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Bidirectional Teleportation of a Two-Qubit State by Using Eight-Qubit Entangled State as a Quantum Channel

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Abstract In this paper, a new scheme of bidirectional quantum teleportation (BQT) making use of an eight-qubit entangled state as the quantum channel is presented. This scheme is the first protocol without controller by which the users can teleport an arbitrary two-qubit state to each other simultaneously. This protocol is based on the ControlledNOT operation, appropriate single-qubit unitary operations and single-qubit measurement in the Z-basis and X-basis.

Keywords Bidirectional quantum teleportation · Two-qubit state · Eight-qubit channel

1 Introduction

Quantum teleportation (QT) [1] is one of the branches of quantum information theory that has been attracting great attention in recent years. In this protocol, an unknown quantum state can be transmitted to a receiver using entanglement and classical information. In 1993, the first protocol of QT using Einstein-Podolsky-Rosen (EPR) pair as a quantum channel was presented by Bennett et al. [2]. After that, several QT protocols [3–11] were proposed using EPR pair, Greenberger, Horne, Zeilinger (GHZ) state, W state and other entangled states as a quantum channel.

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Many experimental approaches have realized QT after the first demonstration in 1997 using entangled photons [12]. Laboratory demonstrations include open destination QT [13], entanglement swapping demonstration [14] and two-qubit composite system QT [15]. Moreover, QT through fiber link has been realized [16, 17]. Recently, QT over 16 km and 100 km has been demonstrated via free space links [18] using single and parametric down conversion sources respectively.

Controlled quantum teleportation (CQT) is one of the types of QT proposed by Karlsson and Bourennane in 1993 [19]. In this protocol, there are three users where one of them is the supervisor or controller. Later, several protocols of CQT with one or more controller were presented [20–24].

In 2013, a bidirectional controlled quantum teleportation (BCQT) by Zha et al. [25] via five-qubit cluster state was proposed. In BCQT or BQT protocol, two users can transmit an unknown quantum state to each other simultaneously. In the same year, Yan [26], Sun and Zha [27], Li and Nie [28], Shukla et al. [29], and Li et al. [30] proposed BCQT protocols by six-qubit cluster state, six-qubit entangled state, five-qubit composite, and two different five-qubit entangled state as a quantum channel, respectively. Also, ref. [29] showed the Li's scheme [28] is not a BQCT scheme.

In 2014, Fu et al. [31] presented a BQT scheme using a four-qubit cluster state as a quantum channel. In this scheme, users can simultaneously exchange their single-qubit states by applying Hadamard operation.

In the same year, Chen [32], Duan et al. [33] and Duan and Zha [34] proposed new schemes of BCQT using five-qubit entangled state, seven-qubit entangled state, and six-qubit entangled state, as a quantum channel, respectively. In Duan et al.'s scheme [33], Charlie improves the security of the protocol by performing single-qubit measurement three times. In the next protocol, Duan and Zha [34] improved the security of their protocol by applying two single-qubit measurements.

In 2015, Chen [35], Wang and Shu [36], Zhang et al. [37], and Hassanpour et al. [38] proposed different schemes of BCQT using six-qubit genuine, GHZ-type state, eight-qubit entangled, and six-qubit entangled state as a quantum channel, respectively. Zhang et al.'s scheme [37] is better than the previous schemes in terms of quantum resource consumptions. In Hassanpour's scheme [38], the quantum channel is prepared easier than the others' presented works. In all the protocols of BCQT or BQT which we mentioned earlier, users can only teleport an arbitrary single-qubit state to each other.

In 2016, Kiktenko et al. [39], proposed a bidirectional modification of the standard one-qubit teleportation protocol. In this scheme, Alice and Bob transfer noisy versions of their qubit states to each other. Then Hong [40] and Sang [41], presented two schemes of BCQT using seven-qubit entangled state as a quantum channel. In those protocols, Bob can teleport an arbitrary two-qubit state to Alice and Alice can teleport an arbitrary single qubit state back to Bob. A little while later, Hassanpour et al. [42] proposed a BQT protocol using six-qubit GHZ state as a quantum channel by which users can teleport a pure EPR state to each other simultaneously. In the same year, Li and Jin [43] proposed a BCQT scheme via a nine-qubit entangled state as a quantum channel, in which users can teleport an unknown two-qubit state to each other. In the last scheme, Li et al. [44] presented a BCQT protocol where Alice can teleport an arbitrary two-qubit state to Bob and Bob can teleport an arbitrary single-qubit state back to Alice via six-qubit cluster state as a quantum channel.

In this paper, we propose a BQT using an eight-qubit entangled state as a quantum channel, through which the users can teleport an unknown two-qubit state to each other. In this protocol, users only perform single-qubit measurements.

The rest of the paper is organized as follows. In Section 2, the proposed protocol is described. In Section 3, comparison with other protocols is presented. Finally, Section 4 concludes the paper.

2 Description of the Presented Protocol

The protocol is a BQT scheme that Alice and Bob can simultaneously transmit an arbitrary two-qubit state to each other described as (1) and (2).

$$|\emptyset\rangle_{A_1A_2} = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle, \tag{1}$$

$$|\emptyset\rangle_{B_1B_2} = \beta_0|00\rangle + \beta_1|01\rangle + \beta_2|10\rangle + \beta_3|11\rangle. \tag{2}$$

where $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1$ and $|\beta_0|^2 + |\beta_1|^2 + |\beta_2|^2 + |\beta_3|^2 = 1$.

The protocol consists of the following steps:

Step1. An eight-qubit state as a quantum channel described as (3) is prepared.

$$\begin{aligned} |G\rangle_{a_1b_1a_2b_2b_3a_3b_4a_4} = \frac{1}{4}[&|00000000\rangle + |00010001\rangle + |001000010\rangle + |00110011\rangle \\ &+|01000100\rangle + |01010101\rangle + |01100110\rangle + |01110111\rangle \\ &+|10001000\rangle + |10011001\rangle + |10101010\rangle + |10111011\rangle \\ &+|11001100\rangle + |11011101\rangle + |11101110\rangle + |11111111\rangle], \end{aligned} \tag{3}$$

where the qubits $a_1a_2a_3a_4$ belong to Alice and qubits $b_1b_2b_3b_4$ belong to Bob, respectively. The state of the whole system can be expressed as (4).

$$|\varphi\rangle_{a_1b_1a_2b_2b_3a_3b_4a_4A_1A_2B_1B_2} = |G\rangle_{a_1b_1a_2b_2b_3a_3b_4a_4} \otimes |\emptyset\rangle_{A_1A_2} \otimes |\emptyset\rangle_{B_1B_2} \tag{4}$$

Step2. In this step, Alice and Bob perform a Controlled-NOT operation with A_1, A_2, B_1 and B_2 as control qubits and qubits a_1, a_2, b_1 and b_2 as target qubits, respectively. After performing Controlled-NOT operation, the state of the whole system will be in the form of (5). In order to save space, the state of qubits are represented in hexadecimal base.

$$\begin{aligned}
&|\varphi\rangle_{a_1b_1a_2b_2b_3a_3b_4a_4A_1A_2B_1B_2}\\
&=\frac{1}{4}[\alpha_0\beta_0(|00\rangle+|11\rangle+|22\rangle+|33\rangle+|44\rangle+|55\rangle+|66\rangle+|77\rangle+|88\rangle+|99\rangle\\
&\quad+|AA\rangle+|BB\rangle+|CC\rangle+|DD\rangle+|EE\rangle+|FF\rangle)|0\rangle\\
&\quad+\alpha_0\beta_1(|10\rangle+|01\rangle+|32\rangle+|23\rangle+|54\rangle+|45\rangle+|76\rangle+|67\rangle+|98\rangle+|89\rangle\\
&\quad+|BA\rangle+|AB\rangle+|DC\rangle+|CD\rangle+|FE\rangle+|EF\rangle)|1\rangle\\
&\quad+\alpha_0\beta_2(|40\rangle+|51\rangle+|62\rangle+|73\rangle+|04\rangle+|15\rangle+|26\rangle+|37\rangle+|C8\rangle+|D9\rangle\\
&\quad+|EA\rangle+|FB\rangle+|8C\rangle+|9D\rangle+|AE\rangle+|BF\rangle)|2\rangle\\
&\quad+\alpha_0\beta_3(|50\rangle+|41\rangle+|72\rangle+|63\rangle+|14\rangle+|05\rangle+|36\rangle+|27\rangle+|D8\rangle+|C9\rangle\\
&\quad+|FA\rangle+|EB\rangle+|9C\rangle+|8D\rangle+|BE\rangle+|AF\rangle)|3\rangle\\
&\quad+\alpha_1\beta_0(|20\rangle+|31\rangle+|02\rangle+|13\rangle+|64\rangle+|75\rangle+|46\rangle+|57\rangle+|A8\rangle+|B9\rangle\\
&\quad+|8A\rangle+|9B\rangle+|EC\rangle+|FD\rangle+|CE\rangle+|DF\rangle)|4\rangle\\
&\quad+\alpha_1\beta_1(|30\rangle+|21\rangle+|12\rangle+|03\rangle+|74\rangle+|65\rangle+|56\rangle+|47\rangle+|B8\rangle+|A9\rangle\\
&\quad+|9A\rangle+|8B\rangle+|FC\rangle+|ED\rangle+|DE\rangle+|CF\rangle)|5\rangle\\
&\quad+\alpha_1\beta_2(|60\rangle+|71\rangle+|42\rangle+|53\rangle+|24\rangle+|35\rangle+|06\rangle+|17\rangle+|E8\rangle+|F9\rangle\\
&\quad+|CA\rangle+|DB\rangle+|AC\rangle+|BD\rangle+|8E\rangle+|9F\rangle)|6\rangle\\
&\quad+\alpha_1\beta_3(|70\rangle+|61\rangle+|52\rangle+|43\rangle+|34\rangle+|25\rangle+|16\rangle+|07\rangle+|F8\rangle+|E9\rangle\\
&\quad+|DA\rangle+|CB\rangle+|BC\rangle+|AD\rangle+|9E\rangle+|8F\rangle)|7\rangle\\
&\quad+\alpha_2\beta_0(|80\rangle+|91\rangle+|A2\rangle+|B3\rangle+|C4\rangle+|D5\rangle+|E6\rangle+|F7\rangle+|08\rangle+|19\rangle\\
&\quad+|2A\rangle+|3B\rangle+|4C\rangle+|5D\rangle+|6E\rangle+|7F\rangle)|8\rangle\\
&\quad+\alpha_2\beta_1(|90\rangle+|81\rangle+|B2\rangle+|A3\rangle+|D4\rangle+|C5\rangle+|F6\rangle+|E7\rangle+|18\rangle+|09\rangle\\
&\quad+|3A\rangle+|2B\rangle+|5C\rangle+|4D\rangle+|7E\rangle+|6F\rangle)|9\rangle\\
&\quad+\alpha_2\beta_2(|C0\rangle+|D1\rangle+|E2\rangle+|F3\rangle+|84\rangle+|95\rangle+|A6\rangle+|B7\rangle+|48\rangle+|59\rangle\\
&\quad+|6A\rangle+|7B\rangle+|0C\rangle+|1D\rangle+|2E\rangle+|3F\rangle)|A\rangle\\
&\quad+\alpha_2\beta_3(|D0\rangle+|C1\rangle+|F2\rangle+|E3\rangle+|94\rangle+|85\rangle+|B6\rangle+|A7\rangle+|58\rangle+|49\rangle\\
&\quad+|7A\rangle+|6B\rangle+|1C\rangle+|0D\rangle+|3E\rangle+|2F\rangle)|B\rangle\\
&\quad+\alpha_3\beta_0(|A0\rangle+|B1\rangle+|82\rangle+|93\rangle+|E4\rangle+|F5\rangle+|C6\rangle+|D7\rangle+|28\rangle+|39\rangle\\
&\quad+|0A\rangle+|1B\rangle+|6C\rangle+|7D\rangle+|4E\rangle+|5F\rangle)|C\rangle\\
&\quad+\alpha_3\beta_1(|B0\rangle+|A1\rangle+|92\rangle+|83\rangle+|F4\rangle+|E5\rangle+|D6\rangle+|C7\rangle+|38\rangle+|29\rangle\\
&\quad+|1A\rangle+|0B\rangle+|7C\rangle+|6D\rangle+|5E\rangle+|4F\rangle)|D\rangle\\
&\quad+\alpha_3\beta_2(|E0\rangle+|F1\rangle+|C2\rangle+|D3\rangle+|A4\rangle+|B5\rangle+|86\rangle+|97\rangle+|68\rangle+|79\rangle\\
&\quad+|4A\rangle+|5B\rangle+|2C\rangle+|3D\rangle+|0E\rangle+|1F\rangle)|E\rangle\\
&\quad+\alpha_3\beta_3(|F0\rangle+|E1\rangle+|D2\rangle+|C3\rangle+|B4\rangle+|A5\rangle+|96\rangle+|87\rangle+|78\rangle+|69\rangle\\
&\quad+|5A\rangle+|4B\rangle+|3C\rangle+|2D\rangle+|1E\rangle+|0F\rangle)|F\rangle].
\end{aligned}\tag{5}$$

Step3. In this step, Alice and Bob apply single-qubit measurement in the Z-basis on qubits a_1, a_2, b_1 and b_2 respectively. The unmeasured qubits collapse into one of the 16 possible states with equal probability as shown in Table 1.

Table 1 The (Z-Basis) measurement results of users and the corresponding collapsed states

Alice's result	Bob's result	The collapsed state of qubits $\mathrm{b}_3\mathrm{b}_4\mathrm{a}_3\mathrm{a}_4\mathrm{A}_1\mathrm{A}_2\mathrm{B}_1\mathrm{B}_2$
00	00	$\alpha_0\beta_0\|00\rangle + \alpha_0\beta_1\|11\rangle + \alpha_0\beta_2\|22\rangle + \alpha_0\beta_3\|33\rangle + \alpha_1\beta_0\|44\rangle + \alpha_1\beta_1\|55\rangle + \alpha_1\beta_2\|66\rangle + \alpha_1\beta_3\|77\rangle + \alpha_2\beta_0\|88\rangle + \alpha_2\beta_1\|99\rangle + \alpha_2\beta_2\|AA\rangle + \alpha_2\beta_3\|BB\rangle + \alpha_3\beta_0\|CC\rangle + \alpha_3\beta_1\|DD\rangle + \alpha_3\beta_2\|EE\rangle + \alpha_3\beta_3\|FF\rangle$
00	01	$\alpha_0\beta_0\|10\rangle + \alpha_0\beta_1\|01\rangle + \alpha_0\beta_2\|32\rangle + \alpha_0\beta_3\|23\rangle + \alpha_1\beta_0\|54\rangle + \alpha_1\beta_1\|45\rangle + \alpha_1\beta_2\|76\rangle + \alpha_1\beta_3\|67\rangle + \alpha_2\beta_0\|98\rangle + \alpha_2\beta_1\|89\rangle + \alpha_2\beta_2\|BA\rangle + \alpha_2\beta_3\|AB\rangle + \alpha_3\beta_0\|DC\rangle + \alpha_3\beta_1\|CD\rangle + \alpha_3\beta_2\|FE\rangle + \alpha_3\beta_3\|EF\rangle$
00	10	$\alpha_0\beta_0\|20\rangle + \alpha_0\beta_1\|31\rangle + \alpha_0\beta_2\|02\rangle + \alpha_0\beta_3\|13\rangle + \alpha_1\beta_0\|64\rangle + \alpha_1\beta_1\|75\rangle + \alpha_1\beta_2\|46\rangle + \alpha_1\beta_3\|57\rangle + \alpha_2\beta_0\|A8\rangle + \alpha_2\beta_1\|B9\rangle + \alpha_2\beta_2\|8A\rangle + \alpha_2\beta_3\|9B\rangle + \alpha_3\beta_0\|EC\rangle + \alpha_3\beta_1\|FD\rangle + \alpha_3\beta_2\|CE\rangle + \alpha_3\beta_3\|DF\rangle$
00	11	$\alpha_0\beta_0\|30\rangle + \alpha_0\beta_1\|21\rangle + \alpha_0\beta_2\|12\rangle + \alpha_0\beta_3\|03\rangle + \alpha_1\beta_0\|74\rangle + \alpha_1\beta_1\|65\rangle + \alpha_1\beta_2\|56\rangle + \alpha_1\beta_3\|45\rangle + \alpha_2\beta_0\|B8\rangle + \alpha_2\beta_1\|A9\rangle + \alpha_2\beta_2\|9A\rangle + \alpha_2\beta_3\|8B\rangle + \alpha_3\beta_0\|FC\rangle + \alpha_3\beta_1\|ED\rangle + \alpha_3\beta_2\|DE\rangle + \alpha_3\beta_3\|CF\rangle$
01	00	$\alpha_0\beta_0\|40\rangle + \alpha_0\beta_1\|51\rangle + \alpha_0\beta_2\|62\rangle + \alpha_0\beta_3\|73\rangle + \alpha_1\beta_0\|04\rangle + \alpha_1\beta_1\|15\rangle + \alpha_1\beta_2\|26\rangle + \alpha_1\beta_3\|37\rangle + \alpha_2\beta_0\|C8\rangle + \alpha_2\beta_1\|D9\rangle + \alpha_2\beta_2\|EA\rangle + \alpha_2\beta_3\|FB\rangle + \alpha_3\beta_0\|8C\rangle + \alpha_3\beta_1\|9D\rangle + \alpha_3\beta_2\|AE\rangle + \alpha_3\beta_3\|BF\rangle$
01	01	$\alpha_0\beta_0\|50\rangle + \alpha_0\beta_1\|41\rangle + \alpha_0\beta_2\|72\rangle + \alpha_0\beta_3\|63\rangle + \alpha_1\beta_0\|14\rangle + \alpha_1\beta_1\|05\rangle + \alpha_1\beta_2\|36\rangle + \alpha_1\beta_3\|27\rangle + \alpha_2\beta_0\|D8\rangle + \alpha_2\beta_1\|C9\rangle + \alpha_2\beta_2\|FA\rangle + \alpha_2\beta_3\|EB\rangle + \alpha_3\beta_0\|9C\rangle + \alpha_3\beta_1\|8D\rangle + \alpha_3\beta_2\|BE\rangle + \alpha_3\beta_3\|AF\rangle$
01	10	$\alpha_0\beta_0\|60\rangle + \alpha_0\beta_1\|71\rangle + \alpha_0\beta_2\|42\rangle + \alpha_0\beta_3\|53\rangle + \alpha_1\beta_0\|24\rangle + \alpha_1\beta_1\|35\rangle + \alpha_1\beta_2\|06\rangle + \alpha_1\beta_3\|17\rangle + \alpha_2\beta_0\|E8\rangle + \alpha_2\beta_1\|F9\rangle + \alpha_2\beta_2\|CA\rangle + \alpha_2\beta_3\|DB\rangle + \alpha_3\beta_0\|AC\rangle + \alpha_3\beta_1\|BD\rangle + \alpha_3\beta_2\|8E\rangle + \alpha_3\beta_3\|9F\rangle$
01	11	$\alpha_0\beta_0\|70\rangle + \alpha_0\beta_1\|61\rangle + \alpha_0\beta_2\|52\rangle + \alpha_0\beta_3\|43\rangle + \alpha_1\beta_0\|34\rangle + \alpha_1\beta_1\|25\rangle + \alpha_1\beta_2\|16\rangle + \alpha_1\beta_3\|07\rangle + \alpha_2\beta_0\|F8\rangle + \alpha_2\beta_1\|E9\rangle + \alpha_2\beta_2\|DA\rangle + \alpha_2\beta_3\|CB\rangle + \alpha_3\beta_0\|BC\rangle + \alpha_3\beta_1\|AD\rangle + \alpha_3\beta_2\|9E\rangle + \alpha_3\beta_3\|8F\rangle$
10	00	$\alpha_0\beta_0\|80\rangle + \alpha_0\beta_1\|91\rangle + \alpha_0\beta_2\|A2\rangle + \alpha_0\beta_3\|B3\rangle + \alpha_1\beta_0\|C4\rangle + \alpha_1\beta_1\|D5\rangle + \alpha_1\beta_2\|E6\rangle + \alpha_1\beta_3\|F7\rangle + \alpha_2\beta_0\|08\rangle + \alpha_2\beta_1\|19\rangle + \alpha_2\beta_2\|2A\rangle + \alpha_2\beta_3\|3B\rangle + \alpha_3\beta_0\|4C\rangle + \alpha_3\beta_1\|5D\rangle + \alpha_3\beta_2\|6E\rangle + \alpha_3\beta_3\|7F\rangle$
10	01	$\alpha_0\beta_0\|90\rangle + \alpha_0\beta_1\|81\rangle + \alpha_0\beta_2\|B2\rangle + \alpha_0\beta_3\|A3\rangle + \alpha_1\beta_0\|D4\rangle + \alpha_1\beta_1\|C5\rangle + \alpha_1\beta_2\|F6\rangle + \alpha_1\beta_3\|E7\rangle + \alpha_2\beta_0\|18\rangle + \alpha_2\beta_1\|09\rangle + \alpha_2\beta_2\|3A\rangle + \alpha_2\beta_3\|2B\rangle + \alpha_3\beta_0\|5C\rangle + \alpha_3\beta_1\|4D\rangle + \alpha_3\beta_2\|7E\rangle + \alpha_3\beta_3\|6F\rangle$
10	10	$\alpha_0\beta_0\|A0\rangle + \alpha_0\beta_1\|B1\rangle + \alpha_0\beta_2\|82\rangle + \alpha_0\beta_3\|93\rangle + \alpha_1\beta_0\|E4\rangle + \alpha_1\beta_1\|F5\rangle + \alpha_1\beta_2\|C6\rangle + \alpha_1\beta_3\|D7\rangle + \alpha_2\beta_0\|28\rangle + \alpha_2\beta_1\|39\rangle + \alpha_2\beta_2\|0A\rangle + \alpha_2\beta_3\|1B\rangle + \alpha_3\beta_0\|6C\rangle + \alpha_3\beta_1\|7D\rangle + \alpha_3\beta_2\|4E\rangle + \alpha_3\beta_3\|5F\rangle$

Table 1 (continued)

Alice's result	Bob's result	The collapsed state of qubits $b_3b_4a_3a_4A_1A_2B_1B_2$
10	11	$\alpha_0\beta_0\|B0\rangle+\alpha_0\beta_1\|A1\rangle+\alpha_0\beta_2\|92\rangle+\alpha_0\beta_3\|83\rangle+\alpha_1\beta_0\|F4\rangle+\alpha_1\beta_1\|E5\rangle+\alpha_1\beta_2\|D6\rangle+\alpha_1\beta_3\|C7\rangle+$ $\alpha_2\beta_0\|38\rangle+\alpha_2\beta_1\|29\rangle+\alpha_2\beta_2\|1A\rangle+\alpha_2\beta_3\|0B\rangle+\alpha_3\beta_0\|7C\rangle+\alpha_3\beta_1\|6D\rangle+\alpha_3\beta_2\|5E\rangle+\alpha_3\beta_3\|4F\rangle$
11	00	$\alpha_0\beta_0\|C0\rangle+\alpha_0\beta_1\|D1\rangle+\alpha_0\beta_2\|E2\rangle+\alpha_0\beta_3\|F3\rangle+\alpha_1\beta_0\|84\rangle+\alpha_1\beta_1\|95\rangle+\alpha_1\beta_2\|A6\rangle+\alpha_1\beta_3\|B7\rangle+$ $\alpha_2\beta_0\|48\rangle+\alpha_2\beta_1\|59\rangle+\alpha_2\beta_2\|6A\rangle+\alpha_2\beta_3\|7B\rangle+\alpha_3\beta_0\|0C\rangle+\alpha_3\beta_1\|1D\rangle+\alpha_3\beta_2\|2E\rangle+\alpha_3\beta_3\|3F\rangle$
11	01	$\alpha_0\beta_0\|D0\rangle+\alpha_0\beta_1\|C1\rangle+\alpha_0\beta_2\|F2\rangle+\alpha_0\beta_3\|E3\rangle+\alpha_1\beta_0\|94\rangle+\alpha_1\beta_1\|85\rangle+\alpha_1\beta_2\|B6\rangle+\alpha_1\beta_3\|A7\rangle+$ $\alpha_2\beta_0\|58\rangle+\alpha_2\beta_1\|49\rangle+\alpha_2\beta_2\|7A\rangle+\alpha_2\beta_3\|6B\rangle+\alpha_3\beta_0\|1C\rangle+\alpha_3\beta_1\|0D\rangle+\alpha_3\beta_2\|3E\rangle+\alpha_3\beta_3\|2F\rangle$
11	10	$\alpha_0\beta_0\|E0\rangle+\alpha_0\beta_1\|F1\rangle+\alpha_0\beta_2\|C2\rangle+\alpha_0\beta_3\|D3\rangle+\alpha_1\beta_0\|A4\rangle+\alpha_1\beta_1\|B5\rangle+\alpha_1\beta_2\|86\rangle+\alpha_1\beta_3\|97\rangle+$ $\alpha_2\beta_0\|68\rangle+\alpha_2\beta_1\|79\rangle+\alpha_2\beta_2\|4A\rangle+\alpha_2\beta_3\|5B\rangle+\alpha_3\beta_0\|2C\rangle+\alpha_3\beta_1\|3D\rangle+\alpha_3\beta_2\|0E\rangle+\alpha_3\beta_3\|1F\rangle$
11	11	$\alpha_0\beta_0\|F0\rangle+\alpha_0\beta_1\|E1\rangle+\alpha_0\beta_2\|D2\rangle+\alpha_0\beta_3\|C3\rangle+\alpha_1\beta_0\|B4\rangle+\alpha_1\beta_1\|A5\rangle+\alpha_1\beta_2\|96\rangle+\alpha_1\beta_3\|87\rangle+$ $\alpha_2\beta_0\|78\rangle+\alpha_2\beta_1\|69\rangle+\alpha_2\beta_2\|5A\rangle+\alpha_2\beta_3\|4B\rangle+\alpha_3\beta_0\|3C\rangle+\alpha_3\beta_1\|2D\rangle+\alpha_3\beta_2\|1E\rangle+\alpha_3\beta_3\|0F\rangle$

Step4. In this step, after users tell their measurement results to each other, they apply suitable unitary operation, according to Table 2.

After Alice and Bob perform unitary operation on their qubits, the state of the unmeasured qubits will be in the form of (6).

$$\begin{aligned}&\alpha_0\beta_0|00000000\rangle+\alpha_0\beta_1|00010001\rangle+\alpha_0\beta_2|00100010\rangle+\alpha_0\beta_3|00110011\rangle\\&+\alpha_1\beta_0|01000100\rangle+\alpha_1\beta_1|01010101\rangle+\alpha_1\beta_2|01100110\rangle+\alpha_1\beta_3|01110111\rangle\\&+\alpha_2\beta_0|10001000\rangle+\alpha_2\beta_1|10011001\rangle+\alpha_2\beta_2|10101010\rangle+\alpha_2\beta_3|10111011\rangle\\&+\alpha_3\beta_0|11001100\rangle+\alpha_3\beta_1|11011101\rangle+\alpha_3\beta_2|11101110\rangle+\alpha_3\beta_3|11111111\rangle.\end{aligned}\tag{6}$$

Step5. In this step, Alice and Bob apply single-qubit measurement in the X-basis on qubits A_1, A_2, B_1 and B_2. The unmeasured qubits collapse into one of the 16 possible states with equal probability. The measurement results can be shown in Table 3.

Step6. After the users tell their measurement results to each other, they apply suitable unitary operation again according to Table 4. After Alice and Bob applied appropriate unitary operation on their qubits, all of the states will be in the form of (7).

$$\begin{aligned}&(\alpha_0\beta_0|0000\rangle+\alpha_0\beta_1|0001\rangle+\alpha_0\beta_2|0010\rangle+\alpha_0\beta_3|0011\rangle+\alpha_1\beta_0|0100\rangle\\&+\alpha_1\beta_1|0101\rangle+\alpha_1\beta_2|0110\rangle+\alpha_1\beta_3|0111\rangle\\&+\alpha_2\beta_0|1000\rangle+\alpha_2\beta_1|1001\rangle+\alpha_2\beta_2|1010\rangle+\alpha_2\beta_3|1011\rangle+\alpha_3\beta_0|1100\rangle\\&+\alpha_3\beta_1|1101\rangle+\alpha_3\beta_2|1110\rangle+\alpha_3\beta_3|1111\rangle)_{b_3b_4a_3a_4}\\&=(\beta_0|00\rangle+\beta_1|01\rangle+\beta_2|10\rangle+\beta_3|11\rangle)_{a_3a_4}\otimes(\alpha_0|00\rangle+\alpha_1|01\rangle+\alpha_2|10\rangle+\alpha_3|11\rangle)_{b_3b_4}\end{aligned}\tag{7}$$

Table 2 Applying suitable unitary operation

Alice's results	Bob's results	Bob's operation on qubit b$_3$	Bob's operation on qubit b$_4$	Alice's operation on qubit a$_3$	Alice's operation on qubit a$_4$
00	00	I	I	I	I
00	01	I	I	I	X
00	10	I	I	X	I
00	11	I	I	X	X
01	00	I	X	I	I
01	01	I	X	I	X
01	10	I	X	X	I
01	11	I	X	X	X
10	00	X	I	I	I
10	01	X	I	I	X
10	10	X	I	X	I
10	11	X	I	X	X
11	00	X	X	I	I
11	01	X	X	I	X
11	10	X	X	X	I
11	11	X	X	X	X

Table 3 The (X-basis) measurement results of users and the corresponding collapsed states

Alice's results	Bob's results	The collapsed state of qubits $b_3b_4a_3a_4$
$++$	$++$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$++$	$+-$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$++$	$-+$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$++$	$--$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$+-$	$++$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$+-$	$+-$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$+-$	$-+$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$+-$	$--$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle + \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$-+$	$++$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$-+$	$+-$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$-+$	$-+$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$-+$	$--$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle + \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 - \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$--$	$++$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$
$--$	$+-$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle + \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle - \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle - \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle + \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$--$	$-+$	$\alpha_0\beta_0\|0000\rangle + \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 - \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle - \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle + \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle - \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle + \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle + \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle - \alpha_3\beta_3\|1011\rangle$
$--$	$--$	$\alpha_0\beta_0\|0000\rangle - \alpha_0\beta_1\|0001\rangle - \alpha_0\beta_2\|0010 + \alpha_0\beta_3\|0011\rangle - \alpha_1\beta_0\|0100\rangle + \alpha_1\beta_1\|0101\rangle + \alpha_1\beta_2\|0110\rangle - \alpha_1\beta_3\|0111\rangle - \alpha_2\beta_0\|1000\rangle + \alpha_2\beta_1\|1001\rangle + \alpha_2\beta_2\|1010\rangle - \alpha_2\beta_3\|1011 + \alpha_3\beta_0\|1100\rangle - \alpha_3\beta_1\|1101\rangle - \alpha_3\beta_2\|1110\rangle + \alpha_3\beta_3\|1011\rangle$

Table 4 Applying suitable unitary operation

Bob's results	Alice's results	Alice's operation on qubit b_3	Alice's operation on qubit b_4	Bob's operation on qubit a_3	Bob's operation on qubit a_4
$++$	$++$	I	I	I	I
$++$	$+-$	I	I	I	Z
$++$	$-+$	I	I	Z	I
$++$	$--$	I	I	Z	Z
$+-$	$++$	I	Z	I	I
$+-$	$+-$	I	Z	I	Z
$+-$	$-+$	I	Z	Z	I
$+-$	$--$	I	Z	Z	Z
$-+$	$++$	Z	I	I	I
$-+$	$+-$	Z	I	I	Z
$-+$	$-+$	Z	I	Z	I
$-+$	$--$	Z	I	Z	Z
$--$	$++$	Z	Z	I	I
$--$	$+-$	Z	Z	I	Z
$--$	$-+$	Z	Z	Z	I
$--$	$--$	Z	Z	Z	Z

Now, users reconstruct the two-qubit states according to (8) and (9) and the BQT is successfully finished. Alice's and Bob's qubits are shown in (8) and (9) respectively.

$$(\beta_0|00\rangle + \beta_1|01\rangle + \beta_2|10\rangle + \beta_3|11\rangle)_{a_3a_4} \tag{8}$$

$$(\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle)_{a3a_4} \tag{9}$$

3 Comparison

In this paper, a new BQT protocol is presented where Alice and Bob can transmit an arbitrary two-qubit state to each other via an eight-qubit entangled state as a quantum channel. This protocol is based on the Control-NOT operation, appropriate unitary operations and single-qubit measurement in the Z-basis and ??-basis. This scheme is the first bidirectional protocol without controller that both users can teleport an arbitrary two-qubit state to each other. Table 5 makes a comparison among all previously presented BQT and BCQT protocols. Then, in Table 6 the proposed protocol is compared with other BQT protocols. The efficiency is defined as the ratio of the number of teleported qubits to the number of channel qubits.

Table 5 Comparision of all bidirectional teleportation protocols

Reference	year	Type of protocol	# Bob's qubit	# Alice's qubit	Quantum channel
[25]	2013	BCQT	1	1	5qubit cluster
[26]	2013	BCQT	1	1	6qubit cluster
[27]	2013	BCQT	1	1	6qubit max ent.
[28]	2013	BCQT	1	1	5qubit composite
[29]	2013	BCQT	1	1	5qubit
[30]	2013	BCQT	1	1	5qubit ent.
[31]	2014	BQT	1	1	4qubit cluster
[32]	2014	BCQT	1	1	5qubit ent.
[33]	2014	BCQT	1	1	6qubit ent.
[34]	2014	BCQT	1	1	6qubit ent.
[35]	2015	BCQT	1	1	6qubit genuine
[36]	2015	BCQT	1	1	GHZ-Type
[37]	2015	BCQT	1	1	8qubit max ent.
[38]	2015	BCQT	1	1	6qubit(3Bell)
[39]	2016	BQT	1	1	8-qubit
[40]	2016	BCQT	2	1	7qubit ent.
[41]	2016	BCQT	2	1	7qubit ent.
[42]	2016	BQT	2(EPR)	2(EPR)	6qubit (2GHZ)
[43]	2016	BCQT	2	2	9qubit
[44]	2016	BCQT	1	2	6qubit cluster
Proposed	2016	BQT	2	2	8-qubit ent.

4 Conclusions

The presented protocol is a BQT one which utilizes an eight-qubit entangled state as a quantum channel. The users can teleport an arbitrary two-qubit state each using an eight-qubit channel and only single-qubit measurements. As a future work, the protocol can be extended such that the users teleport an arbitrary number of qubits to each other simultaneously. We hope that such BQT protocols can be realized experimentally in the future.

Table 6 Comparision of BQT protocols

Reference	year	Efficiency	Type of protocol	# Bob's qubit	# Alice's qubit	Quantum channel
[31]	2014	1/2	BQT	1	1	4qubit cluster
[39]	2016	1/4	BQT	1	1	8-qubit
[42]	2016	1/2	BQT	2(EPR)	2(EPR)	6qubit (2GHZ)
Proposed	2016	1/2	BQT	2	2	8-qubit ent.

References

1. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum information. Cambridge University Press, Cambridge (2010)
2. Bennett, C.H., et al.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
3. Cardoso, W.B.: Teleportation of GHZ-states in QED-cavities without the explicit Bell-state measurement. Int. J. Theor. Phys. **47**, 977 (2008)
4. Shi, B.S., Tomita, A.: Teleportation of an unknown state by W state. Phys. Lett. A, 296 (2002)
5. Yang, K., et al.: Quantum teleportation via GHZ-like state. Int. J. Theor. Phys. **48**, 516 (2009)
6. Tian, D., et al.: Teleportation of an arbitrary two-qudit state based on the non-maximally four-qudit cluster state. Sci. China Ser. G Phys. Mech. Astron. **51**, 1523 (2008)
7. Tang, S.Q., et al.: Quantum teleportation of an unknown two-atom entangled state using four-atom cluster state. Int. J. Theor. Phys. **49**, 1899 (2010)
8. Agrawal, P., Pati, A.: Perfect teleportation and super dense coding with W states. Phys. Rev. A **74**, 062320 (2006)
9. Nie, Y.Y., et al.: Perfect teleportation of an arbitrary three-qubit state by using w-class states. Int. J. Theor. Phys. **50**, 3225 (2011)
10. Tsai, C.W., Hwang, T.: Teleportation of a pure EPR state via GHZ-like state. Int. J. Theor. Phys. **49**, 1969 (2010)
11. Nandi, K., Mazumdar, C.: Quantum teleportation of a two qubit state using GHZ-Like state. Int. J. Theor. Phys. **53**, 1322 (2014)
12. Bouwmeester, D., et al.: Experimental quantum teleportation. Nature **390**, 575 (1997)
13. Zhi Z., et al.: Experimental demonstration of five-photon entanglement and open-destination teleportation. Nature **430**, 54 (2004). ISSN 0028-0836
14. Pan, J.W., et al.: Experimental entanglement swapping: entangling photons that never interacted. Phys. Rev. Lett. **80**, 3891 (1998)
15. Zhang, Q., et al.: Experimental quantum teleportation of a two-qubit composite system. Nat. Phys. **2**, 678 (2006)
16. Marcikic I., et al.: Long-distance teleportation of qubits at telecommunication wavelengths. Nature **421**, 509 (2003)
17. Ursin, R., et al.: Communications: quantum teleportation across the Danube. Nature **430**, 849 (2004)
18. Jin X. M., et al.: Experimental free-space quantum teleportation. Nat. Photon. **4**, 376 (2010)
19. Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A **58**, 4394 (1998)
20. Yang, C.P., Han, S.Y.: A scheme for the teleportation of multi qubit quantum information via the control of many agents in a network. Phys. Lett. A **343**, 267 (2005)
21. Deng, F.G., et al.: Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. Phys. Rev. A **72**, 022338 (2005)
22. Wang, Y.H., Song, H.S.: Preparation of partially entangled W state and deterministic multi-controlled teleportation. Opt. Commun. **281**, 489 (2008)
23. Wang, X.W., et al.: Controlled teleportation against uncooperation of part of supervisors. Quantum Inf. Process. **8**, 319 (2009)
24. Wang, T.Y., Wen, Q.Y.: Controlled quantum teleportation with Bell states. Chin. Phys. B **20**, 5 (2011)
25. Zha, X.W., et al.: Bidirectional quantum controlled celeportation via five-qubit cluster state. Int. J. Theor. Phys. **52**, 1740 (2013)
26. Yan, A.: Bidirectional controlled teleportation via six-qubit cluster state. Int. J. Theor. Phys. **52**, 3870 (2013)
27. Sun, X.M., Zha, X.W.: A scheme of bidirectional quantum controlled teleportation via six-qubit maximally entangled state. Acta. Photonica. Sinica. **48**, 1052 (2013)
28. Li, Y.H., Nie, L.P.: Bidirectional controlled teleportation by using a five-qubit composite GHZ-Bell state. Int. J. Theor. Phys. **52**, 1630 (2013)
29. Shukla, C., et al.: Bidirectional controlled teleportation by using 5-qubit states: a generalized view. Int. J. Theor. Phys. **52**, 3790 (2013)
30. Li, Y.H., et al.: Bidirectional controlled quantum teleportation and secure direct communication using five-qubit entangled state Quantum. Inf. Process. **12**, 3835 (2013)

31. Fu, H.Z., et al.: A general method of selecting quantum channel for bidirectional quantum teleportation. Int. J. Theor. Phys. **53**, 1840 (2014)
32. Chen, Y.: Bidirectional controlled quantum teleportation by using five-qubit entangled state. Int. J. Theor. Phys. **53**, 1454 (2014)
33. Duan, Y.J., et al.: Bidirectional quantum controlled teleportation via a maximally seven-qubit entangled state. Int. J. Theor. Phys. **53**, 2697 (2014)
34. Duan, Y.J., Zha, X.W.: Bidirectional quantum controlled teleportation via a six-qubit entangled state. Int. J. Theor. Phys. **53**, 3780 (2014)
35. Chen, Y.: Bidirectional quantum controlled teleportation by using a genuine six-qubit entangled state. Int. J. Theor. Phys. **54**, 269 (2015)
36. Wang, J.W., Shu, L.: Bidirectional quantum controlled teleportation of qudit state via partially entangled GHZ-type states. Int. J. Modern Phys. B, Condensed Matter Phys. **29** (2015)
37. Zhang, D., et al.: Bidirectional and asymmetric quantum controlled teleportation via maximally eight-qubit entangled state. Quantum Inf. Process. **14**, 3835 (2015)
38. Hassanpour, Sh., Houshmand, M.: Bidirectional quantum controlled teleportation by using EPR states and entanglement swapping. In: 23th Iranian Conference on Electrical Engineering (ICEE) (2015)
39. Kiktenko, E.O., et al.: Bidirectional imperfect quantum teleportation with a single Bell state. Phys. Rev. A **93**, 0623305 (2016)
40. Hong, W.Q.: Asymmetric bidirectional controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. **55**, 384 (2016)
41. Sang, M.H.: Bidirectional quantum controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. **55**, 380 (2016)
42. Hassanpour, Sh., Houshmand, M.: Bidirectional teleportation of a pure EPR state by using GHZ states. Quantum Inf. Process. **15**, 905 (2016)
43. Li, Y.H., Jin, X.M.: Bidirectional controlled teleportation by using nine-qubit entangled state in noisy environments. Quantum Inf. Process. **15**, 929 (2016)
44. Li, Y.H., et al.: Asymmetric bidirectional controlled teleportation by using six-qubit Cluster state. Int. J. Theor. Phys., 553008 (2016)