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Effects of Noise on Joint Remote State Preparation of an Arbitrary Equatorial Two-Qubit State

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Abstract By using a six-qubit cluster state as the quantum channel, we investigat the joint remote state preparation of an arbitrary equatorial two-qubit state. We analytically obtain the fidelities of the joint remote state preparation process in noisy environments, such as the amplitude-damping noise and phase-damping noise. In our scheme, the two different noise including amplitude-damping noise and the phase-damping noise only affect the travel qubits of the quantum channel, and then we show that the fidelities in these two noisy cases only depend on the decoherence noisy rate.

Keywords Joint remote state preparation · Six-qubit cluster state · Amplitude-damping noise · Phase-damping noise

1 Introduction

Remote state preparation (RSP) [1] is a new quantum information processing protocol that uses a classical communication way and a previously shared multi-qubit entangled state to teleport an unknown quantum state. In the RSP protocol, all information of the teleported state is assumed to be completely known by the sender. As ones know, the teleported state is not needed to be known for the sender in quantum teleportation (QT) protocol [2–15]. Moreover, RSP requires less classical communication bits than QT. Due to its interesting properties, several RSP protocols have been widely studied both theoretically [16–22] and experimentally [23–28] in recent years.

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It is known that RSP and QT are two-party teleportation protocols (one sender plus one receiver), and joint remote state preparation (JRSP) is a three-party teleportation protocol (two senders and one receiver). Compared with RSP, in JRSP neither of the senders can know the full information of the quantum state to be prepared. Several novel JRSP protocols have been proposed for different types of quantum states via a variety of quantum entangled channels [29–41]. In 2015, a new and probabilistic JRSP protocol for preparing an arbitrary equatorial two-qubit state $|\Phi\rangle = \frac{1}{2}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)$ was demonstrated by Choudhury and Dhara by using two Greenberger-Horne-Zeilinger (GHZ) states as the shared quantum channel [40], where the two-qubit projective measurements and the appropriate unitary operations are needed. In this protocol, one can obtain the desired state with 25 % success probability, and if and only if one of the parties holding the partial information transmits extra classical communication bits to the receiver, the success probability can be increased to 50 %. In a recent paper [41], Li and Ghose proposed a new protocol for optimal JRSP of an arbitrary equatorial two-qubit state by using the same quantum channel, where the probability of success is 100 % by selecting four different two-qubit measurement bases.

N-qubit ($N > 3$) cluster states are maximally connected with the better persistency than N-qubit GHZ states [42, 43], which also are robust against decoherence [44]. In this paper, we have demonstrated that a six-qubit cluster state could be used to realize the deterministic JRSP of an arbitrary equatorial two-qubit state by performing only the two-qubit projective measurements and the appropriate unitary operations. And the receiver could deterministically obtained the desired state and reconstructed the desired state with 100 % success probability. Moreover, we analytically obtained the fidelities of the JRSP process under the amplitude-damping noise and phase-damping noise. The noisy environments only affected the travel qubits of the quantum channel. Interestingly, we showed that the fidelities in these two cases only depend on the decoherence noisy rate, but had nothing to do with the phase information. At last, we made some discussions for these two cases to show that in which noise more information of the prepared state would be lost.

2 JRSP of an Arbitrary Equatorial Two-Qubit State

Assume the senders Alice and Bob want to help the remote receiver Charlie prepare an arbitrary equatorial two-qubit state, which is described as

$$|\Phi\rangle = \frac{1}{2}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right), \tag{1}$$

where $\delta_i = a_i + b_i$, and a_i's and b_i's are real for $i = 1, 2, 3$. Herein, the phase information of the initial state $|\Phi\rangle$ is known partially to Alice and Bob, and Alice knows complete knowledge of the real coefficients a_i, while Bob has the real coefficients b_i. Charlie cannot obtain any knowledge about the desired state $|\Phi\rangle$ at all. Thus, neither Alice nor Bob alone could help Charlie to obtain the prepared state. In our scheme, the quantum channel is a six-qubit cluster state,

$$|\psi\rangle_{123456} = \frac{1}{2}\left(|000000\rangle + |000111\rangle + |111000\rangle - |111111\rangle\right)_{123456}, \tag{2}$$

where Alice has the qubits 1 and 4, qubits 2 and 5 belong to Bob and Charlie own the qubits 3 and 6, respectively.

In order to help the receiver Charlie to prepare the quantum state $|\Phi\rangle$, Alice and Bob must select some appropriate two-qubit projective measurement bases to measure their qubits, respectively. The forms of Alice's and Bob's measurement bases are shown as follows:

Alice selects $\left\{\left|\varphi^j\right\rangle_{14}\right\}$ $(j = 1, 2, 3, 4)$ as her local measurement basis:

$$\left|\varphi^1\right\rangle = \frac{1}{2}(|00\rangle + e^{-ia_1}|01\rangle + e^{-ia_2}|10\rangle + e^{-ia_3}|11\rangle), \tag{3}$$

$$\left|\varphi^2\right\rangle = \frac{1}{2}(|00\rangle + e^{-ia_1}|01\rangle - e^{-ia_2}|10\rangle - e^{-ia_3}|11\rangle), \tag{4}$$

$$\left|\varphi^3\right\rangle = \frac{1}{2}(|00\rangle - e^{-ia_1}|01\rangle + e^{-ia_2}|10\rangle - e^{-ia_3}|11\rangle), \tag{5}$$

$$\left|\varphi^4\right\rangle = \frac{1}{2}(|00\rangle - e^{-ia_1}|01\rangle - e^{-ia_2}|10\rangle + e^{-ia_3}|11\rangle). \tag{6}$$

And Bob selects $\left\{\left|\phi^k\right\rangle_{25}\right\}$ $(k = 1, 2, 3, 4)$ as his basic measurement basis:

$$\left|\phi^1\right\rangle = \frac{1}{2}(|00\rangle + e^{-ib_1}|01\rangle + e^{-ib_2}|10\rangle - e^{-ib_3}|11\rangle), \tag{7}$$

$$\left|\phi^2\right\rangle = \frac{1}{2}(|00\rangle + e^{-ib_1}|01\rangle - e^{-ib_2}|10\rangle + e^{-ib_3}|11\rangle), \tag{8}$$

$$\left|\phi^3\right\rangle = \frac{1}{2}(|00\rangle - e^{-ib_1}|01\rangle + e^{-ib_2}|10\rangle + e^{-ib_3}|11\rangle), \tag{9}$$

$$\left|\phi^4\right\rangle = \frac{1}{2}(|00\rangle - e^{-ib_1}|01\rangle - e^{-ib_2}|10\rangle - e^{-ib_3}|11\rangle). \tag{10}$$

With the help of these measurement bases, the quantum channel of six-qubit cluster state can be shown as

$$
\begin{aligned}
|\psi\rangle_{123456} = \frac{1}{8}\Big[& \left|\varphi^1\right\rangle_{14}\left|\phi^1\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^1\right\rangle_{14}\left|\phi^2\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^1\right\rangle_{14}\left|\phi^3\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^1\right\rangle_{14}\left|\varphi^4\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^2\right\rangle_{14}\left|\phi^1\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^2\right\rangle_{14}\left|\phi^2\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^2\right\rangle_{14}\left|\phi^3\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^2\right\rangle_{14}\left|\phi^4\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^3\right\rangle_{14}\left|\phi^1\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^3\right\rangle_{14}\left|\phi^2\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^3\right\rangle_{14}\left|\phi^3\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^3\right\rangle_{14}\left|\phi^4\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^4\right\rangle_{14}\left|\phi^1\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^4\right\rangle_{14}\left|\phi^2\right\rangle_{25}\left(|00\rangle - e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^4\right\rangle_{14}\left|\phi^3\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36} \\
& + \left|\varphi^4\right\rangle_{14}\left|\phi^4\right\rangle_{25}\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)_{36}\Big]. \qquad (11)
\end{aligned}
$$

Then Alice first carries out the two-qubit projective measurement on the qubits 1 and 4 under the basis $\left\{\left|\varphi^j\right\rangle_{14}\right\}$ $(j = 1, 2, 3, 4)$ and publicly announces her measured outcome to Bob and Charlie. And Bob should select one of the measuring bases $\left\{\left|\phi^k\right\rangle_{25}\right\}$ $(k = 1, 2, 3, 4)$ to measure his qubits 2 and 5 according to Alice's result, and then inform Charlie of his outcome by the classical channel. In terms of Alice's and Bob's outcomes, Charlie can recover the desired state $|\Phi\rangle$ by a suitable unitary operation.

For instance, without loss of generality, assume Alice's result is $\left|\varphi^j\right\rangle_{14}$ and Bob's result is $\left|\phi^2\right\rangle_{25}$, the qubits 3 and 6 will collapse into the state $\frac{1}{2}\left(|00\rangle + e^{i\delta_1}|01\rangle - e^{i\delta_2}|10\rangle - e^{i\delta_3}|11\rangle\right)_{36}$. According to Alice's and Bob's results, Charlie can perform the unitary operations $\sigma_3^z \otimes I_6$ on his qubits 3 and 6, and then the desired state $|\Phi\rangle$ can be obtained. The relation between the results obtained by Alice and Bob and appropriate unitary operation performed by Charlie is shown in Table 1. It is easily found that, for all the 16 results of Alice and Bob, the receiver Charlie can reconstruct the desired state $|\Phi\rangle$ by performing a suitable unitary operation.

Table 1 Corresponding relation between the measurement outcomes of the senders Alice and Bob and the corresponding unitary operations $\left\{\sigma_{36}^{jk}\right\}$ $(j, k = 1, 2, 3, 4)$ performed by the receiver Charlie

Alice's result	Bob's result	Unitary operation	Alice's result	Bob's result	Unitary operation
$\|\varphi^1\rangle_{14}$	$\|\phi^1\rangle_{25}$	$I_3 \otimes I_6$	$\|\varphi^3\rangle_{14}$	$\|\phi^1\rangle_{25}$	$I_3 \otimes \sigma_6^z$
$\|\varphi^1\rangle_{14}$	$\|\phi^2\rangle_{25}$	$\sigma_3^z \otimes I_6$	$\|\varphi^3\rangle_{14}$	$\|\phi^2\rangle_{25}$	$\sigma_3^z \otimes \sigma_6^z$
$\|\varphi^1\rangle_{14}$	$\|\phi^3\rangle_{25}$	$I_3 \otimes \sigma_6^z$	$\|\varphi^3\rangle_{14}$	$\|\phi^3\rangle_{25}$	$I_3 \otimes I_6$
$\|\varphi^1\rangle_{14}$	$\|\phi^4\rangle_{25}$	$\sigma_3^z \otimes \sigma_6^z$	$\|\varphi^3\rangle_{14}$	$\|\phi^4\rangle_{25}$	$\sigma_3^z \otimes I_6$
$\|\varphi^2\rangle_{14}$	$\|\phi^1\rangle_{25}$	$\sigma_3^z \otimes I_6$	$\|\varphi^4\rangle_{14}$	$\|\phi^1\rangle_{25}$	$\sigma_3^z \otimes \sigma_6^z$
$\|\varphi^2\rangle_{14}$	$\|\phi^2\rangle_{25}$	$I_3 \otimes I_6$	$\|\varphi^4\rangle_{14}$	$\|\phi^2\rangle_{25}$	$I_3 \otimes \sigma_6^z$
$\|\varphi^2\rangle_{14}$	$\|\phi^3\rangle_{25}$	$\sigma_3^z \otimes \sigma_6^z$	$\|\varphi^4\rangle_{14}$	$\|\phi^3\rangle_{25}$	$\sigma_3^z \otimes I_6$
$\|\varphi^2\rangle_{14}$	$\|\phi^4\rangle_{25}$	$I_3 \otimes \sigma_6^z$	$\|\varphi^4\rangle_{14}$	$\|\phi^4\rangle_{25}$	$I_3 \otimes I_6$

In fact, we will generalize the above ideal JRSP protocol to a realistic situation. In our ideal JRSP protocol, the three-party shared a pure six-qubit cluster state before the quantum communication. However, in a realistic situation, Charlie can generate the six-qubit cluster state in experiment. Then he keeps qubits (3, 6) with him, and sends the qubits (1, 4) and (2, 5) to Alice and Bob using the noisy quantum channel, respectively. Due to the interaction with the noisy environment, the initial quantum channel will do some changes. In this paper, we will consider the two different noisy environments including the amplitude damping noise and phase-damping noise. We make a discussion both of these two noisy cases to show that in which noisy environment more information would be lost.

3 Effects of Amplitude-Damping Noise and Phase-Damping Noise

We consider two different noisy environments when the travel qubits couple into either the amplitude-damping noise or phase-damping noise. The amplitude-damping noise is given by [45]

$$E_0^A = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta_A} \end{bmatrix}, E_1^A = \begin{bmatrix} 0 & \sqrt{\eta_A} \\ 0 & 0 \end{bmatrix}, \tag{12}$$

where η_A $(0 \leq \eta_A \leq 1)$ is the probability of error due to amplitude-damping noise when a travel qubit goes through it, and η_A denotes the noisy rate. And the phase-damping noise is [45]

$$E_0^p = \sqrt{1-\eta_P} I, E_1^P = \sqrt{\eta_P} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_2^P = \sqrt{\eta_P} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{13}$$

where η_P $(0 \leq \eta_P \leq 1)$ is the noisy rate.

In Section 2, we have proposed an ideal JRSP protocol by using a pure six-qubit cluster state, and one can easily obtain the corresponding density matrix $\rho = |\psi\rangle\langle\psi|$. The density operator ρ after the noise described by (12) or (13) would become

$$\xi^r(\rho) = \sum_{m,n} \left(E_m^{r1}\right)\left(E_m^{r4}\right)\left(E_n^{r2}\right)\left(E_n^{r5}\right)\rho\left(E_m^{r1}\right)^{\dagger}\left(E_m^{r4}\right)^{\dagger}\left(E_n^{r2}\right)^{\dagger}\left(E_n^{r5}\right)^{\dagger}, \tag{14}$$

where $r \in \{A, P\}$ and "†" is the conjugate transpose of a matrix. For $r = A$, i.e., for amplitude-damping noise $m \in \{0, 1\}$, and at the same times for $r = P$, i.e., for phase-damping noise $m \in \{0, 1, 2\}$, and the superscripts (1,4,2,5) is the operator E, ξ represents an operation which maps from ρ to $\xi^r(\rho)$ depends the noisy environment. In (14), the qubits

3 and 6 are not affected by the noise. Further, the qubits which are sent to Alice and Bob are affected by the same noise. In the noisy environments, we show that the initial quantum channel would become a mixed state

$$\xi^{A}(\rho)=\frac{1}{4}\Big\{\Big[|000000\rangle+(1-\eta_A)|000111\rangle+(1-\eta_A)|111000\rangle+(1-\eta_A)^2|111111\rangle\Big] \times\Big[\langle 000000|+(1-\eta_A)\langle 000111|+(1-\eta_A)\langle 111000|+(1-\eta_A)^2\langle 111111|\Big] +(1-\eta_A)^2\eta_A^2|101101\rangle\langle 101101|+(1-\eta_A)^2\eta_A^2|011011\rangle\langle 011011| +\eta_A^4|001001\rangle\langle 001001|\Big\}. \quad (15)$$

And

$$\xi^{P}(\rho)=\frac{1}{4}\Big\{\Big[(1-\eta_P)^4(|000000\rangle+|000111\rangle+|111000\rangle+|111111\rangle) \times(\langle 000000|+\langle 000111|+\langle 111000|+\langle 111111|)\Big] +\Big[2(1-\eta_P)^2\eta_P^2+\eta_P^4\Big](|000000\rangle\langle 000000|+|111111\rangle\langle 111111|)\Big\}. \quad (16)$$

Charlie can carry out an appropriate local operation on own qubits to obtain the desired state while he knows the both Alice's and Bob's results described in Section 2. Afterwards, the density matrix of the final quantum state is given by

$$\rho_{out}^{r}=\frac{Tr_{14,25}\left[U\xi^{r}(\rho)U^{\dagger}\right]}{Tr_{14,25,36}\left[UU^{\dagger}\xi^{r}(\rho)\right]}, \quad (17)$$

where $Tr_{14,25,36}$ is the trace operation, $Tr_{14,25}$ is the partial trace, with U is a unitary operation expressed as

$$U=\left\{I_{14}\otimes I_{25}\otimes\sigma_{36}^{jk}\right\}\left\{I_{14}\otimes\left|\phi^{k}\right\rangle_{25}\left\langle\phi^{k}\right|_{25}\otimes I_{36}\right\}\left\{\left|\varphi^{j}\right\rangle_{14}\left\langle\varphi^{j}\right|_{14}\otimes I_{25}\otimes I_{36}\right\}, \quad (18)$$

where $j,k\in\{1,2,3,4\}$, and $\left|\varphi^{j}\right\rangle_{14}\left\langle\varphi^{j}\right|_{14}$ is Alice's result, $\left|\phi^{k}\right\rangle_{25}\left\langle\phi^{k}\right|_{25}$ is Bob's result, σ_{36}^{jk} is Charlie's unitary operation which depends on Alice's and Bob's results.

Then we can obtain the density matrix of the final state ρ_{out}^{r} in noisy environments. We can calculate the effect of noise by comparing the quantum state ρ_{out}^{r} with the initial state $|\Phi\rangle$ by using fidelity

$$F=\langle\Phi|\rho_{out}^{r}|\Phi\rangle. \quad (19)$$

It is known that $F=1$ is the perfect JRSP. If F becomes smaller and smaller, it means that we have lost more and more quantum information of the prepared state.

Using (17) and (18), one can obtain the final state ρ_{out}^{r}. We show that ρ_{out}^{r} is independent of three participants' outcomes. And the state ρ_{out}^{r} is given by

$$\rho_{out}^{A}=\frac{1}{4}\Big\{\Big[|00\rangle+(1-\eta_A)e^{i\delta_1}|01\rangle+(1-\eta_A)e^{i\delta_2}|10\rangle+(1-\eta_A)^2e^{i\delta_3}|11\rangle\Big] \times\Big[\langle 00|+(1-\eta_A)e^{-i\delta_1}\langle 01|+(1-\eta_A)e^{-i\delta_2}\langle 10|+(1-\eta_A)^2e^{-i\delta_3}\langle 11|\Big] +\Big[2(1-\eta_A)^2\eta_A^2+\eta_A^4\Big]|11\rangle\langle 11|\Big\}. \quad (20)$$

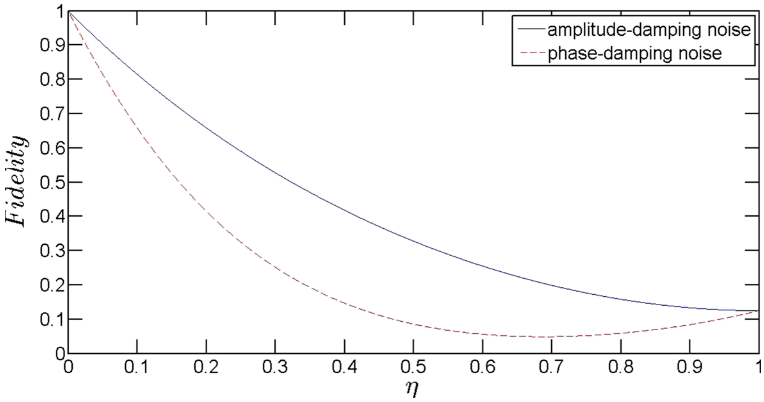

Fig. 1 The fidelities of amplitude-damping noise and phase-damping noise. The *solid line* represents the amplitude-damping noise, and the *dashed line* is the phase-damping noise

And

$$\rho_{out}^{P} = \frac{1}{4}\left\{\left(1-\eta_P\right)^4\left[\left(|00\rangle + e^{i\delta_1}|01\rangle + e^{i\delta_2}|10\rangle + e^{i\delta_3}|11\rangle\right)\right.\right.$$
$$\times\left.\left(\langle 00| + e^{-i\delta_1}\langle 01| + e^{-i\delta_2}\langle 10| + e^{-i\delta_3}\langle 11|\right)\right]$$
$$\left.+\left[2\left(1-\eta_P\right)^2\eta_P^2 + \eta_P^4\right]\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)\right\}. \tag{21}$$

Using (19) and (20), we obtain the fidelity under amplitude-damping noise,

$$F^A = \frac{1}{16}\left\{\left[1 + 2\left(1-\eta_A\right) + \left(1-\eta_A\right)^2\right]^2 + 2\left(1-\eta_A\right)^2\eta_A^2 + \eta_A^4\right\}. \tag{22}$$

Similarly, according to (19) and (21), the fidelity can be obtained under phase-damping noise,

$$F^P = \left(1-\eta_P\right)^4 + \frac{1}{8}\left[2\left(1-\eta_P\right)^2\eta_p^2 + \eta_P^4\right]. \tag{23}$$

From (22) and (23), we find that the fidelities for these two cases only depend on the noisy rate. In Fig. 1, we plot the fidelity with noisy rate η, and we show the fidelity influenced by amplitude-damping noise decreases as η becomes larger, but the fidelity for phase-damping noise decreases first, and then increasing. We note that the minimal fidelity for phase-damping noise does not appear while the noisy rate is maximal, that is true because $dF^p/d\eta_p = 0$ for $\eta_p = 0.687$. When $\eta = 0$, the fidelities for these two noisy cases have the same maximal value, $F = 1$; while $\eta = 1$, the fidelities also have the same value, $F = 1/8$, which is the minimal value for amplitude-damping noise, but not for the phase-damping noise. Moreover, the results show that the fidelity for amplitude-damping noise is always larger than phase-damping noise, which means that the initial quantum state coupled into phase-damping noise will lose more quantum information than amplitude-damping noise.

4 Conclusion

In this work, we have presented a new and practical JRSP scheme of an arbitrary equatorial two-qubit state. As we know, the arbitrary equatorial states have many special quantum properties in quantum information processing and quantum key distribution [46–48], which they have less quantum information than arbitrary states and are easily prepared than arbitrary quantum states. An equatorial single-qubit state has been remotely obtained [16]. We have shown that an arbitrary equatorial two-qubit state can be deterministically prepared by using a six-qubit cluster state. In addition, the six-qubit cluster state has been demonstrated in recent experiment [49], thus our scheme may be realized in the near future.

Furthermore, we have investigated the effect of noise in JRSP process, such as the amplitude-damping noise and phase-damping noise. The effects of these two noisy environments are studied using fidelity, which makes the present work much more realistic. Moreover, the way used here for the discussion of noise is quite general and it is possible to apply this scheme to other similar JRSP schemes. However, how to enhance the efficiency of the JRSP in these two noisy environments should be a more important thing, and it is under way.

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