

Controlled Remote State Preparation of an Arbitrary Two-Qubit State by Using GHZ States

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Abstract In this paper, we demonstrate that two Greenberger-Horne-Zeilinger (GHZ) states can be used to realize the perfect and deterministic controlled remote state preparation of an arbitrary two-qubit state by performing only the two-qubit projective measurements and appropriate unitary operations.

Keywords Controlled remote state preparation · GHZ state · Arbitrary two-qubit state

1 Introduction

Remote state preparation (RSP) is an effective and important way for preparing an arbitrary quantum state, which the sender only knows complete knowledge of phase information and amplitude information of the unknown quantum state [1]. And it is known that RSP requires less classical bits than quantum teleportation [2–20]. In the recent years, various scheme of RSP have been widely reported both theoretically [21–27] and experimentally [28–30]. In order to satisfy different quantum communication tasks, joint remote state preparation (JRSP) were proposed [31], which some senders share all knowledge of the unknown quantum state each other including the phase information and amplitude information. That means that each sender only having partial information about the quantum state in JRSP. In 2015, Choudhury and Dhara argued that two Greenberger-Horne-Zeilinger (GHZ) states are impossible for realizing the perfect and deterministic JRSP of an arbitrary equatorial two-qubit state [32].

In this work, we demonstrate that two three-qubit GHZ states can be used to realize the perfect and deterministic controlled remote state preparation (CRSP) of an arbitrary

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two-qubit state by performing only the two-qubit projective measurements and appropriate unitary operations. In our scheme, the receiver can deterministically obtain the desired unknown two-qubit state with proper unitary operation, at same time the receiver can reconstruct the initial state with 100 % success probability.

2 CRSP of an Arbitrary Two-Qubit State

Suppose Alice and Bob want to help Charlie prepare an arbitrary two-qubit state written as

$$|\Phi\rangle = a_0 |00\rangle + a_1 e^{i\theta_1} |01\rangle + a_2 e^{i\theta_2} |10\rangle + a_3 e^{i\theta_3} |11\rangle, \tag{1}$$

where a_i ($i = 0, 1, 2, 3$) are real with $a_0^2 + a_1^2 + a_2^2 + a_3^2 = 1$, and $\theta_t \in (0, 2\pi)$ ($t = 1, 2, 3$). In this scheme the sender Alice knows complete knowledge of phase information and amplitude information of quantum state, at same time neither Bob nor Charlie has no any information and knowledge of this unknown quantum state.

The quantum channel shared by Alice, Bob and Charlie is two three-qubit GHZ states, which has the form

$$|\psi\rangle_{123456} = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{123} \otimes \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{456}, \tag{2}$$

where Alice possesses the qubit pair (1, 4), Bob holds the qubit pair (2, 5) and qubits 3 and 6 belong to Charlie, respectively.

In order to help Charlie to prepare the arbitrary two-qubit state $|\Phi\rangle$, firstly, Alice and Bob choose four different kinds of two-qubit projective measurement bases to measure their local qubits, respectively. The forms of Alice’s and Bob’s measured bases are given by:

Alice chooses the basis $\{|\varphi^j\rangle_{14}\}$ ($j = 1, 2, 3, 4$) which is related to the computation basis $\{|00\rangle_{14}, |01\rangle_{14}, |10\rangle_{14}, |11\rangle_{14}\}$ as

$$|\varphi^1\rangle_{14} = a_0 |00\rangle + a_1 e^{-i\theta_1} |01\rangle + a_2 e^{-i\theta_2} |10\rangle + a_3 e^{-i\theta_3} |11\rangle, \tag{3}$$

$$|\varphi^2\rangle_{14} = a_1 e^{-i\theta_1} |00\rangle - a_0 |01\rangle - a_3 e^{-i\theta_3} |10\rangle + a_2 e^{-i\theta_2} |11\rangle, \tag{4}$$

$$|\varphi^3\rangle_{14} = a_2 e^{-i\theta_2} |00\rangle + a_3 e^{-i\theta_3} |01\rangle - a_0 |10\rangle - a_1 e^{-i\theta_1} |11\rangle, \tag{5}$$

$$|\varphi^4\rangle_{14} = a_3 e^{-\theta_3} |00\rangle - a_2 e^{-i\theta_2} |01\rangle + a_1 e^{-\theta_1} |10\rangle - a_0 |11\rangle. \tag{6}$$

And at same time Bob chooses $\{|\phi^k\rangle_{25}\}$ ($k = 1, 2, 3, 4$) as his basic measured basis

$$|\phi^1\rangle_{25} = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle), \tag{7}$$

$$|\phi^2\rangle_{25} = \frac{1}{2} (|00\rangle - |01\rangle - |10\rangle + |11\rangle), \tag{8}$$

$$|\phi^3\rangle_{25} = \frac{1}{2} (|00\rangle + |01\rangle - |10\rangle - |11\rangle), \tag{9}$$

$$|\phi^4\rangle_{25} = \frac{1}{2} (|00\rangle - |01\rangle + |10\rangle - |11\rangle). \tag{10}$$

In terms of the basis states $\{|\phi^j\rangle_{14}\}$ and $\{|\phi^k\rangle_{25}\}$, the quantum channel for two three-qubit GHZ states can be expressed as

$$\begin{aligned}
 |\psi\rangle_{123456} = & \frac{1}{4} \left[|\varphi^1\rangle_{14} |\phi^1\rangle_{25} (a_0 |00\rangle + a_1 e^{i\theta_1} |01\rangle + a_2 e^{i\theta_2} |10\rangle + a_3 e^{i\theta_3} |11\rangle)_{36} \right. \\
 & + |\varphi^1\rangle_{14} |\phi^2\rangle_{25} (a_0 |00\rangle - a_1 e^{i\theta_1} |01\rangle - a_2 e^{i\theta_2} |10\rangle + a_3 e^{i\theta_3} |11\rangle)_{36} \\
 & + |\varphi^1\rangle_{14} |\phi^3\rangle_{25} (a_0 |00\rangle + a_1 e^{i\theta_1} |01\rangle - a_2 e^{i\theta_2} |10\rangle - a_3 e^{i\theta_3} |11\rangle)_{36} \\
 & + |\varphi^1\rangle_{14} |\phi^4\rangle_{25} (a_0 |00\rangle - a_1 e^{i\theta_1} |01\rangle + a_2 e^{i\theta_2} |10\rangle - a_3 e^{i\theta_3} |11\rangle)_{36} \\
 & + |\varphi^2\rangle_{14} |\phi^1\rangle_{25} (a_1 e^{i\theta_1} |00\rangle - a_0 |01\rangle - a_3 e^{i\theta_3} |10\rangle + a_2 e^{i\theta_2} |11\rangle)_{36} \\
 & + |\varphi^2\rangle_{14} |\phi^2\rangle_{25} (a_1 e^{i\theta_1} |00\rangle + a_0 |01\rangle + a_3 e^{i\theta_3} |10\rangle + a_2 e^{i\theta_2} |11\rangle)_{36} \\
 & + |\varphi^2\rangle_{14} |\phi^3\rangle_{25} (a_1 e^{i\theta_1} |00\rangle - a_0 |01\rangle + a_3 e^{i\theta_3} |10\rangle - a_2 e^{i\theta_2} |11\rangle)_{36} \\
 & + |\varphi^2\rangle_{14} |\phi^4\rangle_{25} (a_1 e^{i\theta_1} |00\rangle + a_0 |01\rangle - a_3 e^{i\theta_3} |10\rangle - a_2 e^{i\theta_2} |11\rangle)_{36} \\
 & + |\varphi^3\rangle_{14} |\phi^1\rangle_{25} (a_2 e^{i\theta_2} |00\rangle + a_3 e^{i\theta_3} |01\rangle - a_0 |10\rangle - a_1 e^{i\theta_1} |11\rangle)_{36} \\
 & + |\varphi^3\rangle_{14} |\phi^2\rangle_{25} (a_2 e^{i\theta_2} |00\rangle - a_3 e^{i\theta_3} |01\rangle + a_0 |10\rangle - a_1 e^{i\theta_1} |11\rangle)_{36} \\
 & + |\varphi^3\rangle_{14} |\phi^3\rangle_{25} (a_2 e^{i\theta_2} |00\rangle + a_3 e^{i\theta_3} |01\rangle + a_0 |10\rangle + a_1 e^{i\theta_1} |11\rangle)_{36} \\
 & + |\varphi^3\rangle_{14} |\phi^4\rangle_{25} (a_2 e^{i\theta_2} |00\rangle - a_3 e^{i\theta_3} |01\rangle - a_0 |10\rangle + a_1 e^{i\theta_1} |11\rangle)_{36} \\
 & + |\varphi^4\rangle_{14} |\phi^1\rangle_{25} (a_3 e^{i\theta_3} |00\rangle - a_2 e^{i\theta_2} |01\rangle + a_1 e^{i\theta_1} |10\rangle - a_0 |11\rangle)_{36} \\
 & + |\varphi^4\rangle_{14} |\phi^2\rangle_{25} (a_3 e^{i\theta_3} |00\rangle + a_2 e^{i\theta_2} |01\rangle - a_1 e^{i\theta_1} |10\rangle - a_0 |11\rangle)_{36} \\
 & + |\varphi^4\rangle_{14} |\phi^3\rangle_{25} (a_3 e^{i\theta_3} |00\rangle - a_2 e^{i\theta_2} |01\rangle - a_1 e^{i\theta_1} |10\rangle + a_0 |11\rangle)_{36} \\
 & \left. + |\varphi^4\rangle_{14} |\phi^4\rangle_{25} (a_3 e^{i\theta_3} |00\rangle + a_2 e^{i\theta_2} |01\rangle + a_1 e^{i\theta_1} |10\rangle + a_0 |11\rangle)_{36} \right]. \tag{11}
 \end{aligned}$$

In order to realize the CRSP, firstly, Alice applies the two-qubit projective measurement on the qubit pair (1, 4) under the basis $\{|\phi^j\rangle_{14}\}$ ($j = 1, 2, 3, 4$) and publicly tells her measured result to both Charlie and Bob via a classical channel. Next, according to Alice’s measured result, Bob should choose one of the two-qubit projective measurement bases $\{|\phi^k\rangle_{25}\}$ ($j = 1, 2, 3, 4$) to measure his qubit pair (2,5). And then Bob informs Charlie of his measured result by the classical channel. In accord with Alice’s and Bob’s measured results, Charlie can recover the desired state $|\Phi\rangle$ by suitable local unitary operation.

For example, without loss of generality, suppose Alice’s measured outcome is $|\varphi^2\rangle$ and Bob’s measured result is $|\phi^2\rangle_{25}$, the remaining qubits 3 and 6 would collapse into the state $(a_1 e^{i\theta_1} |00\rangle + a_0 |01\rangle + a_3 e^{i\theta_3} |10\rangle + a_2 e^{i\theta_2} |11\rangle)_{36}$. According to Alice’s and Bob’s measured results, Charlie can perform the local unitary operation $I_3 \otimes \sigma_6^x$ on his qubits 3 and 6, and then the desired state $|\Phi\rangle$ can be reconstructed. The relation between the results obtained by Alice and Bob, the corresponding possible states for Charlie, and appropriate unitary operation performed by Charlie is shown in Table 1. It is easily found that, for all

Table 1 Alice’s and Bob’s possible results, the corresponding possible states for Charlie, and the local unitary operations $\{\sigma_3^{jk}\}$ ($j, k = 1, 2, 3, 4$) performed by Charlie

Alice’s result	Bob’s result	State obtained by Charlie	Unitary operation
$ \varphi^1\rangle_{14}$	$ \varphi^1\rangle_{25}$	$(a_0 00\rangle + a_1e^{i\theta_1} 01\rangle + a_2e^{i\theta_2} 10\rangle + a_3e^{i\theta_3} 11\rangle)_{36}$	$I_3 \otimes I_6$
$ \varphi^1\rangle_{14}$	$ \varphi^2\rangle_{25}$	$(a_0 00\rangle - a_1e^{i\theta_1} 01\rangle - a_2e^{i\theta_2} 10\rangle + a_3e^{i\theta_3} 11\rangle)_{36}$	$\sigma_3^z \otimes \sigma_6^z$
$ \varphi^1\rangle_{14}$	$ \varphi^3\rangle_{25}$	$(a_0 00\rangle + a_1e^{i\theta_1} 01\rangle - a_2e^{i\theta_2} 10\rangle - a_3e^{i\theta_3} 11\rangle)_{36}$	$\sigma_3^z \otimes I_6$
$ \varphi^1\rangle_{14}$	$ \varphi^4\rangle_{25}$	$(a_0 00\rangle - a_1e^{i\theta_1} 01\rangle + a_2e^{i\theta_2} 10\rangle - a_3e^{i\theta_3} 11\rangle)_{36}$	$I_3 \otimes \sigma_6^z$
$ \varphi^2\rangle_{14}$	$ \varphi^1\rangle_{25}$	$(a_1e^{i\theta_1} 00\rangle - a_0 01\rangle - a_3e^{i\theta_3} 10\rangle + a_2e^{i\theta_2} 11\rangle)_{36}$	$\sigma_3^z \otimes -i\sigma_6^y$
$ \varphi^2\rangle_{14}$	$ \varphi^2\rangle_{25}$	$(a_1e^{i\theta_1} 00\rangle + a_0 01\rangle + a_3e^{i\theta_3} 10\rangle + a_2e^{i\theta_2} 11\rangle)_{36}$	$I_3 \otimes \sigma_6^x$
$ \varphi^2\rangle_{14}$	$ \varphi^3\rangle_{25}$	$(a_1e^{i\theta_1} 00\rangle - a_0 01\rangle + a_3e^{i\theta_3} 10\rangle - a_2e^{i\theta_2} 11\rangle)_{36}$	$I_3 \otimes -i\sigma_6^y$
$ \varphi^2\rangle_{14}$	$ \varphi^4\rangle_{25}$	$(a_1e^{i\theta_1} 00\rangle + a_0 01\rangle - a_3e^{i\theta_3} 10\rangle - a_2e^{i\theta_2} 11\rangle)_{36}$	$\sigma_3^z \otimes \sigma_6^x$
$ \varphi^3\rangle_{14}$	$ \varphi^1\rangle_{25}$	$(a_2e^{i\theta_2} 00\rangle + a_3e^{i\theta_3} 01\rangle - a_0 10\rangle - a_1e^{i\theta_1} 11\rangle)_{36}$	$-i\sigma_3^y \otimes I_6$
$ \varphi^3\rangle_{14}$	$ \varphi^2\rangle_{25}$	$(a_2e^{i\theta_2} 00\rangle - a_3e^{i\theta_3} 01\rangle + a_0 10\rangle - a_1e^{i\theta_1} 11\rangle)_{36}$	$\sigma_3^x \otimes \sigma_6^z$
$ \varphi^3\rangle_{14}$	$ \varphi^3\rangle_{25}$	$(a_2e^{i\theta_2} 00\rangle + a_3e^{i\theta_3} 01\rangle + a_0 10\rangle + a_1e^{i\theta_1} 11\rangle)_{36}$	$\sigma_3^x \otimes I_6$
$ \varphi^3\rangle_{14}$	$ \varphi^4\rangle_{25}$	$(a_2e^{i\theta_2} 00\rangle - a_3e^{i\theta_3} 01\rangle - a_0 10\rangle + a_1e^{i\theta_1} 11\rangle)_{36}$	$-i\sigma_3^y \otimes \sigma_6^z$
$ \varphi^4\rangle_{14}$	$ \varphi^1\rangle_{25}$	$(a_3e^{i\theta_3} 00\rangle - a_2e^{i\theta_2} 01\rangle + a_1e^{i\theta_1} 10\rangle - a_0 11\rangle)_{36}$	$\sigma_3^x \otimes -i\sigma_6^y$
$ \varphi^4\rangle_{14}$	$ \varphi^2\rangle_{25}$	$(a_3e^{i\theta_3} 00\rangle + a_2e^{i\theta_2} 01\rangle - a_1e^{i\theta_1} 10\rangle - a_0 11\rangle)_{36}$	$-i\sigma_3^y \otimes \sigma_6^x$
$ \varphi^4\rangle_{14}$	$ \varphi^3\rangle_{25}$	$(a_3e^{i\theta_3} 00\rangle - a_2e^{i\theta_2} 01\rangle - a_1e^{i\theta_1} 10\rangle + a_0 11\rangle)_{36}$	$-i\sigma_3^y \otimes -i\sigma_6^y$
$ \varphi^4\rangle_{14}$	$ \varphi^4\rangle_{25}$	$(a_3e^{i\theta_3} 00\rangle + a_2e^{i\theta_2} 01\rangle + a_1e^{i\theta_1} 10\rangle + a_0 11\rangle)_{36}$	$\sigma_3^x \otimes \sigma_6^x$

the 16 measurement outcomes of Alice and Bob, the receiver Charlie can reconstruct the desired state $|\Phi\rangle$ by performing suitable unitary operation.

3 Conclusion

In this paper, a new scheme for CRSP of an arbitrary two-qubit state has been presented by using two sets of three-qubit GHZ states as the quantum channel. This CRSP protocol is perfect and deterministic, that means the probability of success can reach 100 %. In our CRSP scheme, one only needs the two-qubit projective measurements and appropriate local unitary operations. Therefore such a CRSP scheme is experimentally accessible.

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