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Superdense Coding with Uniformly Accelerated Particle

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Abstract We study superdense coding with uniformly accelerated particle in single mode approximation and beyond single mode approximation. We use four different functions, the capacity of superdense coding, negativity, discord and the probability of success for evaluating the final results. In single mode approximation, all the four functions behave as expected, however in beyond single mode approximation, except the probability of success, the other three functions represent peculiar behaviors at least for special ranges where the beyond single mode approximation is strong.

Keywords Superdense coding · Non-inertial frame · Single and beyond single mode approximation

1 Introduction

Two particles, even being far away from each other, can be correlated as a result of existing nonclassical correlation and entanglement in between them. Theoretical studies and experimental investigations of entanglement and nonclassical correlation have been main topics for groups of researchers [1–5]. In the process of so called superdense coding [6] two classical bits of information are transferred by sending only one quantum bit, qubit. The original superdense coding process begins with a pair of entangled two-level particles being shared

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between Alice, sender, and Bob, receiver. An EPR pair [7] is used as a maximally entangled state. We have four orthonormal EPR states which can be written as

$$|\varphi_{\alpha\beta}\rangle_{AB} = \frac{1}{\sqrt{2}}\left\{|0\rangle_A|\alpha\rangle_B + (-1)^{\beta}|1\rangle_A|\overline{\alpha}\rangle_B\right\}, \tag{1}$$

where $\alpha, \beta = \{0, 1\}$, $\overline{\alpha} = 1 - \alpha$ and subscripts A and B denote Alice's qubit and Bob's qubit, respectively.

Let us assume, without loss of generality, that Alice and Bob share the state $|\varphi_{00}\rangle_{AB}$, $\alpha = \beta = 0$. Alice has a two-bit message that she wants to send it to Bob. The classical two-bit message can be one of the forms $ij = \{00, 01, 10, 11\}$. Alice first operates one of the four unitary operators $U_{ij} = Z^j X^i$ on her qubit. X and Z are Pauli operators. Consequently, the initial EPR pair changes to one of the four orthonormal EPR states, $|\varphi_{ij}\rangle$, i.e. the original EPR state is encoded by the message, ij. Then, Alice sends her manipulated qubit to Bob, who performs a measurement in the Bell-basis, that yields the classical message, ij. Superdense coding has been experimentally implemented [8–12].

In this paper, we suppose two particles denoted as Alice and Bob. Alice is accelerated while Bob stays inertial. Therefore, one can say that Alice has constant acceleration with respect to Bob in the z-direction. The accelerated observer's trajectory in Minkowski coordinates is a hyperbola that is indicated in terms of Rindler coordinates (τ, ξ) [13, 14], with the following form

$$(z, t) = \pm\left(\frac{e^{a\xi}}{a}\cosh(a\tau), \frac{e^{a\xi}}{a}\sinh(a\tau)\right), \tag{2}$$

where τ is the Alice's proper time, a is an arbitrary reference acceleration and $\frac{e^{a\xi}}{a}$ is the proper acceleration for Alice. The straight lines passing from origin are obtained by the coordinate constant τ, and hyperbola is obtained by the coordinate ξ as is plotted in Fig. 1. The horizons $H_{\pm}$ that are obtained by the light-like asymptotes, $z^2 = t^2$, represent proper times $\tau = \pm\infty$ in the limit of $\xi \to -\infty$. The right half and the left half of Minkowski plane are two regions that are called Rindler wedges I and II, respectively. Alice and the fictitious observer, anti-Alice, are constrained to move in the Rindler wedges I and II, respectively, as these regions are causally disconnected from each other, i.e. no information can propagate between them.

In a general discussion, we shall study superdense coding with an accelerated particle in single mode approximation and beyond single mode approximation. We cover the discussion in a general manner and find the probability of success for superdense coding with uniformly accelerated particle. We appraise the whole process by means of superdense coding capacity, with definition given below. For the sake of completeness, we also discuss the results in terms of existing entanglement and quantum correlation and the corresponding changes under superdense coding with uniformly accelerated particle.

Superdense coding capacity is the maximum value of classical information that can be conveyed for a primary given state being shared between Alice and Bob. When the encoding operator used in the protocol is a unitary operator and the channel is noiseless, then superdense coding capacity is defined as follow [15–17]

$$C(A:B) = \log_2 d + S(\rho^B) - S(\rho^{AB}). \tag{3}$$

Here, ρ^B is Bob's reduced density matrix, ρ^{AB} is the initial shared state and d is the dimension of Alice's system. $S(\rho)$ is the von Neumann entropy, $S(\rho) = -\sum_i \lambda_i \log_2(\lambda_i)$, where λ_i's are the eigenvalues of ρ.

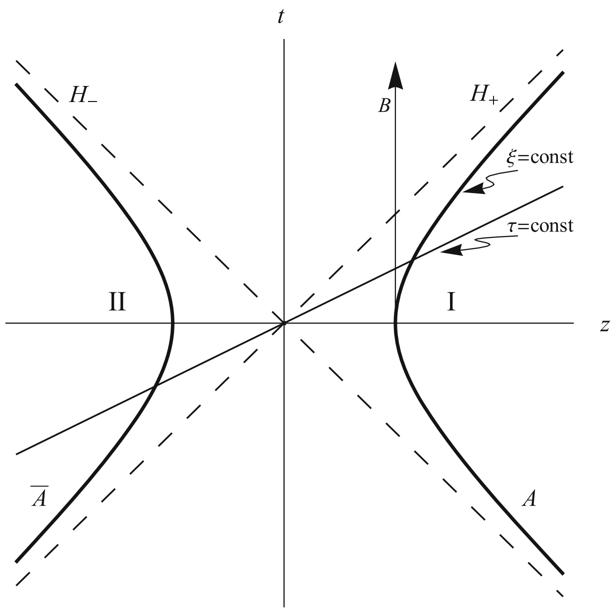

Fig. 1 Minkowski diagram for Alice and Bob. Bob is stationary and Alice travels with constant acceleration and is moving along the hyperbola in region I while fictitious observer anti-Alice moving along a corresponding hyperbola in region II. Bob will cross from the horizons $H_{\pm}$ at his finite Minkowski time t_A. After this time Alice's signals can just across from H_+ and arrives to Bob

Logarithmic negativity [18, 19] that is employed for evaluating entanglement of ρ is defined as

$$N(\rho) = \log_2 \sum_i |\lambda_i(\rho^{\mathrm{pt}})|, \tag{4}$$

where $\lambda_i(\rho^{\mathrm{pt}})$'s are the eigenvalues of the partial transpose of ρ.

Quantum discord is evaluated [20–24] for measuring nonclassical correlation and it is defined as

$$D(A:B) = \mathcal{I}(A:B) - \mathcal{C}(A:B), \tag{5}$$

where $\mathcal{I}(A:B)$ is quantum mutual information. It is determined as

$$\mathcal{I}(A:B) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}). \tag{6}$$

$\mathcal{C}(A:B)$ is the classical correlation given as follow

$$\mathcal{C}(A:B) = \max_{\{\mathcal{B}_k\}}[\mathcal{J}_{\{\mathcal{B}_k\}}(A:B)], \tag{7}$$

where, $\mathcal{J}$ is locally accessible mutual information defined as follow

$$\mathcal{J}_{\{\mathcal{B}_k\}}(A:B) = S(\rho_A) - S_{\{\mathcal{B}_k\}}(A|B). \tag{8}$$

$S_{\{\mathcal{B}_k\}}(A|B)$ is the quantum conditional entropy defined as follow

$$S_{\{\mathcal{B}_k\}}(A|B) = \sum_k p_k S(\rho_{A|k}), \tag{9}$$

where $\{\rho_k, p_k\}$ is the ensemble of the outcome, after von Neumann measurements $\{\mathcal{B}_k\}$ for the subsystem B, and $\rho_{A|k} = \mathrm{Tr}_B(\mathcal{B}_k \rho \mathcal{B}_k)/p_k$, with $p_k = \mathrm{Tr}(\mathcal{B}_k \rho \mathcal{B}_k)$. Calculating quantum discord for a general state can be hard, however for special cases, e.g. where the state is a X-state, there is a standard approach (see appendix A). The resultant states being studied in the process of superdense coding with uniformly accelerated particle are X-states. Therefore, we give precise quantum discord values in addition to logarithmic negativity values and compare them with superdense coding capacities.

2 Superdense Coding in Single-Mode Approximation

Considering a free Minkowski Dirac field in 1+1 dimensions, we assume all modes of the field are in vacuum state except two modes that belong to Alice and Bob. The Minkowski vacuum for Alice can be expanded in terms of the corresponding Rindler vacuum [25], as

$$|0\rangle_A = \cos r|0\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}} + \sin r|1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}, \tag{10}$$

$$|1\rangle_A = |1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}, \tag{11}$$

where $|i\rangle_A$ is the Minkowski particle mode belonging to Alice, $|i\rangle_{\mathrm{I}}$ is the Rindler region I particle modes and $|i\rangle_{\mathrm{II}}$ is the Rindler region II anti-particle modes.

In single mode approximation, the shared state $|\varphi_{00}\rangle_{AB}$ can be rewritten by substituting the relations (10) and (11) in (1) only for Alice, as

$$|\varphi_{00}\rangle_{\mathrm{I,II},B} = \frac{1}{\sqrt{2}}\left\{\cos r|000\rangle + \sin r|110\rangle + |101\rangle\right\}, \tag{12}$$

where $|abc\rangle = |b\rangle_{\mathrm{I}}|c\rangle_{\mathrm{II}}|a\rangle_B$. A unitary operator U_{ij} is applied on Alice, the accelerated qubit. The operator I does not change the state (12), but other operators change the state into another state, as follow

$$\begin{aligned} U_{ij}|\varphi_{00}\rangle_{\mathrm{I,II},B} &= \frac{(-1)^{ij}}{\sqrt{2}}\left\{\cos r|i00\rangle + (-1)^j|\bar{i}01\rangle + (-1)^j \sin r|\bar{i}10\rangle\right\} \\ &= |\varphi_{ij}\rangle_{\mathrm{I,II},B}. \end{aligned} \tag{13}$$

Then, the accelerated particle reaches Bob. If the information to be sent is $ij = 00$ then the resultant density matrix is as follow

$$\begin{aligned} \rho_{00}^{\mathrm{I,II},B} &= \frac{1}{2}\Big\{\cos^2 r|000\rangle\langle 000| + \sin^2 r|110\rangle\langle 110| + |101\rangle\langle 101| + \big(\cos r|000\rangle\langle 101| \\ &\quad + \cos r \sin r|000\rangle\langle 110| + \sin r|110\rangle\langle 101| + \mathrm{h.c.}\big)\Big\}. \end{aligned} \tag{14}$$

Recall that the Rindler regions I and II are causally disconnected. Alice is constrained to move in region I, so by tracing out region II, Bob obtains the shared density matrix, as follow

$$\rho_{00}^{\mathrm{I},B} = \frac{1}{2}\begin{pmatrix} \cos^2 r & 0 & 0 & \cos r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sin^2 r & 0 \\ \cos r & 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

where $|ab\rangle = |b\rangle_{\text{I}}|a\rangle_B$. The density matrix obtained for different cases of ij, the classical message, can be found as

$$\begin{aligned}\rho_{ij}^{\text{I},B} &= \text{Tr}_{\text{II}}(\rho_{ij}^{\text{I,II},B}) \\ &= \frac{1}{2}\left\{\cos^2 r|i0\rangle\langle i0| + \sin^2 r|\bar{i}0\rangle\langle\bar{i}0| + |\bar{i}1\rangle\langle\bar{i}1| + (-1)^j\left(\cos r|i0\rangle\langle\bar{i}1| + \text{h.c.}\right)\right\}.\end{aligned} \tag{16}$$

Equation (16) represents four distinctive states that are X-states. For decoding the classical message, a Bell basis measurement is performed to obtain the following results

$$\begin{aligned}\left\langle\varphi_{ij}|\rho_{ij}^{\text{I},B}|\varphi_{ij}\right\rangle &= \frac{1}{4}(1+\cos r)^2, \\ \left\langle\varphi_{i\bar{j}}|\rho_{ij}^{\text{I},B}|\varphi_{i\bar{j}}\right\rangle &= \frac{1}{4}(1-\cos r)^2, \\ \left\langle\varphi_{\bar{i}j}|\rho_{ij}^{\text{I},B}|\varphi_{\bar{i}j}\right\rangle &= \left\langle\varphi_{\bar{i}\bar{j}}|\rho_{ij}^{\text{I},B}|\varphi_{\bar{i}\bar{j}}\right\rangle = \frac{1}{4}\sin^2 r.\end{aligned} \tag{17}$$

Results of this measurement on the density matrix, after tracing out region II, is dependent on the acceleration parameter, r. In other words, superdense coding is performed with a probability of r. By letting $r = 0$, corresponding to $a = 0$, then superdense coding is run absolutely in accordance with the original scenario [6]. Figure 2 shows probability of success for superdense coding, $P(\rho_{ij}^{\text{I},B}) = \langle\varphi_{ij}|\rho_{ij}^{\text{I},B}|\varphi_{ij}\rangle$, (17), as a function of acceleration parameter, r.

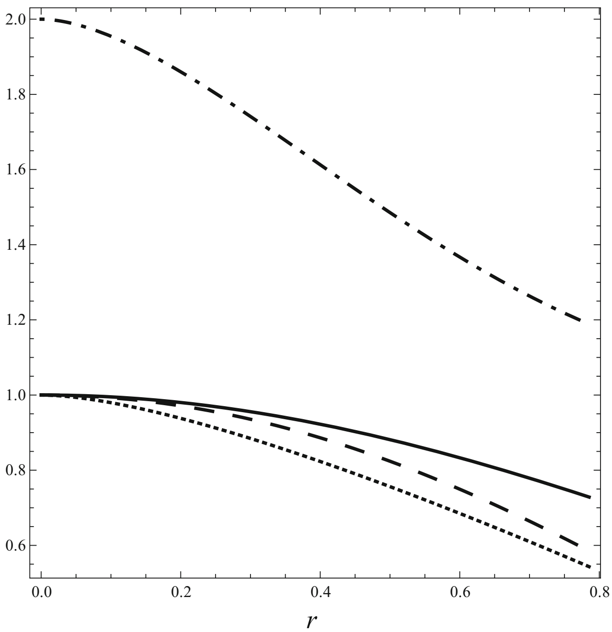

Fig. 2 Probability of success for superdense coding, P, *solid line*, superdense coding capacity, $C(\text{I} : B)$, *dot-dashed line*, logarithmic negativity, N, *dashed line*, and quantum discord, $D(\text{I} : B)$, *dotted line*, as functions of acceleration parameter, r, for $\rho_{00}^{\text{I},B}$, in single mode approximation

In order to evaluate superdense coding capacity and later for quantum discord, we need to calculate the von Neumann entropies as follows

$$\begin{aligned} S(\rho^{\mathrm{I},B}) &= -\frac{1-\cos 2r}{4}\log_2\frac{1-\cos 2r}{4} - \frac{3+\cos 2r}{4}\log_2\frac{3+\cos 2r}{4}, \\ S(\rho^{\mathrm{I}}) &= -\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2} - \frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}, \\ S(\rho^B) &= 1. \end{aligned} \tag{18}$$

Thus, superdense coding capacity, (3), for the state (15), is obtained as follow

$$C(\mathrm{I}:B) = 2 + \frac{1-\cos 2r}{4}\log_2\frac{1-\cos 2r}{4} + \frac{3+\cos 2r}{4}\log_2\frac{3+\cos 2r}{4}. \tag{19}$$

In Fig. 2, superdense coding capacity, (19), is plotted as a function of acceleration parameter, r.

Logarithmic negativity, (4), is calculated for the entanglement of the state, (15). Eigenvalues of the partial transpose of the density matrix $\rho_{00}^{\mathrm{I},B}$, are given by

$$\begin{aligned} \lambda_{1,2}\left(\rho_{\mathrm{I},B}^{\mathrm{pt}}\right) &= \frac{1}{2}, \\ \lambda_{3,4}\left(\rho_{\mathrm{I},B}^{\mathrm{pt}}\right) &= \pm\frac{\cos^2 r}{2}. \end{aligned} \tag{20}$$

Thus, logarithmic negativity can be written as follow

$$N\left(\rho_{00}^{\mathrm{I},B}\right) = \log_2\left(1+\cos^2 r\right). \tag{21}$$

Figure 2 indicates (21) as a function of r.

Quantum discord is given by (5). For the state of (15), after evaluating the corresponding von Neumann entropies, (18), and employing the approach explained in Refs. [27–29], quantum discord is calculated for which Fig. 2 shows its behavior as a function of r. It is clear, that four quantities, probability of success, superdense coding capacity, logarithmic negativity and quantum discord for superdense coding with accelerated particle, in single mode approximation, are descending functions of r.

3 Superdense Coding in Beyond Single-Mode Approximation

In beyond single mode approximation, an accelerated detector can detect a mode in both Rindler wedges I and II, therefore there are different right and left components for the single-particle state denoted as Alice [26],

$$\begin{aligned} |0\rangle_A &= \cos r|0\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}} + \sin r|1\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}}, \\ |1\rangle_A &= q_l|0\rangle_{\mathrm{I}}|1\rangle_{\mathrm{II}} + q_r|1\rangle_{\mathrm{I}}|0\rangle_{\mathrm{II}}, \end{aligned} \tag{22}$$

where q_l and q_r are complex numbers that satisfy $q_l^2 + q_r^2 = 1$. For simplicity, we only consider the cases that q_l and q_r are real. The single mode approximation is found by letting

$q_r = 1$ in the general form (22). The shared state $|\varphi_{00}\rangle_{AB}$ can be rewritten by substituting the relations (10) and (22) for Alice in (1), in beyond single mode approximation, as

$$|\varphi_{00}\rangle_{\mathrm{I,II},B} = \frac{1}{\sqrt{2}}\{\cos r|000\rangle + \sin r|110\rangle + q_l|011\rangle + q_r|101\rangle\}. \tag{23}$$

Alice applies a unitary operator U_{ij} on her qubit. Like the previous section, operator I does not change the state (23), but others do change the state into another state, as follows

$$\begin{aligned} U_{ij}|\varphi_{00}\rangle_{\mathrm{I,II},B} &= \frac{(-1)^{ij}}{\sqrt{2}}\left\{\cos r|i00\rangle + (-1)^j \sin r|\bar{i}10\rangle + q_l|i11\rangle + (-1)^j q_r|\bar{i}01\rangle\right\} \\ &= |\varphi_{ij}\rangle_{\mathrm{I,II},B}. \end{aligned} \tag{24}$$

Now, the state in Bob's possession, after he receives the accelerated particle is $\left|\varphi_{ij}\right\rangle_{\mathrm{I,II},B}$. For the case $ij = 00$, the resultant density matrix is given by

$$\begin{aligned} \rho_{00}^{\mathrm{I,II},B} = \frac{1}{2}\Big\{ &\cos^2 r|000\rangle\langle 000| + \sin^2 r|110\rangle\langle 110| + q_l^2|011\rangle\langle 011| + q_r^2|101\rangle\langle 101| \\ &+\Big(\cos r \sin r|000\rangle\langle 110| + q_l \sin r|110\rangle\langle 011| + q_r \cos r|000\rangle\langle 101| \\ &+q_l \cos r|000\rangle\langle 011| + q_r \sin r|110\rangle\langle 101| + q_l q_r|011\rangle\langle 101| + \mathrm{h.c.}\Big)\Big\}. \end{aligned} \tag{25}$$

The density matrix that is given by tracing out region II, is given by

$$\rho_{00}^{\mathrm{I},B} = \mathrm{Tr}_{\mathrm{II}}(\rho_{00}^{\mathrm{I,II},B}) = \frac{1}{2}\begin{pmatrix} \cos^2 r & 0 & 0 & q_r \cos r \\ 0 & q_l^2 & q_l \sin r & 0 \\ 0 & q_l \sin r & \sin^2 r & 0 \\ q_r \cos r & 0 & 0 & q_r^2 \end{pmatrix}. \tag{26}$$

For other cases for the classical message ij, the density matrix can be obtained as follow

$$\begin{aligned} \rho_{ij}^{\mathrm{I},B} &= \mathrm{Tr}_{\mathrm{II}}\left(\rho_{ij}^{\mathrm{I,II},B}\right) \\ &= \frac{1}{2}\Big\{\cos^2 r|i0\rangle\langle i0| + \sin^2 r|\bar{i}0\rangle\langle\bar{i}0| + q_l^2|i1\rangle\langle i1| + q_r^2|\bar{i}1\rangle\langle\bar{i}1| + (-1)^j \\ &\quad\times\Big(q_r \cos r|i0\rangle\langle\bar{i}1| + q_l \sin r|\bar{i}0\rangle\langle i1| + \mathrm{h.c.}\Big)\Big\}, \end{aligned} \tag{27}$$

which represents four distinctive matrices that are X-forms. Thus, measurement in Bell basis by Bob yields

$$\begin{aligned} \left\langle\varphi_{ij}|\rho_{ij}^{\mathrm{I},B}|\varphi_{ij}\right\rangle &= \frac{1}{4}(q_r + \cos r)^2, \\ \left\langle\varphi_{i\bar{j}}|\rho_{ij}^{\mathrm{I},B}|\varphi_{i\bar{j}}\right\rangle &= \frac{1}{4}(q_r - \cos r)^2, \\ \left\langle\varphi_{\bar{i}j}|\rho_{ij}^{\mathrm{I},B}|\varphi_{\bar{i}j}\right\rangle &= \frac{1}{4}(q_l + \sin r)^2, \\ \left\langle\varphi_{\bar{i}\bar{j}}|\rho_{ij}^{\mathrm{I},B}|\varphi_{\bar{i}\bar{j}}\right\rangle &= \frac{1}{4}(q_l - \sin r)^2. \end{aligned} \tag{28}$$

These results show the probability of success, $P\left(\rho_{ij}^{\mathrm{I},B}\right)$, is $\frac{1}{4}(q_r+\cos r)^2$, and it is illustrated in Figs. 3, 7 and 8. Therefore, measurement by Bob depends on the acceleration parameter, r. If $r = 0$ and $q_r = 1$, corresponding to $a = 0$ and $q_l = 0$, respectively, then the original superdense coding scenario is given [6].

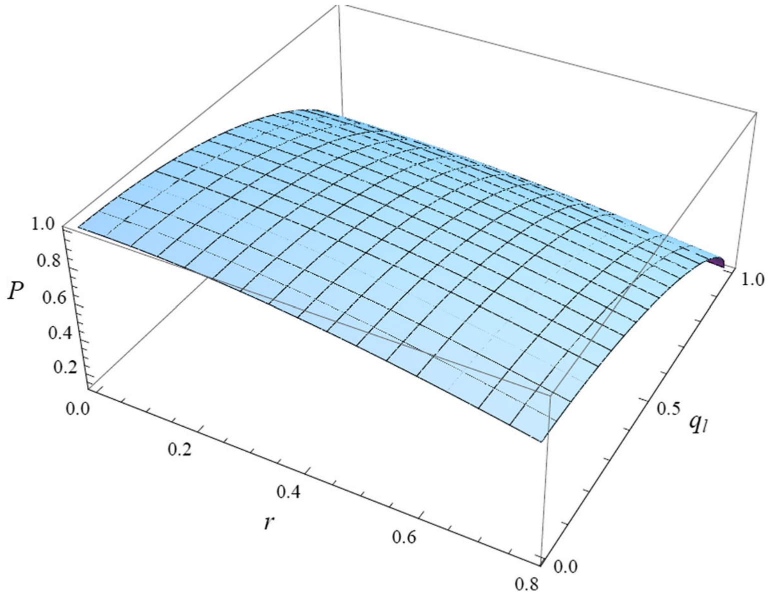

Fig. 3 Probability of success in terms of acceleration parameter, r, and presence possibility of the particle in region II of Rindler region, q_l

For the state (26), the von Neumann entropies are given as follows

$$\begin{aligned} S(\rho^{\mathrm{I},B}) &= -\frac{-1+2q_l^2-\cos 2r}{4}\log_2\frac{-1+2q_l^2-\cos 2r}{4}-\frac{3-2q_l^2+\cos 2r}{4}\log_2\frac{3-2q_l^2+\cos 2r}{4},\\ S(\rho^{\mathrm{I}}) &= -\frac{1-q_l^2+\sin^2 r}{2}\log_2\frac{1-q_l^2+\sin^2 r}{2}-\frac{q_l^2+\cos^2 r}{2}\log_2\frac{q_l^2+\cos^2 r}{2},\\ S(\rho^{B}) &= 1. \end{aligned} \tag{29}$$

Thus, superdense coding capacity $C(\mathrm{I}:B)$, (3), is calculated as follow

$$C(\mathrm{I}:B)=2+\frac{3-2q_l^2+\cos 2r}{4}\log_2\frac{3-2q_l^2+\cos 2r}{4}+\frac{1+2q_l^2-\cos 2r}{4}\log_2\frac{1+2q_l^2-\cos 2r}{4}. \tag{30}$$

Figures 4, 7 and 8 show the behavior of (30) as a function of r and q_l, respectively.

The entanglement of (26) is evaluated by logarithmic negativity, (4). Eigenvalues of the partial transpose of the density matrix $\rho_{00}^{\mathrm{I},B}$, are given by

$$\begin{aligned} \lambda_{1,2}\left(\rho_{\mathrm{I},B}^{\mathrm{pt}}\right) &= \frac{1}{2},\\ \lambda_{3,4}\left(\rho_{\mathrm{I},B}^{\mathrm{pt}}\right) &= \pm\frac{1}{2}\left(\cos^2 r-{q_l}^2\right). \end{aligned} \tag{31}$$

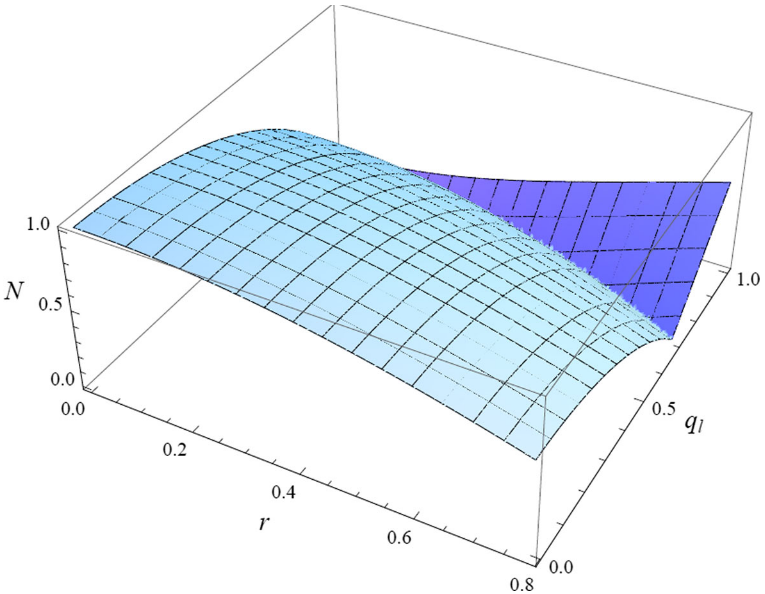

Fig. 4 Capacity of superdense coding in terms of acceleration parameter, r, and presence possibility of the particle in region II of Rindler region, q_l

Thus, logarithmic negativity is calculated as follow

$$N\left(\rho_{00}^{\mathrm{I},B}\right) = \log_2\left(1 + \left|\cos^2 r - q_l^2\right|\right). \tag{32}$$

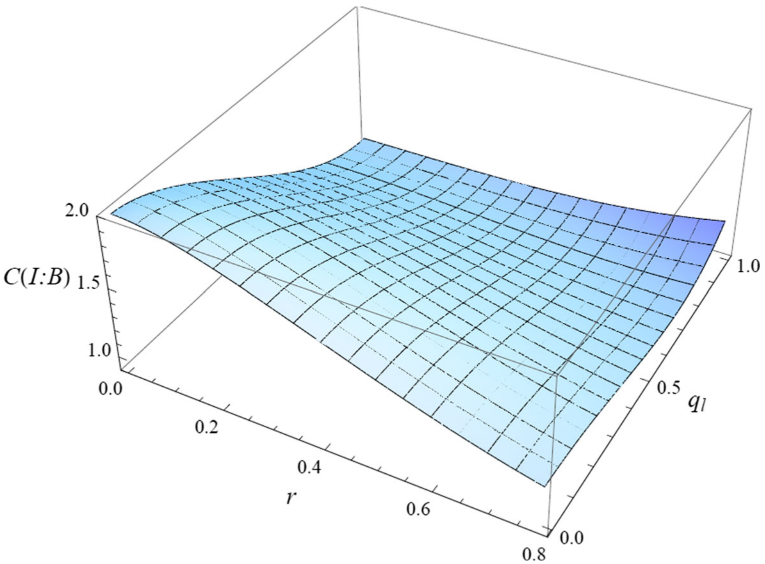

Fig. 5 Logarithmic negativity in terms of acceleration parameter, r, and presence possibility of the particle in region II of Rindler region, q_l

Figures 5, 7 and 8 show the behavior of (3) as a function of r and q_l, respectively.

Quantum discord, (5), is derived by considering the corresponding von Neumann entropies, (29), and following the approach in Refs. [27–29]. Figures 6, 7 and 8 show nonclassical correlation in terms of r and q_l, respectively.

4 Generality of Discussions for all $\alpha, \beta = \{0, 1\}$, (1)

Generally, the initial shared state $|\varphi_{\alpha\beta}\rangle_{A,B}$ can be rewritten by substituting the relations (10) and (22) in beyond single mode approximation, as

$$|\varphi_{\alpha\beta}\rangle_{\mathrm{I},\mathrm{II},B} = \frac{1}{\sqrt{2}}\left\{\cos r|00\alpha\rangle + \sin r|11\alpha\rangle + (-1)^{\beta}q_l|01\bar{\alpha}\rangle + (-1)^{\beta}q_r|10\bar{\alpha}\rangle\right\}. \tag{33}$$

A unitary operator U_{ij} is applied on the accelerated particle. Then the resultant state is sent to Bob. The operator I does not change the state (33), but others do change the state into another state, as follow

$$U_{ij}|\varphi_{\alpha\beta}\rangle_{\mathrm{I},\mathrm{II},B} = \frac{(-1)^{ij}}{\sqrt{2}}\left\{\cos r|i0\alpha\rangle + (-1)^{\beta}q_l|i1\bar{\alpha}\rangle + (-1)^{j}\sin r|\bar{i}1\alpha\rangle + (-1)^{\beta+j}q_r|\bar{i}0\bar{\alpha}\rangle\right\}. \tag{34}$$

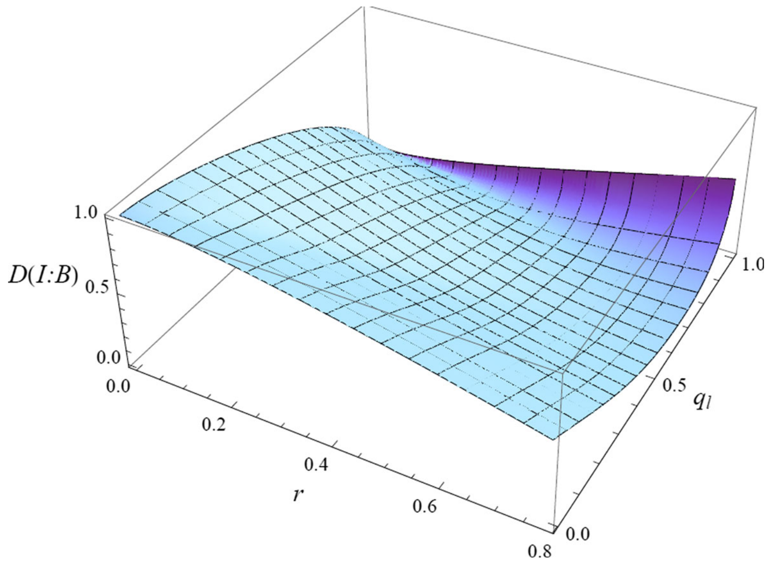

Fig. 6 Quantum discord in terms of acceleration parameter, r, and presence possibility of the particle in region II of Rindler region, q_l

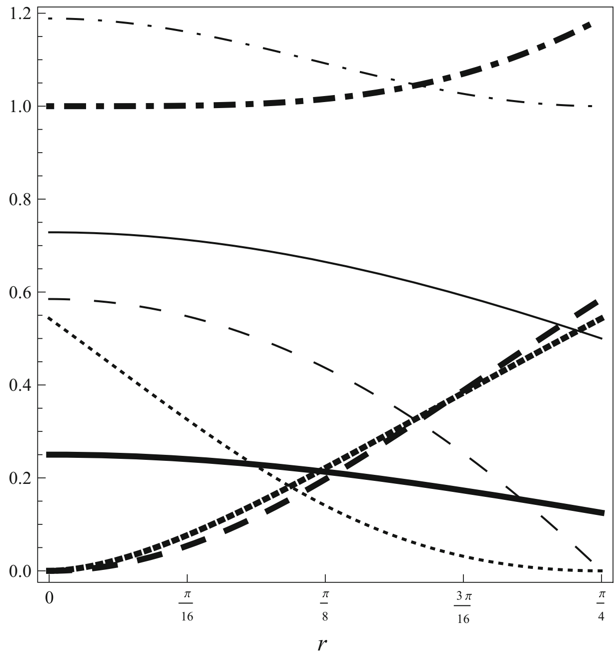

Fig. 7 Probability of success for superdense coding, P, *solid lines*, superdense coding capacity, $C(\mathrm{I}:B)$, *dotdashed lines*, logarithmic negativity, N, *dashed lines*, and quantum discord, $D(\mathrm{I}:B)$, *dotted lines*, for $q_l=\frac{1}{\sqrt{2}}$, *thin lines*, and $q_l=1$, *thick lines*, as functions of r, for $\rho_{00}^{\mathrm{I},B}$, in beyond single mode approximation

This is the state in Bob's possession. The resultant density matrix for superdense coding beyond single mode approximation is given by tracing out region II, as follow

$$\begin{aligned}\rho^{\mathrm{I},B} &= \mathrm{Tr}_{\mathrm{II}}(\rho^{\mathrm{I},\mathrm{II},B}) \\ &= \frac{1}{2}\Big\{\cos^2 r|i\alpha\rangle\langle i\alpha| + \sin^2 r|\bar{i}\alpha\rangle\langle\bar{i}\alpha| + q_l^2|i\bar{\alpha}\rangle\langle i\bar{\alpha}| + q_r^2|\bar{i}\bar{\alpha}\rangle\langle\bar{i}\bar{\alpha}| \\ &\quad +(-1)^{\beta+j}\left(q_r\cos r|i\alpha\rangle\langle\bar{i}\bar{\alpha}| + q_l\sin r|\bar{i}\alpha\rangle\langle i\bar{\alpha}| + \text{h.c.}\right)\Big\},\end{aligned} \tag{35}$$

which represents X-form matrices for all cases of α, β, i, j.

Therefore, our initial assumption of $\alpha=\beta=0$ for the shared entanglement, (1), does not affect the generality of discussions for single mode approximation and beyond single mode approximation. For both of the cases, the resultant states from superdense coding with uniformly accelerated particle can be evaluated for their probabilities of success, superdense coding capacities, negativity values and discord values. The final states, for all four choices of α and β, are X-form states. Therefore, quantum discord can be calculated following the approach discussed in Refs. [27–29].

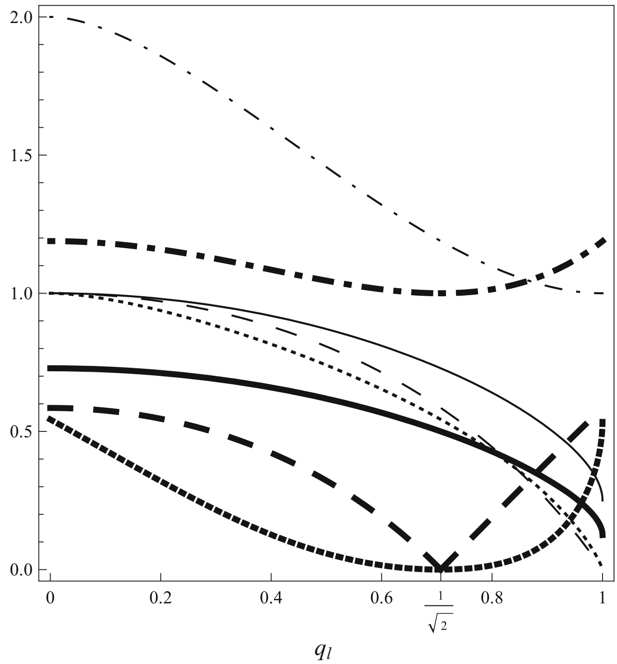

Fig. 8 Probability of success for superdense coding, P, *solid lines*, superdense coding capacity, $C(\mathrm{I} : B)$, *dotdashed lines*, logarithmic negativity, N, *dashed lines*, and quantum discord, $D(\mathrm{I} : B)$, *dotted lines*, for $r = 0$, *thin lines*, and $r = \frac{\pi}{4}$, *thick lines*, as functions of q_l, for $\rho_{00}^{\mathrm{I},B}$, in beyond single mode approximation

5 Discussions and Conclusion

We studied superdense coding with uniformly accelerated particle in single mode approximation and beyond single mode approximation. In single mode approximation, $q_r = 1$ (or equally $q_l = 0$), measurement by Bob on the density matrix after tracing out region II is dependent on the acceleration parameter, r. By letting $r = 0$, corresponding to $a = 0$, superdense coding is performed with absolute probability, (17), in accordance with the original superdense coding [6]. As illustrated in Fig. 2, probability of success, superdense coding capacity, logarithmic negativity and quantum discord are all descending functions of acceleration parameter, r.

In beyond single mode approximation, the situation is more intricate. Figure 7 (Fig. 8) is to show behaviors of probability of success, superdense coding capacity, negativity and quantum discord for the resultant state of superdense coding with uniformly accelerated particle, for distinct values of q_l (r), as functions of r (q_l). q_l is in interval [0,1]. Figure 7 shows the functions for q_l maximum that is $q_l = 1$, and for $q_l = \frac{1}{\sqrt{2}}$. Entanglement and nonclassical correlation are zero for $q_l = \frac{1}{\sqrt{2}}$, with $r = \frac{\pi}{4}$. In Fig. 8, the four functions are shown for r minimum, that is $r = 0$, and for $r = \frac{\pi}{4}$, that is when quantum correlations are zero at $q_l = \frac{1}{\sqrt{2}}$.

Recall that single mode approximation is a special case for beyond single mode approximation for when $q_l = 0$. From Fig. 8, we can see that for $q_l = 0$, and two cases of $r = 0$ and

$r = \frac{\pi}{4}$, the evaluated functions values exactly coincide with the corresponding ones being represented in Fig. 2.

In Fig. 7, when $q_l = 1$, the maximum value for q_l, the maximum probability of success, P, is for $r = 0$, (28). P is decreasing with increasing r. We would expect similar behaviors for entanglement, nonclassical correlation and the capacity, however negativity and discord, as well as the capacity, are representing increasing behaviors. In beyond single mode approximation, (22), if the accelerated object starts from $|1\rangle$, there is some distinct probability for the state to change to $|0\rangle$, and this probability is equal to 1 specifically for when $q_l = 1$, the case illustrated in Fig. 7 with thick lines. Indeed, we do not evaluate the entanglement, nor nonclassical correlation of the original shared entangled state by negativity and discord, and what is illustrated is actually the negativity and discord for the state $|\psi_{\bar{i}j}\rangle$, but not the original state $|\psi_{ij}\rangle$. The same discussion is applied to explain the capacity of superdense coding since this function is evaluated using nonclassical correlations. We, therefore, conclude that the probability of success is the best means for evaluating the process of superdense coding with accelerated particle, specially for a large q_l, i.e. when beyond single mode approximation is strongly used.

In Fig. 7, when $q_l = \frac{1}{\sqrt{2}}$, since q_l is not very large, i.e. even in beyond single mode approximation, the initial state of the accelerated particle only changes to an unbiased superposition of $|0\rangle$ and $|1\rangle$, (22). Therefore, we do not see any peculiar behavior from the studied functions, as the previous paragraph. Here, the capacity of superdense coding, entanglement, discord and the probability of success are all decreasing functions with regard to r.

In Fig. 8, when $r = 0$, with an increase in q_l, the four evaluated functions decrease, which is the expected behavior, consulting the corresponding equations, and specifically (22). In the same figure, when $r = \frac{\pi}{4}$, with an increase in q_l, entanglement and nonclassical correlation decrease until they reach the minimum value $\frac{1}{\sqrt{2}}$. From this point, the behaviors of these two functions are changed. They represent increasing behaviors, which can be explained again by (22), since the state $|\psi_{ij}\rangle$ changes to $|\psi_{\bar{i}j}\rangle$. Correspondingly, the capacity of superdense coding is showing similar peculiar behavior. The capacity of superdense coding generally follows the behavior of quantum correlations, however the relationship is not as simple to give an exact form. The probability of success is presenting behavior as the expectation.

In relativistic regimes, superdense coding with an accelerated particle and its probability of success can be reliably used for evaluating the involved quantum states in terms of their capabilities for being employed and manipulated for quantum information processing purposes. In this regard, negativity, discord and superdense coding capacity definitions are shown to have obstacles at least for specific ranges of acceleration and in a general form where one investigates the process in a general manner, i.e. in beyond single mode approximation.

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