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Quantum Teleportation of Five-qubit State

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Abstract We propose a novel quantum teleportation protocol for certain class of five-qubit state with a seven-qubit cluster state as quantum channel. In our scheme, the sender merely needs to perform a seven-qubit von-Neumann projective measurement, the original state with deterministic probability can be reconstructed by the receiver after a series corresponding unitary transformations. Compared with other schemes proposed before, our scheme has the distinct advantages of requiring fewer quantum channels, possessing higher intrinsic efficiency, and transmitting more quantum information bits.

Keywords Quantum teleportation · Cluster state · Five-qubit state

1 Introduction

In recent years, quantum information science has received extensive attention and development. As one of the main research contents in quantum information science, quantum state transmission develops rapidly. The quantum state transmission includes general single-photon transmission, quantum teleportation and remote state preparation, etc. Among them, quantum teleportation possesses advantages in reliabilities, complexity of communication and potential application prospects. Quantum teleportation has been always highly focused in the region since it was put forward. In 1993 Bennett et al. firstly proposed the concept of quantum teleportation [1], and designed a theoretical scheme of quantum teleportation

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of two-level particles. The primary concept of that is as follow: the sender, “Alice,” and the receiver, “Bob,” must prearrange the sharing of an EPR-correlated pair of particles, which is the quantum channel. Alice performs a projective measurement on her particle and the unknown quantum system, and sends Bob the result of this measurement by the classic channel. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state which Alice destroyed [1]. Since proposing the quantum teleportation, it is starting the theory of quantum teleportation, the experimental analysis [2–6] and the application of assumptions. Recently, quantum teleportation has become a hot spot of present research, and people proposed that using EPR state [7], W-like state [8, 9], GHZ state [10–15], GHZ-like [16–20] state et al. entangled state as quantum channel of quantum teleportation.

Cluster state [21] is a new multi-particle entangled state proposed by H. J. Briegel and R. Raussendorf in 2001. When the number of particles is N>3, cluster state presents their unique characteristics, namely maximum connectedness and a high persistency of entanglement, which means that cluster state is more difficult to be destroyed by single-bit measurement and less susceptible to decoherence. Cluster state has the properties of GHZ and W entangled states. It is harder disentangled than GHZ state by local measurements [22, 23]. In recent years, many quantum teleportation schemes have been proposed by using different types of multiparty cluster states as a quantum channel. Wang et al. [24] presented scheme for quantum teleportation of an arbitrary two-particle state by using one-dimensional cluster state. Transferring a three-particle state by using a five-particle cluster state of QT is proposed by Zhou and Liu [25]. Li et al. [26] proposed teleportation protocols of three and four-qubit entangled state via multi-qubit cluster state. Many teleportation protocols using the multi-particle cluster states are proposed [27–29]. Tan et al. proposed a perfect quantum teleportation of an unknown single-particle pure state or an arbitrary two-particle pure entangled state by using the same four-particle cluster state [30]. Binayak S. Choudhury proposed quantum teleportation protocol of three-qubit state using four-qubit quantum channels [31].

In this paper, we propose a scheme of quantum teleportation with a special five-qubit state by utilizing a seven-qubit cluster state. Alice performs a seven-qubit von-Neumann projective measurement, and informs Bob of her measurement result by the classical channel. Bob gives a corresponding general evolution, and restores the target state. The successful possibility of our scheme is 1. The comparison between our scheme and other schemes [19, 31], our scheme possesses higher intrinsic efficiency.

2 Main Result

Suppose Alice has a five-qubit state, which is given by

$$|\chi\rangle_{abcde} = (\alpha\,|00000\rangle + \beta\,|00011\rangle + \gamma\,|11100\rangle + \delta\,|11111\rangle)_{abcde} \tag{1}$$

where$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Alice and Bob share a seven-qubit cluster state

$$|\phi\rangle_{1234567} = \frac{1}{2}(|0000000\rangle + |0001101\rangle + |1110010\rangle + |1111111\rangle)_{1234567} \tag{2}$$

the qubits a, b, c, d, e, 6 and 7 belong to Alice, qubits 1, 2, 3, 4 and 5 belong to Bob, respectively. The joint state of the five-qubit state and the quantum channel is given by,

$$\begin{aligned}
|\tau\rangle &= |\chi\rangle_{abcde} \otimes |\phi\rangle_{1234567} \\
&= \tfrac{1}{4}\big[\left|\varphi^1\right\rangle_{abcde67} (\alpha\,|00000\rangle + \beta\,|00011\rangle + \gamma\,|11100\rangle + \delta\,|11111\rangle)_{12345} \\
&\quad + \left|\varphi^2\right\rangle_{abcde67} (\alpha\,|00000\rangle - \beta\,|00011\rangle + \gamma\,|11100\rangle - \delta\,|11111\rangle)_{12345} \\
&\quad + \left|\varphi^3\right\rangle_{abcde67} (\alpha\,|00000\rangle + \beta\,|00011\rangle - \gamma\,|11100\rangle - \delta\,|11111\rangle)_{12345} \\
&\quad + \left|\varphi^4\right\rangle_{abcde67} (\alpha\,|00000\rangle - \beta\,|00011\rangle - \gamma\,|11100\rangle + \delta\,|11111\rangle)_{12345} \\
&\quad + \left|\varphi^5\right\rangle_{abcde67} (\alpha\,|00011\rangle + \beta\,|00000\rangle + \gamma\,|11111\rangle + \delta\,|11100\rangle)_{12345} \\
&\quad + \left|\varphi^6\right\rangle_{abcde67} (\alpha\,|00011\rangle - \beta\,|00000\rangle + \gamma\,|11111\rangle - \delta\,|11100\rangle)_{12345} \\
&\quad + \left|\varphi^7\right\rangle_{abcde67} (\alpha\,|00011\rangle + \beta\,|00000\rangle - \gamma\,|11111\rangle - \delta\,|11100\rangle)_{12345} \\
&\quad + \left|\varphi^8\right\rangle_{abcde67} (\alpha\,|00011\rangle - \beta\,|00000\rangle - \gamma\,|11111\rangle + \delta\,|11100\rangle)_{12345} \\
&\quad + \left|\varphi^9\right\rangle_{abcde67} (\alpha\,|11100\rangle + \beta\,|11111\rangle + \gamma\,|00000\rangle + \delta\,|00011\rangle)_{12345} \\
&\quad + \left|\varphi^{10}\right\rangle_{abcde67} (\alpha\,|11100\rangle - \beta\,|11111\rangle + \gamma\,|00000\rangle - \delta\,|00011\rangle)_{12345} \\
&\quad + \left|\varphi^{11}\right\rangle_{abcde67} (\alpha\,|11100\rangle + \beta\,|11111\rangle - \gamma\,|00000\rangle - \delta\,|00011\rangle)_{12345} \\
&\quad + \left|\varphi^{12}\right\rangle_{abcde67} (\alpha\,|11100\rangle - \beta\,|11111\rangle - \gamma\,|00000\rangle + \delta\,|00011\rangle)_{12345} \\
&\quad + \left|\varphi^{13}\right\rangle_{abcde67} (\alpha\,|11111\rangle + \beta\,|11100\rangle + \gamma\,|00011\rangle + \delta\,|00000\rangle)_{12345} \\
&\quad + \left|\varphi^{14}\right\rangle_{abcde67} (\alpha\,|11111\rangle - \beta\,|11100\rangle + \gamma\,|00011\rangle - \delta\,|00000\rangle)_{12345} \\
&\quad + \left|\varphi^{15}\right\rangle_{abcde67} (\alpha\,|11111\rangle + \beta\,|11100\rangle - \gamma\,|00011\rangle - \delta\,|00000\rangle)_{12345} \\
&\quad + \left|\varphi^{16}\right\rangle_{abcde67} (\alpha\,|11111\rangle - \beta\,|11100\rangle - \gamma\,|00011\rangle + \delta\,|00000\rangle)_{12345}\big]
\end{aligned} \tag{3}$$

where $\left|\varphi^i\right\rangle_{adcde67}$ $(i = 1, 2, \cdots, 16)$ are mutually orthonormal seven-qubit state in Alice's possession given by,

$$\left|\varphi^1\right\rangle = \frac{1}{2}(|0000000\rangle + |0001101\rangle + |1110010\rangle + |1111111\rangle)_{abcde67} \tag{4}$$

$$\left|\varphi^2\right\rangle = \frac{1}{2}(|0000000\rangle - |0001101\rangle + |1110010\rangle - |1111111\rangle)_{abcde67} \tag{5}$$

$$\left|\varphi^3\right\rangle = \frac{1}{2}(|0000000\rangle + |0001101\rangle - |1110010\rangle - |1111111\rangle)_{abcde67} \tag{6}$$

$$\left|\varphi^4\right\rangle = \frac{1}{2}(|0000000\rangle - |0001101\rangle - |1110010\rangle + |1111111\rangle)_{abcde67} \tag{7}$$

$$\left|\varphi^5\right\rangle = \frac{1}{2}(|0000001\rangle + |0001100\rangle + |1110011\rangle + |1111110\rangle)_{abcde67} \tag{8}$$

$$\left|\varphi^6\right\rangle = \frac{1}{2}(|0000001\rangle - |0001100\rangle + |1110011\rangle - |1111110\rangle)_{abcde67} \tag{9}$$

$$\left|\varphi^7\right\rangle = \frac{1}{2}(|0000001\rangle + |0001100\rangle - |1110011\rangle - |1111110\rangle)_{abcde67} \tag{10}$$

$$\left|\varphi^8\right\rangle = \frac{1}{2}(|0000001\rangle - |0001100\rangle - |1110011\rangle + |1111110\rangle)_{abcde67} \tag{11}$$

$$\left|\varphi^9\right\rangle = \frac{1}{2}(|0000010\rangle + |0001111\rangle + |1110000\rangle + |1111101\rangle)_{abcde67} \tag{12}$$

$$\left|\varphi^{10}\right\rangle = \frac{1}{2}(|0000010\rangle - |0001111\rangle + |1110000\rangle - |1111101\rangle)_{abcde67} \tag{13}$$

$$\left|\varphi^{11}\right\rangle = \frac{1}{2}(|0000010\rangle + |0001111\rangle - |1110000\rangle - |1111101\rangle)_{abcde67} \tag{14}$$

$$\left|\varphi^{12}\right\rangle = \frac{1}{2}\left(|0000010\rangle - |0001111\rangle - |1110000\rangle + |1111101\rangle\right)_{abcde67} \tag{15}$$

$$\left|\varphi^{13}\right\rangle = \frac{1}{2}\left(|0000011\rangle + |0001110\rangle + |1110001\rangle + |1111100\rangle\right)_{abcde67} \tag{16}$$

$$\left|\varphi^{14}\right\rangle = \frac{1}{2}\left(|0000011\rangle - |0001110\rangle + |1110001\rangle - |1111100\rangle\right)_{abcde67} \tag{17}$$

$$\left|\varphi^{15}\right\rangle = \frac{1}{2}\left(|0000011\rangle + |0001110\rangle - |1110001\rangle - |1111100\rangle\right)_{abcde67} \tag{18}$$

$$\left|\varphi^{16}\right\rangle = \frac{1}{2}\left(|0000011\rangle - |0001110\rangle - |1110001\rangle + |1111100\rangle\right)_{abcde67} \tag{19}$$

After the measurement, Alice informs Bob of her measurement result by the classical channel. Bob gives a corresponding Pauli rotation, and restores the target state. Alice′s measured results, her communicated results to Bob and Bob′s corresponding operations are listed in Table 1.

In our work, we propose a deterministic quantum teleportation. In order to implement it readily in the future, we choose the more general and less requirements demanding quantum state. Based on this, we further compare our scheme with other similar schemes proposed before. To compare the following four aspects: the quantum resource expenditure, the number of quantum channel, the classical resource expenditure and the intrinsic efficiency. They are summarized in Table 2.

From Table 2, comparing our scheme to the Z and B schemes, we can readily discover that our scheme has the remarkable advantages of requiring fewer quantum channels, possessing higher intrinsic efficiency, and transmitting more quantum information bits. In addition, The B scheme and our scheme utilize cluster state as quantum channels, our

Table 1 Strategy for recovering the five-qubit state

Alice's classical results information		Bob's state	Bob's operation
$\|\varphi^1\rangle$	0000	$\alpha\|00000\rangle + \beta\|00011\rangle + \gamma\|11100\rangle + \delta\|11111\rangle$	$I \otimes I \otimes I \otimes I \otimes I$
$\|\varphi^2\rangle$	0001	$\alpha\|00000\rangle - \beta\|00011\rangle + \gamma\|11100\rangle - \delta\|11111\rangle$	$I \otimes I \otimes I \otimes \delta_z \otimes I$
$\|\varphi^3\rangle$	0010	$\alpha\|00000\rangle + \beta\|00011\rangle - \gamma\|11100\rangle - \delta\|11111\rangle$	$\delta_z \otimes I \otimes I \otimes I \otimes I$
$\|\varphi^4\rangle$	0011	$\alpha\|00000\rangle - \beta\|00011\rangle - \gamma\|11100\rangle + \delta\|11111\rangle$	$\delta_z \otimes I \otimes I \otimes \delta_z \otimes I$
$\|\varphi^5\rangle$	0100	$\alpha\|00011\rangle + \beta\|00000\rangle + \gamma\|11111\rangle + \delta\|11100\rangle$	$I \otimes I \otimes I \otimes \delta_x \otimes \delta_x$
$\|\varphi^6\rangle$	0101	$\alpha\|00011\rangle - \beta\|00000\rangle + \gamma\|11111\rangle - \delta\|11100\rangle$	$I \otimes I \otimes I \otimes i\delta_y \otimes \delta_x$
$\|\varphi^7\rangle$	0110	$\alpha\|00011\rangle + \beta\|00000\rangle - \gamma\|11111\rangle - \delta\|11100\rangle$	$I \otimes \delta_z \otimes I \otimes \delta_x \otimes \delta_x$
$\|\varphi^8\rangle$	0111	$\alpha\|00011\rangle - \beta\|00000\rangle - \gamma\|11111\rangle + \delta\|11100\rangle$	$\delta_z \otimes I \otimes I \otimes i\delta_y \otimes \delta_x$
$\|\varphi^9\rangle$	1000	$\alpha\|11100\rangle + \beta\|11111\rangle + \gamma\|00000\rangle + \delta\|00011\rangle$	$\delta_x \otimes \delta_x \otimes \delta_x \otimes I \otimes I$
$\|\varphi^{10}\rangle$	1001	$\alpha\|11100\rangle - \beta\|11111\rangle + \gamma\|00000\rangle - \delta\|00011\rangle$	$\delta_x \otimes \delta_x \otimes \delta_x \otimes \delta_z \otimes I$
$\|\varphi^{11}\rangle$	1010	$\alpha\|11100\rangle + \beta\|11111\rangle - \gamma\|00000\rangle - \delta\|00011\rangle$	$i\delta_y \otimes \delta_x \otimes \delta_x \otimes I \otimes I$
$\|\varphi^{12}\rangle$	1011	$\alpha\|11100\rangle - \beta\|11111\rangle - \gamma\|00000\rangle + \delta\|00011\rangle$	$i\delta_y \otimes \delta_x \otimes \delta_x \otimes \delta_z \otimes I$
$\|\varphi^{13}\rangle$	1100	$\alpha\|11111\rangle + \beta\|11100\rangle + \gamma\|00011\rangle + \delta\|00000\rangle$	$\delta_x \otimes \delta_x \otimes \delta_x \otimes \delta_x \otimes \delta_x$
$\|\varphi^{14}\rangle$	1101	$\alpha\|11111\rangle - \beta\|11100\rangle + \gamma\|00011\rangle - \delta\|00000\rangle$	$\delta_x \otimes \delta_x \otimes \delta_x \otimes i\delta_y \otimes \delta_x$
$\|\varphi^{15}\rangle$	1110	$\alpha\|11111\rangle + \beta\|11100\rangle - \gamma\|00011\rangle - \delta\|00000\rangle$	$i\delta_y \otimes \delta_x \otimes \delta_x \otimes \delta_x \otimes \delta_x$
$\|\varphi^{16}\rangle$	1111	$\alpha\|11111\rangle - \beta\|11100\rangle - \gamma\|00011\rangle + \delta\|00000\rangle$	$i\delta_y \otimes \delta_x \otimes \delta_x \otimes i\delta_y \otimes \delta_x$

Table 2 Comparison between three protocols

S	QRE	QC	CRE	QIBT	η
Z	2 GHZ-like State	2	6	2	2/12
B	3 Four-qubit Cluster State	3	12	3	3/24
Our	1 Seven-qubit Cluster State	1	7	5	5/14

The Z,B, in turn, on behalf of Ref. [19, 31]. The intrinsic efficiency of the communication scheme is defined [32] as $\eta = q_s/(q_u + b_t)$, where q_s is the number of qubits that consist of the quantum information to be exchanged, q_u is the number of the qubits which are used as the quantum channel (except for those chosen for security checking), b_t is the classical bits transmitted

QRE quantum resource expenditure, QC the number of quantum channel, CRE classical resource expenditure, QIBT quantum information bits transmitted

scheme possesses higher intrinsic efficiency, which means our scheme saves the quantum resources effectively. In this sense, our scheme is better.

3 Conclusion

In conclusion, we display a quantum teleportation with cluster state. We propose a quantum teleportation of a special five-qubit state by a seven-qubit cluster state. In our scheme, only requiring a seven-qubit projection measurement and a local single operation, we can recover the target state. We have explicitly given the required measurements for Alice and the specific unit operations required for Bob to reconstruct the target state, and our scheme is a deterministic scheme. The successful possibility of our scheme is 1. Finally, we have compared our scheme with other schemes on quantum and classical resource expenditures, the number of quantum channel and the intrinsic efficiency. We think our scheme is better than other schemes. We also expect to further realize the quantum state transmission of an arbitrary five-particle state.

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