dark energy model by using the Markov chain Monte Carlo approach. Phys. Rev. D 87, 023006 (2013). arXiv:1202.0058
- Feng, C.-J., Li, X.-Z., Shen, X.-Y.: Thermodynamic of the ghost dark energy universe. Mod. Phys. Lett. A 27, 1250182 (2012). arXiv:1105.3253
- Nojiri, S., Odintsov, S.D.: Inhomogeneous equation of state of the universe: Phantom era, future singularity and crossing the phantom barrier. Phys. Rev. D 72, 023003 (2005). arXiv:hep-th/0505215
- Capozziello, S., Cardone, V.F., Elizalde, E., Nojiri, S., Odintsov, S.D.: Observational constraints on dark energy with generalized equations of state. Phys. Rev. D, 73043512 (2006). arXiv:astro-ph/0508350
- Zhitnitsky, A.R.: Contact term, its holographic description in QCD and dark energy. Phys. Rev. D 86, 045026 (2012). arXiv:1112.3365
- Cai, R.G., Tuo, Z.L., Wu, Y.B., Zhao, Y.Y.: More on QCD ghost dark energy. Phys. Rev. D 86, 023511 (2012). arXiv:1201.2494
- Ebrahimi, E., Sheykhi, A., Alavirad, H.: Interacting generalized ghost dark energy in nonflat universe. Cent. Eur. J. Phys 11, 949 (2013). arXiv:1209.3147
- Sheykhi, A., Ebrahimi, E., Yousefi, Y.: Generalized ghost dark energy in Brans-Dicke theory. Can. J. Phys 91, 662 (2013). arXiv:1210.0781 [gr-qc]
- Sievers, J.L., et al.: Cosmological parameters from cosmic background imager observations and comparisons with BOOMERANG, DASI, and MAXIMA. Astrophys. J., 591599 (2003). arXiv:astro-ph/0205387
- Benoit, A., et al.: Cosmological constraints from Archeops. Astron. Astrophys 399, L25 (2003). arXiv:astro-ph/0210306
- Benoit, A., et al.: The cosmic microwave background anisotropy power spectrum measured by Archeops. Astron. Astrophys 399, L19 (2003). arXiv:astro-ph/0210305
- Uzan, J.P., Kirchner, U., Ellis, G.F.R.: Wilkinson Microwave Anisotropy Probe data and the curvature of space. Mon. Not. R. Astron. Soc 344, L65 (2003)
- 79. Linde, A.: Can we have inflation with $\Omega > 1$? JCAP **0305**, 002 (2003). arXiv:astro-ph/0303245v2
- Tegmark, M., de Oliveira-Costa, A., Hamilton, A.: High resolution foreground cleaned CMB map from WMAP. Phys. Rev. D 68, 123523 (2003). arXiv:astro-ph/0302496
- Efstathiou, G.: Is the low cosmic microwave background quadrupole a signature of spatial curvature? Mon. Not. R. Astron. Soc 343, L95 (2003). arXiv:astro-ph/0303127
- Luminet, J.P., Weeks, J., Riazuelo, A., Lehou, R., Uzan, J.: Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background. Nature 425, 593 (2003). arXiv:astro-ph/0310253
- Ellis, G.F.R., Maartens, R.: The emergent universe: Inflationary cosmology with no singularity. Class Quantum Grav. 21, 223 (2004). arXiv:gr-qc/0211082
- Caldwell, R.R., Kamionkowski, M.: Expansion, geometry, and gravity. JCAP 0409, 009 (2004). arXiv:astro-ph/0403003
- Wang, B., Gong, Y.G., Su, R.K.: Probing the curvature of the Universe from supernova measurement. Phys. Lett. B 605, 9 (2005). arXiv:hep-th/0408032
- Olivares, G., Atrio, F., Pavon, D.: Observational constraints on interacting quintessence models. Phys. Rev. D 71, 063523 (2005). arXiv:astro-ph/0503242

- Bertolami, O., Gil Pedro, F., Le Delliou, M.: Dark energy/dark matter interaction and putative violation of the equivalence principle from the Abell cluster A586. Phys. Lett. B 654, 165 (2007). arXiv:astro-ph/0703462
- Costa, A.A., Xu, X.D., Wang, B., Ferreira, E.G.M., Abdalla, E.: Testing the interaction between dark energy and dark matter with Planck data. Phys. Rev. D 89, 103531 (2014). arXiv:1311.7380
- Xu, X.D., Wang, B., Abdalla, E.: Signature of the scattering between dark sectors in large scale cosmic microwave background anisotropies. Phys. Rev. D 85, 083513 (2012). arXiv:1112.1128
- He, J.H., Wang, B., Abdalla, E.: Testing the interaction between dark energy and dark matter via the latest observations. Phys. Rev. D 83, 063515 (2011). arXiv:1012.3904
- Wang, S., Wang, Y.Z., Geng, J.J., Zhang, X.: Effects of time-varying β in SNLS3 on constraining interacting dark energy models. Eur. Phys. J. C 74, 3148 (2014). arXiv:1406.0072
- He, J.H., Wang, B., Abdallab, E., Pavn, D.: The imprint of the interaction between dark sectors in galaxy clusters. JCAP 12, 022 (2010). arXiv:1001.0079
- Abdalla, E., Abramo, L.R., de Souza, J.C.C.: Signature of the interaction between dark energy and dark matter in observations. Phy. Rev. D 82, 023508 (2010). arXiv:0910.5236
- Xu, X.D., Wang, B., Zhang, P., Barandela, F.A.: The effect of Dark Matter and Dark Energy interactions on the peculiar velocity field and the kinetic Sunyaev-Zel'dovich effect. JCAP 12, 001 (2013). arXiv:1308.1475
- Amendola, L.: Scaling solutions in general nonminimal coupling theories. Phys. Rev. D 60, 043501 (1999). arXiv:astro-ph/9904120
- 96. Amendola, L.: Coupled quintessence. Phys. Rev. D 62, 043511 (2000). arXiv:astro-ph/9908023
- Amendola, L., Quercellini, C.: Tracking and coupled dark energy as seen by the Wilkinson microwave anisotropy probe. Phys. Rev. D 68, 023514 (2003). arXiv:astro-ph/0303228
- Amendola, L., Tocchini-Valentini, D.: Stationary dark energy: The present universe as a global attractor. Phys. Rev. D 64, 043509 (2001). arXiv:astro-ph/0011243
- Amendola, L., Valentini, D.T.: Baryon bias and structure formation in an accelerating universe. Phys. Rev. D 66, 043528 (2002). arXiv:astro-ph/0111535
- 100. Zimdahl, W., Pavon, D.: Interacting quintessence. Phys. Lett. B 521, 133 (2001). arXiv:1503.08712
- 101. Zimdahl, W., Pavon, D.: Scaling cosmology. Gen. Rel. Grav. 35, 413 (2003). arXiv:astro-ph/0210484
- Chimento, L.P., Jakubi, A.S., Pavon, D., Zimdahl, W.: Interacting quintessence solution to the coincidence problem. Phys. Rev. D 67, 083513 (2003). arXiv:astro-ph/0303145
- Wang, B., Gong, Y., Abdalla, E.: Transition of the dark energy equation of state in an interacting holographic dark energy model. Phys. Lett. B 624, 141 (2005). arXiv:hep-th/0506069
- Wang, B., Lin, C.Y., Abdalla, E.: Constraints on the interacting holographic dark energy model. Phys. Lett. B 637, 357 (2005). arXiv:hep-th/0509107
- 105. Cai, R.G., Su, Q.P.: On the dark sector interactions. Phys. Rev. D 81, 103514 (2010). arXiv:0912.1943
- 106. Wei, H.: Cosmological Constraints on the Sign-Changeable Interactions. Commun. Theor. Phys. 56, 972 (2011). arXiv:1010.1074
- 107. Wei, H.: Cosmological evolution of quintessence and phantom with a new type of interaction in dark sector. Nucl. Phys. B 845, 381 (2011). arXiv:1008.4968v3
- Chimento, L.P.: Linear and nonlinear interactions in the dark sector. Phys. Rev. D 81, 043525 (2010). arXiv:0911.5687
- Chimento, L.P., Forte, M., Kremer, G.M.: Cosmological model with interactions in the dark sector. Gen. Rel. Grav. 41, 1125 (2009). arXiv:0711.2646
- Khurshudyan, M.: Phase space analysis of some interacting Chaplygin gas Models. Eur. Phys. J. C 77, 65 (2017). arXiv:1509.02263
- 111. Khurshudyan, M.: Interaction between variable Chaplygin gas and tachyonic matter. arXiv:1301.4990v1
- 112. Khurshudyan, M.: Interaction between generalized varying Chaplygin gas and tachyonic fluid. arXiv:1301.1021
- Xi, P., Li, P.: Reexamining generalized Chaplygin gas with the sign-changeable interaction. Astrophys. Space Sci. 360, 3 (2015). arXiv:1510.02859
- 114. Xu, Y.D., Huang, Z.G., Zhai, X.H.: Cosmological evolution of modified Chaplygin gas with the signchangeable interactions. Int. J. Theor. Phys. 51, 3432 (2012)
- 115. Xu, Y.D., Huang, Z.G.: The sign-changeable interaction between variable generalized Chaplygin gas and dark matter. Astrophys. Space Sci **343**, 807 (2013)
- Xu, Y.D.: Statefinder diagnostic for modified Chaplygin gas with the sign-changeable interaction. Int. J. Theor. Phys. 52, 1132 (2013)
- 117. XU, Y.D.: Agegraphic dark energy with the sign-changeable interaction in non-flat universe. Commun. Theor. Phys. 65, 4 (2015). arXiv:1508.06029

- 118. Xu, Y.D., Huang, Z.G.: New agegraphic dark energy model with the sign-changeable interaction. Astrophys. Space Sci. **350**, 855 (2014)
- Abdollahi Zadeh, M., Sheykhi, A., Moradpour, H.: Holographic dark energy with the sign-changeable interaction term. Int. J. Mod. Phys. D, 26(8), 1750080 (2017). arXiv:1610.08093
- Huang, Q.G., Li, M.: The holographic dark energy in a non-flat universe. J. Cosmol. Astropart. Phys. JCAP 08, 013 (2004). arXiv:astro-ph/0404229
- 121. Waterhouse, T.P., Zipin, J.P.: The cosmic variance of Omega. arXiv:0804.1771
- Maggiore, M., Hollenstein, L., Jaccard, M., Mitsou, E.: Early dark energy from zero-point quantum fluctuations. Phys. Lett. B 704, 102 (2011). arXiv:1104.3797 [astro-ph.CO]
- Copeland, E.J., Sami, M., Tsujikawa, S.: Dynamics of dark energy. Int. J. Mod. Phys. D 15, 1753 (2006). arXiv:hep-th/0603057
- 124. Bamba, K., Capozziello, S., Nojiri, S., Odintsov, S.D.: Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests. Astrophys. Space Sci 342, 155 (2012). arXiv:1205.3421