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A Note on Holographic Dark Energy with Varying c^2 Term

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Abstract We reconsider the holographic dark energy (HDE) model with a slowly time varying $c^2(z)$ parameter in the energy density, namely $\rho_D = 3M_p^2 c^2(z)/L^2$, where L is the IR cutoff and z is the redshift parameter. As the system's IR cutoff we choose the Hubble radius and the Granda-Oliveros (GO) cutoffs. The latter inspired by the Ricci scalar curvature. We derive the evolution of the cosmological parameters such as the equation of state and the deceleration parameters as the explicit functions of the redshift parameter z. Then, we plot the evolutions of these cosmological parameters in terms of the redshift parameter during the history of the universe. Interestingly enough, we observe that by choosing $L = H^{-1}$ as the IR cutoff for the HDE with time varying $c^2(z)$ term, the present acceleration of the universe expansion can be achieved, even in the absence of interaction between dark energy and dark matter. This is in contrast to the usual HDE model with constant c^2 term, which leads to a wrong equation of state, namely that for dust $w_D = 0$, when the IR cutoff is chosen the Hubble radius.

Keywords Holographic dark energy · Varying c term · Redshift

1 Introduction

According to nowadays observations, present acceleration of the universe expansion has been well established [1–4]. Within the framework of general relativity (GR), the responsible component of energy for this accelerated expansion is known as *dark energy* (DE) with

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negative pressure. However, the nature of DE is still unknown, and some candidates have been proposed to explain it. The earliest and simplest candidate is the cosmological constant with the time independent equation of state $\omega_\Lambda = -1$ which has some problems like fine-tuning and coincidence problems. Therefore, other theories have been suggested for the dynamical DE scenario to describe the accelerating universe.

An interesting attempt for probing the nature of DE within the framework of quantum gravity, is the so-called HDE proposal. This model which has arisen a lot of enthusiasm recently [5–33], is motivated from the holographic hypothesis [34, 35] and has been tested and constrained by various astronomical observations [36–40]. In holographic principle a short distance cutoff could be related to a long distance cutoff (infrared cutoff) due to the limit set by formation of a black hole. Based on the holographic principle, it was shown by Cohen et al. [5] that the quantum zero-point energy of a system with size L should not exceed the mass of a black hole with the same size, i.e.,

$$L^3 \rho_D \le L M_p^2, \tag{1}$$

where $M_p^2 = 8\pi G$ is the reduced Planck mass and L is the IR cutoff. The largest L allowed is the one saturating this inequality so that we get the

$$\rho_D = 3c^2 M_p^2 / L^2, \tag{2}$$

where c^2 is a dimensionless model parameter. There are many models of HDE, depending on the IR cutoff, that have been studied in literatures [41–47]. The simple choice for IR cutoff is the Hubble radius, i.e., $L = H^{-1}$ which leads to a wrong equation of state (EoS) and the accelerated expansion of the universe cannot be achieved. However, as soon as an interaction between HDE and dark matter is taken into account, the identification of IR cutoff with Hubble radius H^{-1}, in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem [48, 49]. Then, Li [6] showed that taking the particle horizon radius as IR cutoff it is impossible to obtain an accelerated expansion. He also demonstrated that the identification of L with the radius of the future event horizon gives the desired result, namely a sufficiently negative equation of state to obtain an accelerated universe.

It is worth noting that, for the sake of simplicity, very often the c^2 parameter in the HDE model is assumed constant. However, there are no strong evidences to demonstrate that c^2 should be a constant and one should bear in mind that it is more general to consider it a slowly varying function of time. It has been shown that the parameter c^2 can play an essential role in characterizing the model. For example, it was argued that the HDE model in the far future can be like a phantom or quintessence DE model depending whether the parameter c^2 is larger or smaller than 1, respectively. By slowly vary function with time, we mean that $(\dot{c^2})/(c^2)$ is upper bounded by the Hubble expansion rate, i.e., [50]

$$\frac{(c^2)^{\cdot}}{c^2} \le H, \tag{3}$$

where dot indicates derivative with respect to the cosmic time. In this case the time scale of the evolution of c^2 is shorter than H^{-1} and one can be satisfied to consider the time dependency of c^2 [50]. Considering the future event horizon as IR cutoff, the HDE model with time varying parameter c^2, has been studied in [51]. It was argued that depending on the parameter c^2, the phantom regime can be achieved earlier or later compared to the usual HDE with constant c^2 term. In this paper, we reconsider the HDE model with the slowly varying parameter $c^2(z)$ by taking into account the Hubble horizon $L = H^{-1}$ and GO cut-

off, $L = (\alpha H^2 + \beta \dot{H})^{-1/2}$, as the system's IR cutoffs. We shall study four parameterizations of $c(z)$ as follows

$$\text{GHDE1}: \quad c(z) = c_0 + c_1 \frac{z}{(1+z)}, \tag{4}$$

$$\text{GHDE2}: \quad c(z) = c_0 + c_1 \frac{z}{(1+z)^2}, \tag{5}$$

$$\text{GHDE3}: \quad c(z) = \frac{c_0}{1 + c_1 \ln(1+z)}, \tag{6}$$

$$\text{GHDE4}: \quad c(z) = c_0 + c_1 \left(\frac{\ln(2+z)}{1+z} - \ln 2 \right), \tag{7}$$

where GHDE stands for the Generalized HDE model. The above choices for $c(z)$ are inspired by the parameterizations known as Chevallier-Polarski-Linder parametriza-tion (CPL) [52, 53], Jassal-Bagla-Padmanabhan (JBP) parametrization [54], Wetterich parametrization [55, 56], and Ma-Zhang parametrization [57], respectively. Setting $c_1 = 0$ in all these four parameterizations, the original HDE with constant c parameter is recovered.

The paper is organized as follows. In section II, we drive the basic equations for the HDE with time varying c^2 parameter. In this section, we consider the Hubble radius as IR cutoff and derive the evolution of equation of state and deceleration parameters by choosing $c(z)$. In section III, we repeat the study for the GO cutoff and investigate the evolution of the cosmological parameters. The last section is devoted to conclusion and discussions.

2 GHDE in Flat FRW Cosmology with Hubble Radius as IR Cutoff

In the context of flat Friedmann-Robertson-Walker (FRW) cosmology, the Friedmann equation is given by

$$H^2 = \frac{1}{3M_p^2}(\rho_m + \rho_D), \tag{8}$$

where ρ_m and ρ_D are the energy densities of pressureless dark matter and dark energy, respectively. By using the dimensionless energy densities

$$\Omega_m = \frac{\rho_m}{3m_p^2 H^2}, \qquad \Omega_D = \frac{\rho_D}{3m_p^2 H^2}, \tag{9}$$

the Friedmann equation (8) can be written as

$$\Omega_m + \Omega_D = 1. \tag{10}$$

We shall assume there is no interaction between dark matter and GHDE. Therefore, both component obey the equation of conservation respectively. The conservation equations for pressureless dark matter and dark energy, are, respectively, given by

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{11}$$

$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = 0, \tag{12}$$

where $w_D = p_D/\rho_D$ is the EoS parameter of GHDE. In this section we consider the Hubble radius as IR cut off,($L = H^{-1}$), thus the energy density of GHDE model from (2) can be written as

$$\rho_D = 3M_p^2 c^2(z)H^2. \tag{13}$$

Using definition (9), the dimensionless energy density for the GHDE becomes

$$\Omega_D(z) = c^2(z). \tag{14}$$

Taking the time derivative of (13), we find

$$\dot{\rho}_D = 2\rho_D \left(\frac{\dot{c}(z)}{c(z)} + \frac{\dot{H}}{H} \right). \tag{15}$$

Besides, if we take the time derivative of Friedmann equation (8), after using (10), (11) and (15), we find

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} + \frac{c'(z)c(z)}{1 - c^2(z)}, \tag{16}$$

where $c' = \dot{c}/H$ and the prime represents the derivative with respect to $x = \ln a$. Combining (15) and (16) with (12), one can obtain the EoS parameter of GHDE as

$$\omega_D = -\frac{2}{3} \frac{c'(z)}{c(z)[1 - c^2(z)]}, \tag{17}$$

Clearly, for constant parameter we have, $c'(z) = 0$ which leads to a wrong equation of state, namely that for dust with $\omega_D = 0$ [8]. This implies that the HDE with $L = H^{-1}$ as cutoff cannot described an accelerating universe [8]. However, taking the time varying c^2 term in the energy density of the HDE, it is quite possible to reproduce the acceleration of the cosmic expansion.

Another important cosmological parameter for studying the evolution of the universe is the deceleration parameter which is given by

$$q = 1 - \frac{\dot{H}}{H^2}. \tag{18}$$

Substituting (16) into (18) yields

$$q = \frac{1}{2} - \frac{c'(z)c(z)}{1 - c^2(z)}. \tag{19}$$

Again for $c'(z) = 0$ the declaration parameter reduces to $q = 1/2 > 0$ which implies a decelerated universe.

We see from (17) and (19) that the evolution of these cosmological parameters depend on the functional form of $c(z)$. In what follow, we consider four types of parametrization for $c(z)$ as given in (4)–(7).

2.1 GHDE1: the CPL Type

We start with the CPL type for $c(z)$, namely

$$c(z) = c_0 + c_1 \frac{z}{(1 + z)}. \tag{20}$$

When $z \to \infty$ (in the early universe), we see that $c \to c_0 + c_1$ and as $z \to 0$ (at the present time), $c \to c_0$. Thus holographic parameter c varies slowly from $c_0 + c_1$ to c_0 from past to present. On the other hand requiring the fact that the energy density of GHDE1 is positive, we need to have the conditions $c_0 > 0$ and $c_0 + c_1 > 0$.

Using the fact that $a/a_0 = (1+z)^{-1}$, where z is the redshift parameter and prime denotes the derivative with respect to $x = \ln a$, we arrive at $c' = -(1+z)dc/dz$. Taking derivatives of (20), it follows that

$$c' = -(1 + z)\frac{dc}{dz} = -\frac{c_1}{1 + z}. \tag{21}$$

Substituting (20) and (21) into (17), the EoS parameter is obtained as

$$\omega_D(z) = \frac{2}{3} \frac{c_1(1+z)^2}{[c_0(1+z)+c_1z](1+z)^2 - [c_0(1+z)+c_1z]^3}. \tag{22}$$

Combining (20) and (21) with (19), we can obtain the deceleration parameter as

$$q(z) = \frac{1}{2} + \frac{c_0c_1(1+z)+c_1^2z}{(1+z)^2 - [c_0(1+z)+c_1z]^2}. \tag{23}$$

The behavior of $\omega_D(z)$ and $q(z)$ are plotted for different values of model parameters c_0 and c_1 in Fig. 1. From these figures we see that our Universe has a transition from deceleration to the acceleration phase around $z \approx 0.6$ which is consistent with observations [60–65].

2.2 GHDE2: the JBP Model

The second parametrization of $c(z)$ is the JBP parametrization which is written as

$$c(z) = c_0 + c_1 \frac{z}{(1+z)^2} \tag{24}$$

From (24) we see that at the late time where $z \to 0$, we have $c(z) \to c_0$, and as $z \to 1$, we have $c(z) \to c_0 + c_1/4$. Besides, in the early universe where $z \to \infty$, we have $c(z) \to c_0$.

Taking derivative of (24) we find

$$c'(z) = -c_1 \frac{1-z}{(1+z)^2}. \tag{25}$$

Substituting (24) and (25) into (17), we obtain the EoS parameter for GHDE2 model as

$$\omega_D = \frac{2}{3} \frac{c_1(1-z)(1+z)^4}{[c_0(1+z)^2+c_1z][(1+z)^4 - (c_0(1+z)^2+c_1z)^2]}. \tag{26}$$

Combining (24) and (25) with (19) yields

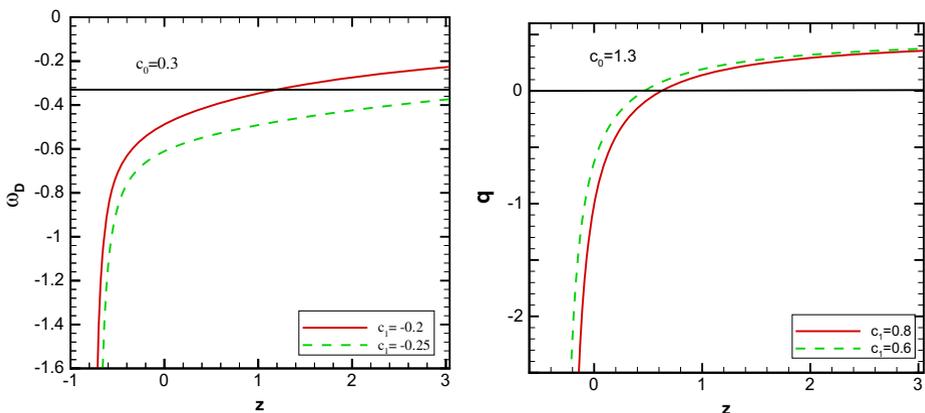$$q = \frac{1}{2} + \frac{c_1(1-z)[c_0(1+z)^2+c_1z]}{(1+z)^4 - [c_0(1+z)^2+c_1z]^2} \tag{27}$$

Fig. 1 The evolution of EoS parameter ω_D and the deceleration parameter q versus redshift z for GHDE1 model with $L = H^{-1}$

The behavior of $\omega_D(z)$ and $q(z)$ are plotted for different values of model parameters c_0 and c_1 in Fig. 2. Again, the universe has a transition from deceleration to the acceleration phase around $z \approx 0.6$ and at the late time the EoS parameter can cross the phantom line $w_D = -1$.

2.3 GHDE3: the Wetterich Type

The third parametrization is Wetterich-type which has the following form of $c(z)$:

$$c(z) = \frac{c_0}{1 + c_1 \ln(1 + z)}. \tag{28}$$

In this model, at the late time where $z \to 0$, we have $c(z) \to c_0$ and at the early universe where $z \to \infty$, we get $c(z) \to 0$. It follows directly that,

$$c'(z) = -(1 + z)\frac{dc(z)}{dz} = \frac{c_0 c_1}{[1 + c_1 \ln(1 + z)]^2}. \tag{29}$$

Inserting (28) and (29) into (17), one can get

$$\omega_D(z) = -\frac{2}{3}\frac{c_1[1 + c_1 \ln(1 + z)]}{[1 + c_1 \ln(1 + z)]^2 - c_0^2}. \tag{30}$$

Combining (19), (28) and (29), we find

$$q(z) = \frac{1}{2} + \frac{c_0^2 c_1}{[1 + c_1 \ln(1 + z)]\left(c_0^2 - [1 + c_1 \ln(1 + z)]^2\right)}. \tag{31}$$

We have plotted the evolutions of $\omega_D(z)$ and $q(z)$ in term of redshift parameter z in Fig. 3. From these figures it is obvious that the present acceleration can be addressed in this model and the transition from deceleration to the acceleration phase occurs around $z \approx 0.6$.

2.4 GHDE4: the Ma-Zhang Type

The last choice for the parametrization of $c(z)$ was proposed in [57] and can be written as

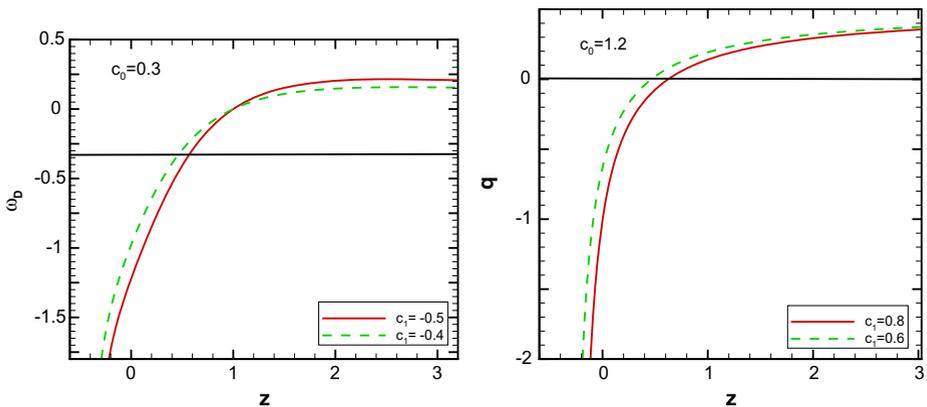$$c(z) = c_0 + c_1\left(\frac{\ln(2 + z)}{1 + z} - \ln 2\right). \tag{32}$$

Fig. 2 The evolution of EoS parameter ω_D and the deceleration parameter q versus redshift z for GHDE2 with $L = H^{-1}$
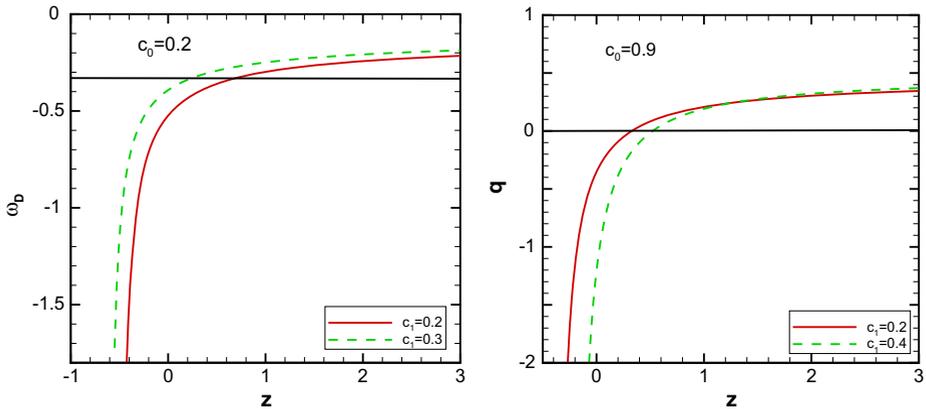
Fig. 3 The evolution of ω_D and q versus redshift z for GHDE3 with $L = H^{-1}$

At the present time where $z \to 0$ we have $c(z) \to c_0$, and at the early time where $z \to \infty$, one gets $c(z) \to c_0 - c_1 \ln 2$. It is worth noting that for the previous choice of $c(z)$, we could not investigate the future behavior of $c(z)$, because it diverges at the future time where $z \to -1$. However, in case of the Ma-Zhang parametrization we have $c(z) \to c_0 + (1 - c_1 \ln 2)$ as $z \to -1$. From (32) it is easy to show that

$$c'(z) = -(1+z)\frac{dc(z)}{dz} = c_1 \frac{(2+z)\ln(2+z) - (1+z)}{(1+z)(2+z)}. \tag{33}$$

Combining (32) and (33) with (17) and (19), we arrive at

$$\omega_D(z) = \frac{2}{3} \frac{c_1(1+z)^2[(1+z) - (2+z)\ln(2+z)]}{(2+z)(c_0(1+z)+c_1\ln(2+z)-c_1(1+z)\ln2)[(1+z)^2-[c_0(1+z)+c_1\ln(2+z)-c_1(1+z)\ln2]^2]}, \tag{34}$$

$$q(z) = \frac{1}{2} + \frac{c_1[c_0(1+z)+c_1\ln(2+z)-c_1(1+z)\ln 2][1+z-(2+z)\ln(2+z)]}{(2+z)\left((1+z)^2 - [c_0(1+z)+c_1\ln(2+z)-c_1(1+z)\ln 2]^2\right)}. \tag{35}$$

In order to have an insight on the behaviour of these functions, we plot them in term of z in Fig. 4. From these figures, we see that the behaviour is similar to the previous parameterizations.

3 GHDE in Flat Universe with GO Cutoff

In this section we consider the GO cutoff as the system's IR cutoff, namely $L = (\alpha H^2 + \beta \dot{H})^{-1/2}$ which first proposed in [59]. With this IR cutoff, the energy density (2) is written

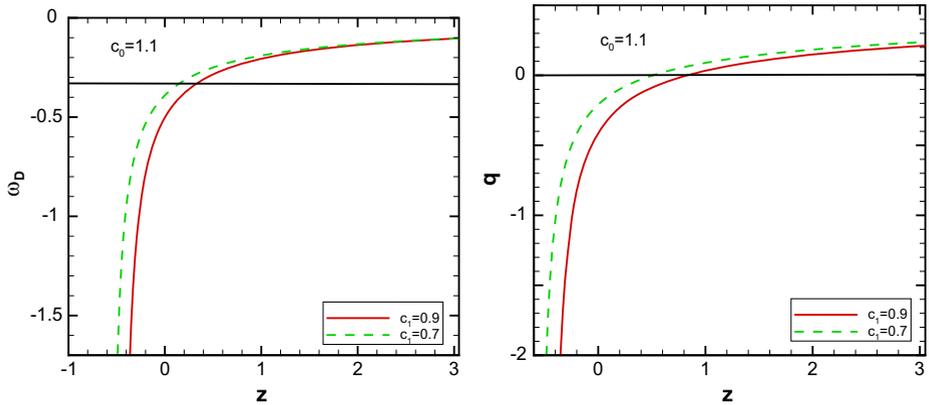$$\rho_D = 3M_p^2 c^2(z)\left(\alpha H^2 + \beta \dot{H}\right), \tag{36}$$

Fig. 4 The evolution of EoS parameter ω_D and the deceleration parameter q versus redshift z for GHDE4 with $L = H^{-1}$

where α and β are constants that should be constrained by observational data. Using the definition of density parameter (10) one can obtain

$$\Omega_D = c^2(z)\left(\alpha + \beta\frac{\dot{H}}{H^2}\right). \tag{37}$$

Taking the time derivative of (37), we get

$$\dot{\Omega}_D = 2\Omega_D\left(\frac{\dot{c}(z)}{c(z)} - \frac{\dot{H}}{H}\right) + \frac{c^2(z)}{H^2}\left(2\alpha\dot{H}H + \beta\ddot{H}\right). \tag{38}$$

Now taking the time derivative of both sides of (8) and using (36) one can obtain

$$c^2H^{-3}(2\alpha\dot{H}H + \beta\ddot{H}) = 2\frac{\dot{H}}{H^2} + 3(1 - \Omega_D) - 2\Omega_D\frac{\dot{c}(z)}{Hc(z)}. \tag{39}$$

Combining (38) and (39) the equation of motion for the dimensionless GHDE density can be written as

$$\dot{\Omega}_D = (1 - \Omega_D)\left(\frac{2\dot{H}}{H} + 3H\right). \tag{40}$$

By help of (37) and using the fact that $\dot{\Omega}_D = H\Omega'_D$, the evolution of dimensionless GHDE density may be rewritten as

$$\Omega'_D(z) = (1 - \Omega_D)\left(\frac{2\Omega_D}{\beta c^2(z)} - \frac{2\alpha}{\beta} + 3\right), \tag{41}$$

where the prime denotes derivative with respect to $x = \ln a$. Taking the time derivative of both sides of (8) and using (12) and (37) we can obtain the EoS parameter of GHDE model as follows

$$\omega_D(z) = \frac{2\alpha}{3\beta\Omega_D} - \frac{2}{3\beta c^2(z)} - \frac{1}{\Omega_D}. \tag{42}$$

Substituting (37) into (18), we get

$$q(z) = -1 - \frac{\Omega_D}{\beta c^2(z)} + \frac{\alpha}{\beta}. \tag{43}$$

Following the previous section, we shall consider four types of parametrization of $c(z)$ listed in (4)–(7), respectively.

3.1 GHDE1: The CPL type

Substituting (4) in (41), the equation of the evolutionary of dimensionless GHDE1 density is obtained as

$$\Omega'_D = (1 - \Omega_D)\left(\frac{2\Omega_D(1+z)^2}{\beta(c_0(1+z) + c_1 z)^2} - \frac{2\alpha}{\beta} + 3\right) \tag{44}$$

The evolution of the dimensionless GHDE density parameter Ω_D as a function of $1 + z = a^{-1}$ is shown in Fig. 5. From this figure we see that at the early universe where $z \to \infty$ we have $\Omega_D \to 0$, while at the late time where $z \to -1$, the DE dominated, namely $\Omega_D \to 1$.

Using (4), (42) and (43) we can obtain the equation of state and deceleration parameteres as

$$\omega_D(z) = \frac{2\alpha}{3\beta\Omega_D} - \frac{2(1+z)^2}{3\beta[c_0(1+z) + c_1 z]^2} - \frac{1}{\Omega_D}, \tag{45}$$

$$q(z) = -1 - \frac{\Omega_D(1+z)^2}{\beta(c_0(1+z) + c_1 z)^2} + \frac{\alpha}{\beta}. \tag{46}$$

The evolution of EoS parameter ω_D and deceleration parameter q are shown numerically in Fig. 6. where we fix $c_0 = 2.7, c_1 = -0.7$ and set different values of α and β. From these figures we clearly see that we have a transition from deceleration to acceleration universe phase around $z \approx 0.6$ which is compatible with observations [63–65].

3.2 GHDE2: the JBP Model

Combining (5) and (41) one can derive he evolution of dimensionless GHDE2 density as

$$\Omega'_D = (1 - \Omega_D)\left(\frac{2\Omega_D(1+z)^4}{\beta(c_0(1+z)^2 + c_1 z)^2} - \frac{2\alpha}{\beta} + 3\right) \tag{47}$$
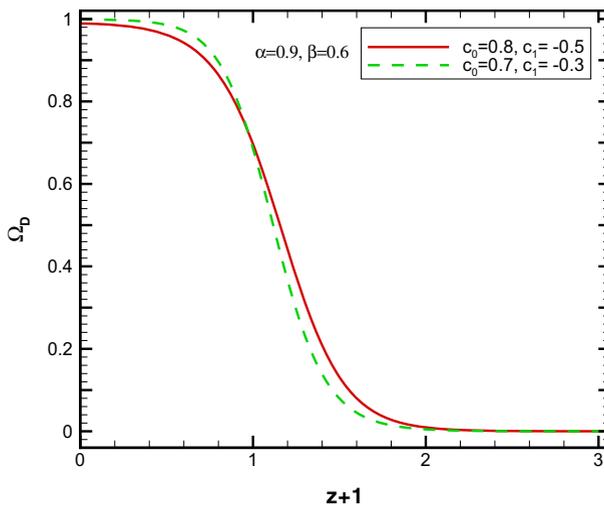

Fig. 5 The evolution of the dimensionless density parameter Ω_D versus redshift z for GHDE1 model with GO cutoff
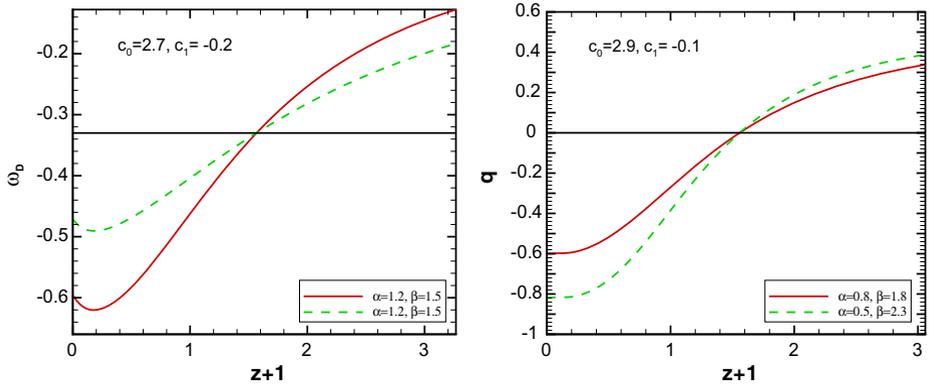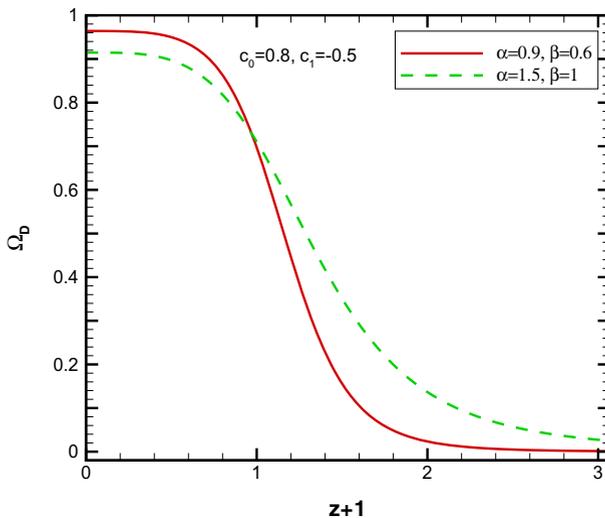
Fig. 6 The evolution of equation of state parameter ω_D and deceleration parameter q versus $1+z$ for GHDE1 model with GO cutoff

The evolution of the dimensionless GHDE density parameter Ω_D as a function of redshift z is shown in Fig. 7. Using (24), (42) and (43) we can obtain the equation of state and deceleration parameters as

$$\omega_D(z) = \frac{2\alpha}{3\beta\Omega_D} - \frac{2(1+z)^4}{3\beta[c_0(1+z)^2 + c_1 z]^2} - \frac{1}{\Omega_D}, \tag{48}$$

$$q(z) = -1 - \frac{\Omega_D(1+z)^4}{\beta[c_0(1+z)^2 + c_1 z]^2} + \frac{\alpha}{\beta}. \tag{49}$$

Fig. 7 The evolution of the dimensionless density parameter Ω_D versus $1 + z$ for GHDE2 model with GO cutoff
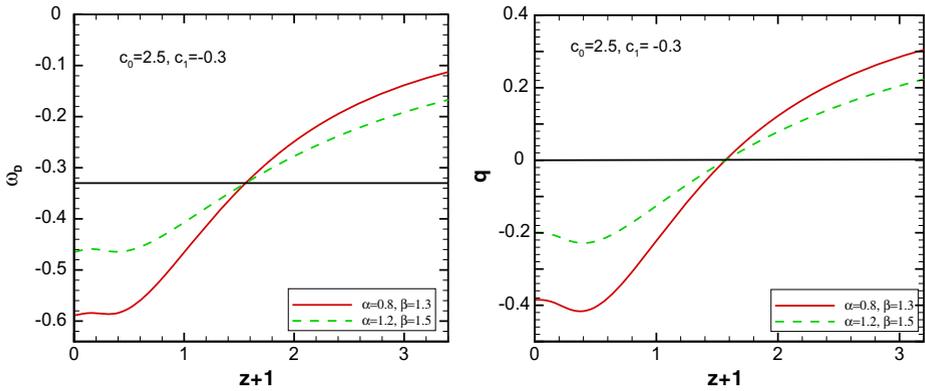
Fig. 8 The evolution of equation of state parameter ω_D and deceleration parameter q versus $1+z$ for GHDE2 model with GO cutoff

The behavior of the equation of state parameter ω_D and deceleration parameter q are plotted in Fig. 8. Again, our universe has a phase transition during its history from a decelerated to an acceleration phase.

3.3 GHDE3: the Wetterich Type

Using (6) and (41) we obtain the evolution of dimensionless GHDE3 density as

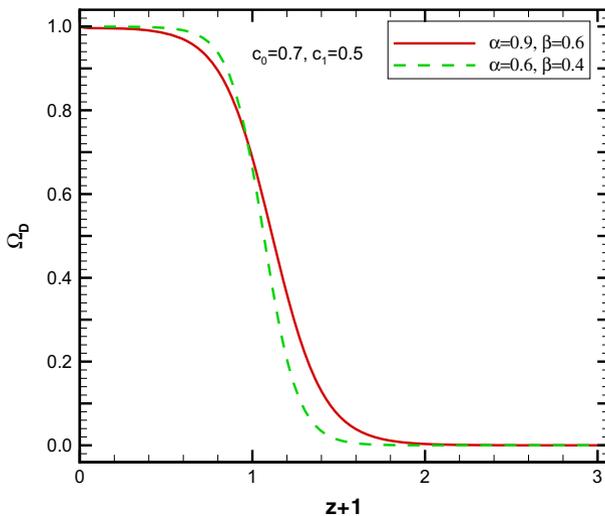$$\Omega_D' = (1 - \Omega_D)\left(\frac{2\Omega_D(1 + c_1 \ln(1+z))^2}{\beta c_0^2} - \frac{2\alpha}{\beta} + 3\right) \qquad (50)$$

Fig. 9 The evolution of the dimensionless density parameter Ω_D versus redshift $1 + z$ for GHDE3 model with GO cutoff
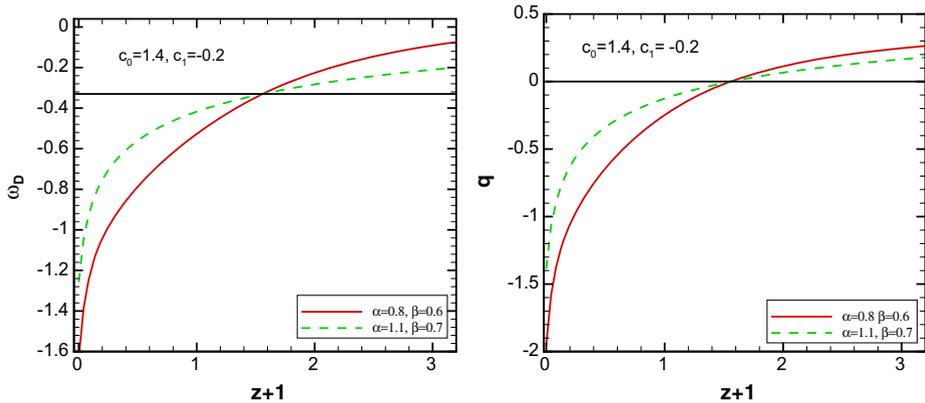
Fig. 10 The evolution of equation of state parameter ω_D and deceleration parameter q versus $1 + z$ for GHDE3 model with GO cutoff

The evolution of the dimensionless GHDE density parameter Ω_D as a function of redshift z is shown in Fig. 9. Inserting (28) in (42) and (43) we can obtain the equation of state and deceleration parameters as follows

$$\omega_D(z) = \frac{2\alpha}{3\beta\Omega_D} - \frac{2[1 + c_1 \ln(1 + z)]^2}{3\beta c_0^2} - \frac{1}{\Omega_D}, \tag{51}$$

$$q(z) = -1 - \frac{\Omega_D(1 + c_1 \ln(1 + z))^2}{\beta c_0^2} + \frac{\alpha}{\beta}. \tag{52}$$

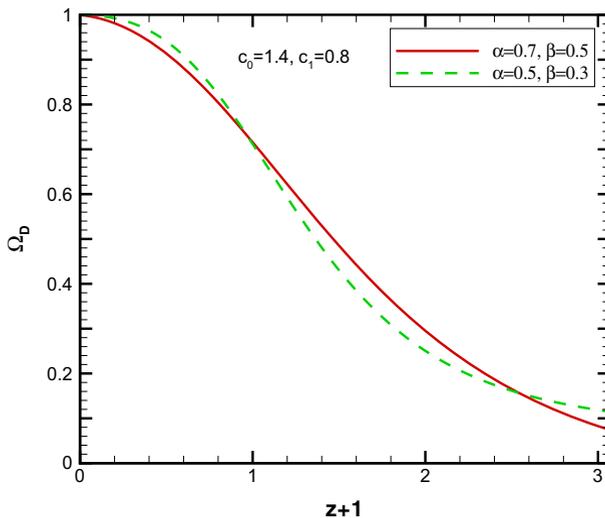The behavior of the EoS parameter ω_D and deceleration parameter q are plotted in Fig. 10.

Fig. 11 The evolution of the dimensionless density parameter Ω_D versus $1 + z$ for GHDE4 model with GO cutoff
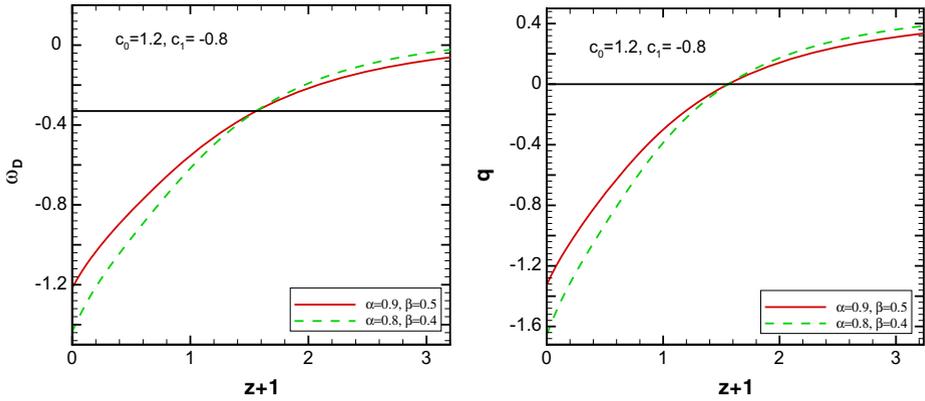
Fig. 12 The evolution of equation of state parameter ω_D and deceleration parameter q versus $1 + z$ for GHDE4 model with GO cutoff

3.4 GHDE4: the Ma-Zhang Type

Using (7) and (41), it is easy to show that the evolution of the dimensionless GHDE4 density can be obtained as

$$\Omega'_D = (1 - \Omega_D)\left(\frac{2\Omega_D(1+z)^2}{\beta[c_0(1+z) + c_1\ln(2+z) - c_1(1+z)\ln 2]^2} - \frac{2\alpha}{\beta} + 3\right). \quad (53)$$

Figure 11 shows the evolution of the dimensionless GHDE density parameter Ω_D. Again, at the early universe where $z \to \infty$ we have $\Omega_D \to 0$, while at the late time where $z \to -1$,
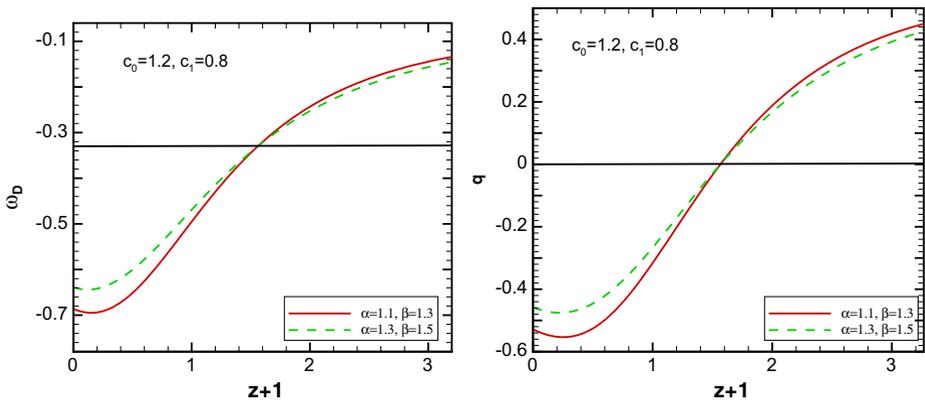

Fig. 13 The evolution of equation of state parameter ω_D and deceleration parameter q versus $1 + z$ for GHDE4 model with GO cutoff

the DE dominated, namely $\Omega_D \to 1$. Inserting (7) in (42) and (43) we can obtain the EoS and the deceleration parameters as follows

$$\omega_D(z) = \frac{2\alpha}{3\beta\Omega_D} - \frac{2(1+z)^2}{3\beta[c_0(1+z) + c_1\ln(2+z) - c_1(1+z)\ln 2]^2} - \frac{1}{\Omega_D}, \quad (54)$$

$$q(z) = -1 - \frac{\Omega_D(1+z)^2}{\beta[c_0(1+z) + c_1\ln(2+z) - c_1(1+z)\ln 2]^2} + \frac{\alpha}{\beta}. \quad (55)$$

The behavior of the EoS parameter ω_D and the deceleration parameter q are plotted in Figs. 12 and 13. When we fix $1 \leq c_0 \leq 2$ and $-1 \leq c_1 \leq 0$ and vary the parameter $0 \leq \alpha, \beta \leq 1$ the GHDE4 can behave as phantom DE model at the future, while for $c_0, c_1 > 0$ and $1 < \alpha, \beta < 2$ the EoS parameter cannot cross the phantom line and is always larger than -1.

4 Conclusion and Discussion

In this paper, we have studied HDE with time varying parameter c^2, the so-called generalized holographic dark energy (GHDE), in a spatially flat universe. It is important to note that, for the sake of simplicity, very often the c^2 parameter in the HDE model is assumed constant. However, in general it can be regarded as a function of redshift parameter z during the history of the universe. By choosing four parameterizations of $c(z)$, including the CPL type, JBP type, Wetterich type and Ma-Zhang type parameterizations for $c(z)$, we have investigated the effects of varying $c^2(z)$ term on the cosmological evolutions of the HDE model. As system's IR cutoff we considered the Hubble radius $L = H^{-1}$ and the GO cutoff $L = (\alpha H^2 + \beta \dot{H})^{-1/2}$ inspired by the Ricci scalar curvature. We have investigated the evolution of EoS and deceleration parameters for all these parameterizations. We found that all GHDE models, with both Hubble and GO cutoffs, can lead to an accelerated expanding Universe.

The simple and natural choice for the system's IR cutoff in the HDE model is the Hubble radius $L = H^{-1}$. However, it was argued that this choice for the IR cutoff leads to a wrong equation od state for dark energy, namely $\omega_D = 0$ [8], unless the interaction between two dark components of the universe is taken into account [48, 49]. In this paper, we demonstrated that by taking into account the time varying parameter $c(z)$ can leads to an accelerated universe for $L = H^{-1}$ IR cutoff, even in the absence of the interaction between two dark components of energy of the universe. Besides, by suitably choosing of the parameter, not only the accelerated universe can be achieved, but also the EoS parameter can cross the phantom line $\omega_D = -1$, even in the absence of interaction. As far as we know, this is a new result, which has not been reported already. In order to more investigate the behavior of the EoS and deceleration parameters, we plotted the evolution of these parameters versus redshift parameter z. From these figures we see that has a decelerated phase at the early time ($z \to \infty$) and encounters a phase transition to an accelerated phase around $z \approx 0.6$ which is consistent with recent observations [60–65].

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