Int J Theor Phys (2017) 56:3400–3404
DOI 10.1007/s10773-017-3502-8

Bidirectional Controlled Quantum Information Transmission by Using a Five-Qubit Cluster State

Zhi-wen Sang[1]

Received: 19 March 2017 / Accepted: 4 August 2017 / Published online: 11 August 2017

Abstract We demonstrate that an entangled five-qubit cluster state can be used to realize the deterministic bidirectional controlled quantum information transmission by performing only Bell-state measurement and single-qubit measurements. In our protocol, Alice can teleport an arbitrary unknown single-qubit state to Bob and at the same time Bob can remotely prepare an arbitrary known single-qubit state for Alice via the control of the supervisor Charlie.

Keywords Controlled quantum information transmission · Five-qubit cluster state · Bell-state measurement

1 Introduction

Quantum entanglement is an important quantum resource, which can be used to implement various quantum information processing tasks [1–10], such as quantum teleportation [11] and quantum remote state preparation [12]. As known, if a sender (Alice) wants to transmit an arbitrary unknown quantum state to a receiver (Bob), they can use the protocol of the standard quantum teleportation. However, if Alice wants to prepare an arbitrary known quantum state to Bob, they can use the protocol of the standard quantum remote state preparation. In recent years, various theoretical quantum teleportation schemes [13–31] and remote state preparation schemes [32–38] have been proposed. In 2013, Zha et al. [39] proposed the original bidirectional quantum controlled teleportation protocol by using an entangled five-qubit cluster state. In their scheme, Alice can teleport an arbitrary unknown single-qubit state to Bob and at the same time Bob can teleport an arbitrary unknown single-qubit state to Alice via the control of the supervisor Charlie.

✉ Zhi-wen Sang
zhiwensang@163.com

1 School of Physics and Electronic Information, Shangrao Normal University, Shangrao 334001, China

In this work, we propose a new protocol for bidirectional controlled quantum information transmission by using an entangled five-qubit cluster state, where Alice can teleport an arbitrary unknown single-qubit state to Bob and at the same time Bob can remotely prepare an arbitrary known single-qubit state for Alice via the control of the supervisor Charlie. In our protocol, only Bell-state measurement and single-qubit measurements are used. Our results make the five-qubit cluster state multipurpose, i.e., no matter whether the transmitted state is known or unknown, the quantum information state can be transmitted with each other by using an entangled five-qubit cluster state under the control of the third party as a supervisor.

2 Bidirectional Controlled Quantum Information Transmission

Our protocol can be described as follows. Assume that Alice has an arbitrary unknown single-qubit state, which is written as

$$|\psi\rangle_a = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1}$$

where α and β are complex numbers and satisfy that $|\alpha|^2 + |\beta|^2 = 1$. Now Alice wants to teleport the arbitrary unknown single-qubit state $|\psi\rangle_a$ of qubit a to Bob. At the same time, Bob must remotely prepare an arbitrary known single-qubit state of qubit b for Alice. The known single-qubit state of qubit b can be expressed as

$$|\psi\rangle_b = b_0\,|0\rangle + b_1 e^{i\phi_1}\,|1\rangle\,, \tag{2}$$

where b_0, b_1, ϕ_1 are real number, and satisfy that $\phi_1 \in [0, 2\pi]$ with $|b_0|^2 + |b_1|^2 = 1$. Assume that Alice, Bob and Charlie share an entangled five-qubit cluster state, which has the form

$$|C_5\rangle_{12345} = \frac{1}{2}\left(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle\right)_{12345}\,, \tag{3}$$

where the qubits 1 and 5 belong to Alice, the qubits 2 and 3 belong to Bob, and the qubit 4 belongs to Charlie, respectively. Therefore, the combined quantum state of the six qubits can be expressed as

$$|\Omega\rangle_{a12345} = |\psi\rangle_a \otimes |C_5\rangle_{12345}\,. \tag{4}$$

To achieve the purpose of bidirectional controlled quantum information transmission, firstly, Bob introduces one auxiliary qubit 6 with an initial state $|0\rangle_6$. Therefore, the system state of the seven qubits can be expressed as

$$|\mathrm{X}\rangle_{a123456} = |\psi\rangle_a \otimes |C_5\rangle_{12345} \otimes |0\rangle_6\,. \tag{5}$$

Secondly, Bob performs a C-NOT operation on qubits 3 and 6, where qubit 3 works as controlling qubit and auxiliary qubit 6 as target qubit. After that, the above seven-qubit combined state will become

$$|\Pi\rangle_{a123456} = |\psi\rangle_a \otimes \frac{1}{2}\left(|000000\rangle + |001111\rangle + |111011\rangle + |110100\rangle\right)_{123456}\,. \tag{6}$$

Thirdly, Alice has to carry out a Bell-state measurement on her qubit pair (a, 1), and the Bell-state measurement bases are

$$\left|\Phi^{\pm}\right\rangle_{a1} = \frac{\sqrt{2}}{2}\left(|00\rangle \pm |11\rangle\right)_{a1}\,, \tag{7}$$

and

$$\left|\Psi^{\pm}\right\rangle_{a1} = \frac{\sqrt{2}}{2}\left(|01\rangle \pm |10\rangle\right)_{a1}. \quad (8)$$

Without loss of generality, if Alice's Bell-state measurement result is $\left|\Phi^{+}\right\rangle_{a1}$, then the state of other qubits 2, 3, 4, 5, 6 are collapsed into the following state

$$|\Xi\rangle_{23456} = \frac{\sqrt{2}}{2}\left(\alpha\,|00000\rangle + \alpha\,|01111\rangle + \beta\,|11011\rangle + \beta\,|10100\rangle\right)_{23456}. \quad (9)$$

Fourthly, Bob must perform a single-qubit measurement on his qubit 3, and the single-qubit measurement bases are given by

$$\left|\xi^{+}\right\rangle_{3} = \left(b_0\,|0\rangle + b_1\,|1\rangle\right)_3, \quad (10)$$

$$\left|\xi^{-}\right\rangle_{3} = \left(b_1\,|0\rangle - b_0\,|1\rangle\right)_3. \quad (11)$$

If Bob's single-qubit measurement result is $\left|\xi^{+}\right\rangle_{3}$, then the state of remaining qubits 2, 4, 5 and 6 are collapsed into the state

$$|\varphi\rangle_{2456} = \left(\alpha b_0\,|0000\rangle + \alpha b_1\,|0111\rangle + \beta b_1\,|1011\rangle + \beta b_0\,|1100\rangle\right)_{2456}. \quad (12)$$

Fifthly, Bob then perform a single-qubit measurement on his qubit 6, and the single-qubit measurement bases are given by

$$\left|\zeta^{+}\right\rangle_{6} = \frac{\sqrt{2}}{2}\left(|0\rangle + e^{-i\phi_1}\,|1\rangle\right)_6, \quad (13)$$

$$\left|\zeta^{-}\right\rangle_{6} = \frac{\sqrt{2}}{2}\left(|0\rangle - e^{-i\phi_1}\,|1\rangle\right)_6. \quad (14)$$

If Bob's single-qubit measurement result is $\left|\zeta^{+}\right\rangle_{6}$, then the state of remaining qubits 2, 4 and 5 are collapsed into the state

$$|\tau\rangle_{245} = \left(\alpha b_0\,|000\rangle + \alpha b_1 e^{i\phi_1}\,|011\rangle + \beta b_1 e^{i\phi_1}\,|101\rangle + \beta b_0\,|110\rangle\right)_{245}. \quad (15)$$

Finally, Charlie has to make out a single-qubit measurement on his qubit 4 under the basis $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. If Charlie's single-qubit measurement result is $|+\rangle_4$, then the state of qubits 2 and 5 will be collapsed into the following state

$$\begin{aligned}|\omega\rangle_{25} &= \left(\alpha b_0\,|00\rangle + \alpha b_1 e^{i\phi_1}\,|01\rangle + \beta b_1 e^{i\phi_1}\,|11\rangle + \beta b_0\,|10\rangle\right)_{25}\\ &= \left(\alpha\,|0\rangle + \beta\,|1\rangle\right)_2 \otimes \left(b_0\,|0\rangle + b_1 e^{i\phi_1}\,|1\rangle\right)_5.\end{aligned}$$

Therefore, the bidirectional controlled quantum information transmission is successfully realized. That is to say Alice has teleported an arbitrary unknown single-qubit state to Bob and at the same time Bob has remotely prepared an arbitrary known single-qubit state for Alice via the control of the supervisor Charlie.

Analogously, for other measurement results by Alice, Bob and Charlie, the receivers both Alice and Bob can perform an appropriate unitary transformation according to the appropriate measurement results by Alice, Bob and Charlie. At last, the bidirectional controlled quantum information transmission task is easily fulfilled.

3 Conclusions

In summary, we have demonstrated that an entangled five-qubit cluster state can be used to implement the deterministic bidirectional controlled quantum information transmission task by performing one Bell-state measurement and two single-qubit measurements. In our work, the sender Alice can teleport an arbitrary unknown single-qubit state of qubit a to the receiver Bob, and at the same time the sender Bob can remotely prepare an arbitrary known single-qubit state of qubit b for the receiver Alice via the control of the supervisor Charlie. The receivers can operate an appropriate unitary transformation to obtain the desired state according to the measured result by Alice, Bob and Charlie. Without the help of Charlie, the receivers both Alice and Bob cannot fully obtain the desired state.

References

1. Li, Y.H., Xiang, T., Nie, Y.Y., Sang, M.H., Chen, X.F.: Spectral compression of single-photon-level laser pulse. Sci. Rep. **7**, 43494 (2017)
2. Xiong, J.G., Sang, M.H.: Splitting an arbitrary two-qubit state via a genuine six-qubit entangled state. Int. J. Theor. Phys. **54**, 1578 (2015)
3. Li, Y.H., Sang, M.H., Wang, X.P., Nie, Y.Y.: Quantum teleportation of a four-qubit state by using six-qubit cluster state. Int. J. Theor. Phys. **55**, 3547 (2016)
4. Li, G.Z., Chen, Y.P., Jiang, H.W., Chen, X.F.: Enhanced Kerr electro-optic nonlinearity and its application in controlling second-harmonic generation. Photon. Res. **3**, 168 (2015)
5. An, N., Zheng, Y.L., Ren, H.J., Zhao, X.H., Deng, X.W., Chen, X.F.: Normal, degenerated, and anomalous-dispersion-like Cerenkov sum-frequency generation in one nonlinear medium. Photon. Res. **3**, 106 (2015)
6. Zhang, M., Li, H.: Weak blind quantum signature protocol based on entanglement swapping. Photon. Res. **3**, 324 (2015)
7. Feng, X.L., Wu, Z.H., Wang, X.Y., He, S.L., Gao, S.M.: All-optical two-channel polarization-multiplexing format conversion from QPSK to BPSK signals in a silicon waveguide. Photon. Res. **4**, 245 (2016)
8. Xia, J.F., Serna, S., Zhang, W.W., Vivien, L., Cassan, É.: Hybrid silicon slotted photonic crystal waveguides: how does third order nonlinear performance scale with slow light? Photon. Res. **4**, 257 (2016)
9. Yang, G.F., Chen, P., Gao, S.M., Chen, G.Q., Zhang, R.: Zheng. Y.D.: White-light emission from InGaN/GaN quantum well microrings grown by selective area epitaxy. Photon. Res. **4**, 17 (2016)
10. Wen, X., Xu, K., Song, Q.H.: Design of a barcode-like waveguide nanostructure for efficient chip–fiber coupling. Photon. Res. **4**, 209 (2016)
11. Bennett, C.H., Brassard, G., Crpeau, C., et al.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
12. Lo, H.K.: Classical-communication cost in distributed quantum-information processing: a generalization of quantum-communication complexity. Phys. Rev. A **62**, 012313 (2000)
13. Nie, Y.Y., Li, Y.H., Liu, J.C., Sang, M.H.: Quantum information splitting of an arbitrary three-qubit state by using two four-qubit cluster states. Quantum. Inf. Process. **10**, 297 (2011)
14. Nie, Y.Y., Li, Y.H., Liu, J.C., Sang, M.H.: Quantum information splitting of an arbitrary three-qubit state by using a genuinely entangled five-qubit state and a Bell-state. Quantum. Inf. Process. **11**, 563 (2012)
15. Muralidharan, S., Panigrahi, P.K.: Quantum-information splitting using multipartite cluster states. Phys. Rev. A **78**, 062333 (2008)
16. Li, Y.H., Li, X.L., Nie, L.P., Sang, M.H.: Quantum teleportation of three and four-qubit state using multi-qubit cluster states. Int. J. Theor. Phys. **55**, 1820 (2016)
17. Deng, F.G., Li, C.Y., Li, Y.S., et al.: Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. Phys. Rev. A **72**, 022338 (2005)
18. Li, Y.H., Nie, L.P., Li, X.L.: Controlled teleportation of an arbitrary three-ion state in ion-trap systems. Int. J. Theor. Phys. **53**, 3756 (2014)
19. Li, Y.H., Li, X.L., Sang, M.H., Nie, Y.Y., Wang, Z.S.: Bidirectional controlled quantum teleportation and secure direct communication using five-qubit entangled state. Quantum. Inf. Process. **12**, 3835 (2013)
20. Chen, Y.: Bidirectional quantum controlled teleportation by using a genuine six-qubit entangled state. Int. J. Theor. Phys. **54**, 269 (2015)
21. Yan, A.: Bidirectional controlled teleportation via six-qubit cluster state. Int. J. Theor. Phys. **52**, 3870 (2013)

22. Fu, H.-Z., Tian, X.-L., Hu, Y.: A general method of selecting quantum channel for bidirectional quantum teleportation. Int. J. Theor. Phys. **53**, 1840 (2014)
23. Li, Y.H., Jin, X.M.: Bidirectional controlled teleportation by using nine-qubit entangled state in noisy environments. Quantum Inf. Process. **15**, 929 (2016)
24. Nie, Y.Y., Sang, M.H.: Effects of noise on asymmetric bidirectional controlled teleportation. Int. J. Theor. Phys. **55**, 4759 (2016)
25. Sang, M.H.: Bidirectional quantum teleportation by using five-qubit cluster state. Int. J. Theor. Phys. **55**, 1333 (2016)
26. Sang, M.H.: Bidirectional quantum controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. **55**, 380 (2016)
27. Hu, A.R.: Splitting an arbitrary three-qubit state by using an eight-qubit entangled state. Int. J. Theor. Phys. **55**, 396 (2016)
28. Hong, W.Q.: Asymmetric bidirectional controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. **55**, 384 (2016)
29. Huang, Z.H.: Quantum state sharing of an arbitrary three-qubit state by using a seven-qubit entangled state. Int. J. Theor. Phys. **54**, 3438 (2015)
30. Zhu. H.P.: Perfect teleportation of an arbitrary two-qubit state via GHZ-like states. Int. J. Theor. Phys. **53**, 4095 (2014)
31. Zhu. H.P.: Quantum state sharing of an arbitrary single-atom state by using a genuine six-atom entangled state in cavity QED. Int. J. Theor. Phys. **52**, 1588 (2013)
32. Zhang, D., Zha, X.W., Duan, Y.J., Yang, Y.Q.: Deterministic controlled bidirectional remote state preparation via a six-qubit entangled state. Quantum. Inf. Process. **15**, 2169 (2016)
33. Zha, X.W., Song, H.Y.: Remote preparation of a two-particle state using a four-qubit cluster state. Opt. Commun. **284**, 1472 (2011)
34. Hou, K., Li, Y.B., Liu, G.H., Sheng, S.Q.: Joint remote preparation of an arbitrary two-qubit state via GHZ-type states. J. Phys. A **44**, 255304 (2011)
35. Wang, D., Ye, L.: Probabilistic joint remote preparation of four-particle cluster-type states with quaternate partially entangled channels. Int. J. Theor. Phys. **51**, 3376 (2012)
36. Zhan, Y.B., Hu, B.L., Ma, P.C.: Joint remote preparation of four-qubit cluster-type states. J. Phys. B **44**, 095501 (2011)
37. Long, L.R., Zhou, P., Li, Z., Yin, C.L.: Multiparty joint remote preparation of an arbitrary GHZ-class state via positive operator-valued measurement. Int. J. Theor. Phys. **51**, 2438 (2012)
38. Nie, Y.Y., Sang, M.H., Nie, L.P.: Controlled remote state preparation of an arbitrary four-qubit entangled cluster-type state using seven-qubit cluster state. Int. J. Theor. Phys. **56**, 1883 (2017)
39. Zha, X.W., Zou, Z.C., Qi, J.X., Song, H.Y.: Bidirectional quantum controlled teleportation via five-qubit cluster state. Int. J. Theor. Phys. **52**, 1740 (2013)