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Quantum Teleportation of A Four-qubit State by Using Six-qubit Cluster State

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Abstract We propose a scheme for perfect quantum teleportation of a special form of four-qubit state by using a six-qubit cluster state as quantum channel. In our scheme, the sender only needs six-qubit von-Neumann projective measurements, and the receiver can reconstruct the original four-qubit state by applying the appropriate unitary operation.

Keywords Quantum teleportation; Six-qubit cluster state; Four-qubit state

1 Introduction

Quantum communication is one of the most striking applications of quantum information science [1]. As one branch of quantum communication, quantum teleportation aims to transport an unknown single-qubit state from a sender to a receiver with the help of a Bell state and classical communication. The original quantum teleportation protocol was presented by Bennett et al. [2] in 1993. A key ingredient in quantum teleportation is a quantum channel connecting Alice and Bob that is supposed to be a maximally entangled pure state [3–17]. Subsequently, many quantum teleportation schemes have been proposed by using different types of multiparty entangled state as a quantum channel [18–26]. In 2008, it was demonstrated that the cluster state may be useful in perfect teleportation of an arbitrary single and two qubit states [27]. In 2011, Nie et al. [28] had shown that the cluster state can also be used successfully to teleport a three-qubit GHZ state.

In this work, we demonstrate that a six-qubit cluster state can be used to realize the perfect teleportation of a special form of four-qubit state based on the six-qubit von-Neumann projective measurements and local unitary operations.

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2 Quantum Teleportation of a Four-qubit State

Suppose Alice has a four-qubit state, which is given by

$$\left|\psi\right\rangle_{abcd} = (\alpha\,|0000\rangle + \beta\,|0011\rangle + \gamma\,|1100\rangle + \delta\,|1111\rangle)_{abcd}\,, \tag{1}$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. And Alice and Bob share a six-qubit cluster state

$$|\mathcal{C}\rangle_{123456} = \frac{1}{2}\,(|000000\rangle + |001101\rangle + |110010\rangle - |111111\rangle)_{123456}\,, \tag{2}$$

the qubits a, b, c, d, 5 and 6 belong to Alice, qubits 1, 2, 3 and 4 belong to Bob, respectively. The joint state of the four-qubit state and the quantum channel is given by,

$$\begin{aligned}
|\Psi\rangle &= \left|\psi\right\rangle_{abcd} \otimes |\mathcal{C}\rangle_{123456} \\
&= \tfrac{1}{4}[\left|\varphi^1\right\rangle_{abcd56}\,(\alpha\,|0000\rangle + \beta\,|0011\rangle + \gamma\,|1100\rangle + \delta\,|1111\rangle)_{1234} \\
&+ \left|\varphi^2\right\rangle_{abcd56}\,(\alpha\,|0000\rangle - \beta\,|0011\rangle + \gamma\,|1100\rangle - \delta\,|1111\rangle)_{1234} \\
&+ \left|\varphi^3\right\rangle_{abcd56}\,(\alpha\,|0000\rangle + \beta\,|0011\rangle - \gamma\,|1100\rangle - \delta\,|1111\rangle)_{1234} \\
&+ \left|\varphi^4\right\rangle_{abcd56}\,(\alpha\,|0000\rangle - \beta\,|0011\rangle - \gamma\,|1100\rangle + \delta\,|1111\rangle)_{1234} \\
&+ \left|\varphi^5\right\rangle_{abcd56}\,(\alpha\,|0011\rangle + \beta\,|0000\rangle + \gamma\,|1111\rangle + \delta\,|1100\rangle)_{1234} \\
&+ \left|\varphi^6\right\rangle_{abcd56}\,(\alpha\,|0011\rangle - \beta\,|0000\rangle + \gamma\,|1111\rangle - \delta\,|1100\rangle)_{1234} \\
&+ \left|\varphi^7\right\rangle_{abcd56}\,(\alpha\,|0011\rangle + \beta\,|0000\rangle - \gamma\,|1111\rangle - \delta\,|1100\rangle)_{1234} \\
&+ |\varphi 8\rangle_{abcd56}\,(\alpha\,|0011\rangle - \beta\,|0000\rangle - \gamma\,|1111\rangle + \delta\,|1100\rangle)_{1234} \\
&+ |\varphi 9\rangle_{abcd56}\,(\alpha\,|1100\rangle + \beta\,|1111\rangle + \gamma\,|1100\rangle + \delta\,|0011\rangle)_{1234} \\
&+ |\varphi 10\rangle_{abcd56}\,(\alpha\,|1100\rangle - \beta\,|1111\rangle + \gamma\,|1100\rangle - \delta\,|0011\rangle)_{1234} \\
&+ |\varphi 11\rangle_{abcd56}\,(\alpha\,|1100\rangle + \beta\,|1111\rangle - \gamma\,|1100\rangle - \delta\,|0011\rangle)_{1234} \\
&+ |\varphi 12\rangle_{abcd56}\,(\alpha\,|1100\rangle - \beta\,|1111\rangle - \gamma\,|1100\rangle + \delta\,|0011\rangle)_{1234} \\
&+ |\varphi 13\rangle_{abcd56}\,(\alpha\,|1111\rangle + \beta\,|1100\rangle + \gamma\,|0011\rangle + \delta\,|0000\rangle)_{1234} \\
&+ |\varphi 14\rangle_{abcd56}\,(\alpha\,|1111\rangle - \beta\,|1100\rangle + \gamma\,|0011\rangle - \delta\,|0000\rangle)_{1234} \\
&+ |\varphi 15\rangle_{abcd56}\,(\alpha\,|1111\rangle + \beta\,|1100\rangle - \gamma\,|0011\rangle - \delta\,|0000\rangle)_{1234} \\
&+ |\varphi 16\rangle_{abcd56}\,(\alpha\,|1111\rangle - \beta\,|1100\rangle - \gamma\,|0011\rangle + \delta\,|0000\rangle)_{1234}],
\end{aligned} \tag{3}$$

where $\left|\varphi^i\right\rangle_{abcd56}$ $(i = 1, 2, \cdots, 16)$ are mutually orthonormal six-qubit states in Alice's possession given by,

$$\left|\varphi^1\right\rangle = \frac{1}{2}(|\,000000\rangle + |\,001101\rangle + |\,110010\rangle - |\,111111\rangle)_{abcd56}, \tag{4}$$

$$\left|\varphi^2\right\rangle = \frac{1}{2}(|\,000000\rangle - |\,001101\rangle + |\,110010\rangle + |\,111111\rangle)_{abcd56}, \tag{5}$$

$$\left|\varphi^3\right\rangle = \frac{1}{2}(|\,000000\rangle + |\,001101\rangle - |\,110010\rangle + |\,111111\rangle)_{abcd56}, \tag{6}$$

$$\left|\varphi^4\right\rangle = \frac{1}{2}(|\,000000\rangle - |\,001101\rangle - |\,110010\rangle - |\,111111\rangle)_{abcd56}, \tag{7}$$

$$\left|\varphi^5\right\rangle = \frac{1}{2}(|\,000001\rangle + |\,001100\rangle + |\,110011\rangle - |\,111110\rangle)_{abcd56}, \tag{8}$$

$$\left|\varphi^6\right\rangle = \frac{1}{2}(|\,000001\rangle - |\,001100\rangle + |\,110011\rangle + |\,111110\rangle)_{abcd56}, \tag{9}$$

$$\left|\varphi^7\right\rangle = \frac{1}{2}(|\,000001\rangle + |\,001100\rangle - |\,110011\rangle + |\,111110\rangle)_{abcd56}, \tag{10}$$

$$\left|\varphi^8\right\rangle = \frac{1}{2}(|\,000001\rangle - |\,001100\rangle - |\,110011\rangle - |\,111110\rangle)_{abcd56}, \tag{11}$$

$$\left|\varphi^9\right\rangle = \frac{1}{2}(|\,000010\rangle + |\,001111\rangle + |\,110000\rangle - |\,111101\rangle)_{abcd56}, \tag{12}$$

Table 1 Strategy for recovering the four-qubit state

Alice's results	Classical information	Bob's state	Bob's operation
$\lvert\varphi^1\rangle$	0000	$\alpha\lvert 0000\rangle+\beta\lvert 0011\rangle+\gamma\lvert 1100\rangle+\delta\lvert 1111\rangle$	$I\otimes I\otimes I\otimes I$
$\lvert\varphi^2\rangle$	0001	$\alpha\lvert 0000\rangle-\beta\lvert 0011\rangle+\gamma\lvert 1100\rangle-\delta\lvert 1111\rangle$	$I\otimes I\otimes\sigma_z\otimes I$
$\lvert\varphi^3\rangle$	0010	$\alpha\lvert 0000\rangle+\beta\lvert 0011\rangle-\gamma\lvert 1100\rangle-\delta\lvert 1111\rangle$	$\sigma_z\otimes I\otimes I\otimes I$
$\lvert\varphi^4\rangle$	0011	$\alpha\lvert 0000\rangle-\beta\lvert 0011\rangle-\gamma\lvert 1100\rangle+\delta\lvert 1111\rangle$	$\sigma_z\otimes I\otimes\sigma_z\otimes I$
$\lvert\varphi^5\rangle$	0100	$\alpha\lvert 0011\rangle+\beta\lvert 0000\rangle+\gamma\lvert 1111\rangle+\delta\lvert 1100\rangle$	$I\otimes I\otimes\sigma_x\otimes\sigma_x$
$\lvert\varphi^6\rangle$	0101	$\alpha\lvert 0011\rangle-\beta\lvert 0000\rangle+\gamma\lvert 1111\rangle-\delta\lvert 1100\rangle$	$I\otimes I\otimes i\sigma_y\otimes\sigma_x$
$\lvert\varphi^7\rangle$	0110	$\alpha\lvert 0011\rangle+\beta\lvert 0000\rangle-\gamma\lvert 1111\rangle-\delta\lvert 1100\rangle$	$\sigma_z\otimes I\otimes\sigma_x\otimes\sigma_x$
$\lvert\varphi^8\rangle$	0111	$\alpha\lvert 0011\rangle-\beta\lvert 0000\rangle-\gamma\lvert 1111\rangle+\delta\lvert 1100\rangle$	$\sigma_z\otimes I\otimes i\sigma_y\otimes\sigma_x$
$\lvert\varphi^9\rangle$	1000	$\alpha\lvert 1100\rangle+\beta\lvert 1111\rangle+\gamma\lvert 0000\rangle+\delta\lvert 0011\rangle$	$\sigma_x\otimes\sigma_x\otimes I\otimes I$
$\lvert\varphi^{10}\rangle$	1001	$\alpha\lvert 1100\rangle-\beta\lvert 1111\rangle+\gamma\lvert 0000\rangle-\delta\lvert 0011\rangle$	$\sigma_x\otimes\sigma_x\otimes\sigma_z\otimes I$
$\lvert\varphi^{11}\rangle$	1010	$\alpha\lvert 1100\rangle+\beta\lvert 1111\rangle-\gamma\lvert 0000\rangle-\delta\lvert 0011\rangle$	$i\sigma_y\otimes\sigma_x\otimes I\otimes I$
$\lvert\varphi^{12}\rangle$	1011	$\alpha\lvert 1100\rangle-\beta\lvert 1111\rangle-\gamma\lvert 0000\rangle+\delta\lvert 0011\rangle$	$i\sigma_y\otimes\sigma_x\otimes\sigma_z\otimes I$
$\lvert\varphi^{13}\rangle$	1100	$\alpha\lvert 1111\rangle+\beta\lvert 1100\rangle+\gamma\lvert 0011\rangle+\delta\lvert 0000\rangle$	$\sigma_x\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x$
$\lvert\varphi^{14}\rangle$	1101	$\alpha\lvert 1111\rangle-\beta\lvert 1100\rangle+\gamma\lvert 0011\rangle-\delta\lvert 0000\rangle$	$\sigma_x\otimes\sigma_x\otimes i\sigma_y\otimes\sigma_x$
$\lvert\varphi^{15}\rangle$	1110	$\alpha\lvert 1111\rangle+\beta\lvert 1100\rangle-\gamma\lvert 0011\rangle-\delta\lvert 0000\rangle$	$i\sigma_y\otimes\sigma_x\otimes\sigma_x\otimes\sigma_x$
$\lvert\varphi^{16}\rangle$	1111	$\alpha\lvert 1111\rangle-\beta\lvert 1100\rangle-\gamma\lvert 0011\rangle+\delta\lvert 0000\rangle$	$i\sigma_y\otimes\sigma_x\otimes i\sigma_y\otimes\sigma_x$

$$\left|\varphi^{10}\right\rangle=\frac{1}{2}(\mid 000010\rangle-\mid 001111\rangle+\mid 110000\rangle+\mid 111101\rangle)_{abcd56}, \tag{13}$$

$$\left|\varphi^{11}\right\rangle=\frac{1}{2}(\mid 000010\rangle+\mid 001111\rangle-\mid 110000\rangle+\mid 111101\rangle)_{abcd56}, \tag{14}$$

$$\left|\varphi^{12}\right\rangle=\frac{1}{2}(\mid 000010\rangle-\mid 001111\rangle-\mid 110000\rangle-\mid 111101\rangle)_{abcd56}, \tag{15}$$

$$\left|\varphi^{13}\right\rangle=\frac{1}{2}(\mid 000011\rangle+\mid 001110\rangle+\mid 110001\rangle-\mid 111100\rangle)_{abcd56}, \tag{16}$$

$$\left|\varphi^{14}\right\rangle=\frac{1}{2}(\mid 000011\rangle-\mid 001110\rangle+\mid 110001\rangle+\mid 111100\rangle)_{abcd56}, \tag{17}$$

$$\left|\varphi^{15}\right\rangle=\frac{1}{2}(\mid 000011\rangle+\mid 001110\rangle-\mid 110001\rangle+\mid 111100\rangle)_{abcd56}, \tag{18}$$

$$\left|\varphi^{16}\right\rangle=\frac{1}{2}(\mid 000011\rangle-\mid 001110\rangle-\mid 110001\rangle-\mid 111100\rangle)_{abcd56}, \tag{19}$$

After the measurement, Alice communicates her measured result via four classical bits to Bob. Bob then applies appropriate Pauli rotations to recover the original unknown four-qubit state. Alice's measured results, her communicated results to Bob and Bob's corresponding operations are listed in Table 1.

Table 1: Strategy for recovering the four-qubit state

3 Conclusion

In summary, we have shown that a restricted class of four-qubit state can be teleported by a six-qubit cluster state. In our scheme, only six-qubit von-Neumann projective measurements and local unitary operations are needed. We have explicitly calculated Alice's measurement bases and the unitary operations required by Bob to reconstruct the unknown four-qubit state. We are also looking forward to generalize our schemes for quantum information splitting of an arbitrary four-qubit state by using eight-qubit cluster state as a quantum channel.

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