Int J Theor Phys (2017) 56:3935–3940
DOI 10.1007/s10773-017-3383-x

The Quantum Logical Challenge: Peter Mittelstaedt's Contributions to Logic and Philosophy of Science

E. Beltrametti[1] · **M. L. Dalla Chiara**[2] · **R. Giuntini**[3]

Received: 26 January 2017 / Accepted: 7 April 2017 / Published online: 21 April 2017

Abstract Peter Mittelstaedt's contributions to quantum logic and to the foundational problems of quantum theory have significantly realized the most authentic spirit of the *International Quantum Structures Association*: an original research about hard technical problems, which are often "entangled" with the emergence of important changes in our general world-conceptions. During a time where both the logical and the physical community often showed a skeptical attitude towards Birkhoff and von Neumann's quantum logic, Mittelstaedt brought into light the deeply innovating features of a quantum logical thinking that allows us to overcome some strong and unrealistic assumptions of classical logical arguments. Later on his intense research on the unsharp approach to quantum theory and to the measurement problem stimulated the increasing interest for unsharp forms of quantum logic, creating a fruitful interaction between the work of quantum logicians and of many-valued logicians. Mittelstaedt's general views about quantum logic and quantum theory seem to be inspired by a conjecture that is today more and more confirmed: there is something universal in the quantum theoretic formalism that goes beyond the limits of microphysics, giving rise to interesting applications to a number of different fields.

Keywords Quantum logic · Peter Mittelstaedt

✉ R. Giuntini
giuntini@unica.it

E. Beltrametti
enrico.beltrametti@ge.infn.it

M. L. Dalla Chiara
dallachiara@unifi.it

[1] Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, I-16146, Genova, Italy

[2] Dipartimento di Lettere e Filosofia, Università di Firenze, Via Bolognese 52, I-50139 Firenze, Italy

[3] Dipartimento di Pedagogia, Psicologia, Filosofia, Università di Cagliari, Via Is Mirrionis 1, I-09123 Cagliari, Italy

1 The Quantum Logical Challenge

With deep emotion we write this article devoted to the memory of Peter Mittelstaedt who has been bound to us by a warm friendship and by a long scientific interaction. Mittelstaedt's contributions to logical, foundational and philosophical problems of quantum theory have realized the most "authentic spirit" of the *International Quantum Structures Association*, developing a constant search for original investigations on hard technical problems, which are often "entangled" with the emergence of important conceptual revolutions in different fields, even far from microphysics.

As is well known, Birkhoff and von Neumann's pioneering article "The Logic of Quantum Mechanics" did not immediately arise any strong interest either in the logical or in the physical community. For different reasons both physicists and logicians did not seem inclined to accept that what had been termed *quantum logic* could represent a "genuine" form of logic that in some situations should force us to assume *rules of reasoning* different from the classical ones. One of the arguments used by the "quantum logical Skeptics" was: quantum logic does not have any well-behaved implication-connective; while according to a common view "no logic is possible without an acceptable conditional connective". Interestingly enough, Mittelstaedt, who was not a professional logician, realized very early that this argument was wrong: a natural quantum logical implication is there and is represented by the "Sasaki-conditional" (also called the "Sasaki-hook"). In the language of quantum logic this connective can be defined as follows:

$$A \to B := \neg A \vee (A \wedge B),$$

where A and B represent generic sentences, while $\neg$, $\wedge$, $\vee$ are the quantum logical negation, conjunction and disjunction. Of course, in the case of classical (distributive) logic we have:

$$\neg A \vee (A \wedge B) \leftrightarrow \neg A \vee B \leftrightarrow \neg(A \wedge \neg B).$$

Hence, the Sasaki-conditional coincides with the standard material implication of classical logic.

As is well known, the Sasaki-conditional plays an important role in the algebraic semantics of quantum logic. For, the validity of the *Modus-Ponens Principle*

$$A \wedge (A \to B) \to B$$

corresponds to one of the possible formulations of the *orthomodular property* for a lattice whose elements represent possible *meanings* of quantum logical sentences. At first sight the quantum logical implication may appear somewhat "pathological" in the perspective of a "classical logical thinking". Some basic principles that characterize the behavior of the conditional connective either in classical logic or in some important alternative logics (like intuitionistic logic) turn out to be violated. For instance both the *a fortiori principle* ($A \to (B \to A)$) and the *import-export law* ($[(A \wedge B) \to C] \leftrightarrow [A \to (B \to C)]$) are not valid in quantum logic. One should be aware, however, that violations of some strong classical logical arguments may represent an advantage, whenever we are looking for a formal representation of some *natural* forms of reasoning in different situations, even far from microphysics. The "dialog semantics" for quantum logic (proposed and investigated by Mittelstaedt and by Stachow[1]) has interestingly shown how quantum logic can

---

[1] See [1, 2, 6].

be justified in terms of some general reasons that do not necessarily depend on quantum phenomena.

A non-classical attitude in logic can be naturally connected with a general view that Mittelstaedt has developed with strong arguments in his last book *Rational Reconstruction of Modern Physics*:

> *"The three leading theories of modern physics, Special Relativity, General Relativity, Quantum Mechanics cannot be adequately understood as an increase of knowledge about various empirical facts. In contrast, the very progress of these transitions consists of a stepwise reduction of prejudices, i.e. of quite general hypothetical assumptions of classical mechanics, that can be traced back to the metaphysics of the 17th and 18th centuries."*[2]

2 The Unsharp Approaches to Quantum Theory

In the Nineties Mittelstaedt devoted an intense research-activity on the *unsharp approaches* to quantum theory. The wonderful book *The Quantum Theory of Measurement* [3], written in collaboration with Paul Busch and Pekka Lahti, had a great impact on the investigations about *quantum structures.* The transition from *sharp* to *unsharp quantum theory* has been described by Mittelstatedt as the result of a *progressive weakening* of some basic assumptions concerning the *ontology of physical objects*:

> *"The classical ontology assumes that there are individual objects S_i and that these objects possess elementary properties P_λ. An elementary property P_λ refers to an object such that either P_λ or the counterproperty $\bar{P}_\lambda$ pertains to the system."*[3]

Elementary physical properties correspond to *physical events* (or *propositions*) that can be either *verified* or *falsified* by the physical systems under investigation. As is well known, according to a standard mathematical formalism, it is customary to represent *classical physical events* as subsets of convenient sets (the *phase-spaces* of the systems in question). Consequently, the algebraic structure of *a classical physical event-system* turns out to be a Boolean algebra. However, as noticed by Mittelstaedt:

> *"The strict postulates of classical ontology are neither both intuitive and plausible nor can they be confirmed and justified by experimental means."*[4]

The *quantum ontology* is based on a characteristic weakening of the basic assumptions of classical ontology. One shall distinguish the *sharp quantum ontology* from the *unsharp* one. In both cases:

> *"Quantum objects are not thoroughgoingly determined. They possess only a few elementary properties, either positive or negative. Properties that pertain simultaneously to an object are called "objective" and "mutually commensurable"."*[5]

At the same time, a characteristic feature of sharp quantum theory is the following:

---

[2] [5], Introduction, p.x.

[3] [5], p. 50–51.

[4] [5], p. 51.

[5] [5], p. 51.

> *"Any arbitrary property P can be tested at a given object with the result that either P or the counter property $\bar{P}$ pertains to the object system."*[6]

As is well known, in the standard quantum-theoretical formalism *sharp quantum events* are mathematically represented as projection-operators of convenient Hilbert spaces. Their "sharp character" depends on the fact that projections satisfy the *non-contradiction principle*:

$$P \sqcap P^{\perp} = \mathbb{O}.$$

The *infimum* between a projection P and its orthogonal projection $P^{\perp}$ is always the null projection $\mathbb{O}$. Hence, contradictory events are impossible! Accordingly, the algebraic structure of a *sharp quantum event-system* turns out to be an *orthomodular lattice*, where distributivity and other important Boolean laws are generally violated.

The unsharp quantum ontology is based on a weakening of some assumptions of the sharp quantum ontology. The requirement according to which "any arbitrary property P can be tested at a given object with the result that either P or the counter property $\bar{P}$ pertains to the object system" is considered too strong and is no longer accepted:

> *"More detailed investigations of the quantum theory of measurement have shown in recent years that after a unitary measurement process a definite value of the measured property cannot be attributed to the object system and no definite value can be attributed to the pointer of the measuring apparatus. ...... We could weaken the ontological presuppositions by considering unsharp properties and unsharp elementary propositions, which are not value definite. This idea is strongly supported by physics, since the quantum mechanics of unsharp observables (POV-measures) was developed recently and is now well established."*[7]

In the framework of unsharp quantum theory quantum events are mathematically represented as *effects*, which are natural generalizations of projection-operators. In fact, the set of all effects of a Hilbert space $\mathcal{H}$ can be characterized as the largest set of linear bounded operators E for which a *Born-probability* can be defined:

$$\mathtt{Tr}(\rho E) \in [0, 1]$$

(where ρ is any density operator of $\mathcal{H}$ and `Tr` is the trace-functional). Unlike projections, effects can represent *unsharp physical events*, since they may violate the non-contradiction principle. We may have:

$$E \sqcap E^{\perp} \neq \mathbb{O}.$$

We know how the unsharp approaches to quantum theory have stimulated intense research-activities in the domain of quantum structures. New algebraic structures have been created and investigated: *effect algebras*, *D-posets*, *quantum MV algebras*, etc. . The transition from sharp to unsharp quantum structures has even been ironically set to music in the "Lattice Song", composed by Dirk Aerts during the Liptowsky Jan conference (1998), when the IQSA-community sang:

"You were so sweet and sharp,
and now you are so unsharp!"

---

[6] [5], p. 51.

[7] [5], p. 65.

The research on unsharp quantum structures has naturally stimulated new logical ideas. Quantum logic has been transformed into different forms of *paraconsistent logic*, where the non-contradiction principle

$$\neg(A \wedge \neg A)$$

is generally violated. In the framework of this new logical environment Birkhoff and von Neumann have naturally met Łukasiewicz, the "father" of modern many-valued and fuzzy logics. As a consequence, the "population" of quantum logics has become more and more "crowded". Are we perhaps dealing with "too many" logics for a single physical theory? Does such a situation determine a sense of "logical uneasiness"? In this connection we should recall that the plurality of logics is no longer regarded as a "danger" in contemporary logical researches. Logicians are aware that different *contexts* can suggest *the most convenient logics* to be used in particular situations that are described by specific languages. And a logical principle of *context-dependence* seems to be perfectly in agreement with the basic features of quantum theory, where different forms of *contextuality* play a relevant role. As observed by Mittelstaedt:

> *"We cannot expect that unsharp quantum logic is the "final logic" of physics, which is in accordance with the universal "final theory of everything". However, unsharp quantum logic is closer to the "final logic" than orthomodular logic and classical logic. This means, in addition, that due to the more adequate relaxation of non-empirical ontological hypotheses quantum logic of unsharp properties is more intuitive and more plausible than quantum logic of sharp properties and classical logic."*[8]

3 Quantum Objects, Individuality and Identity

Some important contributions given by Mittelstaedt concern the logical and philosophical debates about two crucial questions of the foundations of quantum theory:

- What exactly are quantum objects?
- To what extent do quantum objects violate some traditional logical and philosophical views about *individuality* and *identity*?

His clear and rigorous arguments have contributed to confirm the thesis that the quantum world is strongly *non-Leibnizian.* Quantum superpositions and entanglement are incompatible with Leibniz' view according to which each *individual* is characterized by a *complete concept* that, at least *in mente Dei*, can *semantically decide* all relevant properties thereof. In fact, quantum uncertainties and *No-go theorems* forbid the possibility of *complete concepts* for quantum objects!

Quantum theory gives also rise to essential violations of *Leibniz' Law*, which can be formalized as follows in the framework of a second-order logical language:

$$a = b \leftrightarrow \forall P(Pa \leftrightarrow Pb).$$

In other words, objects that share *all* properties are one and the same object. But quantum statistics have shown that *indiscernibility* and *identity* are, in principle, two different rela-

---

[8] [5], p. 69.

tions: there are systems consisting of two objects (say, two bosons in the same state) that are *indistinguishable*, sharing *all* relevant properties.

As observed by Mittelstaedt:

> *“Individuals in the strict sense do not exist in quantum physics. However, unsharp observables, almost repeatable and weakly disturbing measurements allow for the definition of unsharp individuals which is sufficient for all practical purposes. Many quantum physical experiments and the obvious existence of individuals in the classical world can be explained in this way. On the other hand, if quantum mechanics is considered as universally valid, then there is no classical world in the strict sense. Consequently, the deficiency of individuals in quantum physics implies that there are no individuals at all. The Leibniz project of a universal language which allows for a unique characterization of each individual system would then turn out to be a great illusion.”*[9]

A significant aspect of Mittelstaedt’s philosophical ideas is his deep interest for Kant’s theories, which may have a weight even in the foundational questions of quantum theory. Following Kant, natural sciences deal with *phenomena*, characterized by an essential interaction with subjective observers and researchers. *Noumena* cannot be grasped! What Bas van Fraassen has once termed “a pre-Kantian” attitude may sometimes lead to naive positions in philosophy of science. One is dealing with a “danger” that is not always avoided in the current debates about the foundations of quantum theory. We need only think of many discussions about the meaning of the expression “element of reality” (used in the celebrated paper by Einstein-Podolsky-Rosen) or of some debates about the question “to what extent does the wave-function mirror *real* properties of quantum objects, quite independently of human knowledge?”

Peter Mittelstaedt has represented a rare figure of humanist-scientist, in the spirit of the most significant tradition of European thought. His research-activity has been always characterized by a deep general culture and by a constant curiosity towards a number of different fields. We will miss Peter’s noble presence among us for a very long time.

References

1. Mittelstaedt, P.: Philosophical Problems of Modern Physics. Reidel, Dordrecht (1976)
2. Mittelstaedt, P.: Quantum Logic. Reidel, Dordrecht (1978)
3. Busch, P., Lahti, P., Mittelstaedt, P.: The Quantum Theory of Measurement. Springer, Heidelberg (1996)
4. Mittelstaedt, P.: Individual Objects, Kinds and the Identity of Indiscernibles in Physics. In: Waves, Information and Foundations of Physics. A Tribute to Giuliano Toraldo di Francia on his 80th Birthday, Florence 1996, Proceedings of the Italian Physical Society, Florence, 1996
5. Mittelstaedt, P.: Rational Reconstruction of Modern Physics. Springer, Heidelberg (2011)
6. Stachow, E.-W.: Completeness of Quantum Logic. J. Philos. Log. **5**, 237–280 (1976)

---

[9] [4], p. 258.