Int J Theor Phys (2016) 55:5233–5238
DOI 10.1007/s10773-016-3144-2

On Puthoff's Semiclassical Electron and Vacuum Energy

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Received: 29 February 2016 / Accepted: 22 August 2016 / Published online: 15 September 2016

Abstract A possible connection between a point electron and vacuum energy was recently claimed by Puthoff (Int. J. Theor. Phys. **46**, 3005 (2007)). He envisions a point electron as an ideally conducting spherical shell with a distributed charge on the surface, in equilibrium with the radiation pressure from electromagnetic vacuum fluctuations on the outside, and claims that his analysis demonstrates the reality of high-energy-density vacuum fluctuation fields. The present paper finds, instead, that the analysis is meaningless without specific knowledge on the cutoff frequency that is a free parameter in the model.

Keywords Classical electron · Vacuum energy

1 Motivation

A few years after deriving the minute pressure between two conducting plates close to each other now known as the Casimir force, Casimir [2, 3] speculates on a possible connection between the zero-point energy of the vacuum and one of the early models for an electron. This particular model assumes a negatively charged conducting shell, whose outward pressure from electrostatic forces is counteracted by the Casimir force. Such classical electron models continue to be of interest for their possible insight into various conceptual problems [4].

As is well-known, the Casimir force originates in a slightly lowered energy density in the vacuum due to the exclusion of certain electromagnetic oscillations, hence the Casimir effect is a fixed point in discussions of zero-point vacuum fluctuations beyond the

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electromagnetic interactions as well. In particular, the Casimir force is sometimes hinted at in support of a concept known as Quantum Vacuum Energy [5], in which energy-demanding applications such as space travel [6, 7] could become possible by extracting a minute fraction of the Planck energy density $\rho_P^E \simeq 5 \times 10^{114}$ J/m^3) that the vacuum would contain.

The primary motivation for the analysis in this paper is to show explicitly why the discussion in Ref. [1] has no bearing on the reality of vacuum fluctuations (as indeed stated elsewhere: [8]). Instead, the conclusion is that such a model has no meaning without a compelling choice of an upper frequency Ω that is a crucial parameter in the analysis. In particular, there is no reason to set Ω equal to the Planck frequency $\omega_P \simeq 2 \times 10^{43}$/s, which could indeed suggest that the vacuum might contain a substantial amount of energy.

2 Analysis

The analysis in Ref. [1] elaborates on Casimir's musing [3] about the possible relevance of the vacuum's zero-point energy to an early model for the electron. This concept envisions the electron as a hollow, infinitely conducting shell with radius R, charged uniformly over its surface. Such a shell could attain an equilibrium radius, R_0, through a postulated surface tension that could conceivably come from the pressure associated with the zero-point energy density w_v of the electromagnetic field outside. Making the radius of the shell vanish would result in a point-like electron when relevant quantities, e.g., the system's energy, behave properly when taking the limit. The analysis here follows Ref. [1], except that any limits are not taken upfront but deferred to the end of the calculation.

That such a model has an equilibrium radius is easy to see. An electrically charged shell with radius R pressurized on the outside has an energy $W(R)$ that consists of two terms. The Coulomb energy $W_c(R) \propto 1/R$ describes the repulsion between charges on the shell. For a vacuum-tight shell the pressure on its outside could come from evacuating the volume, but for the conducting shell here the vacuum is assumed to carry an energy density w_v that is missing on the inside. The energy from the vacuum $W_v(R) = w_v V(R)$ is then proportional to the shell's volume $V(R) = 4\pi R^3/3$, so that the energy decreases rapidly with shell radius. In between these two dependencies on radius there must be a minimum in the energy, at the equilibrium radius R_0. Distortions in the shell are unstable, but they can not occur in a spherically symmetric model.

In Ref. [1] the energy density outside the shell is thought to be from the same electromagnetic vacuum fluctuations that give rise to the Casimir force. The corresponding energy density is usually very small, comparable to a rough vacuum. In other circumstances the electromagnetic radiation pressure can be substantial; the pressure from black-body radiation is $w_v \simeq 10^{17}$J/m^3 or $w_v \simeq 10^8$ GPa, inside a closed cavity with walls $k_B T \simeq 100$ eV, somewhat like in inertial confinement fusion (ICF)).

The energy density in vacuum fluctuations is discussed in the original derivation of the Casimir force [2], and more extensively in modern accounts [9, 10]. One derivation considers a box with length scale L, and assigns the zero-point energy $\hbar\omega/2$ to each wave that fits in. The spectral energy density of electromagnetic waves in such a box is

$$\rho(\omega) d\omega = \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \tag{1}$$

where $\hbar \simeq 10^{-34}$ Js is the reduced Planck quantum, and ω is the (angular) frequency. Qualitatively, the spectral energy density $\rho(\omega)$ is the differential photon energy $\hbar d\omega$

averaged over a cubic wave length λ^3, where $\lambda = 2\pi c/\omega$. When integrated over $d\omega$, the corresponding energy density is

$$w_v = \int_0^{\Omega} \rho(\omega) d\omega = \frac{\hbar\Omega^4}{8\pi^2 c^3} = \frac{\pi\hbar\Omega}{\Lambda^3}. \tag{2}$$

Here $\hbar\Omega$ is an upper limit to the photon energy, and Λ^3 is the volume corresponding with the wave length $\Lambda = c/2\pi\Omega$: it is the maximum photon energy $\hbar\Omega$ averaged over its typical volume (as mentioned by Casimir as 'a curiosity'). So far the upper limit to the frequency is undefined.

Casimir explicitly recognizes that the electromagnetic energy density in (2) would diverge if the upper limit on the frequency were unbounded, but in Casimir's considerations the upper limit is irrelevant. The Casimir pressure $p_c = \hbar c\pi^3/240 d_0^4$ follows from a reduced energy density due to the exclusion of waves longer than $2d_0$ in between two ideally conducting plates a distance d_0. Ref. [2] explicitly dismisses any concern with an upper frequency limit in this situation with "... the upper limit for the frequency is x-rays, for which our plate is hardly an obstacle at all...". Instead, Ref. [1] assumes that none of the relevant radiation penetrates the boundary.

Two examples suffice to illustrate how much the boundaries affect the vacuum energy density stated in (1), hence they should be discussed explicitly as done in Ref. [2] but not in Ref. [1]. In thermal equilibrium the spectral energy density is not (1) but instead the well-known black-body formula, with an additional exponential factor $\exp(-\hbar\omega/k_B T)$ that is a consequence of the quantized emission of radiation: emitting photons with energies $\hbar\omega$ higher than about $k_B T$ is strongly suppressed. The opposite case is illustrated by a cavity at liquid helium temperature ($\simeq 4.2$ K), with a small hole in the boundary through which a laser shines in. In this case the spectral density of (1) is negligible compared to that in the neighborhood of the laser's nominal photon energy: the energy density in equilibrium is given by photon absorption into the cold boundary (or by leakage through cracks).

To emphasize the purely theoretical nature of what follows, specifically not intended as an attempt to construct a model for an electron unlike Ref. [1], the analysis here spreads a charge $Q = -Ne$ equivalent to N point electrons uniformly over a hollow sphere with radius R. As in Ref. [1], the electric field inside the shell vanishes, and spherical symmetry ensures that the electric field $\mathbf{E}(\mathbf{x})$ outside the sphere is radial, with magnitude $E = Q/4\pi\varepsilon_0 r^2$ at radius r. The electrostatic potential is $\phi = -Q/4\pi\varepsilon_0 r$, the energy density of the electrostatic field outside the shell is $w_c = \varepsilon_0 E^2/2$; integrating $w_c(r)$ from ∞ to R gives the electrostatic energy outside the shell as

$$W_c = \frac{Q^2}{8\pi\varepsilon_0 R}. \tag{3}$$

Ref. [1] states the equivalent to (3) but one essential step further, viz., already in the limit for $R \to 0$ and together with the disclaimer that the expression formally diverges. As will become clear in the following, it is better to defer the limiting process to the end of the calculation.

As in Ref. [1], it is convenient to normalize the variables. At this point in the analysis, the spatial variable R is best normalized with the classical electron radius $r_e = e^2/4\pi\varepsilon_0 mc^2$, energies W with the relativistic electron energy mc^2. In the normalized variable $s = R/r_e$, (3) becomes

$$\frac{W_c}{mc^2} = \frac{N^2}{2}\frac{1}{s}. \tag{4}$$

Chosen in Ref [1] is another normalization that already anticipates a connection with quantum mechanics, by using the fine structure constant $\alpha = e^2/4\pi\varepsilon_0\hbar c$. This comes in through the factor $\hbar$. However, at this point the charged shell has nothing quantum mechanical about it so that the normalization should not yet know about $\hbar$. The (reduced) Compton wave length of the electron $\lambda_e = \hbar/mc$ turns out to be an interesting normalization as well, except for the premature connection to quantum physics by the factor α that connects λ to r_e.

In the model of Ref. [1], $\hbar$ comes into the problem through the energy density of the vacuum fluctuations, $w_v = \hbar\Omega^4/8\pi^2c^3$. This energy density is assumed to exist only outside the shell because this is a perfect conductor. Excluding the vacuum energy density from inside the shell represents an energy W_v that is proportional to the volume inside the shell, $4\pi R^3/3$. Hence, W_v is

$$\frac{W_v}{mc^2} = \frac{\hbar\Omega^4R^3}{(6\pi c^3)mc^2} = \frac{s^3p^4}{6\alpha\pi^2} = W_v/mc^2 = P^2s^3/6. \tag{5}$$

Here the energy W_v is again normalized with the electron's rest energy, while $p = \Omega r_e/c$ is the maximum frequency Ω normalized with the time scale r_e/c corresponding to the classical electron radius.

At this point the maximum frequency Ω, or its normalized version p, is a free parameter that lacks any connection with quantum mechanics, whose signature factor $\hbar$ in (1) is absorbed in the fine structure constant α. Setting $P^2 = p^4/\alpha\pi^2$ gives the last expression in (5): the free parameter is now $P \propto \Omega^2$.

At the equilibrium radius s the total energy $W(s) = W_c(s) + W_v(s)$ is a minimum. This happens when $\partial W/\partial s = 0$, or when

$$-\frac{N^2}{2s^2} + \frac{s^2P^2}{2} = 0. \tag{6}$$

Therefore, the shell's equilibrium radius s_0 in normalized units is $s_0 = \sqrt{N/P}$; unnormalized, the equilibrium radius $R_0 = s_0r_e$ scales as $R_0 \propto \sqrt{N}/\Omega$. Any desired value for the equilibrium radius can therefore be obtained by changing the two free parameters, the number of electron charges N on the shell, and the maximum frequency Ω, in whichever ratio gives the appropriate outcome.

The same is true for the energy of the charged shell. As a function of the equilibrium radius, this is

$$\frac{W}{mc^2} = \frac{2Ns_0}{3} \propto \frac{N^{3/2}}{\Omega}. \tag{7}$$

The energy of the shell depends not only on the number of electrons on the shell, but also the maximum frequency.

For a comparison with the electron model in Ref [1], it seems appropriate to assume that the charge on the shell corresponds to that of a single electron only, hence $N = 1$. Likewise, it seems reasonable to make the energy the rest mass energy, i.e., $W = mc^2$. The normalized frequency is then determined, by $c/\Omega r_e 3/\sqrt{\pi\sqrt{\alpha}} \simeq 2.9$ and the maximum frequency Ω is fixed too. However, determining Ω like this seems entirely arbitrary. The exercise can be done in reverse as well: any specific choice of Ω gives a specific value for the energy. Then, $c/\Omega r_e = 1$ gives for the normalized energy $\sqrt{\pi\sqrt{\alpha}}/3 \simeq 0.345$, and the actual energy 176 keV.

As already stated in Ref. [1], the energy of the charged shell vanishes when the equilibrium radius $R_0 \propto \sqrt{N}/\Omega$ goes to zero, but there is no limit to the number of ways to accomplish the limit. One way is to shrink the equilibrium radius while keeping the maximum frequency the same, and to reduce the number N of electrons on the shell in unison. Doing so would be impossible if the charge on the shell were to consist of discrete electrons, but within the model it is perfectly fine for N to be continuous and have N vanish smoothly toward zero. The other way is to keep the charge at a single electron, and to increase the radiation pressure by taking the limit $\Omega \to \infty$. But, if these two free parameters were to vanish while keeping the same ratio N/Ω^2, the equilibrium radius would remain the same.

The charged-shell model as evaluated here has two parameters that seem to remain unconstrained by the requirement of an equilibrium radius, even when this radius is allowed to vanish as in Ref. [1]. In particular, the analysis does not seem to imply anything about the reality of high energy density radiation vacuum fluctuations, or about the various other conclusion stated in Ref. [1]: just about any desirable outcome can be gotten by a suitable choice of the maximum frequency Ω in the expression for the energy density of the radiation. However, delaying the limit process to the end gives some insights that may seem obscure when the limiting processes are done as in Ref. [1].

3 Conclusion

A reexamination of Ref. [1] suggests that a premature limiting process may have obscured the freedom of choice in the two free parameters ,N and Ω. They do not seem to have a compelling choice, or even a connection that seems the most logical. Therefore, nothing much can be deduced about vacuum energy from this electron model, in contrast to a statement in Ref. [1], such as ”... the reality of high-energy-density vacuum fluctuation fields at the fundamental particle level is buttressed, ...”. Even though the model in Ref. [1] does not demonstrate the utility of vacuum fluctuation pressure in understanding electrons, the quantum vacuum energy concept is a scientifically fascinating field [9, 10] that will reward careful study.

Acknowledgments We acknowledge useful discussions with Dr. H. E. Puthoff, and encouragement from Dr. T. Zheleva at the Army Research Laboratory. We also thank Dr. E. W. Davis who made us aware of Ref. [1]. This work was supported in part by contract W911QX09D0016-0004 with the Army Research Laboratory.

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