



Neutrinoless Double Beta Decay and Lepton Flavour Violation in Broken $\mu - \tau$ Symmetric Neutrino Mass Models

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Abstract We have studied neutrinoless double beta decay and charged lepton flavour violation in broken $\mu - \tau$ symmetric neutrino masses in a generic left-right symmetric model (LRSM). The leading order $\mu - \tau$ symmetric mass matrix originates from the type I (II) seesaw mechanism, whereas the perturbations to $\mu - \tau$ symmetry in order for generation of non-zero reactor mixing angle θ_{13} , as required by latest neutrino oscillation data, originates from the type II (I) seesaw mechanism. In our work, we considered four different realizations of $\mu - \tau$ symmetry, viz. Tribimaximal Mixing (TBM), Bimaximal Mixing (BM), Hexagonal Mixing (HM) and Golden Ratio Mixing (GRM). We then studied the new physics contributions to neutrinoless double beta decay (NDBD) ignoring the left-right gauge boson mixing and the heavy-light neutrino mixing within the framework of LRSM. We have considered the mass of the gauge bosons and scalars to be around TeV and studied the effects of the new physics contributions on the effective mass and the NDBD half life and compared with the current experimental limit imposed by KamLAND-Zen. We further extended our analysis by correlating the lepton flavour violation of the decay processes, $(\mu \rightarrow 3e)$ and $(\mu \rightarrow e\gamma)$ with the lightest neutrino mass and atmospheric mixing angle θ_{23} respectively.

Keywords Neutrino masses and mixings · Physics beyond standard model

1 Introduction

The milestone discovery of neutrino oscillation and the corresponding realization that neutrinos are massive particles has been one of the compelling revelation which suggests

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physics beyond the Standard Model (SM). The recent neutrino experiments MINOS [1], T2K [2], Double Chooz [3], Daya Bay [4] and RENO [5] have not only confirmed the earlier observations but also measured the neutrino parameters more accurately. The 3σ global fit values of the neutrino oscillation parameters according to recent analysis are shown in the Table 1.

Notwithstanding, the absolute neutrino mass scale is still unperceived. However, the Planck experiment has given an upper bound on the sum of the light neutrino mass to be $\sum_i |m_i| < 0.23$ eV [7] in 2012 and recently the bound has been constrained to $\sum_i |m_i| < 0.17$ eV [8]. The simplest hypothesis (way) to account for a neutrino mass is to introduce atleast two right handed (RH) neutrino in the Standard Model (SM). This will allow a Dirac coupling with the Higgs, like other fermions in the SM. However, corresponding Yukawa coupling has to be fine tuned around 10^{-12} which is quite unnatural. This kind of fine tuning can be avoided to explain the neutrino masses in the seesaw mechanism, a mechanism beyond SM (BSM) physics which is categorised into type I [9–12], type II [13–16], type III [17], inverse [18] seesaw mechanism. The BSM physics also unveils various phenomenon like Baryon Asymmetry of the Universe (BAU), Lepton Number Violation (LNV), Lepton Flavour Violation (LFV), existence of dark matter etc. One of the theoretical framework to make the first three processes observable is the left-right symmetric model (LRSM) [19–23] which is considered to be an appealing candidate for physics BSM. Here, the gauge group is very simple extension of the SM gauge group. It provides a natural framework to understand the spontaneous breaking of parity and origin of small neutrino mass via seesaw mechanism.

Furthermore, the physics community worldwide is embarking on the next challenging problem in finding out the nature of the neutrinos, whether they are four component Dirac particles possessing a conserved lepton number or two component Majorana particles, along with the absolute scale of neutrino mass. This problem is directly related to the issue of LN conservation, which is one of the most obscure sides of the SM not supported by an underlying principle. One of such process of fundamental importance in particle physics which pops up almost in any extension of the SM is neutrinoless double beta decay (NDBD) [24, 25]. It is defined as a second order, slow radioactive process that transforms a nuclide of atomic number Z into its isobar with atomic number $Z+2$,

$$N(A, Z) \rightarrow N(A, Z + 2) + e^- + e^-, \tag{1}$$

Table 1 Global fit 3σ values of ν oscillation parameters [6]

Parameters	3σ Ranges	Best Fit $\pm 1\sigma$
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	7.11–8.18	$7.60^{+0.19}_{-0.18}$
$\Delta m_{31}^2 [10^{-3} \text{eV}^2](\text{NH})$	2.30–2.65	$2.48^{+0.05}_{-0.07}$
$\Delta m_{23}^2 [10^{-3} \text{eV}^2](\text{IH})$	2.26–2.48	$2.38^{+0.05}_{-0.06}$
$\sin^2 \theta_{12}$	0.278–0.375	0.323 ± 0.016
$\sin^2 \theta_{23}(\text{NH})$	0.392–0.643	$0.567^{+0.032}_{-0.128}$
(IH)	0.403 – 0.640	$0.573^{+0.025}_{-0.043}$
$\sin^2 \theta_{13}(\text{NH})$	0.177–0.294	0.234 ± 0.020
(IH)	0.183 – 0.297	0.240 ± 0.019
δ	0- 2π (NH)	254^0
	0- 2π (IH)	266^0

thereby violating the total LN conservation. Its existence is directly linked to that of the Majorana neutrinos [26] (i.e., identical to its own anti particle).

The general expression for the total decay width of $0\nu\beta\beta$ taking into account the coulomb interaction of the electrons and the final nucleus is given by,

$$\Gamma^{0\nu} = \frac{1}{T_{\frac{1}{2}}^{0\nu}} = G^{0\nu}(Q, Z) \left| M^{0\nu} \right|^2 \frac{|m_{\beta\beta}|^2}{m_e^2}. \tag{2}$$

The numerical values of $G^{0\nu}(Q,Z)$, Q and the natural abundance of several nuclei of experimental interest are given in the Table 2 which are adopted from reference [27].

The main aim of the experiment on the search for $0\nu\beta\beta$ decay is the measurement of the effective Majorana neutrino mass, which is a combination of the neutrino mass eigenstates and neutrino mixing matrix terms, given by,

$$m_{\beta\beta} = \sum_i U_{ej}^2 m_j, j = 1, 2, 3, \tag{3}$$

where, U_{ej} are the elements of the first row of the neutrino mixing matrix, U_{PMNS} (dependent on the known parameters θ_{13}, θ_{12} and the unknown Majorana phases α and β [28–31]). U_{PMNS} is the diagonalizing matrix of the light neutrino mass matrix, m_ν given by (4).

$$U_{PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & -c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} U_{Maj}. \tag{4}$$

The abbreviations used are $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, δ is the Dirac CP phase while the diagonal phase matrix, $U_{Maj} = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$ [32] contains the Majorana phases α and β . The Majorana phases α and β have an effect in the process, which are allowed only if massive neutrinos are Majorana particles and are characterized by a violation of total LN, such as NDBD. In the standard parameterization of the mixing matrix, $m_{\beta\beta}$ is given by,

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}. \tag{5}$$

A huge amount of experimental and theoretical activity is pursued in order to detect and predict the decay process. Although no convincing experimental evidence of the decay exists till date, but new generation of experiments that are already running or about to run assures to expedite the current limits exploring the degenerate-hierarchy region of neutrino masses. In addition, from the life time of this decay combined with sufficient knowledge of the nuclear matrix elements (NME), one can set a constraint involving the neutrino masses.

Table 2 The values of $G^{0\nu}(Q,Z)$, Q of the initial isotope for several NDBD processes of experimental interest

$\beta\beta - \text{decay}$	$G^{0\nu}[10^{-14}y^{-1}]$	$Q[KeV]$	Experiments
$48Ca \rightarrow 48Ti$	6.3	4273.7	CANDLES
$76Ge \rightarrow 76Se$	0.63	2039.1	GERDA, Majorana
$82Se \rightarrow 82Kr$	2.7	2995.5	SuperNEMO, Lucifer
$100Mo \rightarrow 100Ru$	4.4	3035.0	MOON, AMoRe
$116Cd \rightarrow 116Sn$	4.6	2809	Cobra
$130Te \rightarrow 130Xe$	4.1	2530.3	CUORE
$136Xe \rightarrow 136Ba$	4.3	2461.9	EXO, KamLAND-Zen, NEXT, XMASS
$150Nd \rightarrow 150Sm$	19.2	3367.3	SNO+, DCBA/MTD

Moreover, if one incorporates the recent results of neutrino oscillation experiments, one can set a stringent limit on the neutrino mass scale. The latest experiments [33, 34] that have improved the lower bound of the half life of the decay process include KamLAND-Zen [35] and GERDA [36] which uses Xenon-136 and Germanium-76 nuclei respectively. Incorporating the results from first and second phase of the experiment, KamLAND-Zen imposes the best lower limit on the decay half life using Xe-136 as $T_{1/2}^{0\nu} > 1.07 \times 10^{26}$ yr at 90% CL and the corresponding upper limit of effective Majorana mass in the range (0.061–0.165)eV.

Again one of the most important BSM framework to understand the origin of neutrino mass and large leptonic mixing is to identify the possible underlying symmetries. Symmetries can relate two or more free parameters of the model or make them vanish, making the model more predictive. The widely studied $\mu - \tau$ symmetric [37–56] neutrino mass matrix giving zero θ_{13} is one such scenario where discrete flavor symmetries can relate two or more terms in the neutrino mass matrix. The neutrino oscillation data before the discovery of non zero θ_{13} were in perfect agreement with $\mu - \tau$ symmetric neutrino mass matrix. The four different realizations of neutrino mixing pattern generally found in literature which can generate from $\mu - \tau$ symmetric mass matrices are tribimaximal mixing (TBM), bimaximal mixing (BM), hexagonal mixing (HM), golden ratio mixing (GRM) matrices. But, after discovery of non zero θ_{13} , one needs to go beyond these $\mu - \tau$ symmetric framework. Since the experimental value of θ_{13} is still much smaller than the other two mixing angles, $\mu - \tau$ symmetry can still be a valid approximation and the non zero θ_{13} [57–59] can be accounted for by incorporating the presence of small perturbation to $\mu - \tau$ symmetry.

The discovery of neutrino oscillation has provided clear evidence of the fact that neutrinos are massive as well as the violation of the lepton flavour [60, 61] during the propagation of the neutrinos. Lepton flavour is consequently a broken symmetry and the SM has to be adapted to incorporate massive neutrinos and thus we can also hope that lepton flavour violation (LFV) will be visible in the charged lepton sector [62]. The exact mechanism of LFV being unknown, its study is of large interest as it is linked to neutrino mass generation, CP violation and new physics BSM. The LFV effects from new particles at TeV scale are naturally generated in many models and therefore considered to be a prominent signature for new physics. In LRSM, where electroweak symmetry is broken dynamically, an experimentally accessible amount of LFV is predicted in a large region of parameter space. In a wide range of models for physics BSM, highest sensitivity in terms of BR is expected for $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ decay processes.

To study these phenomenon theoretically or phenomenologically, many works have been performed in LRSM based framework [63–73]. In most of these works, authors mostly considered the TBM like neutrino mass as leading order contribution and arising from type I seesaw and using the type II seesaw as a perturbation to generate non zero θ_{13} [74, 75]. More recently, the authors of [76, 77] studied the new physics contribution to NDBD with prominent type I and type II as well as equally dominating type I and type II seesaw. Again, many works have been done in charged lepton flavour violation sector in literature considering type I and type II dominant cases as well as equally dominant type I and type II in the TeV scale LRSM framework which is within the presently accessible reach of the colliders and implements the two seesaw mechanisms naturally [62].

In this context, we present a phenomenological study of different $\mu - \tau$ symmetric [37–56] neutrino mass models to check their consistency with the stringent constraints from cosmology, with various processes like LNV, LFV etc. We have taken the leading order

mass matrices obeying μ - τ symmetry originating from type I (II) seesaw then incorporating type II (I) seesaw as perturbations to generate non zero θ_{13} . Then we studied the LFV in the LRSM framework and further correlated the LFV of the processes ($\mu \rightarrow e\gamma$) and ($\mu \rightarrow 3e$) with lightest neutrino mass and atmospheric mixing angle, θ_{23} in different neutrino mass models favouring $\mu - \tau$ symmetry. In NDBD, we discuss the different contributions [76] from right handed (RH) neutrinos and RH gauge bosons, triplet Higgs [78, 79] as well as light heavy neutrino mixing that can contribute to the effective mass governing the process and identify the significant ones. In this work, we have considered only the dominant new physics contribution as coming from the diagrams containing purely RH current mediated by the heavy gauge boson, W_R by the exchange of heavy right handed neutrino, N_R and another from the charged Higgs scalar Δ_R mediated by the heavy gauge boson W_R [77]. We have ignored the contributions coming from the left-right gauge boson mixing and heavy light neutrino mixing.

This paper is structured as follows. In Section 2 we briefly discuss the left-right symmetric model framework and the origin of neutrino mass and summarize the NDBD process in this framework in Section 3. We also discuss the different feynmann diagrams contributing to the amplitude of the decay process (the new physics contribution) in this section. In Section 4, we briefly discuss lepton flavor violating processes, mainly ($\mu \rightarrow 3e$) and ($\mu \rightarrow e\gamma$). In Section 5, we present our numerical analysis and results and then in Section 6, we conclude by giving a brief overview of our work.

2 Left Right Symmetric Model(LRSM) and Neutrino Mass

The explanation of the smallness of neutrino mass and the profile of its mixing as required by the recent experiments has been taken as a great puzzle in particle physics. The fact that neutrino has mass implies the requirement of new physics beyond the $SU(3)_c \times SU(2)_L \times U(1)_Y$ SM [80]. One possibility to introduce neutrino mass is the so called seesaw mechanism wherein we introduce right handed heavy singlet neutrino, ν_R (type I seesaw), scalar Higgs triplet (type II seesaw) and hypercharge-less fermion triplets (type III seesaw). Left-right symmetric model (LRSM) [19–23] can be considered to be very appealing model for Physics beyond the Standard model. The seesaw mechanisms can be realized in the context of left-right symmetric model or GUTs where seesaw scale might be related to other physical scales.

In LRSM, the gauge group is a very simple extension of the standard model gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$. Most of the problems like parity violation of weak interaction, massless neutrinos, CP problems, hierarchy problems etc can be explained in the framework of LRSM, based on the gauge group, $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [19–23, 81]. In this model, the electric charge is related to the generators of the group as

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2} = T_{3L} + Y, \quad (6)$$

where T_{3L} and T_{3R} are the 3rd components of isospin under $SU(2)_L$ and $SU(2)_R$. In LRSM, the left and right handed components of the fields are treated on the same footing. If the Higgs sector of the model is chosen so that RH symmetry is spontaneously broken by triplets, the model gives rise to tiny neutrino masses naturally via seesaw mechanism. Herein, there are 2 sources of lepton number violation, the Majorana masses of neutrinos

and Yukawa interaction of triplet Higgs. The Quarks and leptons transform under the L-R symmetric gauge group as,

$$q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix} \equiv \left(3, 2, 1, \frac{1}{3} \right), q_R = \begin{bmatrix} u_R \\ d_R \end{bmatrix} \equiv \left(3, 1, 2, \frac{1}{3} \right) \tag{7}$$

$$l_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \equiv (1, 2, 1, -1), l_R = \begin{bmatrix} \nu_R \\ e_R \end{bmatrix} \equiv (1, 1, 2, -1) \tag{8}$$

We consider the general class of left-right symmetric model which are invariant under $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry with the Higgs content, $\phi(1, 2, 2, 0)$, $\Delta_L(1, 2, 1, -1)$, $\Delta_R(1, 1, 2, -1)$. A convenient representation of fields is given by 2×2 matrices for the Higgs bidoublets and the $SU(2)_{L,R}$ triplets as,

$$\phi = \begin{bmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{bmatrix} \equiv (\phi_1, \tilde{\phi}_2), \tag{9}$$

$$\Delta_{L,R} = \begin{bmatrix} \delta_{L,R}^{++} & \delta_{L,R}^{++} \\ \delta_{L,R}^{+} & -\delta_{L,R}^{+} \end{bmatrix} \tag{10}$$

The neutral Higgs fields $\delta_{L,R}^0, \phi_1^0, \phi_2^0$ can potentially acquire VEVs v_R, v_L, k_1, k_2 respectively.

$$\langle \phi \rangle = \begin{bmatrix} \frac{k_1}{\sqrt{2}} & 0 \\ 0 & \frac{k_2}{\sqrt{2}} \end{bmatrix} \tag{11}$$

$$\langle \Delta_{L,R} \rangle = \begin{bmatrix} 0 & 0 \\ \frac{v_{L,R}}{\sqrt{2}} & 0 \end{bmatrix}. \tag{12}$$

The VEV v_R breaks the $SU(2)_R$ symmetry and sets the mass scale for the extra gauge bosons (W_R and Z') and for right handed neutrino field (ν_R). The VEVs k_1 and k_2 serves the twin purpose of breaking the remaining the $SU(2)_L \times U(1)_{B-L}$ symmetry down to $U(1)_{em}$, thereby setting the mass scales for the observed W_L and Z bosons and providing Dirac masses for the quarks and leptons. Clearly, v_R must be significantly larger than k_1 and k_2 in order for W_R and Z' to have greater masses than the W_L and Z bosons. v_L is the VEV of Δ_L , it plays a significant role in the seesaw relation which is the characteristics of the LR model and can be written as,

$$\langle \Delta_L \rangle = v_L = \frac{\gamma k^2}{v_R}. \tag{13}$$

The acceptable breaking pattern is, $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle \Delta_R \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle \phi \rangle} U(1)_{em}$.

The Yukawa lagrangian in the lepton sector is given by,

$$\mathcal{L} = h_{ij} \bar{\Psi}_{L,i} \phi \Psi_{R,j} + \tilde{h}_{ij} \bar{\Psi}_{L,i} \tilde{\phi} \Psi_{R,j} + f_{L,ij} \Psi_{L,i}^T C i \sigma_2 \Delta_L \Psi_{L,j} + f_{R,ij} \Psi_{R,i}^T C i \sigma_2 \Delta_R \Psi_{R,j} + h.c. \tag{14}$$

Where the family indices i, j are summed over, the indices $i, j=1, 2, 3$ represents the three generations of fermions. $C = i\gamma_2\gamma_0$ is the charge conjugation operator, $\tilde{\phi} = \tau_2 \phi^* \tau_2$ and γ_μ are the Dirac matrices. Considering discrete parity symmetry, the Majorana Yukawa couplings $f_L = f_R$ (for left-right symmetry) gives rises to Majorana neutrino mass after

electroweak symmetry breaking when the triplet Higgs Δ_L and Δ_R acquires non zero vacuum expectation value. Then (14) leads to 6×6 neutrino mass matrix as shown in reference 2 of [63–73]

$$M_\nu = \begin{bmatrix} M_{LL} & M_D \\ M_D^T & M_{RR} \end{bmatrix}, \tag{15}$$

where

$$M_D = \frac{1}{\sqrt{2}}(k_1 h + k_2 \tilde{h}), M_{LL} = \sqrt{2}v_L f_L, M_{RR} = \sqrt{2}v_R f_R. \tag{16}$$

Where M_D , M_{LL} and M_{RR} are the Dirac neutrino mass matrix, left handed and right handed mass matrix respectively. Assuming $M_L \ll M_D \ll M_R$, the light neutrino mass, generated within a type I+II seesaw can be written as,

$$M_\nu = M_\nu^I + M_\nu^{II}, \tag{17}$$

$$M_\nu = M_{LL} + M_D M_{RR}^{-1} M_D^T = \sqrt{2}v_L f_L + \frac{k^2}{\sqrt{2}v_R} h_D f_R^{-1} h_D^T. \tag{18}$$

Where the first and second terms in (18) corresponds to type II seesaw and type I seesaw mediated by RH neutrino respectively. Here,

$$h_D = \frac{(k_1 h + k_2 \tilde{h})}{\sqrt{2}k}, k = \sqrt{|k_1|^2 + |k_2|^2} \tag{19}$$

In the context of LRSM both type I and type II seesaw terms can be written in terms of M_{RR} which arises naturally at a high energy scale as a result of spontaneous parity breaking. In LRSM the Majorana Yukawa couplings f_L and f_R are same (i.e, $f_L = f_R$) and the VEV for left handed triplet v_L can be written as,

$$v_L = \frac{\gamma M_W^2}{v_R}. \tag{20}$$

Thus (18) can be written as,

$$M_\nu = \gamma \left(\frac{M_W}{v_R} \right)^2 M_{RR} + M_D M_{RR}^{-1} M_D^T. \tag{21}$$

In literature, (reference [76, 82]) author define the dimensionless parameter γ as,

$$\gamma = \frac{\beta_1 k_1 k_2 + \beta_2 k_1^2 + \beta_3 k_2^2}{(2\rho_1 - \rho_3)k^2}. \tag{22}$$

Here the terms β, ρ are the dimensionless parameters that appears in the expression of the Higgs potential.

3 $0\nu\beta\beta$ Decay in LRSM

Many theoretical groups has studied NDBD in connection with LRSM [83]. In the context of LRSM, there are several contributions to NDBD in addition to the standard contribution via light Majorana neutrino exchange owing to the presence several heavy additional scalar, vector and fermionic fields . Many of the earlier works have explained it in details with the corresponding feynmann diagrams (see ref. [76]). The various contributions to $0\nu\beta\beta$ decay transition rate in LRSM are briefly summarized below.

- Standard Model contribution to NDBD where the intermediate particles are the W_L^- bosons and light neutrinos. The amplitude of this process depends upon the leptonic mixing matrix elements and light neutrino masses.
- Heavy right handed neutrino contribution to NDBD in which the mediator particles are the W_L^- bosons. The amplitude of this process depends upon the mixing between light and heavy neutrinos as well the mass of the heavy neutrino, N_i .
- Light neutrino contribution to NDBD in which the intermediate particles are W_R^- bosons. The amplitude of this process depends upon the mixing between light and heavy neutrinos as well as the mass of the right handed gauge boson, W_R^- boson.
- Heavy right handed neutrino contribution to NDBD in which the mediator particles are the W_R^- bosons. The amplitude of this process depends upon the elements of the right handed leptonic mixing matrix and the mass of the right handed gauge boson, W_R^- boson as well as the mass of the heavy right handed Majorana neutrino, N_i .
- Light neutrino contribution from the Feynman diagram mediated by both W_L^- and W_R^- . The amplitude of this process depends upon the mixing between light and heavy neutrinos, leptonic mixing matrix elements, light neutrino masses and the mass of the gauge bosons, W_L^- and W_R^- .
- Heavy neutrino contribution from the Feynman diagram mediated by both W_L^- and W_R^- . The amplitude of the process depends upon the right handed leptonic mixing matrix elements, mixing between the light and heavy neutrinos as well as the mass of the gauge bosons, W_L^- and W_R^- and the mass of the heavy right handed neutrino, M_i .
- Triplet Higgs Δ_L contribution to NDBD in which the mediator particles are W_L^- bosons. The amplitudes for the process depends upon the masses of the W_L^- bosons, left handed triplet Higgs, Δ_L as well as their coupling to leptons, f_L .
- Right handed triplet Higgs Δ_R contribution to NDBD in which the mediator particles are W_R^- bosons. The amplitude for the process depends upon the masses of the W_R^- bosons, right handed triplet Higgs, Δ_R as well as their coupling to leptons, f_R .

However in our present work, we have considered only three of the above mentioned contributions to NDBD. One from the standard light neutrino contribution through exchange of W_L^- as shown in Fig. 1a and the other two are the new physics contributions to NDBD which corresponds to Fig. 1b and c, that is the ones mediated by W_R^- and Δ_R respectively. The amplitudes of the contributions are given in several earlier works like [76]. For simple approximations, an assumption of similar mass scales for the heavy particles has been made in the LRSM, where, $M_R \approx M_{W_R} \approx M_{\Delta_{L++}} \approx M_{\Delta_{R++}} \approx \text{TeV}$, at a scale accessible at the LHC. Under these assumptions, the amplitude for the light-heavy mixing contribution

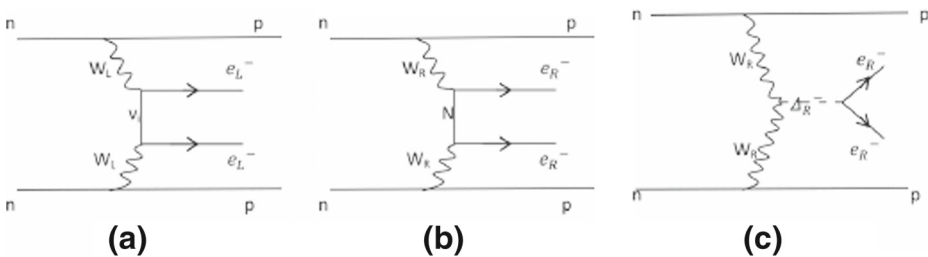


Fig. 1 Standard light neutrino contribution and new physics contribution (from heavy RH neutrino and scalar Higgs triplet) to NDBD in LRSM

which is proportional to $\frac{m_D^2}{M_R}$ remains very small (since $m_\nu \approx \frac{m_D^2}{M_R} \approx (0.01 - 0.1)\text{eV}$, $m_D \approx (10^5 - 10^6)\text{eV}$ which implies $\frac{m_D}{M_R} \approx (10^{-7} - 10^{-6})\text{eV}$). Thus, we ignore the contributions involving the light and heavy neutrino mixings. For a simplified approach, we have also ignored the mixing between W_L^- and W_R^- bosons owing to the above mentioned assumptions, which would cause a further suppression in the amplitude of the process (for reference see [77]). Again, the contribution from Δ_L^-, W_L^- is suppressed by the type II seesaw contribution to light neutrino mass and hence neglected here.

Considering these contributions we have studied the NDBD. Different neutrino mass satisfying the mixing criteria namely, TBM, BM, HM and GRM are considered as leading contribution in either type I or type II seesaw. The perturbation is added for generation of non zero θ_{13} [84] in either of the seesaw terms.

The amplitude of the corresponding processes which we have considered in our work are given by,

- Standard light neutrino contribution,

$$A_\nu^{LL} \cong \frac{1}{M_{W_L}^4} \sum \frac{U_{L_{e_i}}^2 m_i}{p^2}. \tag{23}$$

where, $|p| \sim 100\text{ MeV}$ [85] is the typical momentum transfer at the leptonic vertex, $m_{ee} = \sum U_{L_{e_i}}^2 m_i$ is the effective neutrino mass. $U_{L_{e_i}}$ represents the elements of the first row of the neutrino mixing matrix, U_{PMNS} .

- Heavy RH neutrino contribution,

$$A_N^{RR} \propto \frac{1}{M_{W_L}^4} \frac{U_{R_{e_i}}^{*2}}{M_i}. \tag{24}$$

- Scalar triplet contribution,

$$A_{\Delta_R}^{RR} \propto \frac{1}{M_{W_R}^4} \frac{1}{M_{\Delta_R}^2} f_{R\nu R}. \tag{25}$$

Here, $U_{R_{e_i}}^*$ denotes the first row of the unitary matrix diagonalizing the right handed neutrino mass matrix, M_{RR} with mass eigen values, M_i .

4 Lepton Flavour Violation (LFV)

There have been various attempts to observe and predict theoretically the manifestation of LFV involving various modes of muon decay since long. The most promising LFV low energy channels are probably $\mu \rightarrow e\gamma, \mu \rightarrow 3e, \mu \rightarrow e$ conversion in nuclei which occur in rates accessible in recent experiments. Defining the decay rates (from reference [60, 61]) as,

$$\Gamma_\mu \equiv \Gamma(\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e), \Gamma_{\text{capt}}^Z \equiv \Gamma(\mu^- + A(Z, N) \rightarrow \nu_\mu + A(Z - 1, N + 1)). \tag{26}$$

The relevant branching ratios (BR) for the processes are,

$$\text{BR}_{\mu \rightarrow e\gamma} \equiv \frac{\Gamma(\mu^+ \rightarrow e^+ \gamma)}{\Gamma_\mu}, \tag{27}$$

$$\text{BR}_{\mu \rightarrow e}^Z \equiv \frac{\Gamma(\mu^- + A(N, Z) \rightarrow e^- + A(N, Z))}{\Gamma_{\text{capt}}^Z}, \tag{28}$$

$$BR_{\mu \rightarrow 3e} \equiv \frac{\Gamma(\mu^+ \rightarrow e^+e^-e^+)}{\Gamma_\nu} \tag{29}$$

The selected limits for lepton flavour violating muon decays and muon to electron conversion experiments are shown in Table 3.

In the SM seesaw, the LFV decay rates induced by neutrino mixing are suppressed by tiny neutrino masses, $\left(\frac{\Delta m_\nu^2}{M_W^2}\right) \sim 10^{-50}$ and hence are well below the current experimental limits and even the distant future sensitivities. New physics beyond the standard model is required to make the process observable, there are several theoretical frameworks BSM that could provide the necessary operators. One of those theories is the LRSM in which several new contributions appear due to the additional RH current interactions, which could lead to sizeable LFV rates for TeV scale ν_R that occur at rates observable in current experiments. LFV in the LRSM has been studied in many previous works. There are various LFV processes providing constraints on the masses of the right handed neutrinos and doubly charged scalars. It turns out that the process $\mu \rightarrow 3e$ induced by doubly charged bosons Δ_L^{++} and Δ_R^{++} and $\mu \rightarrow e\gamma$ provides the most relevant constraint. In our present work, we consider these processes in minimal left-right symmetric model (MLRSM). The limit of branching ratio of the process $\mu \rightarrow 3e$ as shown in Table 3 is $< 1.0 \times 10^{-12}$ at 90% CL was obtained at the Paul Scherrer institute (PSI) over 20 years ago by the SINDRUM experiment [87]. Presently the Mu3e collaboration has submitted a letter of intent to PSI to perform a new improved search for the decay $\mu \rightarrow 3e$ with a sensitivity of 10^{-16} at 95% CL [88, 89] which corresponds to an improvement by four orders of magnitude compared to the former SINDRUM experiment. Whereas the new upper limit for BR of the process $\mu \rightarrow e\gamma$ is established to be $< 4.2 \times 10^{-13}$ at 90% CL by the MEG collaboration. Taking into account the contributions from heavy righthanded neutrinos and Higgs scalars, the expected branching ratios and conversion rates of the above processes have been calculated in the LRSM in the work (first reference in [90, 91]).

The BR for the process $(\mu \rightarrow 3e)$ is given by,

$$BR(\mu \rightarrow 3e) = \frac{1}{2} |h_{\mu e} h_{ee}^*|^2 \left(\frac{m_{W_L}^4}{M_{\Delta_L^{++4}}^4} + \frac{m_{W_R}^4}{M_{\Delta_R^{++4}}^4} \right) \tag{30}$$

Where h_{ij} describes the lepton Higgs coupling in LRSM and is given by,

$$h_{ij} = \sum_{n=1}^3 V_{in} V_{jn} \left(\frac{M_n}{M_{W_R}} \right), \quad i, j = e, \mu, \tau. \tag{31}$$

For $\mu \rightarrow e\gamma$, the relevant BR is given by, [60, 61]

$$BR(\mu \rightarrow e\gamma) = 1.5 \times 10^{-7} |g_{lfv}|^2 \left(\frac{1\text{TeV}}{M_{W_R}} \right)^4, \tag{32}$$

Table 3 Experimental limits on LFV muon decays

Decay Channel	Experiment	Branching Ratio Limit
$\mu \rightarrow e\gamma$	MEG	$< 4.2 \times 10^{-13}$ [86]
$\mu \rightarrow eee$	SINDRUM	$< 1.0 \times 10^{-12}$ [87]
$\mu Au \rightarrow e Au$	SINDRUM II	$< 7 \times 10^{-13}$ [88, 89]

where, g_{lfv} is defined as,

$$g_{lfv} = \sum_{n=1}^3 V_{\mu n} V_{en}^* \left(\frac{M_n}{M_{W_R}} \right)^2 = \frac{[M_R M_R^*]_{\mu e}}{M_{W_R}^2} \tag{33}$$

The sum is over the heavy neutrinos only. $M_{\Delta_{L,R}}^{++}$ are the masses of the doubly charged bosons, $\Delta_{L,R}^{++}$, V is the mixing matrix of the right handed neutrinos with the electrons and muons. M_n ($n = 1, 2, 3$) are the right handed neutrino masses.

5 Numerical Analysis and Results

In our present work we have studied LNV (NDBD) for standard as well as non standard contributions for the effective mass as well as the half life governing the decay process in the framework of LRSM. We have also correlated the LFV of the process, $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ with the lightest neutrino mass and atmospheric mixing angle, θ_{23} respectively for both normal and inverted mass hierarchies. In this section we present a detailed analysis of our work and we have divided it into different subsections, firstly the standard light neutrino contribution to NDBD and then the new physics contribution to NDBD considering perturbation in type II and then type I seesaw. Lastly we have shown the analysis of correlating LFV with m_{lightest} and θ_{23} .

5.1 Standard Light Neutrino Contribution

For NDBD mediated by the light Majorana neutrinos, the half life of the decay process is given by (1) and the effective mass governing the process is as given in (5). In our present work, we first evaluated the effective light neutrino mass within the standard mechanism using the formula (3) where, U_{ej} are the elements of the first row of the neutrino mixing matrix, U_{PMNS} (dependent on the known parameters θ_{13}, θ_{12} and the unknown Majorana phases α and β). U_{PMNS} is the diagonalizing matrix of the light neutrino mass matrix, m_ν , such that

$$m_\nu = U_{PMNS} M_\nu^{(\text{diag})} U_{PMNS}^T, \tag{34}$$

where $M_\nu^{(\text{diag})} = \text{diag}(m_1, m_2, m_3)$. In the case of 3 neutrino mixing, 2 ν mass spectra are possible,

- Normal Hierachy (NH) which corresponds to $m_1 < m_2 \ll m_3 ; \Delta m_{12}^2 \ll \Delta m_{23}^2$.
- Inverted Hierachy (IH) which corresponds to $m_3 \ll m_1 \sim m_2 ; \Delta m_{12}^2 \ll |\Delta m_{13}^2|$.

In both the spectra, $\Delta m_{12}^2 = \Delta m_{\text{solar}}^2$. For NH, $\Delta m_{23}^2 = \Delta m_{\text{atm}}^2$ and for IH, $|\Delta m_{13}^2| = \Delta m_{\text{atm}}^2$. In the case of NH, the neutrino masses m_2 and m_3 are connected with the lightest mass m_1 by the relation,

$$m_2 = \sqrt{m_1^2 + \Delta m_{\text{sol}}^2}, m_3 = \sqrt{m_1^2 + \Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2}. \tag{35}$$

In IH, m_3 is the lightest mass and we have,

$$m_1 = \sqrt{m_3^2 + \Delta m_{\text{atm}}^2}, m_2 = \sqrt{m_3^2 + \Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2}. \tag{36}$$

For both the normal and inverted hierarchies, (5) can be written in terms of lightest neutrino mass as,

for NH,

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + \sqrt{(m_1^2 + \Delta m_{\text{sol}}^2 s_{12}^2 c_{13}^2 e^{2i\alpha})} + \sqrt{(m_1^2 + \Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2 s_{13}^2 e^{2i\beta})}, \tag{37}$$

for IH,

$$m_{\beta\beta} = \sqrt{(m_3^2 + \Delta m_{\text{atm}}^2 c_{12}^2 c_{13}^2)} + \sqrt{(m_3^2 + \Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta})}. \tag{38}$$

The 3σ ranges of the mass squared differences and mixing angles from global analysis of oscillation data are outlined as in the Table 1. Using the best fit values of the mass squared differences and the 3σ ranges of the three mixing angles from a global analysis of oscillation data (as shown in Table 1), we have shown the variation of the effective Majorana mass as a function of the lightest neutrino mass m_1 (for NH) and m_3 (for IH). During our calculation, we have varied the Majorana phase α and β from 0 to 2π . The effective mass assumes different values depending on whether the neutrino mass states follows normal hierarchy (NH) or inverted hierarchy (IH). We have used (37) and (38) in evaluating the effective mass in terms of the lightest neutrino mass. The variation is shown in Fig. 2. It is seen from the figure that the light neutrino contribution to neutrinoless double beta decay ($0\nu\beta\beta$) can saturate the bound imposed by KamLAND-ZEN ($\leq 0.061 - 0.165\text{eV}$) (reference [35]) only for the higher values of lightest neutrino masses which is disallowed by the Planck data (lightest mass for NH ~ 0.07 and lightest mass for IH ~ 0.065).

Again, we have evaluated the effective majorana mass for different leptonic mixing patterns possessing $\mu - \tau$ symmetry, namely, tribimaximal, golden ratio and hexagonal mixing [37–56] using (37) and (38). In all the different $\mu - \tau$ symmetric mixing patterns which we have considered, i.e., TBM, HM, GRM, the reactor mixing angle θ_{13} is 0 and θ_{23} is 45° . Whereas $\theta_{12} = 35.5^\circ$ (for TBM), $\theta_{12} = 30^\circ$ (for HM), $\theta_{12} = 31.71^\circ$ (for GRM). Since, $\theta_{12} = 45^\circ$, i.e, BM has been ruled out by experiments, we have ignored this case for the standard contributions. Again, it is to be noted that there are two values of θ_{12} for GRM, which are, 31.7° and 35.96° [92]. In our present study, we have considered the first value which is allowed as mentioned in reference [92–94].

The variations of m_{ν}^{eff} for the different mixing patterns for NH and IH in terms of lightest neutrino mass are shown as in Fig. 3.

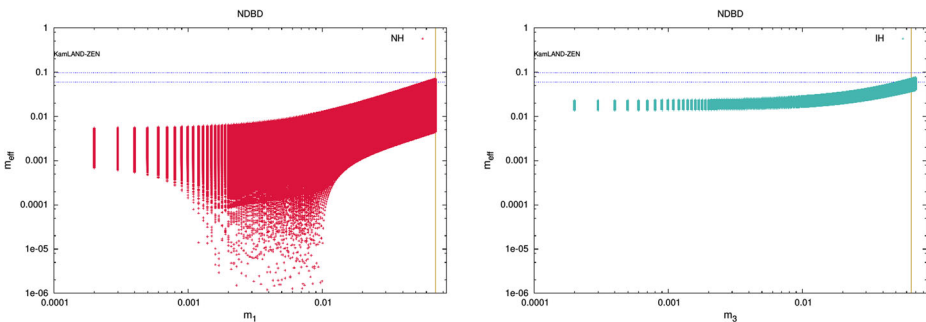


Fig. 2 Effective Majorana mass for $0\nu\beta\beta$ as a function of lightest neutrino mass, m_1 (in eV) for NH (as shown in figure left) and m_3 (in eV) for IH (as shown in figure right) within the standard mechanism. The blue dashed line and the yellow solid line represents the KamLAND-Zen bound on the effective mass and the Planck bound on the sum of the absolute neutrino mass respectively

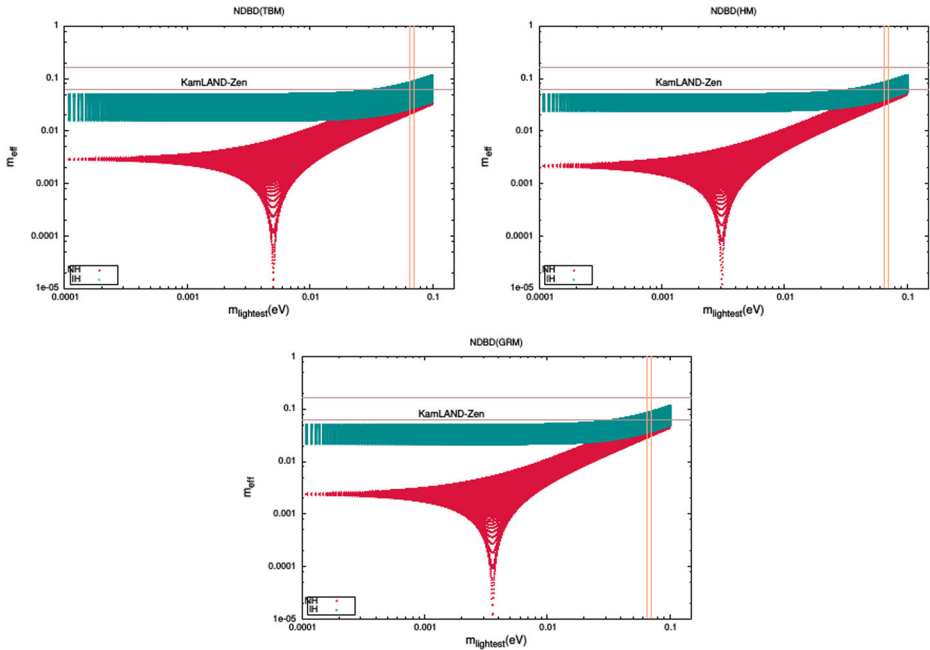


Fig. 3 Standard light neutrino contribution to effective mass for $0\nu\beta\beta$ for different neutrino mass models (TBM, HM and GRM) as a function of lightest neutrino mass (in eV) for NH/IH (m_1/m_3). The horizontal lines represents the upper limit of effective mass propounded by kamLAND-Zen and vertical line represents the plancks bound on lightest neutrino mass for NH and IH

5.2 New Physics Contribution to NDBD Considering Perturbation in Type II Seesaw

For the new Physics contribution, we have considered the contributions of $0\nu\beta\beta$ from the right handed current and from the triplet Higgs (Δ_R). The contributions from the left handed Higgs triplet, Δ_L is suppressed by the light neutrino mass. Also we consider the mixing between LH and RH sector to be so small that their contributions to $0\nu\beta\beta$ can be neglected. The total effective mass is thus given by the formula, (as in [77])

$$m_{N+\Delta_R}^{\text{eff}} = p^2 \frac{M_{W_L}^4}{M_{W_R}^4} \frac{U_{\text{Rei}}^* 2}{M_i} + p^2 \frac{M_{W_L}^4}{M_{W_R}^4} \frac{U_{\text{Rei}}^2 M_i}{M_{\Delta_R}^2}. \tag{39}$$

Here, $\langle p^2 \rangle = m_e m_p \frac{M_N}{M_N}$ is the typical momentum exchange of the process, where m_p and m_e are the mass of the proton and electron respectively and M_N is the NME corresponding to the RH neutrino exchange. We know that TeV scale LRSM plays an important role in $0\nu\beta\beta$ decay. We have considered the values $M_{W_R} = 3.5 \text{ TeV}$, $M_{W_L} = 80 \text{ GeV}$, $M_{\Delta_R} \approx 3 \text{ TeV}$, the heavy RH neutrino $\approx \text{TeV}$ which are within the recent collider limits [95]. The allowed value of p (the virtuality of the exchanged neutrino) is in the range $\sim (100\text{-}200) \text{ MeV}$. In our analysis, we have taken $p \approx 180 \text{ MeV}$ [76].

Thus,

$$p^2 \frac{M_{W_L}^4}{M_{W_R}^4} \simeq 10^{10} \text{ eV}^2. \tag{40}$$

However, (39) is valid only in the limit $M_1^2 \gg |\langle p^2 \rangle|$ and $M_\Delta^2 \gg |\langle p^2 \rangle|$.

The formula for light ν masses in the presence of both type I and type II seesaw can be written as,

$$M_\nu = M_\nu^I + M_\nu^{II}, \tag{41}$$

$$U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T = M_\nu^{II} + U_{(\mu-\tau)} U_{Maj} M_\nu^{I(diag)} U_{Maj}^T U_{(\mu-\tau)}^T, \tag{42}$$

where, U_{PMNS} and $U_{(\mu-\tau)}$ represents the diagonalizing matrix of M_ν and M_ν^I . The Majorana phases have been taken in the type I seesaw term [77]. From (42) we can evaluate M_ν^{II} . We have considered the case when M_ν^I possess $\mu-\tau$ symmetry, with the various choices for mixing matrices such as TBM, BM, HM, GRM, with uniquely predicting $\theta_{13} = 0$. We have considered $M_\nu^{I(diag)} = X M_\nu^{(diag)}$, where we have introduced the parameter X to describe the relative strength of the type I and II seesaw terms. The parameter X can take any numerical value provided the two seesaw terms gives rise to correct light neutrino mass matrix. In our case, we have considered $X = 0.5$, i.e., equal contributions from both the seesaw terms. The required correction to $\mu-\tau$ type ν mass matrix for generation of non zero reactor mixing angle (θ_{13}) can be obtained from the perturbation matrix, M_ν^{II} mass matrix. M_ν^{II} can be constructed as,

$$M_\nu^{II} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}. \tag{43}$$

It can be derived using (41). The type II seesaw mass matrix is evaluated in terms of light neutrino mass matrix, constructed using the best fit neutrino data and $\mu-\tau$ symmetric type I mass matrices (TBM, BM, HM, GRM). The elements are shown in Appendix.

To evaluate $m_{N+\Delta_R}^{eff}$, we need the diagonalizing matrix of the heavy right handed Majorana mass matrix M_{RR} , U_{Rei} and its mass eigenvalues, M_i .

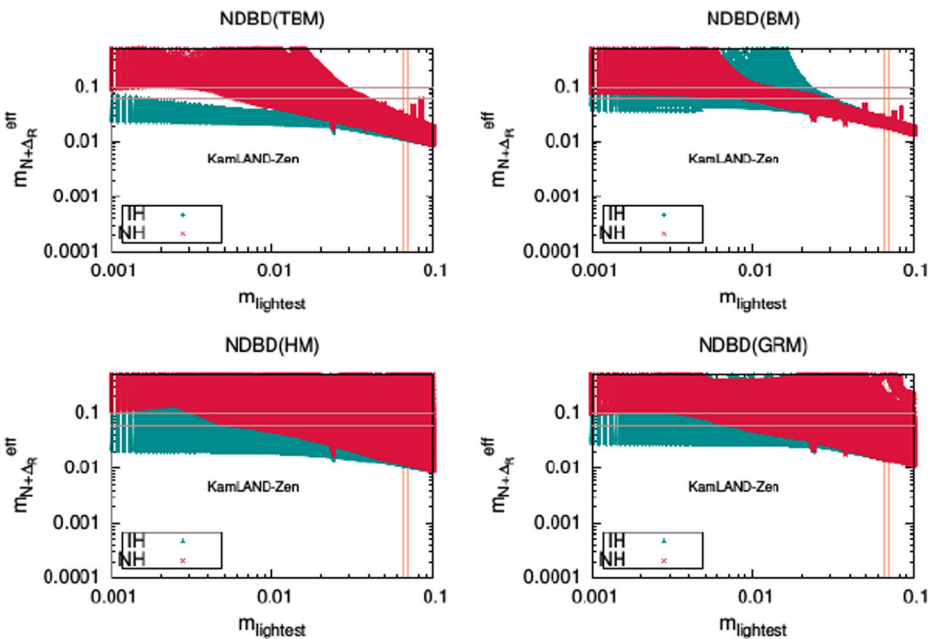


Fig. 4 New Physics contribution to effective mass for $0\nu\beta\beta$ considering perturbation in type II seesaw for different mass models (TBM, BM, HM and GRM)

M_{RR} can be written in the form(from reference [96]) and is evident from (21)

$$M_{RR} = \frac{1}{\gamma} \left(\frac{v_R}{M_{WL}} \right)^2 M_\nu^{II}, \tag{44}$$

$$M_\nu^{II} = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T - U_{(\mu-\tau)} U_{Maj} M_\nu^{I(diag)} U_{Maj}^T U_{(\mu-\tau)}^T. \tag{45}$$

In the above equation, $U_{(\mu-\tau)}$ represents U_{TBM} , U_{BM} , U_{HM} , U_{GRM} [37–56], i.e, the diagonalizing matrices of the TBM, BM, HM and GRM mass matrices. For TeV scale type I + type II seesaw, we have fine tuned the dimensionless parameter, $\gamma \sim 10^{-10}$, we have considered $v_R \sim TeV$. Thus after obtaining M_{RR} , we diagonalized it and obtained the eigenvalues, M_i and its diagonalizing matrix in terms of the lightest neutrino mass (m_1 or m_3) for (NH/IH) and the Majorana phases (α and β). We have varied the Majorana phases α and β from 0 to 2π and evaluated the effective mass for new physics contribution using formula (39) in terms of lightest neutrino mass. This is shown in Fig. 4. We have imposed the KamLAND-Zen bound on the new physics contribution to effective mass and the Planck bound on the sum of the absolute neutrino mass.

5.3 New Physics Contribution to NDBD Considering Perturbation in Type I Seesaw

Alternatively, we have again considered the type II seesaw to give rise to $\mu - \tau$ type neutrino mass matrix and the necessary correction to obtain non-zero θ_{13} is obtained from the type I seesaw term. Thus, M_ν^{II} in (44) can be written as,

$$M_\nu^{II} = U_{(\mu-\tau)} U_{Maj} M_\nu^{II(diag)} U_{Maj}^T U_{(\mu-\tau)}^T, \tag{46}$$

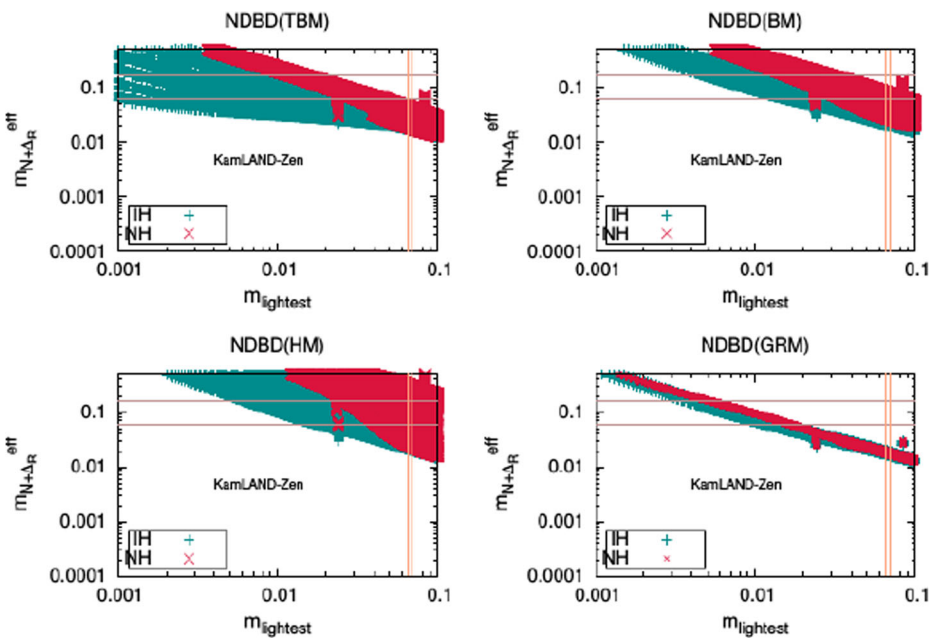


Fig. 5 New Physics contribution to effective mass for $0\nu\beta\beta$ considering perturbation in type I seesaw for different mass models (TBM, BM, HM and GRM)

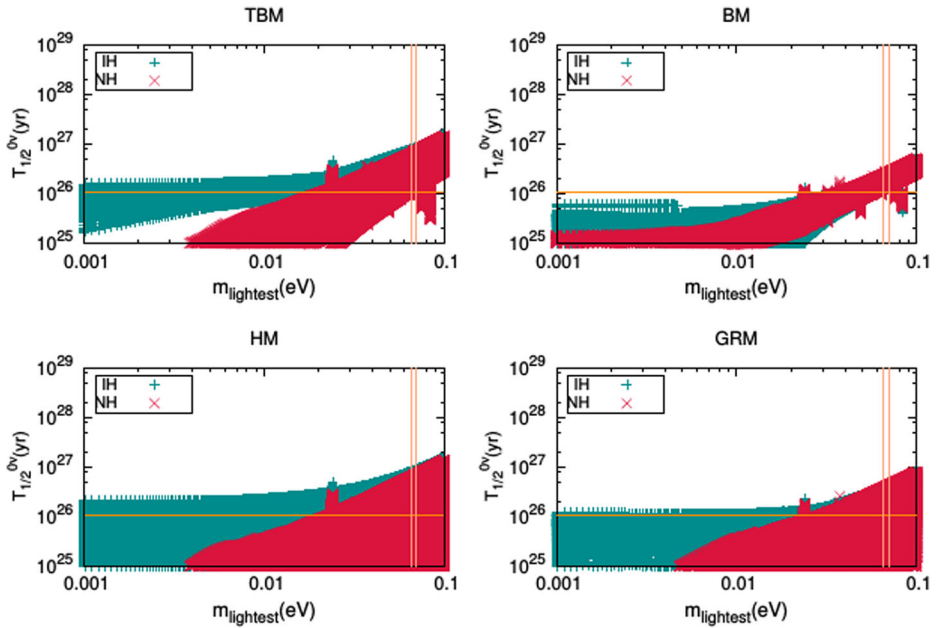


Fig. 6 New Physics contributions to half life of $0\nu\beta\beta$ considering perturbation in type II seesaw in different mass models (TBM, BM, GRM, HM) for normal and inverted hierarchies. The horizontal line represents the lower limit on $0\nu\beta\beta$ half life imposed by KamLAND-ZEN projected sensitivity respectively

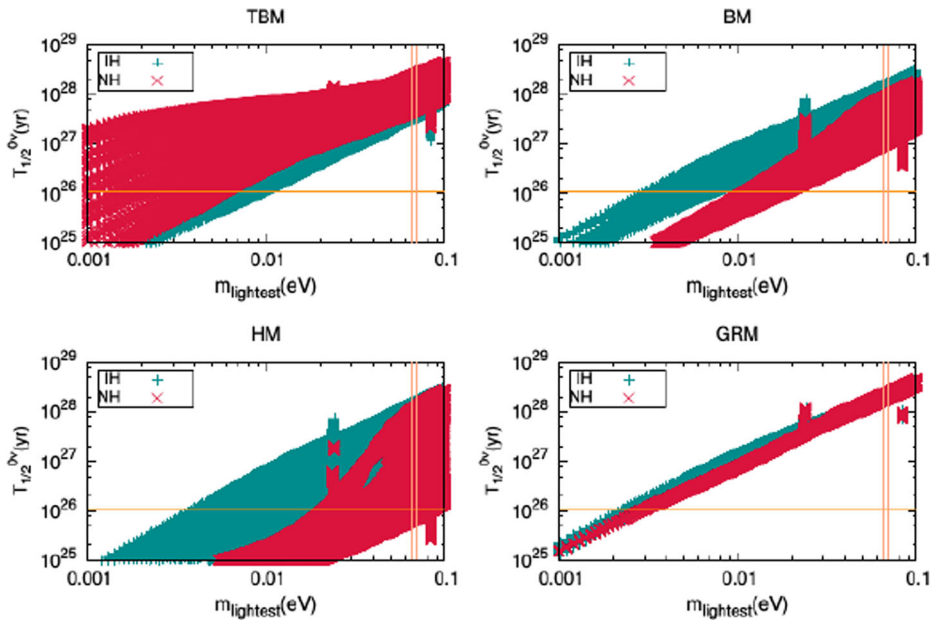


Fig. 7 New Physics contributions to half life of $0\nu\beta\beta$ considering perturbation in type I seesaw in different mass models (TBM, BM, GRM, HM) for normal and inverted hierarchies. The horizontal line represents the lower limit on $0\nu\beta\beta$ half life imposed by KamLAND-ZEN projected sensitivity respectively

where, $U_{(\mu-\tau)}$ represents U_{TBM} , U_{BM} , U_{HM} , U_{GRM} .

$$M_\nu^I = M_\nu - M_\nu^{II}, \tag{47}$$

$$M_\nu^I = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T - M_\nu^{II}. \tag{48}$$

Like in the previous case, we have again evaluated the right handed Majorana mass matrix using (44). We have fine tuned the dimensionless parameter γ and then by diagonalizing the right handed Majorana mass matrix M_{RR} , we have obtained U_{Rei} and the eigenvalues, M_i (i.e. $M_{RR}^{(diag)}$) where,

$$M_{RR} = U_{Rei} M_{RR}^{(diag)} U_{Rei}^T \tag{49}$$

We then evaluated the effective Majorana mass, $m_{N+\Delta R}^{eff}$ using (39) as a function of the lightest left handed neutrino mass. This is shown in Fig. 5. When we consider the type II seesaw term to be $\mu - \tau$ symmetric and the perturbation from the type I seesaw term, the type I seesaw mass matrix can be derived as in the previous case and is shown in Appendix.

For the new physics contribution in which the type II term acts as the perturbation, we have also evaluated the half life of the $0\nu\beta\beta$ decay process using (2), where

$$|m_\nu^{eff}|^2 = |m_N^{eff} + m_{\Delta R}^{eff}|^2. \tag{50}$$

By substituting the values of the phase factors (G_0^ν) [97, 98], nuclear matrix element(NME) [98, 99] and mass of electron, we have obtained the half life as a function of the lightest mass in the different mixing patterns for both NH and IH, as shown in Fig. 6. In the similar process, we have also computed the half life for new physics contribution to NDBD in which the type I term acts as the perturbation, for generation of non zero θ_{13} . It is shown in Fig. 7.

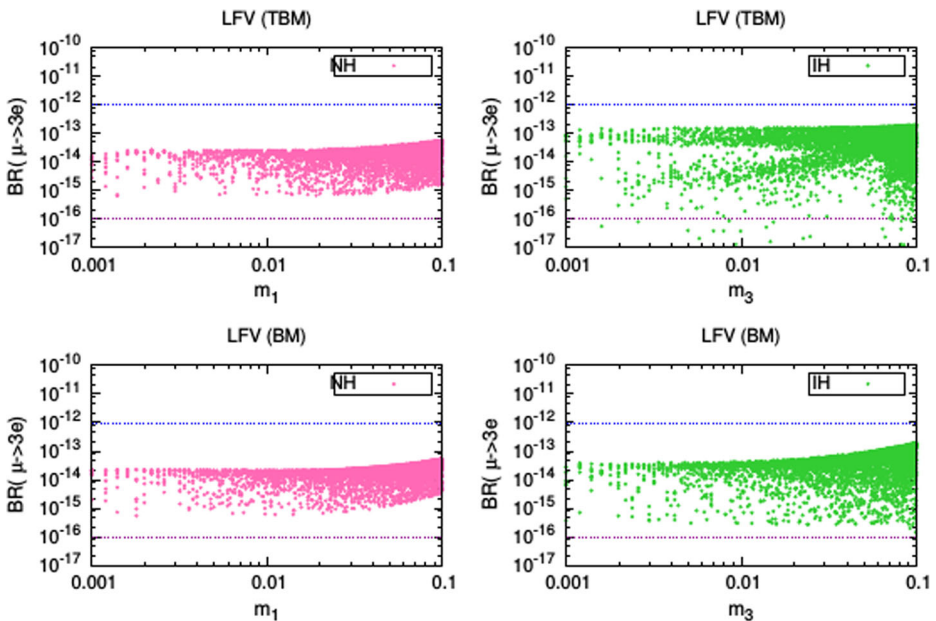


Fig. 8 Total contribution to lepton flavour violation as a function of the lightest neutrino mass for the TBM and BM neutrino mass models for normal and inverted hierarchies. The blue and violet dashed line shows the limit of BR as given by SINDRUM experiment and the recently proposed limit of $\mu 3e$ experiment respectively

5.4 Correlating LFV with Lightest Neutrino Mass and θ_{23}

To correlate LFV with neutrino mass in our analysis, we have considered the LFV processes, $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$. The BR for both the processes have a strong flavour dependence on the RH mixing matrix. Since the process $\mu \rightarrow 3e$ is controlled by $h_{\mu e} h_{ee}^*$ whereas $\mu \rightarrow e\gamma$ is controlled by the factor $[M_R M_R^*]_{\mu e}$, the later is independent of the Majorana CP phases and the lightest neutrino mass, m_j . We have correlated the BR of the process $\mu \rightarrow 3e$ with the lightest neutrino mass (m_1/m_3) for (NH/IH). The BR of the process $\mu \rightarrow e\gamma$ is correlated with the atmospheric mixing angle, θ_{23} , since the other two mixing angles θ_{12} and θ_{13} are measured precisely. For calculating the BR, we used the expression given in (30) and (32). The lepton Higgs coupling h_{ij} in (31) can be computed explicitly for a given RH neutrino mass matrix as shown in (44) by diagonalizing the RH neutrino mass matrix and obtaining the mixing matrix element, V_i and the eigenvalues M_i . For evaluating M_{RR} , we need to know M_ν^{II} , as evident from (44). We computed M_ν^{II} from (42). For determining the BR for $\mu \rightarrow 3e$, we imposed the best fit values of the parameters, Δm_{sol}^2 , Δm_{atm}^2 , δ , θ_{13} , θ_{23} , θ_{12} in M_ν . The numerical values of M_ν^I can be computed as before for different mixing patterns, TBM, BM, HM, GRM. Thus, we get M_ν^{II} as a function of the parameters α , β and $m_{lightest}$. Then varying both the Majorana phases, α, β from 0 to 2π , we obtained M_ν^{II} as a function of $m_{lightest}$. Similarly, for $\mu \rightarrow e\gamma$ we substituted the values of the lightest mass (m_1/m_3) for (NH/IH) as (0.07eV/0.065eV) and best fit values for the parameters Δm_{sol}^2 , Δm_{atm}^2 , δ , θ_{13} , while varying both the Majorana phases, α, β from 0 to 2π and thus obtained M_ν^{II} and hence M_{RR} as a function of the atmospheric mixing angle θ_{23} . Thus BR can be obtained as a function of $\sin^2 \theta_{23}$ from (32). We have varied the value

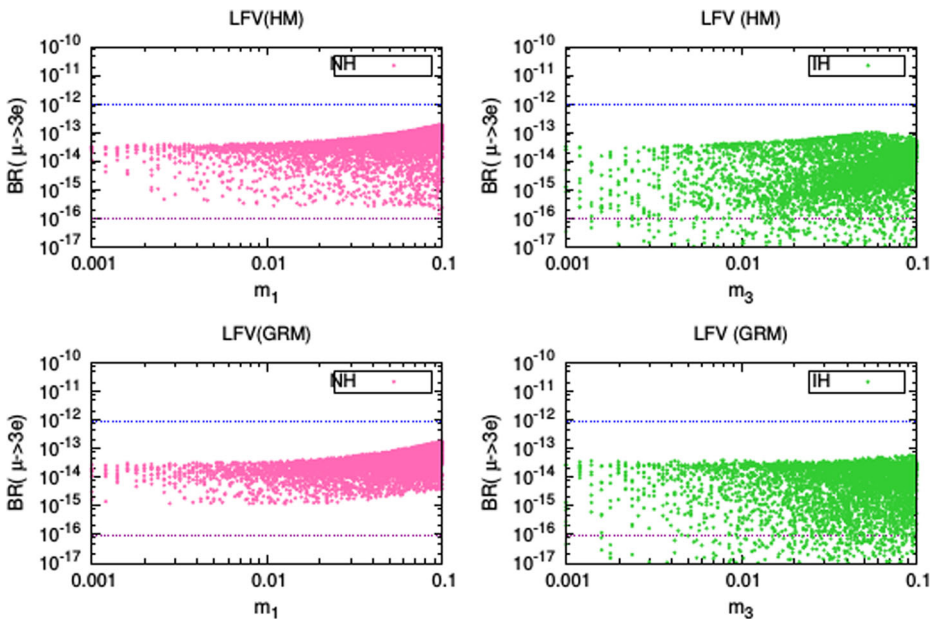


Fig. 9 Total contribution to lepton flavour violation with type (I+II) seesaw shown as a function of the lightest neutrino mass for the HM and GRM neutrino mass models for normal and inverted hierarchies. The blue and violet dashed line shows the limit of BR as given by SINDRUM experiment and the recently proposed limit of $\mu 3e$ experiment respectively

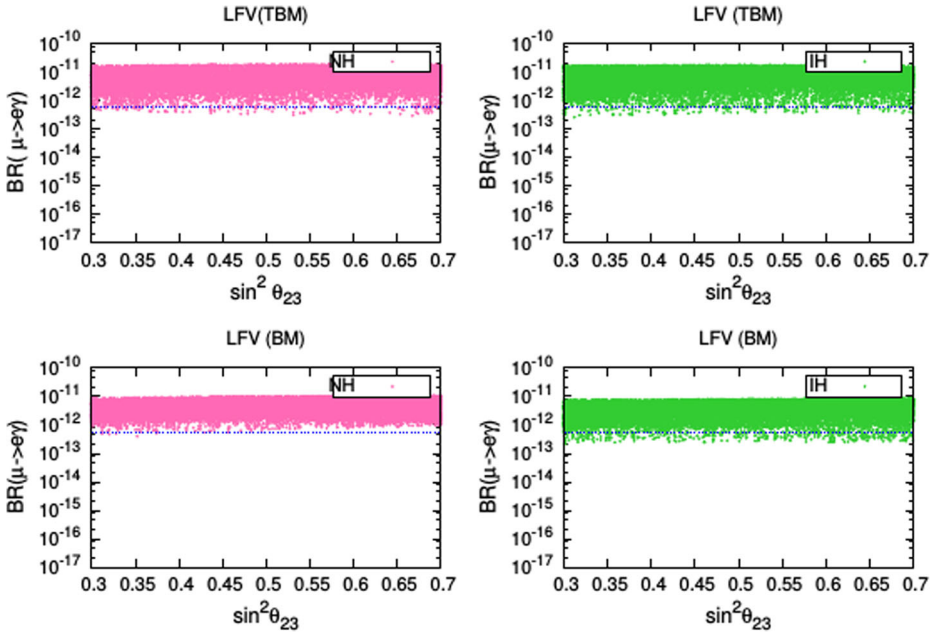


Fig. 10 Total contribution to lepton flavour violation with type (I+II) seesaw shown as a function of the atmospheric mixing angle θ_{23} for TBM and BM neutrino mass models for normal and inverted hierarchies. The blue dashed line shows the limit of BR

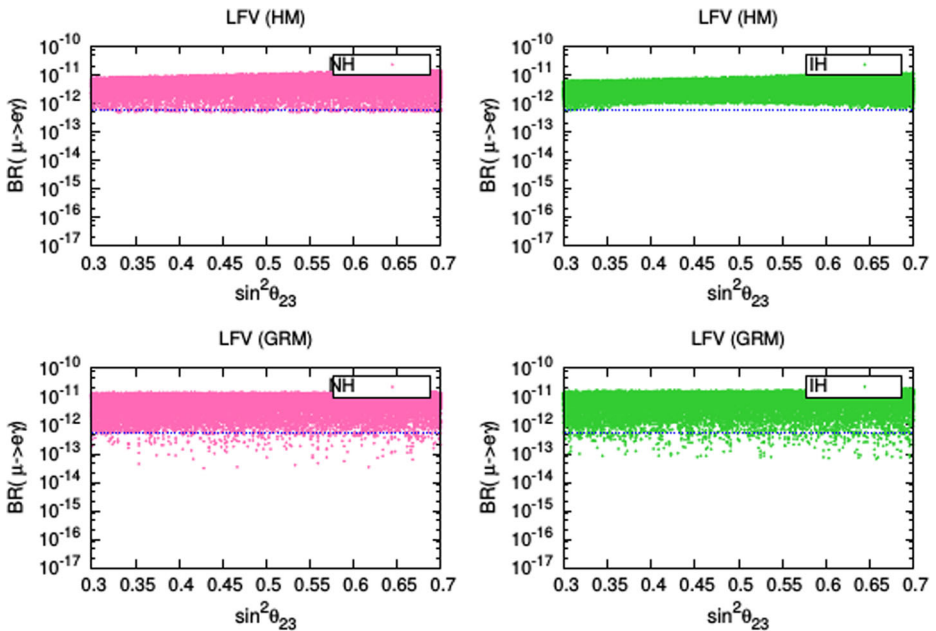


Fig. 11 Total contribution to lepton flavour violation μ with type (I+II) seesaw shown as a function of the atmospheric mixing angle θ_{23} for the HM and GRM neutrino mass models for normal and inverted hierarchies. The blue dashed line shows the limit of BR

of $\sin^2 \theta_{23}$ in its 3σ range as in Table 1 and the lightest neutrino mass from 10^{-3} to 10^{-1} and obtained the values of BR for different mixing patterns, TBM, BM, HM, GRM. The variation is shown in Figs. 8, 9, 10 and 11 for both NH and IH.

6 Conclusion

The quest for NDBD and its interrelation with neutrino mass makes it a very interesting and enthralling topic of research at present time. Its existence would not only confirm the intrinsic nature of the neutrinos but also would provide a stringent limit on the absolute scale of the neutrino mass. In this paper, we contemplated the implications of NDBD and LFV in LRSM framework. Owing to the presence of new scalars and gauge bosons in this model, various additional sources would give rise to contributions to NDBD process, which involves RH neutrinos, RH gauge bosons, scalar Higgs triplets as well as the mixed LH-RH contributions. For a simplified analysis we have ignored the left-right gauge boson mixing and heavy light neutrino mixing. We have considered the extra gauge bosons and scalars to be of the order of TeV. Again the existence of non zero θ_{13} has many implications in neutrino sector beyond SM. A simple way to accommodate non zero θ_{13} is by adding a perturbation matrix to the neutrino mass matrix. A well known neutrino mass mixing pattern is the one obeying $\mu - \tau$ symmetry. In our present analysis, we have considered the different realizations of the $\mu - \tau$ symmetric mass matrices, namely, TBM, BM, HM, GRM matrices. The perturbation to these matrices to generate non zero θ_{13} is obtained from either of the seesaw terms, type I and type II. We have considered two different approaches, type I giving $\mu - \tau$ symmetry and type II as perturbation, type II giving $\mu - \tau$ symmetry and type I as perturbation, for generation of non zero θ_{13} . We analysed the standard as well as new physics contribution to the effective mass m_{eff} governing NDBD as well as the half life considering both type I and type II seesaw. We have shown the variations of the effective mass as well as the half life with the lightest neutrino mass which corresponds to the standard as well as the non standard contributions. We have seen from our analysis that both the approaches yields different consequences in NDBD. The various parameters we have chosen for our numerical analysis are consistent with constraints from ν oscillation experiments. We have also discussed the impacts of the lightest neutrino mass and not so precisely known atmospheric mixing angle, θ_{23} on the behaviour of LFV of the decay process, $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ respectively. Based on our observations, the following conclusions could be arrived at,

- In the standard light neutrino contribution to NDBD, it is observed that all the mass patterns (TBM, HM, GRM) yields almost similar results for NH mass spectrum. The effective mass governing NDBD is found to be of the order of 10^{-3} eV and are within and much below the current experimental limit [35]. Whereas in case of IH mass spectrum, for TBM, HM and GRM, the values of effective mass are found to be within and close to the experimental limit and are of the order of 10^{-2} eV. However, in all the cases, the light neutrino contribution can saturate the experimental limit for lightest neutrino mass (m_1/m_3) for (NH/IH) of around 0.1 eV.
- In new physics contribution considering perturbation in type II seesaw, for IH, TBM, HM and GRM shows results within the recent experimental bound for lightest mass varying from (0.001-0.1) eV. Whereas, for NH the effective mass lies within experimental limit for lightest mass in the range (0.01-0.1) eV. In case of half life also, except BM mass pattern, TBM, HM and GRM schemes shows better results. In all the cases, both NH and IH seems to be more compatible with the experimental results.

- In new physics contribution considering perturbation in type I seesaw, the values that are consistent with experimental bound imposed by KamLAND-Zen are found for lightest mass (0.001-0.1) eV for TBM and about (0.01-0.1 eV) for all other cases. Whereas for half life, TBM shows better results. In all other mixing patterns, half life lies within experimental bound for values of lightest mass lying from (0.005-0.1) eV for IH.
- It is observed from our analysis that the BR for the process $\mu \rightarrow 3e$ in the LRSM remains consistent with the experimental bound for a wide range of light neutrino mass. However, it depends on the neutrino mass spectrum as evident from Figs. 8 and 9. In case of IH, the BR is spread over a wide range and lies even in the range of the recently proposed limit with a sensitivity of 10^{-16} . For the process, $\mu \rightarrow e\gamma$, the results for BR are found to be consistent with the experimental limit for all the mixing patterns, except for HM and BM (NH) in the 3σ range of θ_{23} . In this case, the dependence of LFV on the neutrino mass spectrum is not much significant as seen in Figs. 10 and 11.

The effective neutrino mass depends on the character of the neutrino mass spectrum. In most of our analysis in case of NDBD as well as LFV, we have observed that both the hierarchical patterns shows almost equal dominance. However, it is easier to observe the process if we consider the leading order mass matrices obeying μ - τ symmetry originating from type I seesaw and using type II seesaw as perturbations to generate non zero θ_{13} . Nevertheless, a more detailed analysis considering the presence of all the mechanisms which can generate the process in the LRSM framework should be persued to give a general conclusion.

Appendix

Elements of the Type II Seesaw Mass Matrix (Case B)

Type I Seesaw Mass Matrix (Case C)

$$S_{11} = \left(c_{12}^2 c_{13}^2 - X c_{12}^2 \mu\tau \right) m_1 + e^{2i(\beta-\delta)} s_{13}^2 m_3 + \left(c_{13}^2 s_{12}^2 - X s_{12}^2 \mu\tau \right) e^{2i\alpha} m_2 \tag{51}$$

$$S_{12} = \left(-c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} \right) m_1 + \left(-c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - c_{13} s_{12}^2 s_{13} s_{23} e^{i(2\alpha+\delta)} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} e^{2i\alpha} \right) m_2 + \left(c_{13} s_{13} s_{23} e^{i(2\beta-\delta)} \right) m_3 \tag{52}$$

$$S_{13} = \left(c_{12}^2 c_{13} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} c_{12} c_{13} - X c_{12}^{\mu\tau} s_{12}^{\mu\tau} s_{23}^{\mu\tau} \right) m_1 + \left(-c_{13} s_{12} c_{23} s_{12} s_{13} e^{i(2\alpha+\delta)} - X c_{12}^{\mu\tau} s_{12}^{\mu\tau} s_{23}^{\mu\tau} e^{2i\alpha} \right) m_2 + \left(e^{i(2\beta-\delta)} c_{13} c_{23} s_{13} \right) m_3 \tag{53}$$

$$S_{21} = \left(-c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} \right) m_1 + \left(c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - s_{12}^2 s_{13} s_{23} c_{13} e^{i(2\alpha+\delta)} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} e^{2i\alpha} \right) m_2 + \left(e^{i(2\beta-\delta)} c_{13} s_{23} s_{13} \right) m_3 \tag{54}$$

$$S_{22} = \left((c_{23} s_{12} - e^{i\delta} c_{12} s_{13} s_{23})^2 - X c_{23}^2 \mu\tau s_{12}^2 \mu\tau \right) m_1 + \left(-X c_{12}^2 \mu\tau c_{23}^2 \mu\tau + (-c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23})^2 \right) m_2 e^{2i\alpha} + \left(c_{13}^2 s_{23}^2 - X s_{23}^2 \mu\tau e^{2i\beta} \right) m_3 \tag{55}$$

$$S_{23} = \left((-c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23}) (-c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23}) + Xc_{23}^{\mu\tau} s_{12}^2 s_{23}^{\mu\tau} \right) m_1 + \left((-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23}) (-c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23}) + Xc_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{23}^{\mu\tau} \right) m_2 e^{2i\alpha} + \left(c_{13}^2 c_{23} s_{23} e^{2i\beta} - c_{23}^{\mu\tau} s_{23}^{\mu\tau} \right) m_3 \tag{56}$$

$$S_{31} = \left(c_{12}^2 c_{13} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} c_{12} c_{13} - Xc_{12}^{\mu\tau} s_{12}^2 s_{23}^{\mu\tau} \right) m_1 + \left(c_{13} s_{12}^2 e^{i\delta} c_{23} s_{13} + c_{12} s_{23} c_{13} s_{12} e^{2i\alpha} - Xc_{12}^{\mu\tau} s_{12}^2 s_{23}^{\mu\tau} \right) m_2 e^{2i\alpha} + \left(e^{2i\beta-i\delta} c_{13} c_{23} s_{13} \right) m_3 \tag{57}$$

$$S_{32} = \left((-e^{i\delta}c_{12}c_{23}s_{13} + s_{12}s_{23}) (-c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23}) + c_{23}^{\mu\tau} s_{12}^2 s_{23}^{\mu\tau} \right) m_1 + \left((-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23}) (-c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23}) + Xc_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{23}^{\mu\tau} \right) e^{2i\alpha} m_2 + \left(c_{13}^2 c_{23} s_{23} - Xc_{23}^{\mu\tau} s_{23}^{\mu\tau} \right) e^{2i\beta} m_3 \tag{58}$$

$$S_{33} = \left((-e^{i\delta}c_{12}c_{23}s_{13} + s_{12}s_{23})^2 - Xs_{12}^2 s_{23}^2 s_{23}^{\mu\tau} \right) m_1 + \left((-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23})^2 - Xc_{12}^2 s_{23}^2 s_{23}^{\mu\tau} \right) e^{2i\alpha} m_2 + \left(c_{13}^2 c_{23}^2 - c_{23}^{\mu\tau} \right) e^{2i\beta} m_3 \tag{59}$$

Where, $c_{ij}^{\mu\tau} = \cos \theta_{ij}^{\mu\tau}$, $s_{ij}^{\mu\tau} = \sin \theta_{ij}^{\mu\tau}$ represents the mixing angles for $\mu - \tau$ symmetric neutrino mass matrix (TBM, BM, HM, GRM).

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