Int J Theor Phys (2016) 55:5344–5347
DOI 10.1007/s10773-016-3154-0

Controlling Thermal Entanglement in a Three-qubit Spin System

Jianping Li[1]

Received: 2 July 2016 / Accepted: 29 August 2016 / Published online: 6 September 2016

Abstract In this paper, thermal entanglement in three-qubit spin system has been addressed. The results show that spin-spin exchange interaction, the effective external magnetic field, next-nearest-neighbouring interaction have notable effects on the time evolution of the state and thermal entanglement So we can control thermal entanglement by changing the above parameters.

Keywords Thermal entanglement · Thermal distribution function · Next-nearest-neighbouring interaction

1 Introduction

Quantum entangled plays a fundamental role in various fields of quantum computation and quantum information such as quantum cryptography and quantum teleportation [1–4]. Since the entanglement is very fragile, the question of how to create stable entanglement remains a main focus of recent studies in the field of quantum information processing. The quantum entanglement in solid state systems s is an important emerging field, spin chains are natural candidates for the realization of entanglement and has been researched [5–10].

In this paper, the eigenvalues, eigenstates of the Hamiltonian, thermal distribution function of the state and the density matrix and reduced density matrix in spin chain systems are addressed. The results show that spin-spin exchange interaction, the effective external

✉ Jianping Li
leejianping16@163.com

1 School of Computer and Information Technology, Northeast Petroleum University, Daqing, Heilongjiang, 163318, China

magnetic field, next-nearest-neighbouring interaction have notable effects on the time evolution of the state, the density matrix, and thermal entanglement So we can control thermal entanglement by changing the above parameters.

2 Controlling Thermal Entanglement in Three-qubit Spin Systems

The Hamiltonian for a three-qubit anisotropic Heisenberg spin chain is

$$H = J \sum_{i=1,2} \left(\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+ \right) + J\gamma \left(\sigma_1^+ \sigma_3^- + \sigma_1^- \sigma_3^+ \right) + B \sum_{i=1,2,3} \sigma_i^z \tag{1}$$

where

$$\sigma^+ = {}^{(\sigma^x + i\sigma^y)}\!\big/_2 ,\ \sigma^- = {}^{(\sigma^x - i\sigma^y)}\!\big/_2 \tag{2}$$

$\sigma_i^x, \sigma_i^y, \sigma_i^z$ are the Pauli operators. In the space of states

$$|000\rangle , |001\rangle , |010\rangle , |100\rangle , |011\rangle , |101\rangle , |110\rangle , |111\rangle$$

The Hamilton has the following form

$$H = \begin{pmatrix} -3B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B & J & J\gamma & 0 & 0 & 0 & 0 \\ 0 & J & -B & J & 0 & 0 & 0 & 0 \\ 0 & J\gamma & J & -B & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B & J & J\gamma & 0 \\ 0 & 0 & 0 & 0 & J & B & J & 0 \\ 0 & 0 & 0 & 0 & J\gamma & J & B & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3B \end{pmatrix} \tag{3}$$

According to $H = \sum E_i \left|\varphi_i\right\rangle \left\langle\varphi_i\right|$, the eigenvalue E_i of Hamilton are:

$$\begin{aligned} E_0 &= -3B \\ E_1 &= 3B \\ E_2 &= -B - J\gamma \\ E_3 &= B - J\gamma \\ E_4 &= -B + J\zeta_2 \\ E_5 &= B + J\zeta_2 \\ E_6 &= -B + J\zeta_1 \\ E_7 &= B + J\zeta_1 \end{aligned} \tag{4}$$

$$\zeta_1 = \frac{\gamma + \sqrt{\gamma^2 + 8}}{2}, \zeta_2 = \frac{\gamma - \sqrt{\gamma^2 + 8}}{2}$$

The corresponding eigenstate $|\varphi_i\rangle$ is:

$$\begin{aligned}
|\varphi_0\rangle &= |000\rangle \\
|\varphi_1\rangle &= |111\rangle \\
|\varphi_2\rangle &= -|001\rangle + |100\rangle \\
|\varphi_3\rangle &= -|011\rangle + |110\rangle \\
|\varphi_4\rangle &= |001\rangle - \frac{3\gamma - \sqrt{8+\gamma^2}}{-2-\gamma^2+\gamma\sqrt{8+\gamma^2}}|010\rangle + |100\rangle \\
|\varphi_5\rangle &= |011\rangle - \frac{3\gamma - \sqrt{8+\gamma^2}}{-2-\gamma^2+\gamma\sqrt{8+\gamma^2}}|101\rangle + |110\rangle \\
|\varphi_6\rangle &= |001\rangle - \frac{-3\gamma - \sqrt{8+\gamma^2}}{2+\gamma^2+\gamma\sqrt{8+\gamma^2}}|010\rangle + |100\rangle \\
|\varphi_7\rangle &= |011\rangle - \frac{-3\gamma - \sqrt{8+\gamma^2}}{2+\gamma^2+\gamma\sqrt{8+\gamma^2}}|101\rangle + |110\rangle
\end{aligned} \tag{5}$$

Normalized $|\varphi_i\rangle$ has the following form

$$\begin{aligned}
|\varphi_0\rangle &= |000\rangle \\
|\varphi_1\rangle &= |111\rangle \\
|\varphi_2\rangle &= \frac{1}{\sqrt{2}}(-|001\rangle + |100\rangle) \\
|\varphi_3\rangle &= \frac{1}{\sqrt{2}}(-|011\rangle + |110\rangle) \\
|\varphi_4\rangle &= \frac{1}{\eta_1}(|001\rangle - \zeta_2|110\rangle + |100\rangle) \\
|\varphi_5\rangle &= \frac{1}{\eta_1}(|011\rangle - \zeta_2|101\rangle + |110\rangle) \\
|\varphi_6\rangle &= \frac{1}{\eta_2}(|001\rangle - \zeta_1|010\rangle + |100\rangle) \\
|\varphi_7\rangle &= \frac{1}{\eta_2}(|011\rangle - \zeta_1|101\rangle + |110\rangle)
\end{aligned} \tag{6}$$

Where $\eta_1 = \sqrt{4+\gamma\zeta_2}$, $\eta_2 = \sqrt{4+\gamma\zeta_1}$

For a system in equilibrium at temperature T, the density operator is

$$\rho = \frac{1}{Z}e^{-\beta H} = \sum_j e^{-\beta H_j}\,|\rangle \tag{7}$$

Entanglement of two qubits can be measured by the concurrence [11]

$$C = \max\left\{0, 2\left(\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\right), 2\left(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}\right)\right\} \tag{8}$$

Acknowledgments This work is supported by the National Natural Science Foundation of China (Grant No.61402099).

References

1. Nielsen, M.A., Chuang, I.L.: Quantum computation and quantum information. Cambridge University Press, Cambridge (2000)
2. Shan, C.-J., Liu, J.-B., Liu, T.-K., Huang, Y.-X., Li, H.: Int. J. Theor. Phys. **48**, 1516 (2009)
3. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
4. Zheng, S.B.: Phys. Rev. A **69**, 064302 (2004)
5. Wang, X.G.: Phys. Rev. A **64**, 012313 (2001)
6. Zhou, L., Song, H.S., Guo, Y.Q., Li, C.: Phys. Rev. A **68**, 024301 (2003)
7. Zhang, G.F.: Phys. Rev. A **72**, 034302 (2005)
8. Shan, C.J., Chen, T., Liu, J.B., Cheng, W.W., Liu, T.K., Huang, Y.X., Li, H.: Int. J. Theor. Phys. **49**, 717 (2010)
9. Zeng, H.F., Shao, B., Yang, L.G., Li, J., Zou, J.: Chin. Phys. B **18**, 3265 (2009)
10. Shan, C.J., Cheng, W.W., Liu, T.K., Liu, J.B., Wei, H.: Chin. Phys. Lett. **25**, 3115 (2008)
11. Wootters, W.K.: Phys. Rev. Lett. **80**, 2245 (1998)